Complete Experiments in pseudoscalar meson photoproduction

- Recent results from fitting CBELSA/TAPS data -

Yannick Wunderlich

HISKP, University of Bonn

16.02.2016

Details on the multipole Fit procedure I

Two step method:

1. Fit the angular distributions of observables, parametrized by

$$\check{\Omega}^{\alpha}\left(W,\theta\right)=\sum_{k=\beta_{\alpha}}^{2\ell_{\mathrm{max}}+\beta_{\alpha}+\gamma_{\alpha}}\left(a_{L}\right)_{k}^{\alpha}\left(W\right)P_{k}^{\beta_{\alpha}}\left(\cos\theta\right)$$

$\Rightarrow$ Angular fit parameters $\left(a_{L}^{\mathrm{Fit}}\right)_{k}^{\alpha}$

Details on the multipole Fit procedure I

Two step method:

1. Fit the angular distributions of observables, parametrized by

$$\check{\Omega}^{\alpha}\left(W,\theta\right)=\sum_{k=\beta_{\alpha}}^{2\ell_{\mathrm{max}}+\beta_{\alpha}+\gamma_{\alpha}}\left(a_{L}\right)_{k}^{\alpha}\left(W\right)P_{k}^{\beta_{\alpha}}\left(\cos\theta\right)$$

   $\Rightarrow$ Angular fit parameters $\left(a_{L}^{\mathrm{Fit}}\right)_{k}^{\alpha}$

2. Minimize the functional ("multi-indices" $(i,j)=(\{\alpha,k\},\{\alpha',k'\})$):

$$\chi^{2}=\sum_{i,j}\left[\left(a_{L}^{\mathrm{Fit}}\right)_{i}-\langle\mathcal{M}_{\ell}|\left(C_{L}\right)_{i}|\mathcal{M}_{\ell}\rangle\right]\mathrm{C}_{ij}^{-1}\left[\left(a_{L}^{\mathrm{Fit}}\right)_{j}-\langle\mathcal{M}_{\ell}|\left(C_{L}\right)_{j}|\mathcal{M}_{\ell}\rangle\right],$$

   using the MATHEMATICA method

$$\mathrm{FindMinimum}\left[\chi^{2}\left(\mathcal{M}_{\ell}\right),\{\{\mathrm{Re}\left[E_{0+}\right],(x_{1})_{0}\},\ldots,\{\mathrm{Im}\left[M_{\ell_{\mathrm{max}}-}\right],(y_{n})_{0}\}\}\right]$$

   and varying the real and imaginary parts of the (possibly phase constrained) multipoles in the fit.

C_{ij} is the covariance matrix stemming from the fit results of step 1.

Details on the multipole fit procedure II

Ansatz: Use the total cross section $\sigma(W)$. Example: $\ell \leq \ell_{\mathrm{max}} = 1$, phase constraint $\mathrm{Im}\left[\tilde{E}_{0+}\right] = 0$ & $\mathrm{Re}\left[\tilde{E}_{0+}\right] > 0$:

$$\sigma(W) \approx 4\pi \frac{q}{k} \Big(\mathrm{Re}\left[\tilde{E}_{0+}\right]^2 + 6\,\mathrm{Re}\left[\tilde{E}_{1+}\right]^2 + 6\,\mathrm{Im}\left[\tilde{E}_{1+}\right]^2 + 2\,\mathrm{Re}\left[\tilde{M}_{1+}\right]^2 + 2\,\mathrm{Im}\left[\tilde{M}_{1+}\right]^2 + \mathrm{Re}\left[\tilde{M}_{1-}\right]^2 + \mathrm{Im}\left[\tilde{M}_{1-}\right]^2 \Big)$$

Details on the multipole fit procedure II

Ansatz: Use the total cross section $\sigma\left(W\right)$. Example: $\ell \leq \ell_{\mathrm{max}} = 1$, phase constraint $\mathrm{Im}\left[\tilde{E}_{0+}\right] = 0$ & $\mathrm{Re}\left[\tilde{E}_{0+}\right] > 0$:

$$\sigma(W) \approx 4\pi \tfrac{q}{k} \Big(\mathrm{Re}\left[\tilde{E}_{0+}\right]^2 + 6\,\mathrm{Re}\left[\tilde{E}_{1+}\right]^2 + 6\,\mathrm{Im}\left[\tilde{E}_{1+}\right]^2 + 2\,\mathrm{Re}\left[\tilde{M}_{1+}\right]^2 + 2\,\mathrm{Im}\left[\tilde{M}_{1+}\right]^2 + \mathrm{Re}\left[\tilde{M}_{1-}\right]^2 + \mathrm{Im}\left[\tilde{M}_{1-}\right]^2 \Big)$$

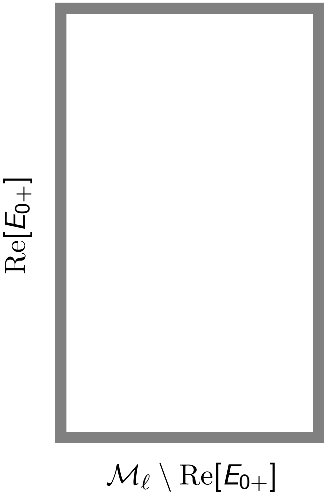
- $\sigma\left(W\right)$ constrains the intervals of the multipoles:

$$\mathrm{Re}\left[\tilde{E}_{0+}\right] \in \left[0, \sqrt{\frac{k}{q}\frac{\sigma(W)}{4\pi}}\right], \ldots, \mathrm{Im}\left[\tilde{M}_{1-}\right] \in \left[-\sqrt{\frac{k}{q}\frac{\sigma(W)}{4\pi}}, \sqrt{\frac{k}{q}\frac{\sigma(W)}{4\pi}}\right]$$

- The total cross section, being quadratic form in the multipoles, also defines an ellipsoid in the multipole space.

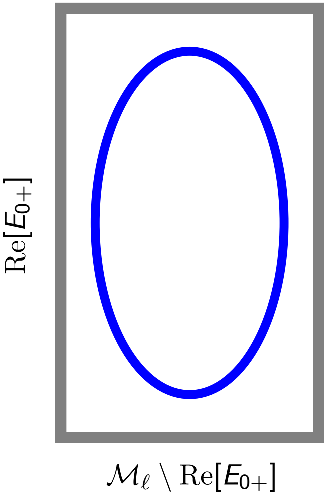
Generation of start values for FindMinimum

1. The total cross section $\sigma(W)$ constrains the $(8\ell_{\max} - 1)$-dimensional multipole space $\mathcal{M}_\ell$.



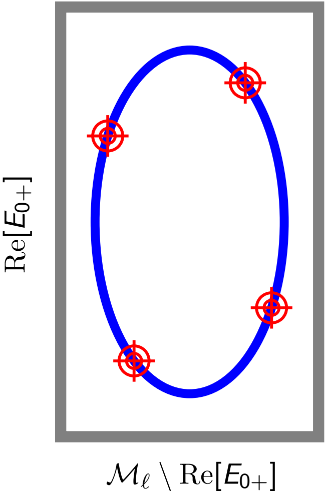
Generation of start values for FindMinimum

1. The total cross section $\sigma(W)$ constrains the $(8\ell_{\mathrm{max}} - 1)$-dimensional multipole space $\mathcal{M}_{\ell}$.
2. $\sigma(W)$ defines an $(8\ell_{\mathrm{max}} - 2)$-dimensional ellipsoid in $\mathcal{M}_{\ell}$.



Generation of start values for FindMinimum

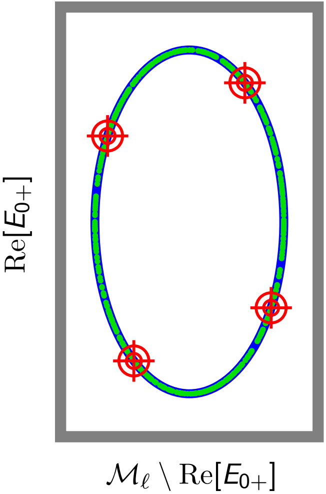
1. The total cross section $\sigma(W)$ constrains the $(8\ell_{\max} - 1)$-dimensional multipole space $\mathcal{M}_\ell$.
2. $\sigma(W)$ defines an $(8\ell_{\max} - 2)$-dimensional ellipsoid in $\mathcal{M}_\ell$.
3. Solutions to the TPWA problem lie on the ellipsoid defined by $\sigma(W)$.



Generation of start values for FindMinimum

1. The total cross section $\sigma(W)$ constrains the $(8\ell_{\mathrm{max}} - 1)$-dimensional multipole space $\mathcal{M}_\ell$.
2. $\sigma(W)$ defines an $(8\ell_{\mathrm{max}} - 2)$-dimensional ellipsoid in $\mathcal{M}_\ell$.
3. Solutions to the TPWA problem lie on the ellipsoid defined by $\sigma(W)$.
4. The start values for the $\mathrm{FindMinimum}$-Fit are chosen randomly on the $\sigma(W)$-ellipsoid.

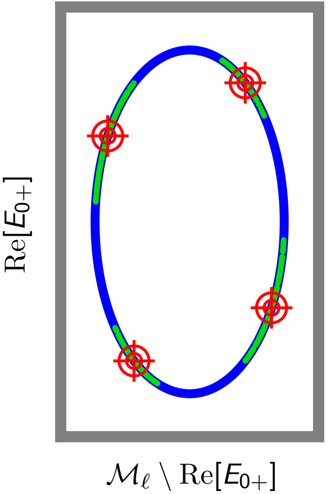
   $\Rightarrow$ Monte Carlo sampling of the multipole space.



Generation of start values for FindMinimum

5. An attempt was made to use the $\chi^2 (\mathcal{M}_\ell)$ for the generation of the start values.
   $\Rightarrow$ Clustering of start configurations near the minima (cf. [Sandorfi, Hoblit, Kamano and Lee (2011)]).

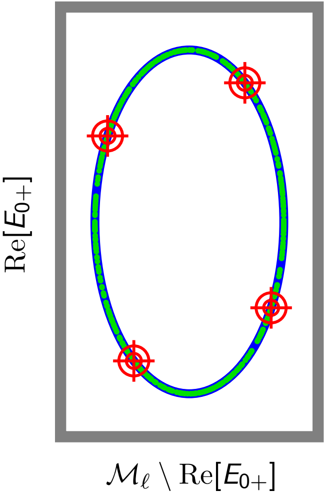
   However, this approach has not yet been usable due to interminable calculation times (using MATHEMATICA).



Generation of start values for FindMinimum

6. A FindMinimum-minimization is performed for each of the randomly generated start configurations.

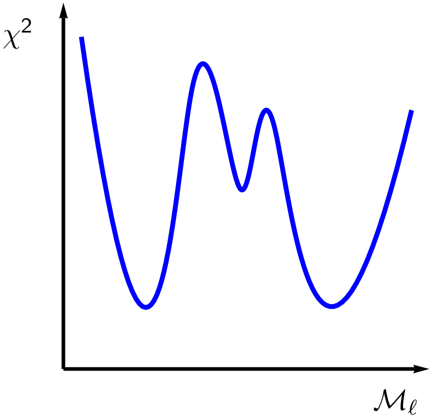
   $\Rightarrow N_{MC}$ = # of M.C. start configurations
   = # of (possibly redundant) solutions



Cut selections for solution "data"

Cut on solutions χ_j^2 with $\frac{\chi_j^2 - \chi_{\mathrm{best}}^2}{\chi_{\mathrm{best}}^2} < \epsilon$

Mathematical ambiguity | Unique best solution

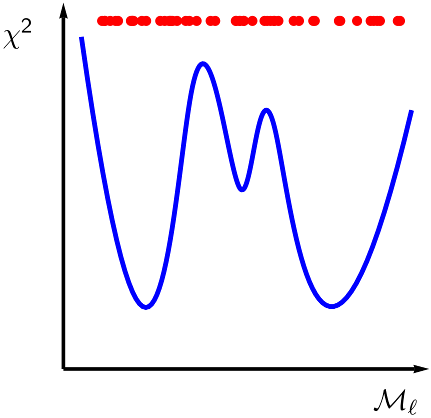


The χ^2 is defined by the fitted Legendre coefficients $\left(a_L^{\mathrm{Fit}}\right)_k^{\alpha}$.

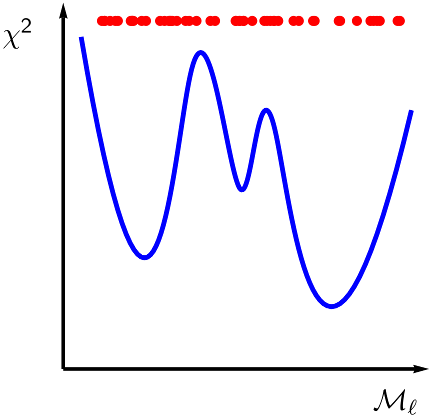
Cut selections for solution "data"

Cut on solutions χ_j^2 with $\frac{\chi_j^2 - \chi_{\text{best}}^2}{\chi_{\text{best}}^2} < \epsilon$

Mathematical ambiguity



Unique best solution

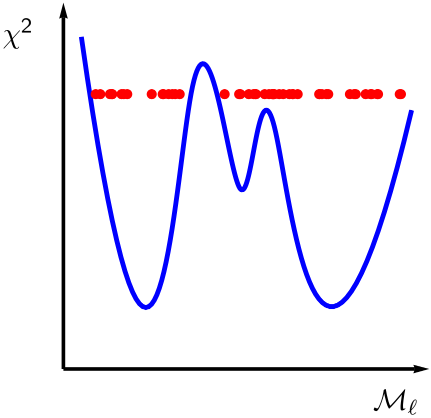


Start values have been distributed on the relevant part of the space $\mathcal{M}_\ell$.

