Dispersion relations

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Overview

- Formalism.
- DR results for B-amplitudes.
- CGLN-amplitudes.
- Multipoles.
- Conclusions.

Formalism

For real part we have:

$$Ref(z) = \frac{1}{\pi} \fint_{-\infty}^{+\infty} \frac{Imf(z')}{z' - z} dz' \quad (1)$$

For imaginary part we have:

$$Imf(z) = -\frac{1}{\pi} \fint_{-\infty}^{+\infty} \frac{Ref(z')}{z' - z} dz' \quad (2)$$

Where $\fint$ is a Principle Value integral and $Imf(z) \to 0, z \to \infty$.

Even functions

$$\boxed{f(-z) = f^*(z)}$$

$$Ref(z) = \frac{2}{\pi}\int_0^{+\infty} \frac{zImf(z')}{z'^2 - z^2} dz'$$

$$Imf(z) = -\frac{2}{\pi}\int_0^{+\infty} \frac{z'Ref(z')}{z'^2 - z^2} dz'$$

Odd functions

$$\boxed{f(-z) = -f^*(z)}$$

$$Ref(z) = \frac{2}{\pi} \fint_0^{+\infty} \frac{z' Imf(z')}{z'^2 - z^2} dz'$$

$$Imf(z) = -\frac{2}{\pi} \fint_0^{+\infty} \frac{z Ref(z')}{z'^2 - z^2} dz'$$

Dispersion Relations for Invariant amplitudes

$$\mathrm{Re}B^{(I)}_{1,2,6}(\nu,t) = B_i^{(I)\,pole}(\nu,t) + \frac{2}{\pi}\,-\!\!\!\!\!\!\int_{\nu_{thr}}^{\infty} \mathrm{d}\nu'\ \frac{\nu'\,\mathrm{Im}B^{(I)}_{1,2,6}(\nu',t)}{\nu'^2-\nu^2}\,,$$

$$\mathrm{Re}B^{(I)}_{8}(\nu,t) = B_i^{(I)\,pole}(\nu,t) + \frac{2\nu}{\pi}\,-\!\!\!\!\!\!\int_{\nu_{thr}}^{\infty} \mathrm{d}\nu'\ \frac{\mathrm{Im}B^{(I)}_{8}(\nu',t)}{\nu'^2-\nu^2}\,.$$

B_1, B_2, B_6 are crossing even functions B_8 is crossing odd for π^0 and η photoproduction.

Dispersion Relations for Invariant amplitudes

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B_1, B_2, B_6 are crossing even functions B_8 is crossing odd for π^0 and η photoproduction.

Resonance content of CGLN Amplitudes

Qualitatively up to lmax=3 (F-waves)

$$F_1(W,x) = \begin{cases} 0: E_{0+} & S_{11}(1535) \\ 1: (M_{1+}+E_{1+})x + M_{1-} & P_{13}(1720), P_{11}(1440), P_{11}(1710) \\ 2: (M_{2+}+E_{2+})x^2 + (M_{2-}+E_{2-}) & D_{15}(1675), D_{13}(1520), D_{13}(1700) \\ 3: (M_{3+}+E_{3+})x^3 + (M_{3-}+E_{3-})x & F_{15}(1680) \end{cases}$$

Resonance content of CGLN Amplitudes

Qualitatively up to lmax=3 (F-waves)

$$F_2(W,x) = \begin{cases} 1 : (M_{1+} + M_{1-}) & P_{13}(1720), P_{11}(1440) \\ 2 : (M_{2+} + M_{2-})x & D_{15}(1675), D_{13}(1520), D_{13}(1700) \\ 3 : (M_{3+} + M_{3-})x^2 & F_{15}(1680) \end{cases}$$

Resonance content of CGLN Amplitudes

Qualitatively up to lmax=3 (F-waves)

$$F_3(W,x) = \begin{cases} 1 : (E_{1+} - M_{1+}) & P_{13}(1720) \\ 2 : (E_{2+} - M_{2-})x + (E_{2-} + M_{2-}) & D_{15}(1675), D_{13}(1520), D_{13}(1700) \\ 3 : (M_{3+} - M_{3+})x^2 + (E_{3-} + M_{3-})x & F_{15}(1680) \end{cases}$$

