7 Flexure of the lithosphere and basin subsidence

The earth's crust can be considered as an elastic (or viscoelastic) plate that floats on top of a weak substratum (the asthenosphere). The crust bends under applied loads (think of the bending of the Pacific plate under the weight of the Hawaiian volcanoes). The equation relating the vertical deflection w of a plate to the applied load q (e.g. the weight of the volcano), is called the flexure equation [see Turcotte and Schubert 2001]

$$D\frac{\partial^4 w}{\partial x^4} + \Delta\rho g w = q(x) \tag{1}$$

where D is the flexural rigidity ($D = Eh^3/(12[1-\nu]^2)$). $\Delta\rho$ is the density difference between air and rocks below the elastic plate (asthenosphere) and g is the gravitational acceleration.

The bending equation has two new features, we didn't encounter sofar. First it involves fourth-order derivatives. Second, the equation is time-independent (this type of equation is called elliptical).

Since we have 4th order derivatives, we need to set two boundary conditions at each side of the plate. Commonly used conditions are $\frac{\partial w}{\partial x} = 0$ and $w = 0$ on each side.

7.0.1 Exercise

- Write a program that solved the bending equation, using an implicit finite difference discretization scheme.
- Google for some reasonable values of D and $\Delta\rho$. How much bending is to be expected due to the load of Hawaii?