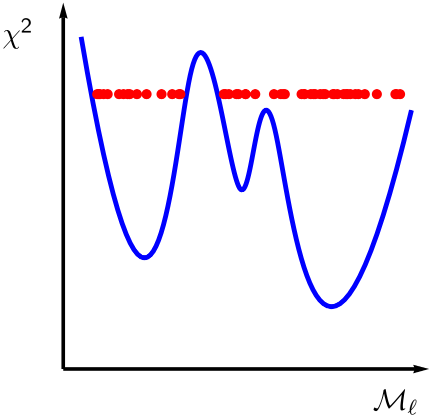
Cut selections for solution "data"

Cut on solutions χ_j^2 with $\frac{\chi_j^2 - \chi_{\mathrm{best}}^2}{\chi_{\mathrm{best}}^2} < \epsilon$

Mathematical ambiguity



Unique best solution

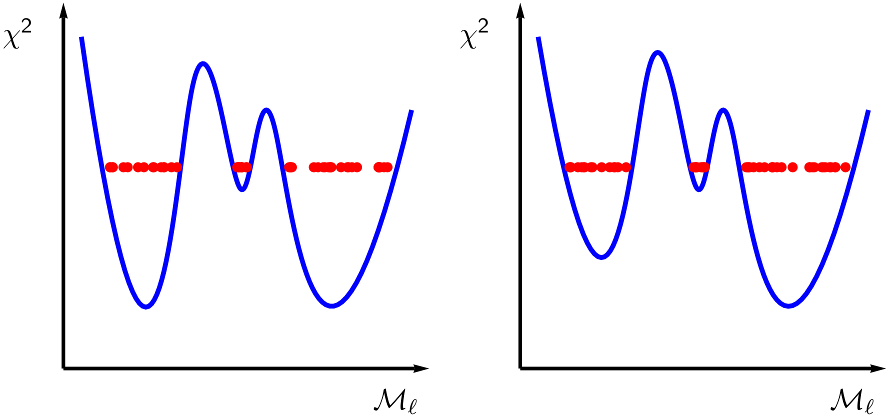


Minimizations of χ^2 converge within several iterations.

Cut selections for solution “data”

Cut on solutions χ_j^2 with $\frac{\chi_j^2 - \chi_{\text{best}}^2}{\chi_{\text{best}}^2} < \epsilon$

Mathematical ambiguity | Unique best solution

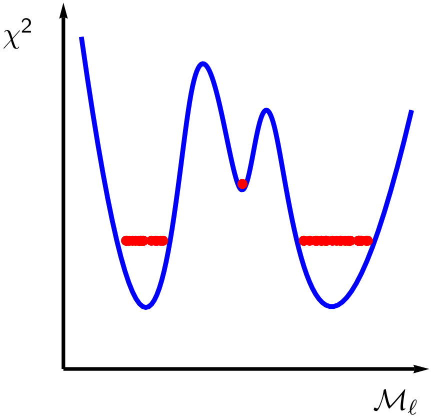


Minimizations of χ^2 converge within several iterations.

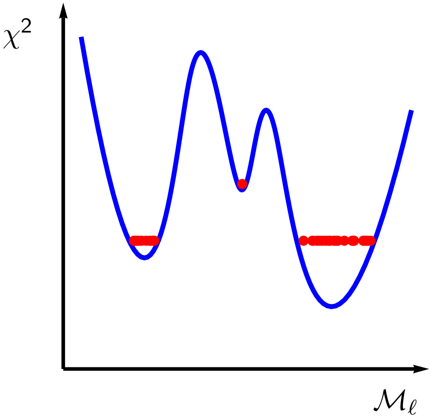
Cut selections for solution “data”

Cut on solutions χ_j^2 with $\frac{\chi_j^2 - \chi_{\text{best}}^2}{\chi_{\text{best}}^2} < \epsilon$

Mathematical ambiguity



Unique best solution

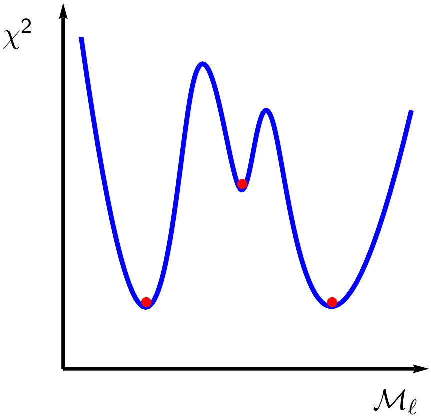


Minimizations of χ^2 converge within several iterations.

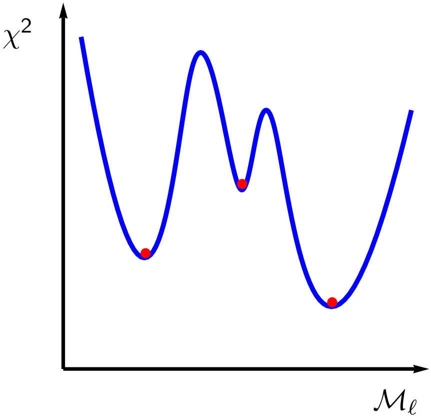
Cut selections for solution "data"

Cut on solutions χ_j^2 with $\frac{\chi_j^2 - \chi_{\mathrm{best}}^2}{\chi_{\mathrm{best}}^2} < \epsilon$

Mathematical ambiguity



Unique best solution

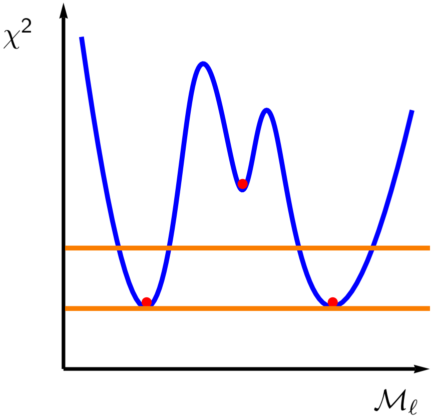


Minimizations of χ^2 converge within several iterations.

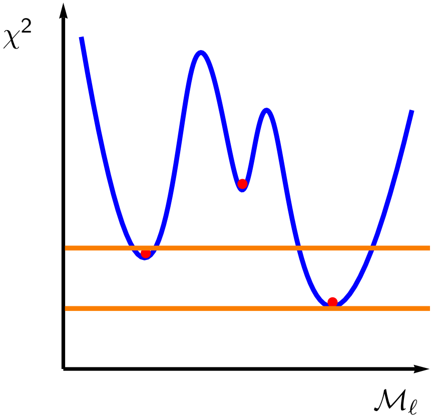
Cut selections for solution "data"

Cut on solutions χ_j^2 with $\frac{\chi_j^2 - \chi_{\text{best}}^2}{\chi_{\text{best}}^2} < \epsilon$

Mathematical ambiguity



Unique best solution

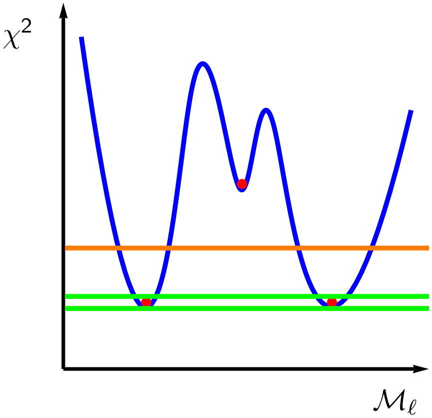


Cut selection using $\epsilon = 1$

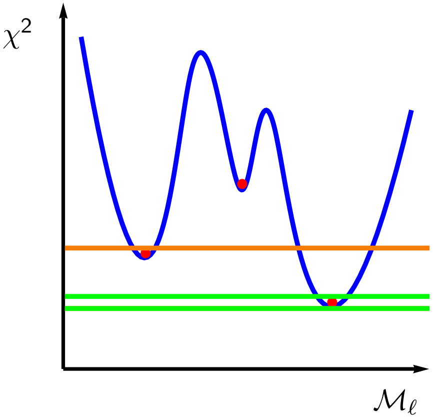
Cut selections for solution "data"

Cut on solutions χ_j^2 with $\frac{\chi_j^2 - \chi_{\mathrm{best}}^2}{\chi_{\mathrm{best}}^2} < \epsilon$

<u>Mathematical ambiguity</u>



<u>Unique best solution</u>



Cut selection using $\epsilon = 1$ / Cut selection using $\epsilon \sim \mathrm{num.\,precision}$

Description of the fitted datasets

The following datasets were investigated for $\gamma p \rightarrow \pi^0 p$:

I. Data taken at the MAMI facility:
   - σ_0: 266 energy points for $E_\gamma^{\mathrm{LAB}} \in [218, 1573]$ MeV
     [P. Adlarson et al., arXiv:1506.08849 [hep-ex]]

Description of the fitted datasets

The following datasets were investigated for $\gamma p \rightarrow \pi^0 p$:

I. Data taken at the MAMI facility:
   - σ_0: 266 energy points for $E_{\gamma}^{\mathrm{LAB}} \in [218, 1573]$ MeV
     [P. Adlarson et al., arXiv:1506.08849 [hep-ex]]

II. Data taken at the GRAAL facility:
   - Σ: 31 energy points for $E_{\gamma}^{\mathrm{LAB}} \in [551, 1450]$ MeV
     [O. Bartalini et al., Eur. Phys. J. A 26, 399 (2005)]

Description of the fitted datasets

The following datasets were investigated for $\gamma p \rightarrow \pi^0 p$:

I. Data taken at the MAMI facility:
   - σ_0: 266 energy points for $E_{\gamma}^{\mathrm{LAB}} \in [218, 1573]$ MeV
     [P. Adlarson et al., arXiv:1506.08849 [hep-ex]]

II. Data taken at the GRAAL facility:
   - Σ: 31 energy points for $E_{\gamma}^{\mathrm{LAB}} \in [551, 1450]$ MeV
     [O. Bartalini et al., Eur. Phys. J. A 26, 399 (2005)]

III. Data from CBELSA/TAPS:
   - T: 24 energy points for $E_{\gamma}^{\mathrm{LAB}} \in [700, 1900]$ MeV
   - P: 8 (!) energy points, i.e. $E_{\gamma}^{\mathrm{LAB}} \in [650, 950]$ MeV
   - H: 8 (!) energy points, i.e. $E_{\gamma}^{\mathrm{LAB}} \in [650, 950]$ MeV
     for all 3 obs. cf. [J. Hartmann et al., Phys. Lett. B 748 (2015)]
   - E: 33 energy points for $E_{\gamma}^{\mathrm{LAB}} \in [600, 2300]$ MeV
     [M. Gottschall et al., Phys. Rev. Lett. 112 no. 1, 012003 (2014)]
   - G: 19 energy points for $E_{\gamma}^{\mathrm{LAB}} \in [630, 1950]$ MeV
     [A. Thiel et al., Phys. Rev. Lett. 109, 102001 (2012)]

Description of the fitted datasets

The following datasets were investigated for $\gamma p \rightarrow \pi^0 p$:

I. Data taken at the MAMI facility:
   - σ_0: 266 energy points for $E_\gamma^{\mathrm{LAB}} \in [218, 1573]$ MeV
     [P. Adlarson et al., arXiv:1506.08849 [hep-ex]]

II. Data taken at the GRAAL facility:
   - Σ: 31 energy points for $E_\gamma^{\mathrm{LAB}} \in [551, 1450]$ MeV
     [O. Bartalini et al., Eur. Phys. J. A 26, 399 (2005)]

III. Data from CBELSA/TAPS:
   - T: 24 energy points for $E_\gamma^{\mathrm{LAB}} \in [700, 1900]$ MeV
   - P: 8 (!) energy points, i.e. $E_\gamma^{\mathrm{LAB}} \in [650, 950]$ MeV
   - H: 8 (!) energy points, i.e. $E_\gamma^{\mathrm{LAB}} \in [650, 950]$ MeV
     for all 3 obs. cf. [J. Hartmann et al., Phys. Lett. B 748 (2015)]
   - E: 33 energy points for $E_\gamma^{\mathrm{LAB}} \in [600, 2300]$ MeV
     [M. Gottschall et al., Phys. Rev. Lett. 112 no. 1, 012003 (2014)]
   - G: 19 energy points for $E_\gamma^{\mathrm{LAB}} \in [630, 1950]$ MeV
     [A. Thiel et al., Phys. Rev. Lett. 109, 102001 (2012)]

$\rightarrow$ Datasets overlap on 8 (!) energy-points $E_\gamma^{\mathrm{LAB}} \in [650, 950]$ MeV!