Resonance content of CGLN Amplitudes

Qualitatively up to lmax=3 (F-waves)

$$F_4(W,x) = \begin{cases} 2 : (M_{2+} - E_{2+} - E_{2-} - M_{2-}) & D_{15}(1675), D_{13}(1520), D_{13}(1700) \\ 3 : (M_{3+} - E_{3+} - E_{3-} - M_{3-})x & F_{15}(1680) \end{cases}$$

Resonance content of the Invariant Amplitudes

- $B_1(F_1, F_2) \rightarrow S_{11}, P_{13}, P_{11}, D_{13}, D_{15}, F_{15}$
- $B_2(F_3, F_4) \rightarrow P_{13}, D_{13}, D_{15}, F_{15}$
- $B_6(F_1, F_2, F_3, F_4) \rightarrow$ All resonances
- $B_8(F_1, F_2, F_3, F_4) \rightarrow$ All resonances

Results for $t = -0.2 GeV^2$

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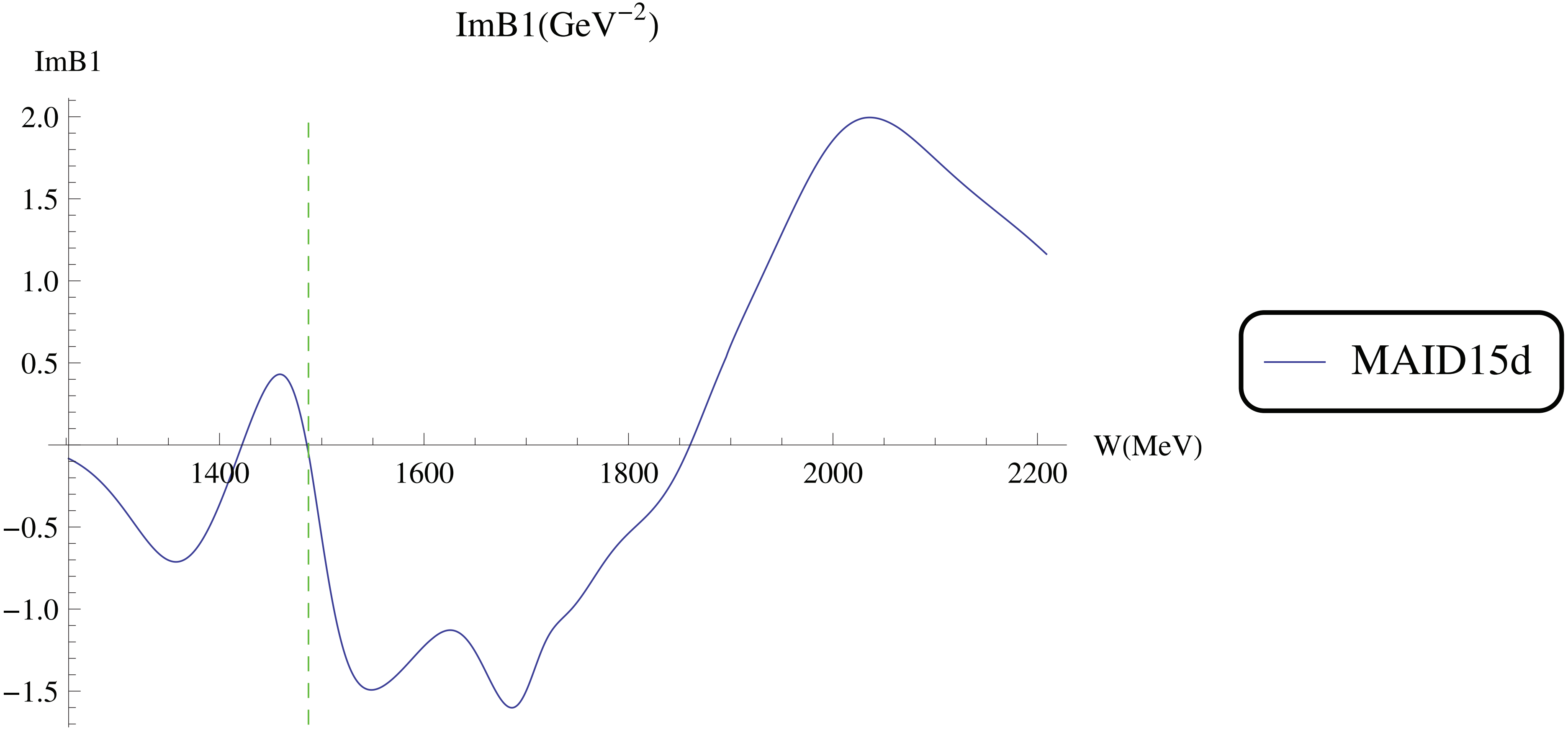