Moment-analysis/ "LFit-method"

∗) Utilize the parametrization of the angular distributions of polarization observables $\check{\Omega}^{\alpha}$ as expansions into $P_{\ell}^{m}(\cos\theta)$ for fixed energy:

$$\check{\Omega}^{\alpha}(W,\theta) = \sum_{k=\beta_{\alpha}}^{2\ell_{\mathrm{max}}+\beta_{\alpha}+\gamma_{\alpha}} (a_L)_k^{\alpha}(W)\, P_k^{\beta_{\alpha}}(\cos\theta)\,.$$

Moment-analysis/ "LFit-method"

$*$) Utilize the parametrization of the angular distributions of polarization observables $\check{\Omega}^{\alpha}$ as expansions into $P_{\ell}^{m}(\cos\theta)$ for fixed energy:

$$\check{\Omega}^{\alpha}(W,\theta) = \sum_{k=\beta_{\alpha}}^{2\ell_{\mathrm{max}}+\beta_{\alpha}+\gamma_{\alpha}} (a_L)_k^{\alpha}(W)\, P_k^{\beta_{\alpha}}(\cos\theta)\,.$$

$\rightarrow$ Fit angular distributions with some low initial ℓ_{max} ($\ell_{\mathrm{max}} = 1$ most commonly) and see if χ^2/ndf is satisfactory. If not:

$\rightarrow$ Raise truncation order by 1 and do new fit until $\left(\chi^2/\mathrm{ndf}\right) \approx 1$.

$\rightarrow$ Hint for dominant partial waves by the order ℓ_{max} at which this procedure terminates.

Moment-analysis/ "LFit-method"

$*)$ Utilize the parametrization of the angular distributions of polarization observables $\check{\Omega}^{\alpha}$ as expansions into $P_{\ell}^{m}(\cos\theta)$ for fixed energy:

$$\check{\Omega}^{\alpha}(W,\theta) = \sum_{k=\beta_{\alpha}}^{2\ell_{\mathrm{max}}+\beta_{\alpha}+\gamma_{\alpha}} (a_L)_k^{\alpha}(W)\, P_k^{\beta_{\alpha}}(\cos\theta) \, .$$

$\rightarrow$ Fit angular distributions with some low initial ℓ_{max} ($\ell_{\mathrm{max}} = 1$ most commonly) and see if χ^2/ndf is satisfactory. If not:

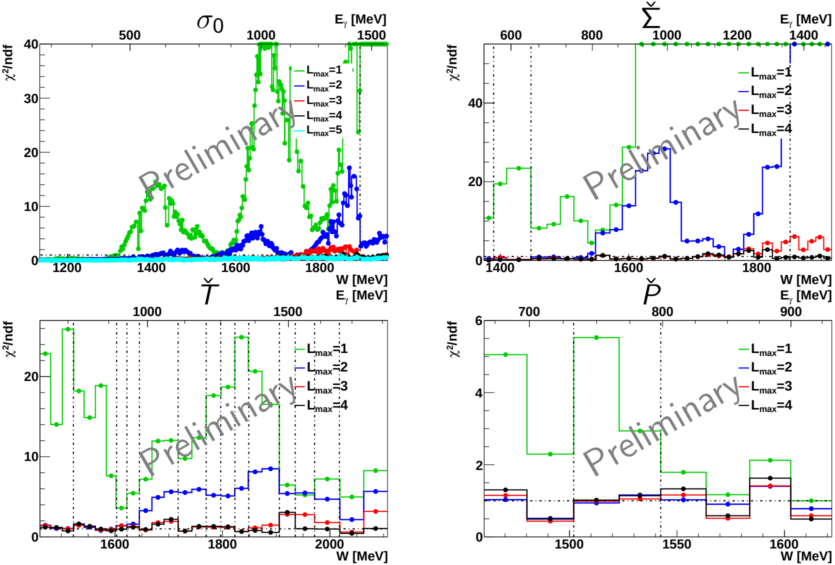
$\rightarrow$ Raise truncation order by 1 and do new fit until $\left(\chi^2/\mathrm{ndf}\right) \approx 1$.

$\rightarrow$ Hint for dominant partial waves by the order ℓ_{max} at which this procedure terminates.

$*)$ Nice: Procedure is simple, model-independent and furthermore reliably reflects the capability of the data to give infomation on higher partial wave contributions.

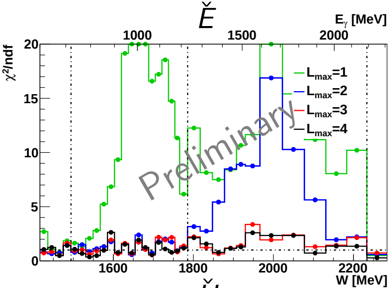
LFits to $\{\sigma_0, \Sigma, T, P, E, G, H\}$

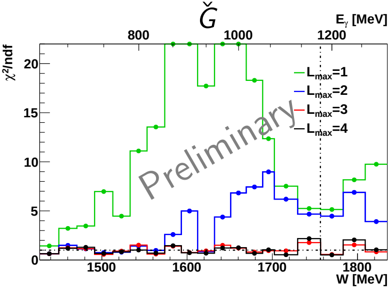
To be published in [Y. W., F. Afzal, A. Thiel and R. Beck, (2016)]

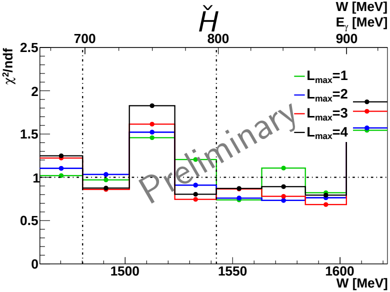


LFits to $\{\sigma_0, \Sigma, T, P, E, G, H\}$

To be published in [Y. W., F. Afzal, A. Thiel and R. Beck, (2016)]





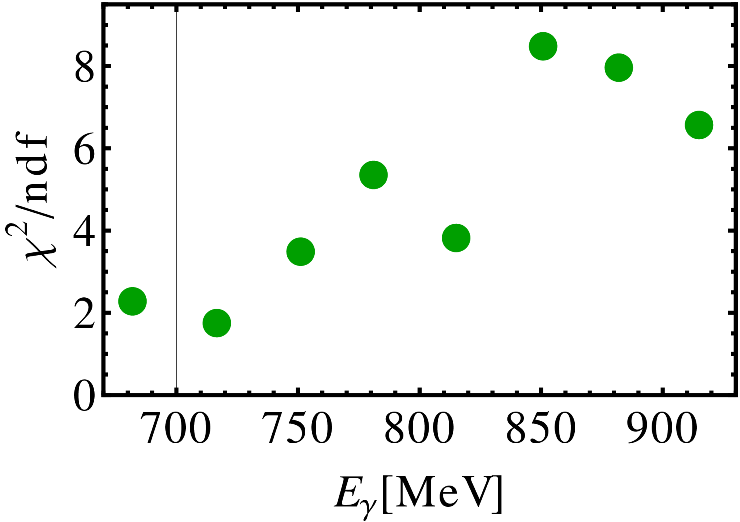


Overall, $\ell_{\mathrm{max}} = 2$ should be OK in all energy bins

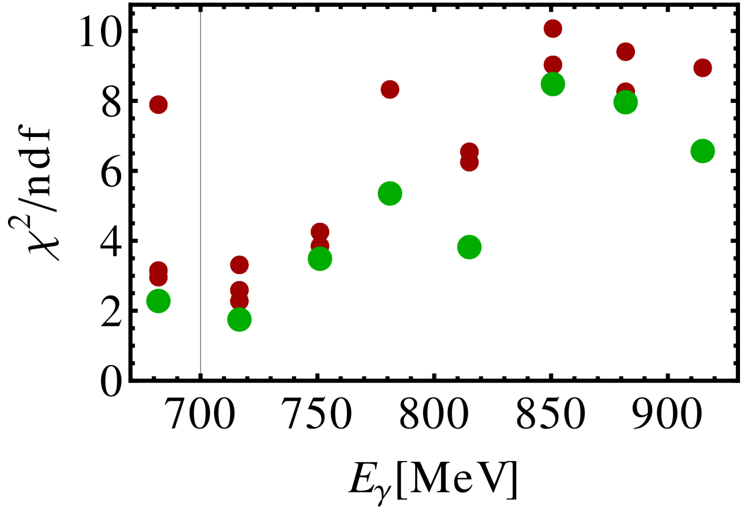
$$E_\gamma^{\mathrm{LAB}} \in [650, 950] \text{ MeV}$$

except maybe the last 2 bins.

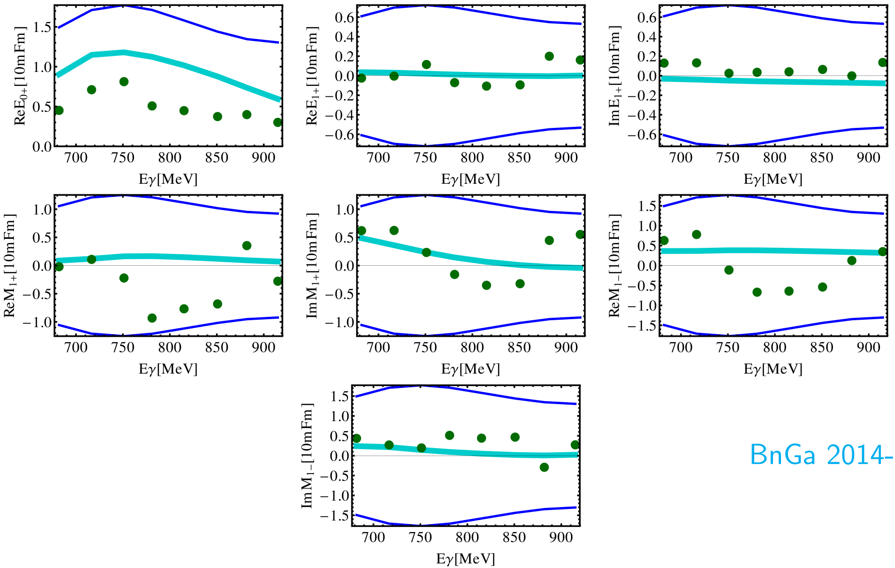
χ^2_{best} vs. E_γ for the $\ell_{\max} = 2$-fit



χ^2_{best} vs. E_γ for the $\ell_{\max} = 2$-fit

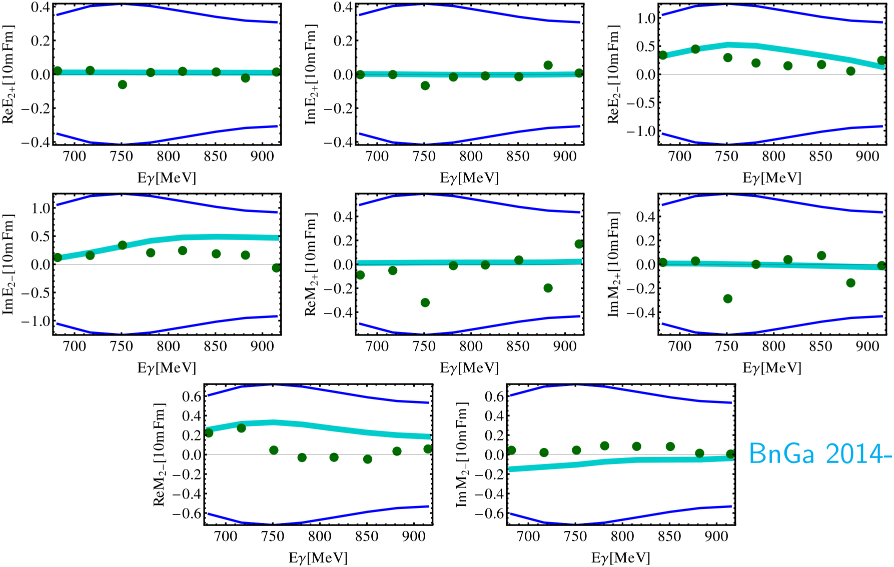


The best solution for *S*-, *P*- and *D*-waves



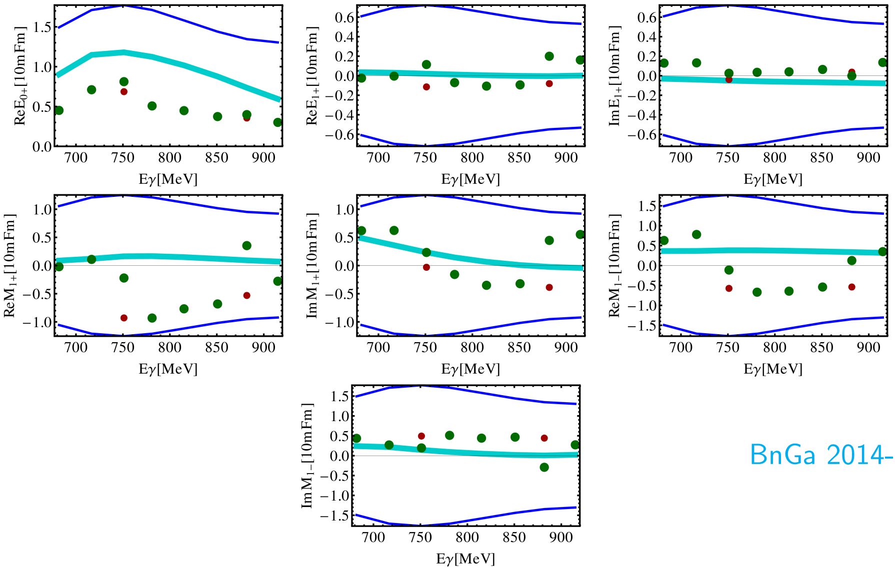
BnGa 2014-02

The best solution for *S*-, *P*- and *D*-waves



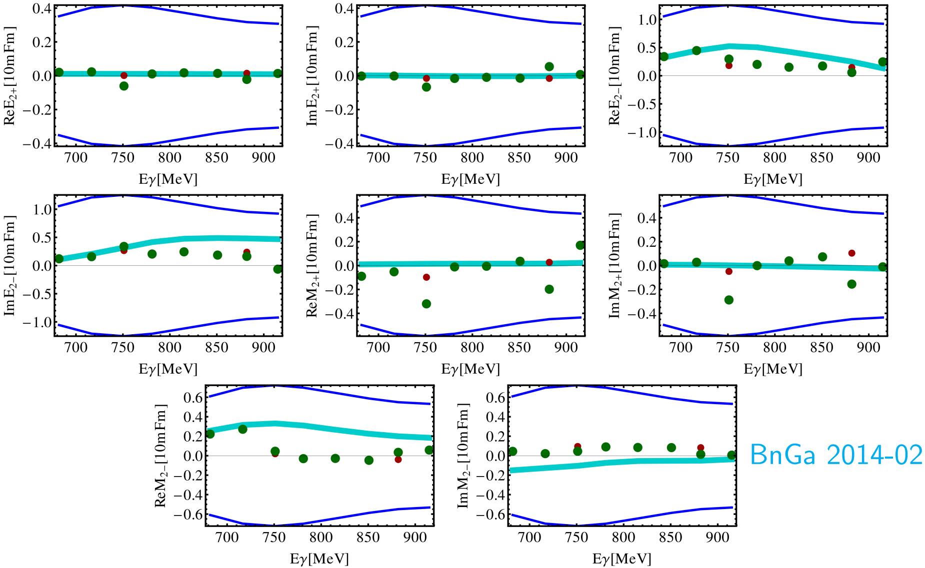
BnGa 2014-02

S-, P- and D-waves in the interval $\left(\chi^2_{\text{best}} + 0.5\right)$

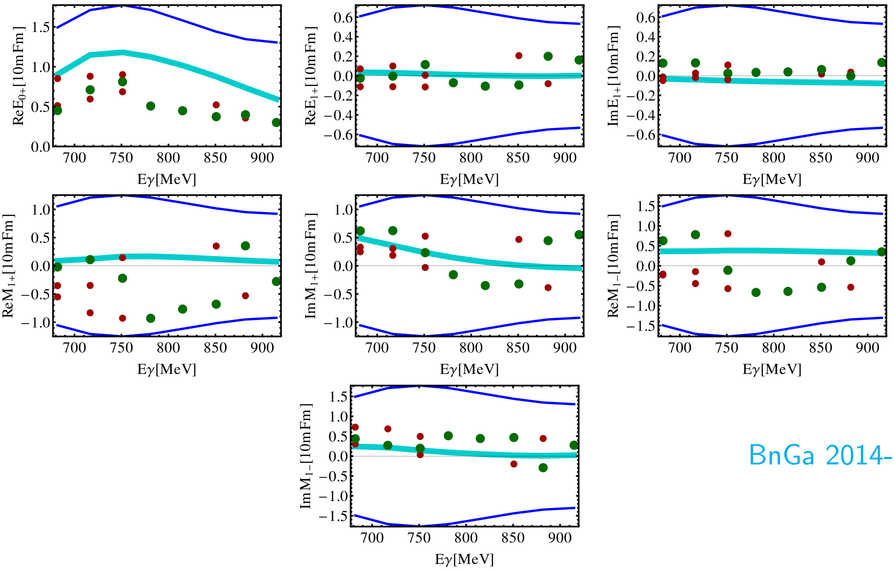


BnGa 2014-02

S-, P- and D-waves in the interval $\left(\chi^2_{\mathrm{best}} + 0.5\right)$

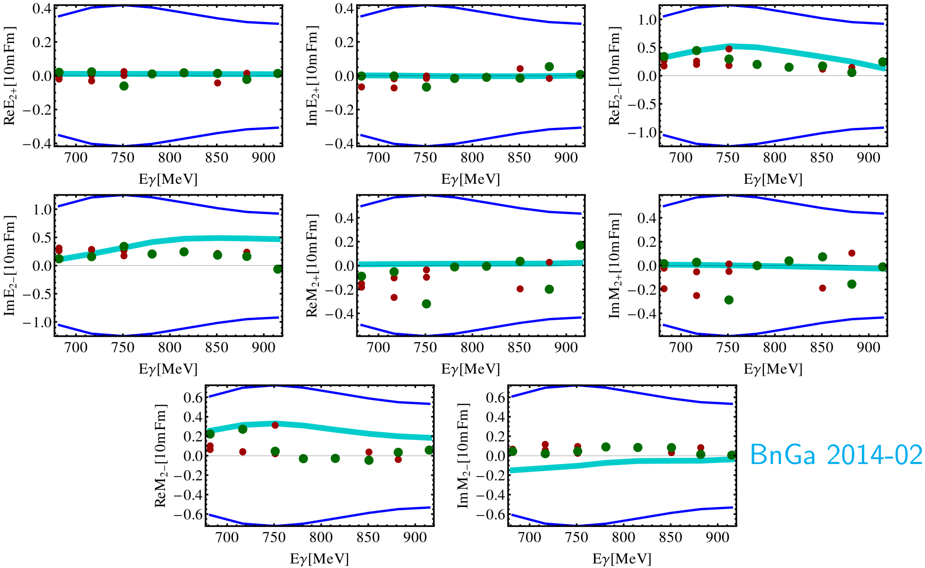


S-, P- and D-waves in the interval $\left(\chi^2_{\mathrm{best}} + 1.0\right)$

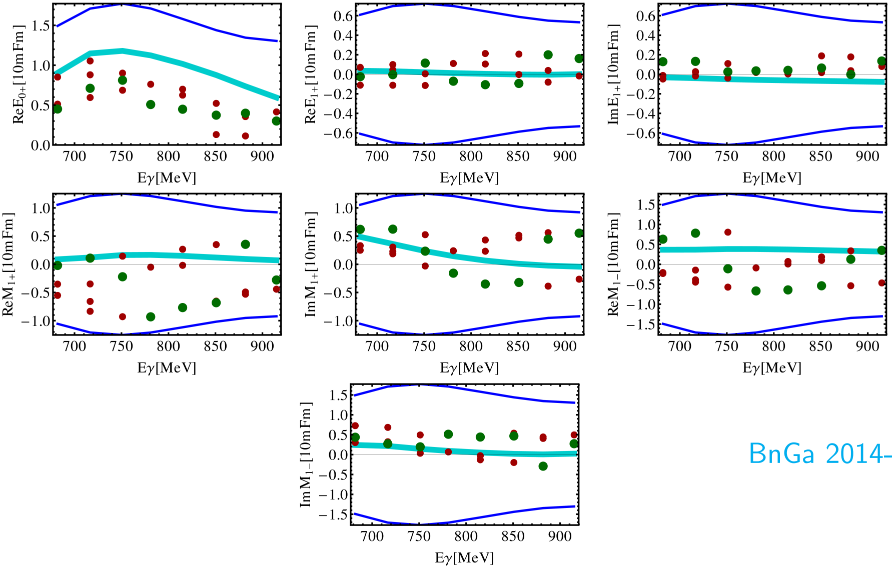


BnGa 2014-02

S-, P- and D-waves in the interval $\left(\chi^2_{\text{best}} + 1.0\right)$

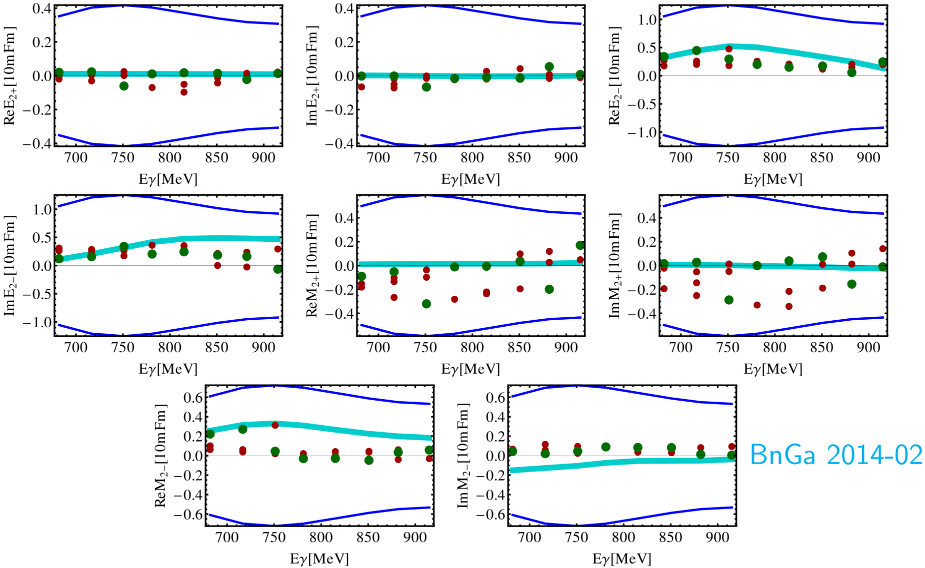


S-, P- and D-waves in the interval $\left(\chi^2_{\mathrm{best}} + 4.0\right)$



BnGa 2014-02

S-, P- and D-waves in the interval $\left(\chi^2_{\mathrm{best}} + 4.0\right)$



Problems with the $\ell_{\max} = 2$ multipole fit

There exists a unique, well-separated (in χ^2) solution for $\ell_{\max} = 2$,

however:

(i) χ^2/ndf is too large for all energy bins except the first 2-4.

(ii) Solution does not make sense compared to models (more precisely, to BnGa 2014-02).

Partial wave interferences in Legendre coefficients

$$\left(a_L\right)_k^{\alpha} = \left[\; \mathcal{M}^{*}_{\ell \leq \ell_{\mathrm{max}}} \; \middle| \; \mathcal{M}^{*}_{\ell > \ell_{\mathrm{max}}} \; \right] \left[\begin{array}{c|c} \left(\mathcal{C}_L\right)_k^{\alpha} & \left(\tilde{\mathcal{C}}_L\right)_k^{\alpha} \\ \hline \left[\left(\tilde{\mathcal{C}}_L\right)_k^{\alpha}\right]^{\dagger} & \left(\hat{\mathcal{C}}_L\right)_k^{\alpha} \end{array} \right] \left[\begin{array}{c} \mathcal{M}_{\ell \leq \ell_{\mathrm{max}}} \\ \hline \mathcal{M}_{\ell > \ell_{\mathrm{max}}} \end{array} \right]$$

$*$) In the $\left(a_L\right)_k^{\alpha}$, partial waves with $\ell_{\mathrm{max}} \geq 3$ may interfere with those having $\ell_{\mathrm{max}} < 3$ but the LFits may only hint at this, or not show this at all!

Partial wave interferences in Legendre coefficients

$$\left(a_L\right)_k^{\alpha} = \left[\ \mathcal{M}^{*}_{\ell \leq \ell_{\mathrm{max}}} \ \middle|\ \mathcal{M}^{*}_{\ell > \ell_{\mathrm{max}}}\ \right] \left[\begin{array}{c|c} \left(\mathcal{C}_L\right)_k^{\alpha} & \left(\tilde{\mathcal{C}}_L\right)_k^{\alpha} \\ \hline \left[\left(\tilde{\mathcal{C}}_L\right)_k^{\alpha}\right]^{\dagger} & \left(\hat{\mathcal{C}}_L\right)_k^{\alpha} \end{array} \right] \left[\begin{array}{c} \mathcal{M}_{\ell \leq \ell_{\mathrm{max}}} \\ \hline \mathcal{M}_{\ell > \ell_{\mathrm{max}}} \end{array} \right]$$

*) In the $\left(a_L\right)_k^{\alpha}$, partial waves with $\ell_{\mathrm{max}} \geq 3$ may interfere with those having $\ell_{\mathrm{max}} < 3$ but the LFits may only hint at this, or <u>not</u> show this at all!

*) In case the multipole fit has all partial waves $\mathcal{M}_{\ell}$ with $\ell \geq 3$ set equal to zero, it has no chance to take into account the interferences and modify the results for S-, P-, and D-waves accordingly.

Partial wave interferences in Legendre coefficients

$$(a_L)_k^{\alpha} = \left[\; \mathcal{M}^{*}_{\ell \leq \ell_{\mathrm{max}}} \; \middle| \; \mathcal{M}^{*}_{\ell > \ell_{\mathrm{max}}} \; \right] \left[\begin{array}{c|c} \left(\mathcal{C}_L\right)_k^{\alpha} & \left(\tilde{\mathcal{C}}_L\right)_k^{\alpha} \\ \hline \left[\left(\tilde{\mathcal{C}}_L\right)_k^{\alpha}\right]^{\dagger} & \left(\hat{\mathcal{C}}_L\right)_k^{\alpha} \end{array} \right] \left[\begin{array}{c} \mathcal{M}_{\ell \leq \ell_{\mathrm{max}}} \\ \hline \mathcal{M}_{\ell > \ell_{\mathrm{max}}} \end{array} \right]$$

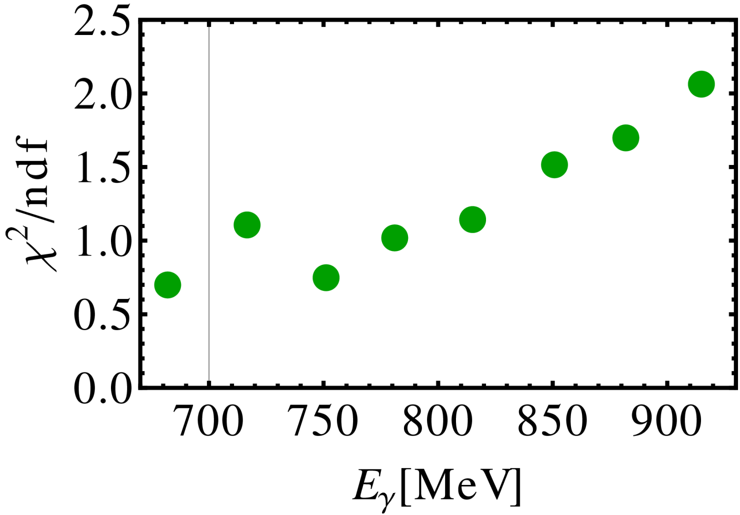
*) In the $(a_L)_k^{\alpha}$, partial waves with $\ell_{\mathrm{max}} \geq 3$ may interfere with those having $\ell_{\mathrm{max}} < 3$ but the LFits may only hint at this, or not show this at all!

*) In case the multipole fit has all partial waves $\mathcal{M}_{\ell}$ with $\ell \geq 3$ set equal to zero, it has no chance to take into account the interferences and modify the results for S-, P-, and D-waves accordingly.

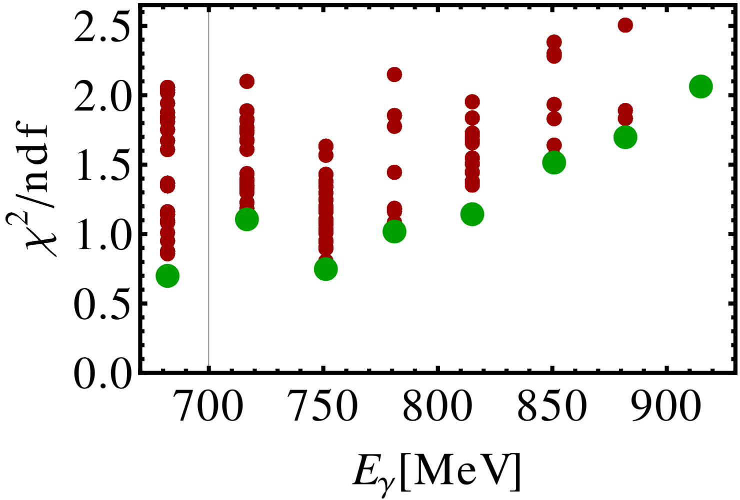
→ One has to at least take into account F-waves into the fitting in some way!

→ Fit a truncation at $\ell_{\mathrm{max}} = 3$ and let the F-waves run freely in the fit.

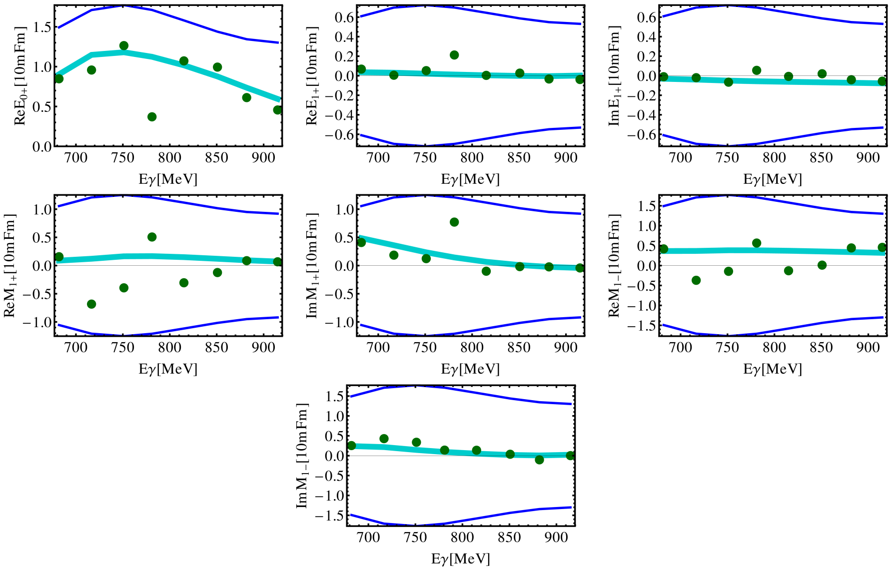
χ^2_{best} vs. E_γ for the $\ell_{\mathrm{max}} = 3$-fit



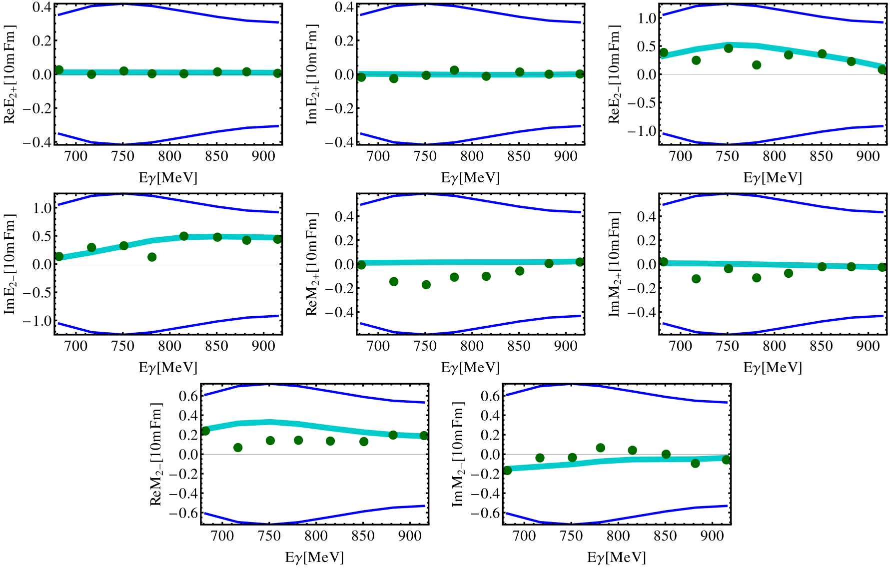
χ^2_{best} vs. E_γ for the $\ell_{\mathrm{max}} = 3$-fit



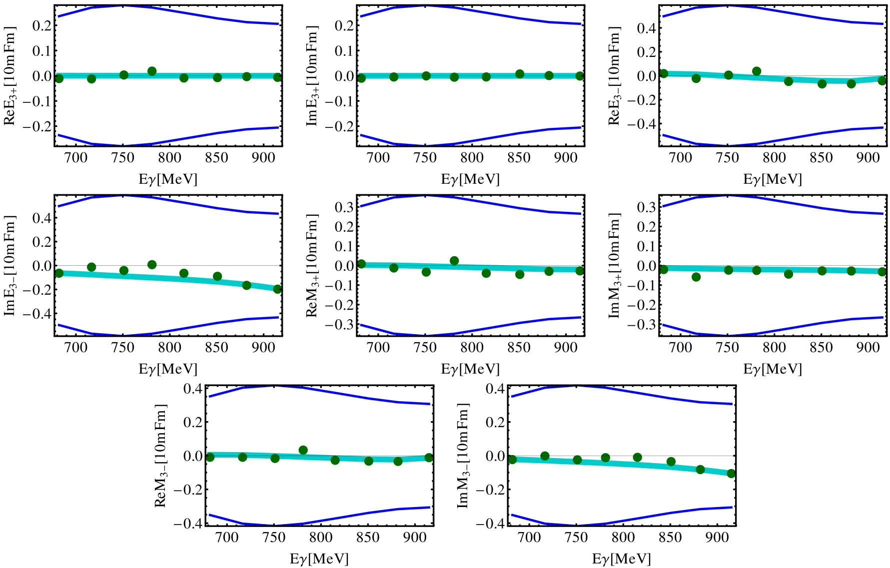
The best solution for *S*-, *P*-, *D*- and *F*-waves



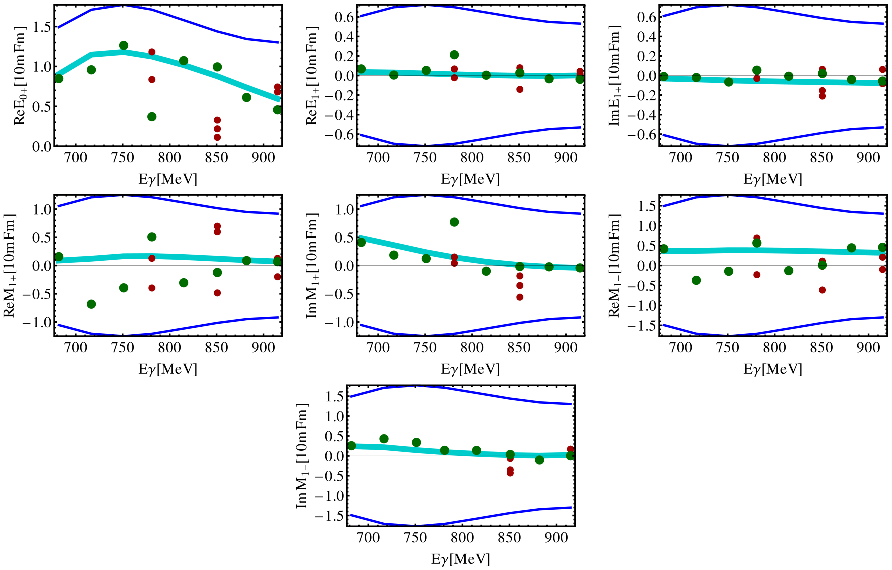
The best solution for *S*-, *P*-, *D*- and *F*-waves



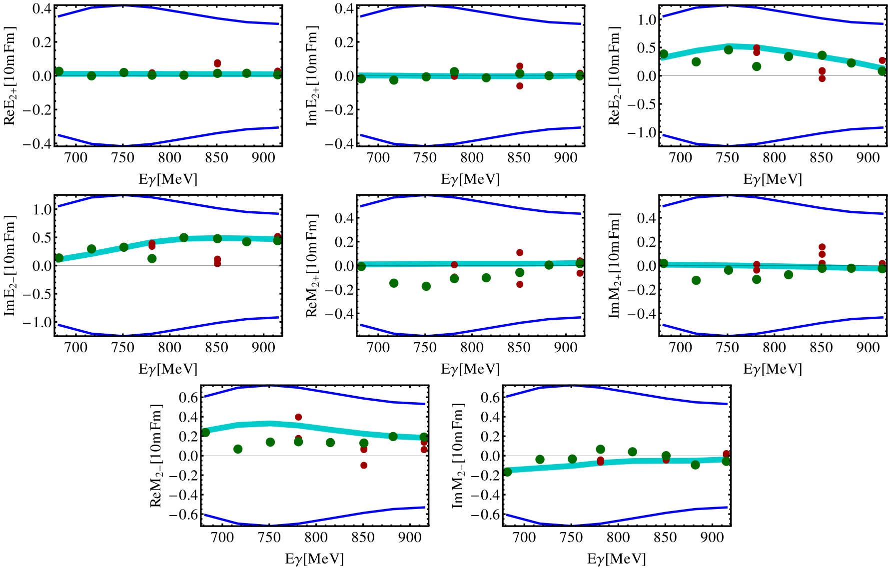
The best solution for *S*-, *P*-, *D*- and *F*-waves



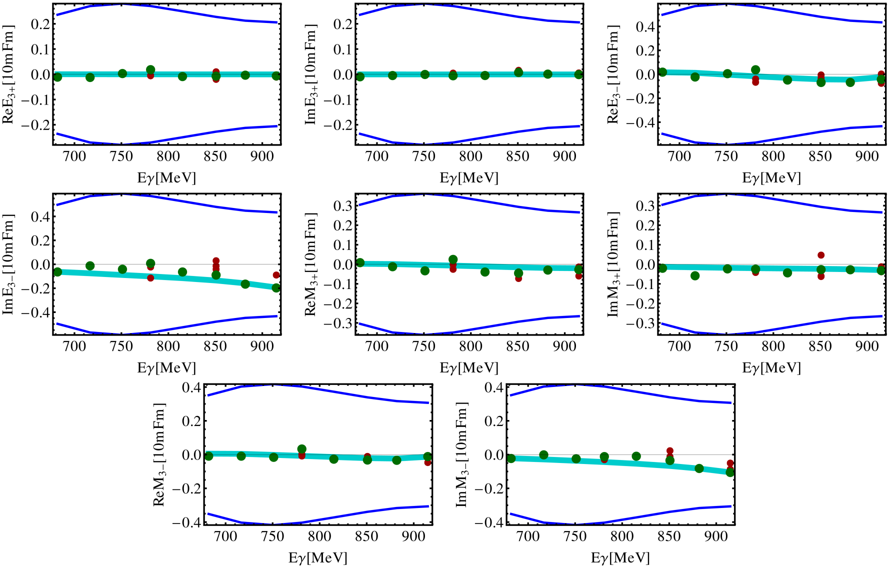
S-, P-, D- and F-waves in the interval $\left(\chi^2_{\text{best}} + 0.05\right)$



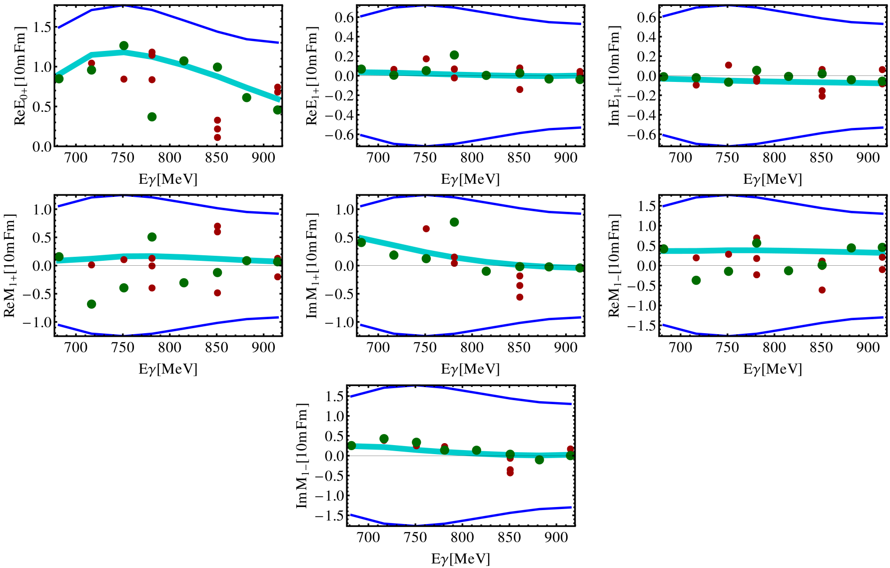
S-, P-, D- and F-waves in the interval $\left(\chi^2_{\text{best}} + 0.05\right)$



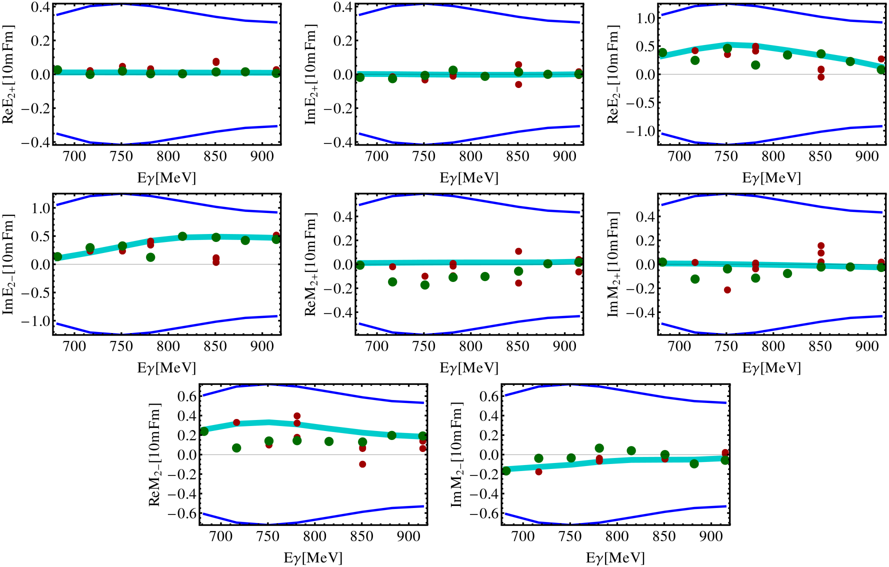
S-, P-, D- and F-waves in the interval $\left(\chi^2_{\mathrm{best}} + 0.05\right)$



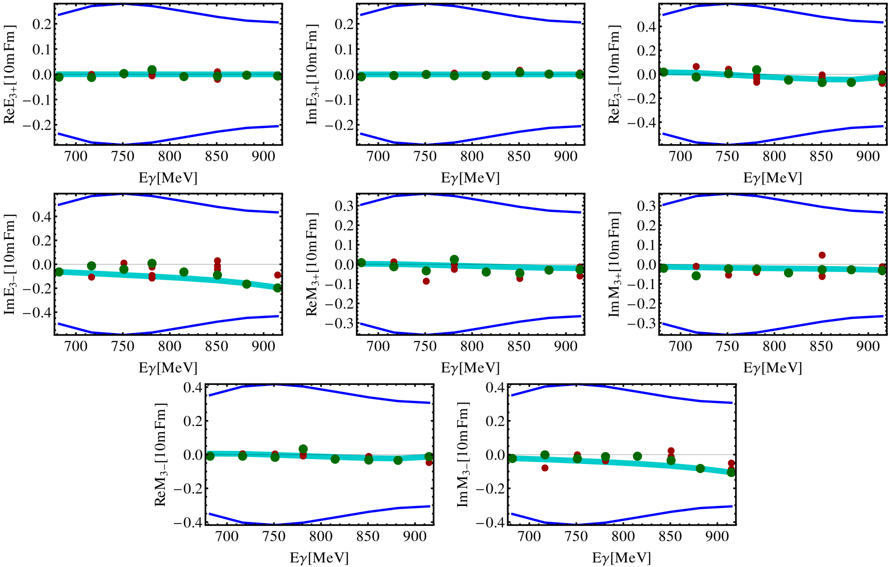
S-, P-, D- and F-waves in the interval $\left(\chi^2_{\mathrm{best}} + 0.1\right)$



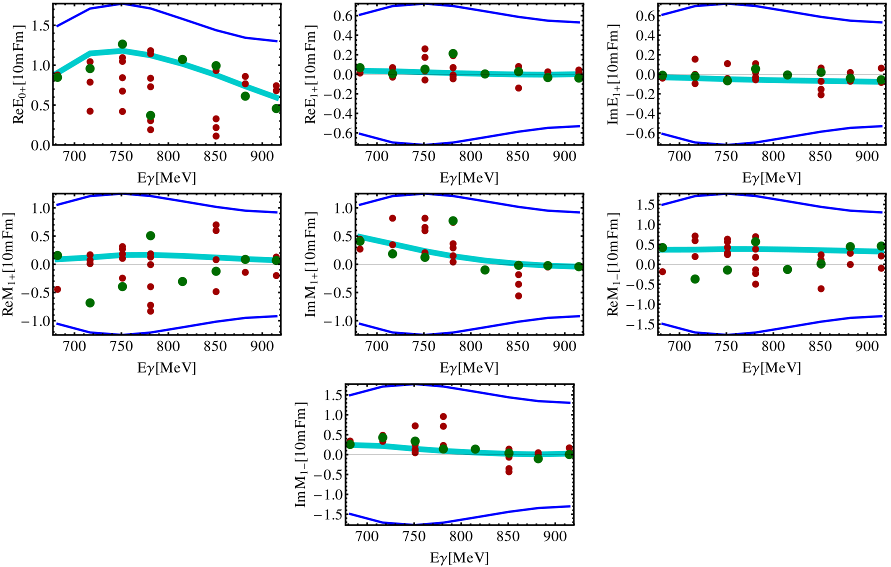
S-, P-, D- and F-waves in the interval $\left(\chi^2_{\mathrm{best}} + 0.1\right)$



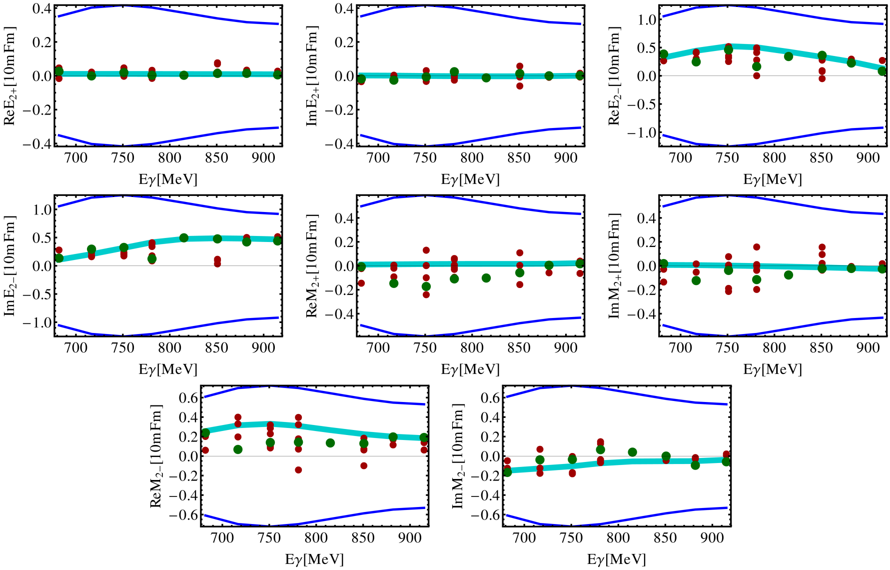
S-, P-, D- and F-waves in the interval $\left(\chi^2_{\text{best}} + 0.1\right)$



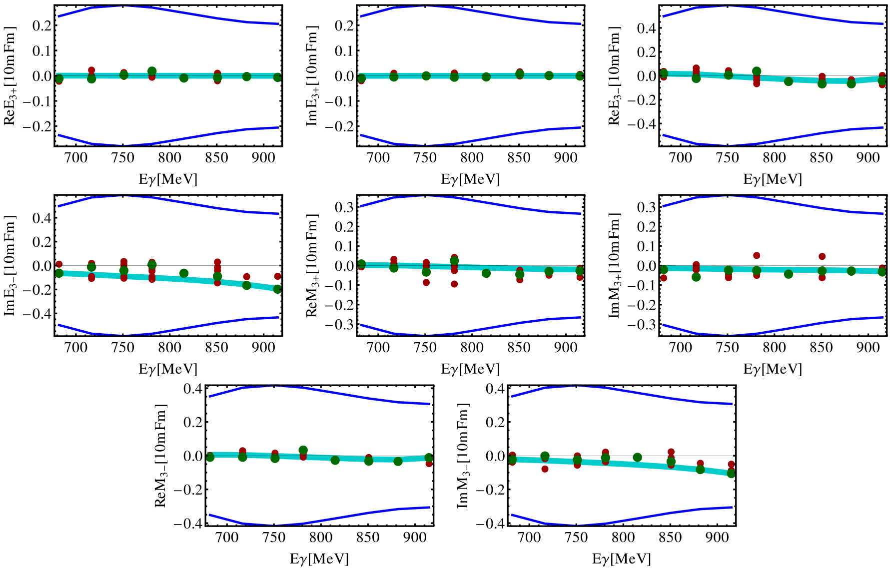
S-, P-, D- and F-waves in the interval $\left(\chi^2_{\mathrm{best}} + 0.2\right)$



S-, P-, D- and F-waves in the interval $\left(\chi^2_{\mathrm{best}} + 0.2\right)$



S-, P-, D- and F-waves in the interval $\left(\chi^2_{\text{best}} + 0.2\right)$



Problems with the $\ell_{\mathrm{max}} = 3$ multipole fit

There exists a global minimum, which is however not well separated from the other local minima of χ^2!

Problems with the $\ell_{\mathrm{max}} = 3$ multipole fit

There exists a global minimum, which is however not well separated from the other local minima of χ^2!

Reasons:

(i) Equation set defined by $\left(a_L^{\mathrm{Fit}} \right)_k^{\alpha}$ is not "compatible" ($\equiv$ exactly solvable).

Problems with the $\ell_{\mathrm{max}} = 3$ multipole fit

There exists a global minimum, which is however not well separated from the other local minima of χ^2!

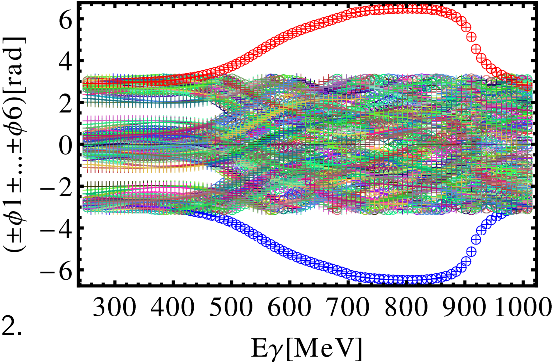
Reasons:

(i) Equation set defined by $\left(a_L^{\mathrm{Fit}}\right)_k^{\alpha}$ is not "compatible" ($\equiv$ exactly solvable).

(ii) There exist

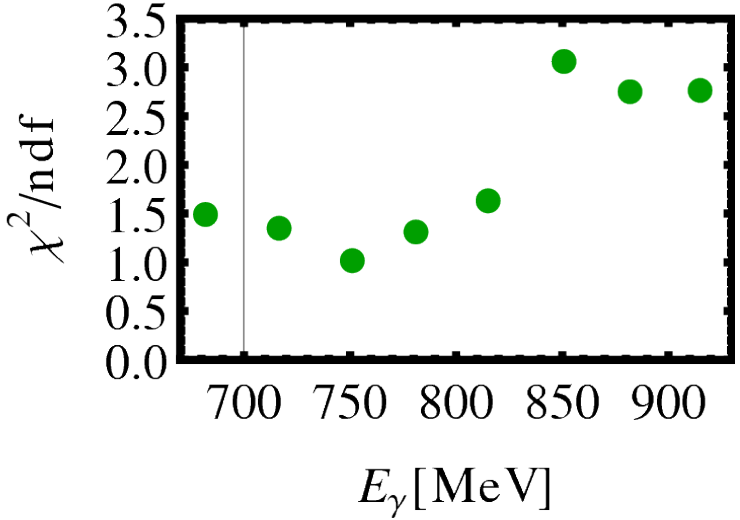
$$\frac{1}{2}\left(4^{2\ell_{\mathrm{max}}} - 2\right) = 2047$$

candidates for accidential ambiguities for $\ell_{\mathrm{max}} = 3$!

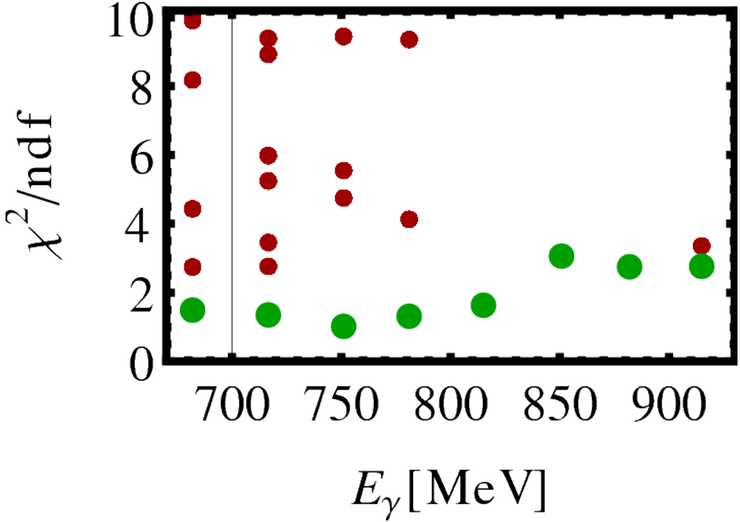


Way out: fix F-waves to a model, here: BnGa 2014-02.

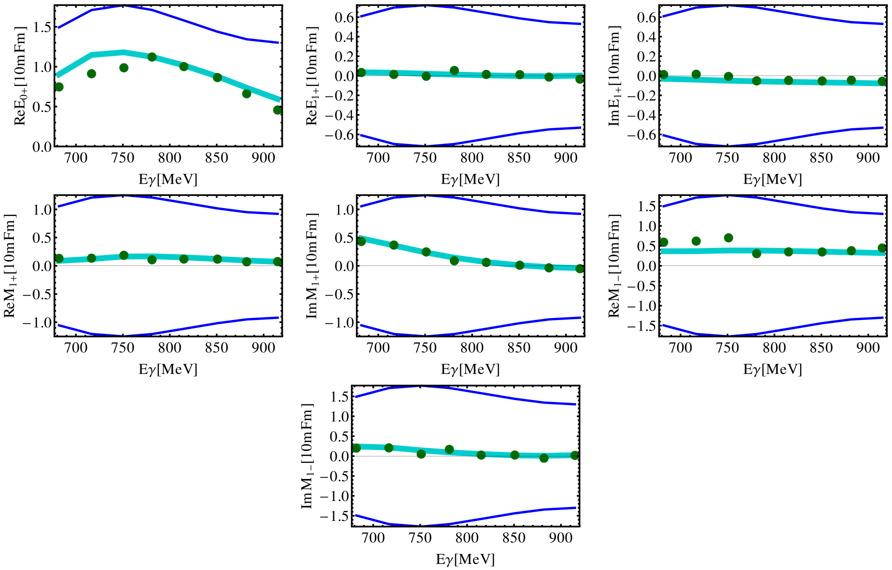
χ^2_{best} vs. E_γ for the fit including BnGa-F-waves



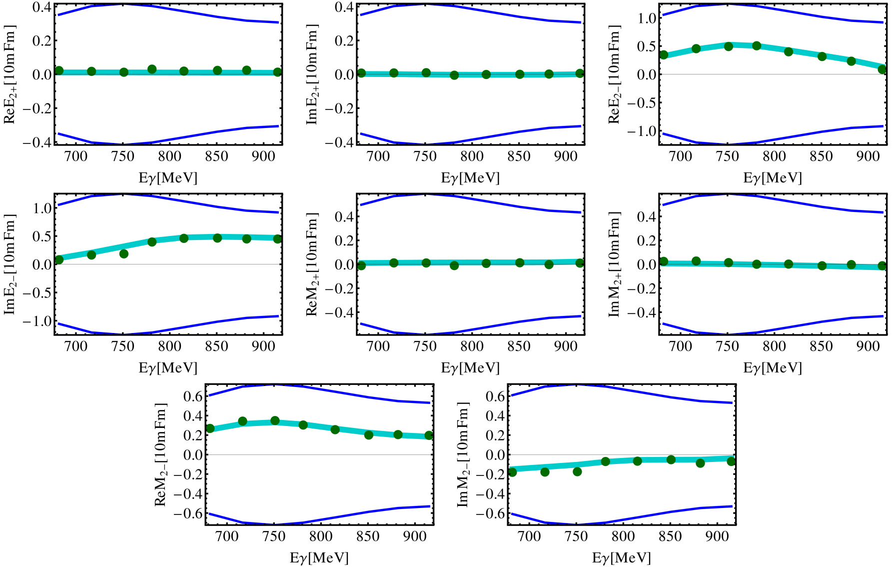
χ^2_{best} vs. E_γ for the fit including BnGa-F-waves



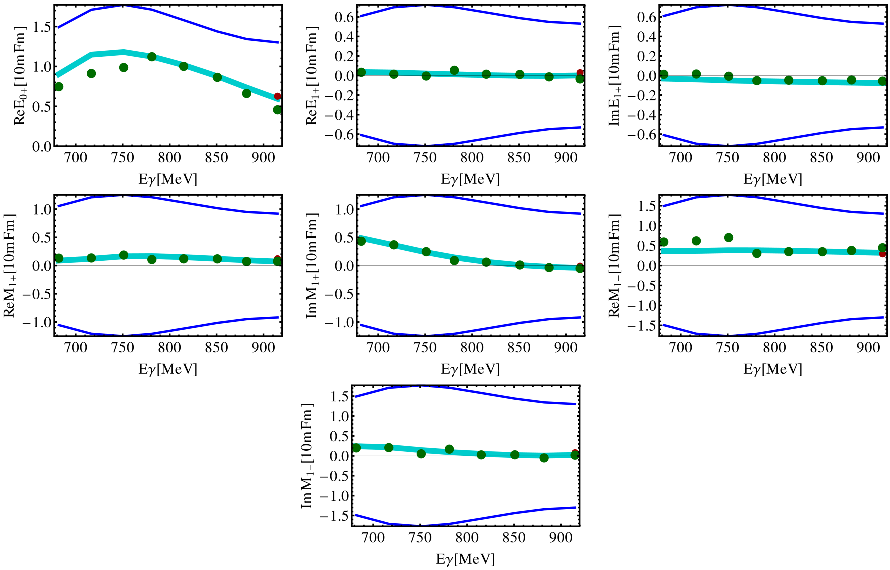
The best solution for *S*-, *P*- and *D*-waves



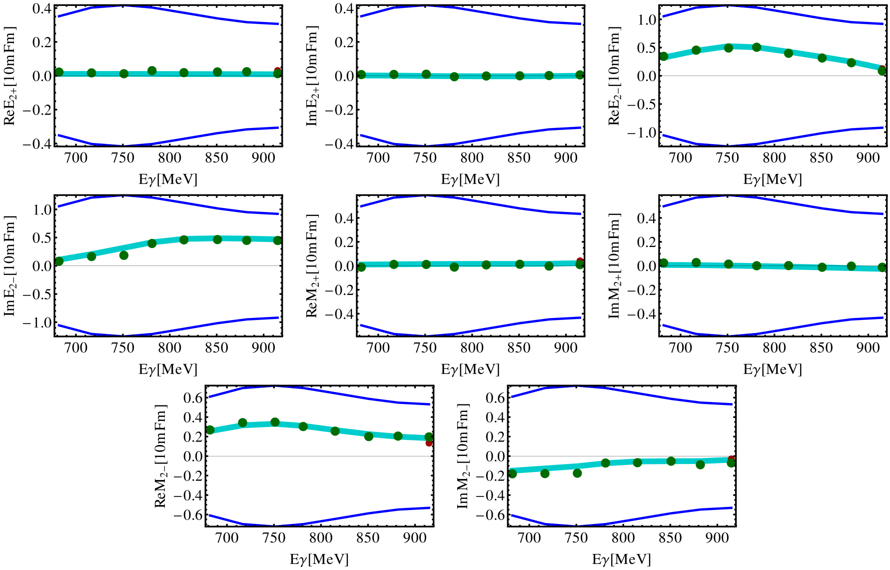
The best solution for S-, P- and D-waves



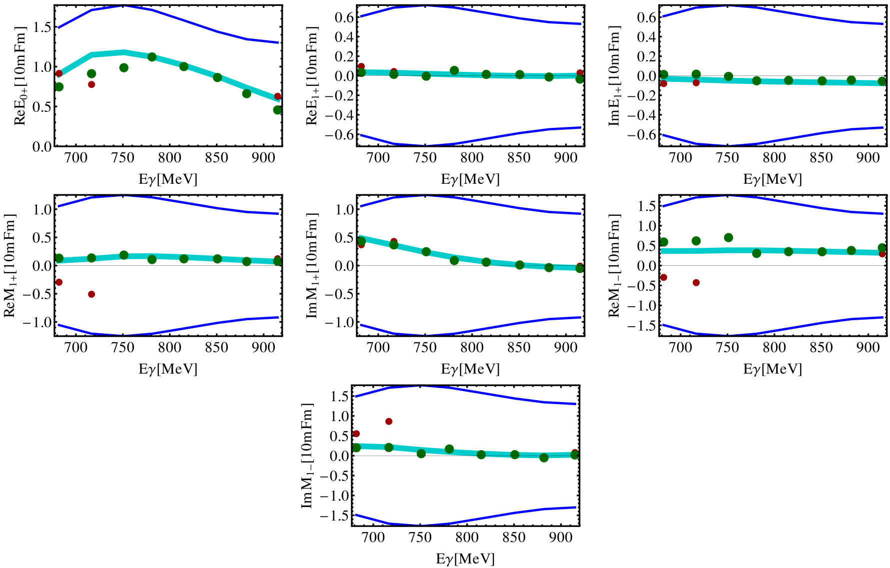
S-, P- and D-waves in the interval $\left(\chi^2_{\mathrm{best}} + 0.6\right)$



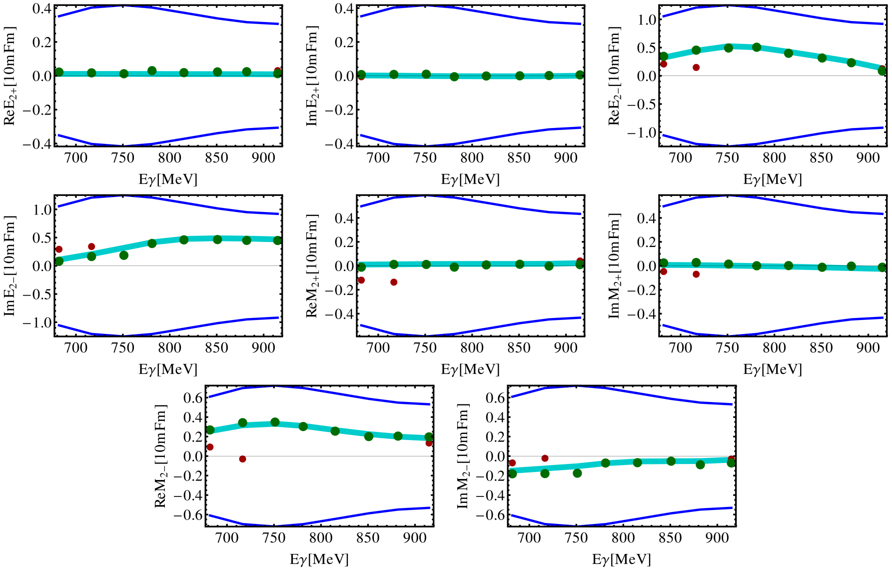
S-, P- and D-waves in the interval $\left(\chi^2_{\text{best}} + 0.6\right)$



S-, P- and D-waves in the interval $\left(\chi^2_{\mathrm{best}} + 1.5\right)$



S-, P- and D-waves in the interval $\left(\chi^2_{\text{best}} + 1.5\right)$



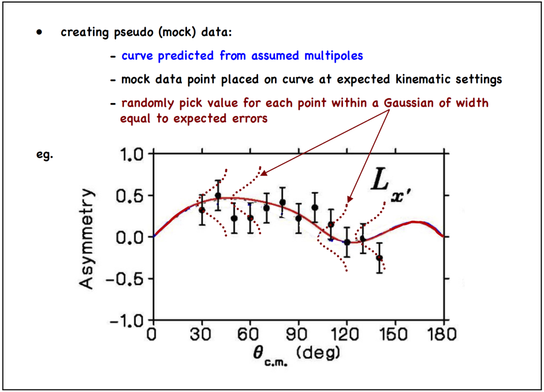
Bootstrapping

$*$) [B. Efron, The Annals Of Statistics 7 no. 1, 1 (1979)]: Estimate an unknown distribution function of a statistic $T(X_1, \ldots, X_n)$, by generating bootstrap random samples $x_b = (x_1^*, \ldots, x_n^*)$ from a given set of data $(x_1, \ldots, x_n)$ and approximating the T-distribution-fct. by $T_b(x_b)$.

Bootstrapping

$*)$ [B. Efron, The Annals Of Statistics 7 no. 1, 1 (1979)]: Estimate an unknown distribution function of a statistic $T\left(X_1, \ldots, X_n\right)$, by generating bootstrap random samples $x_b = \left(x_1^*, \ldots, x_n^*\right)$ from a given set of data $\left(x_1, \ldots, x_n\right)$ and approximating the T-distribution-fct. by $T_b\left(x_b\right)$.

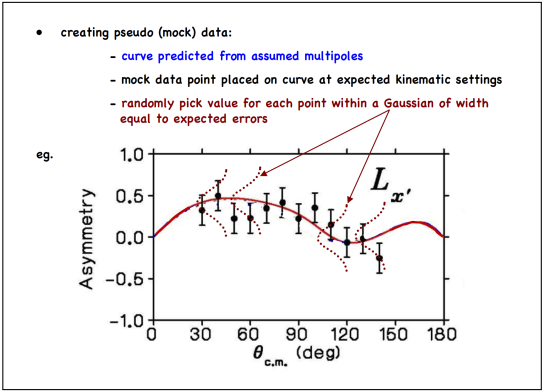
Similar ideas were already introduced into TPWA-fitting by Sandorfi in the talk [A. Sandorfi, Trento, 2014].



Bootstrapping

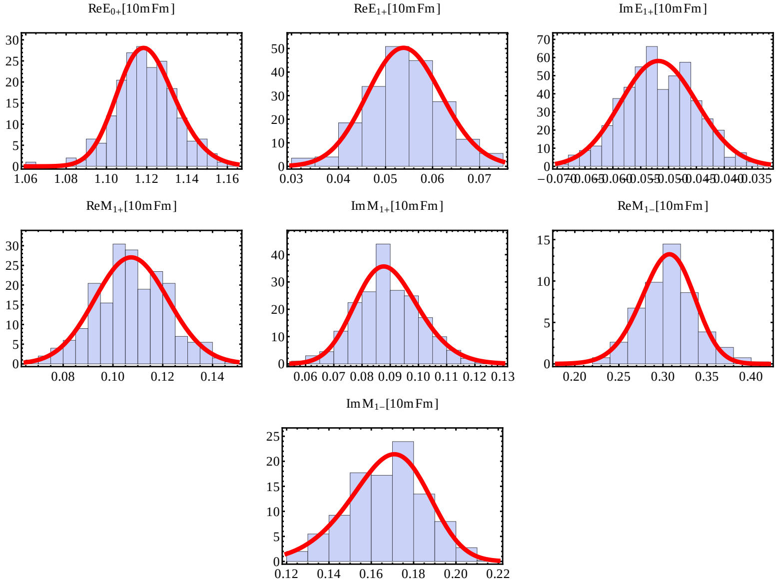
$*$) [B. Efron, The Annals Of Statistics 7 no. 1, 1 (1979)]: Estimate an unknown distribution function of a statistic $T(X_1, \ldots, X_n)$, by generating bootstrap random samples $x_b = (x_1^*, \ldots, x_n^*)$ from a given set of data $(x_1, \ldots, x_n)$ and approximating the T-distribution-fct. by $T_b(x_b)$.

Similar ideas were already introduced into TPWA-fitting by Sandorfi in the talk [A. Sandorfi, Trento, 2014].

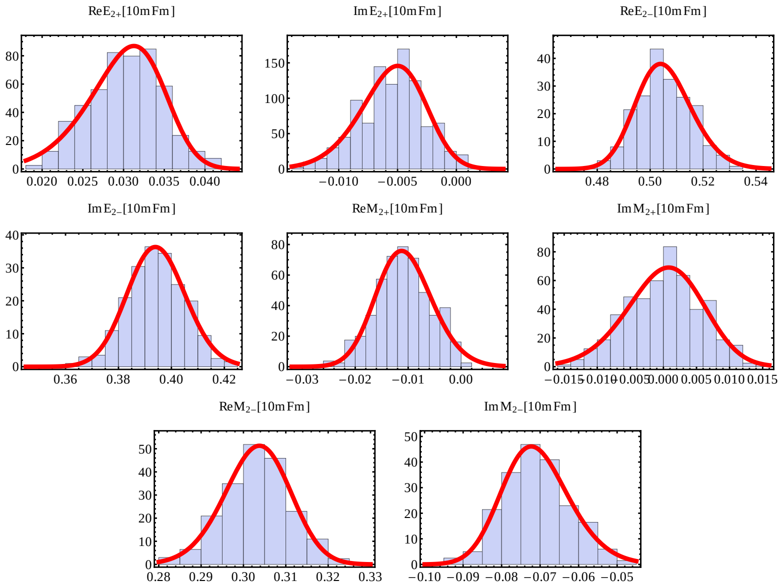


$\rightarrow$ Ensemble of $(1 + N_{\mathrm{Ens.}})$ equivalent datasets. Do TPWA for each. Distribute away from datapoint, not from a fit-curve!

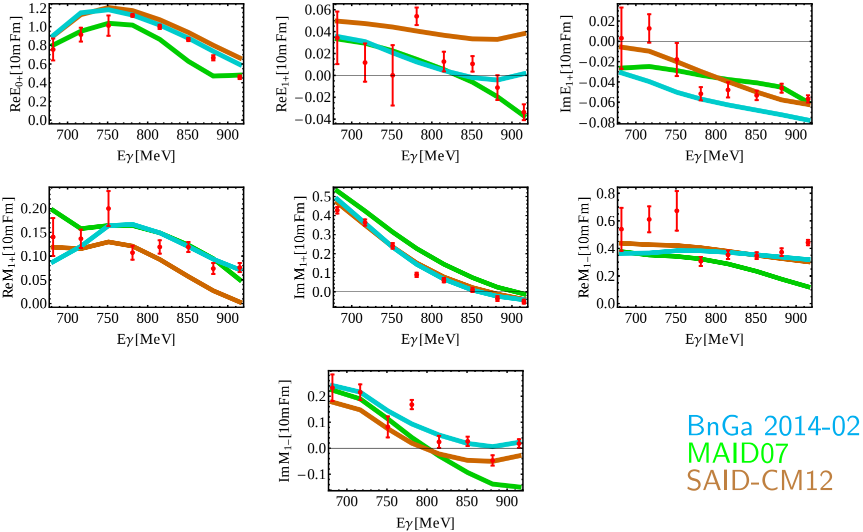
Examples for multipole bootstrap histograms



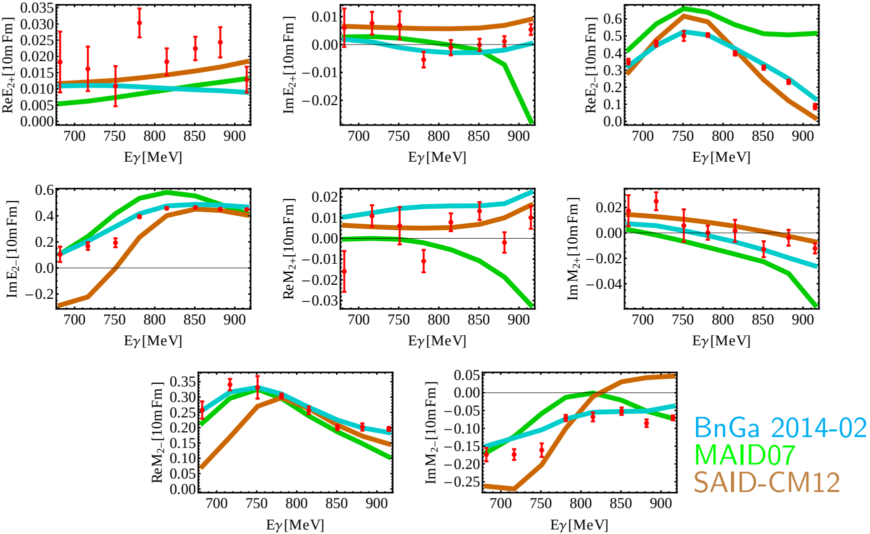
Examples for multipole bootstrap histograms



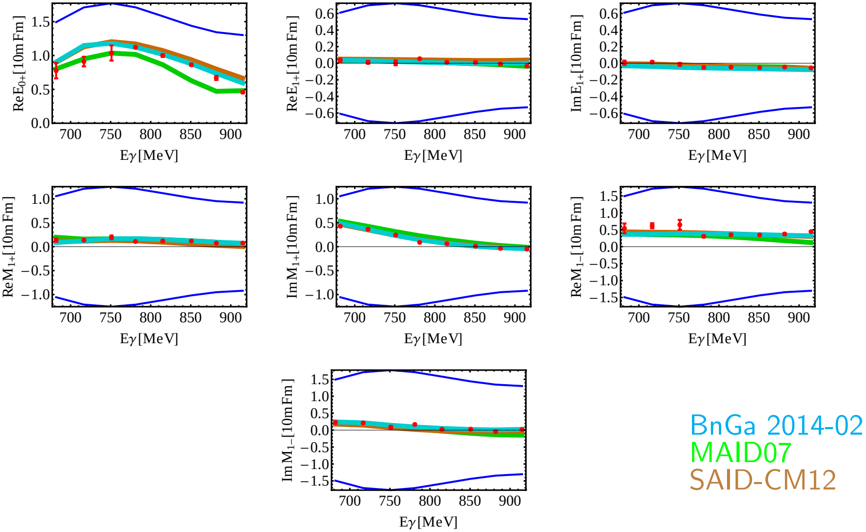
Bootstrap results for the *S*-, *P*- and *D*-waves



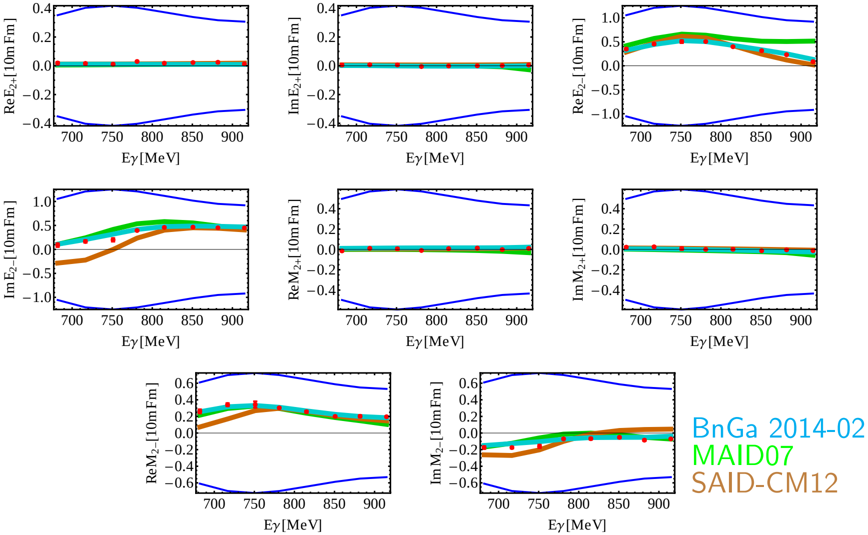
Bootstrap results for the *S*-, *P*- and *D*-waves



Bootstrap results for the *S*-, *P*- and *D*-waves - Whole plot interval



Bootstrap results for the S-, P- and D-waves - Whole plot interval



Summary & Outlook

$*$) The monte-carlo sampling fit method was applies to $\{\sigma_0, \Sigma, T, P, E, G, H\}$-data for $\gamma p \rightarrow \pi^0 p$ in the 2^{nd} resonance region.

Summary & Outlook

*) The monte-carlo sampling fit method was applies to $\{\sigma_0, \Sigma, T, P, E, G, H\}$-data for $\gamma p \to \pi^0 p$ in the 2^{nd} resonance region.

- → "LFits" suggest an $\ell_{\mathrm{max}} = 2$-truncation to describe the data.
- → $\ell_{\mathrm{max}} = 2$ multipole fit: the best solution is "unique" but χ^2 too large (high-low partial wave interferences!)
- → $\ell_{\mathrm{max}} = 3$ multipole fit: a "unique" global minimum exists, however there are many side-minima (ambiguities!)
- → S-, P-wave multipoles varied, F-waves fixed to BnGa:
  Monte Carlo method yields a global minimum, well separated from other local minima.
  χ^2/ndf and the behaviour of the solution are reasonable.

Summary & Outlook

$\ast$) The monte-carlo sampling fit method was applies to $\{\sigma_0, \Sigma, T, P, E, G, H\}$-data for $\gamma p \to \pi^0 p$ in the 2^{nd} resonance region.

- $\to$ "LFits" suggest an $\ell_{\mathrm{max}} = 2$-truncation to describe the data.
- $\to$ $\ell_{\mathrm{max}} = 2$ multipole fit: the best solution is "unique" but χ^2 too large (high-low partial wave interferences!)
- $\to$ $\ell_{\mathrm{max}} = 3$ multipole fit: a "unique" global minimum exists, however there are many side-minima (ambiguities!)
- $\to$ S-, P-wave multipoles varied, F-waves fixed to BnGa: Monte Carlo method yields a global minimum, well separated from other local minima. χ^2/ndf and the behaviour of the solution are reasonable.

$\ast$) What to do with the obtained solution?

- $\to$ L+P-fitting: already done by Alfred ✓
- $\to$ Iteration of multipole-fitting with BnGa-code applied to SE-results: under construction.
- $\to$ SE-solutions as initial conditions for Tuzla-Mainz approach: ?