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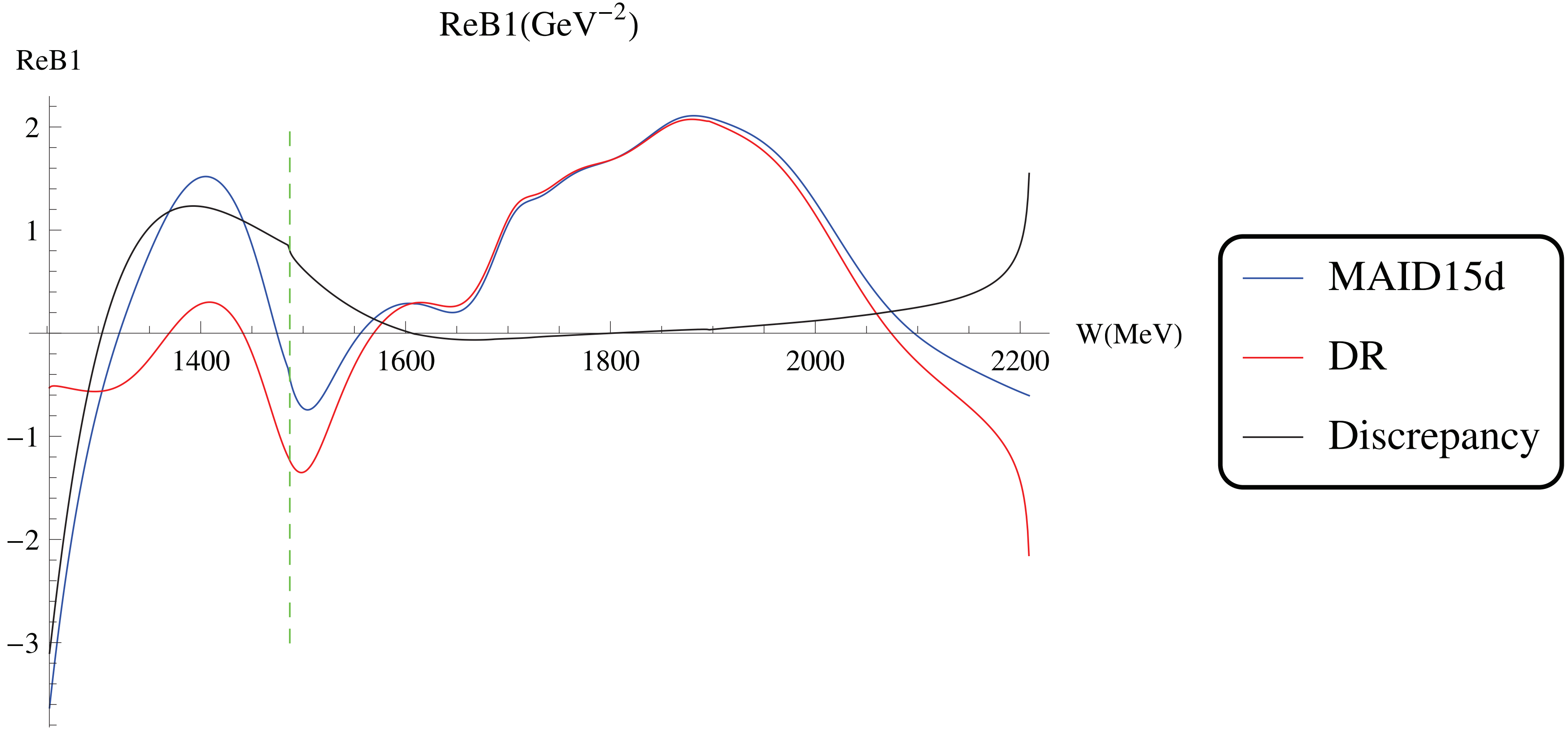

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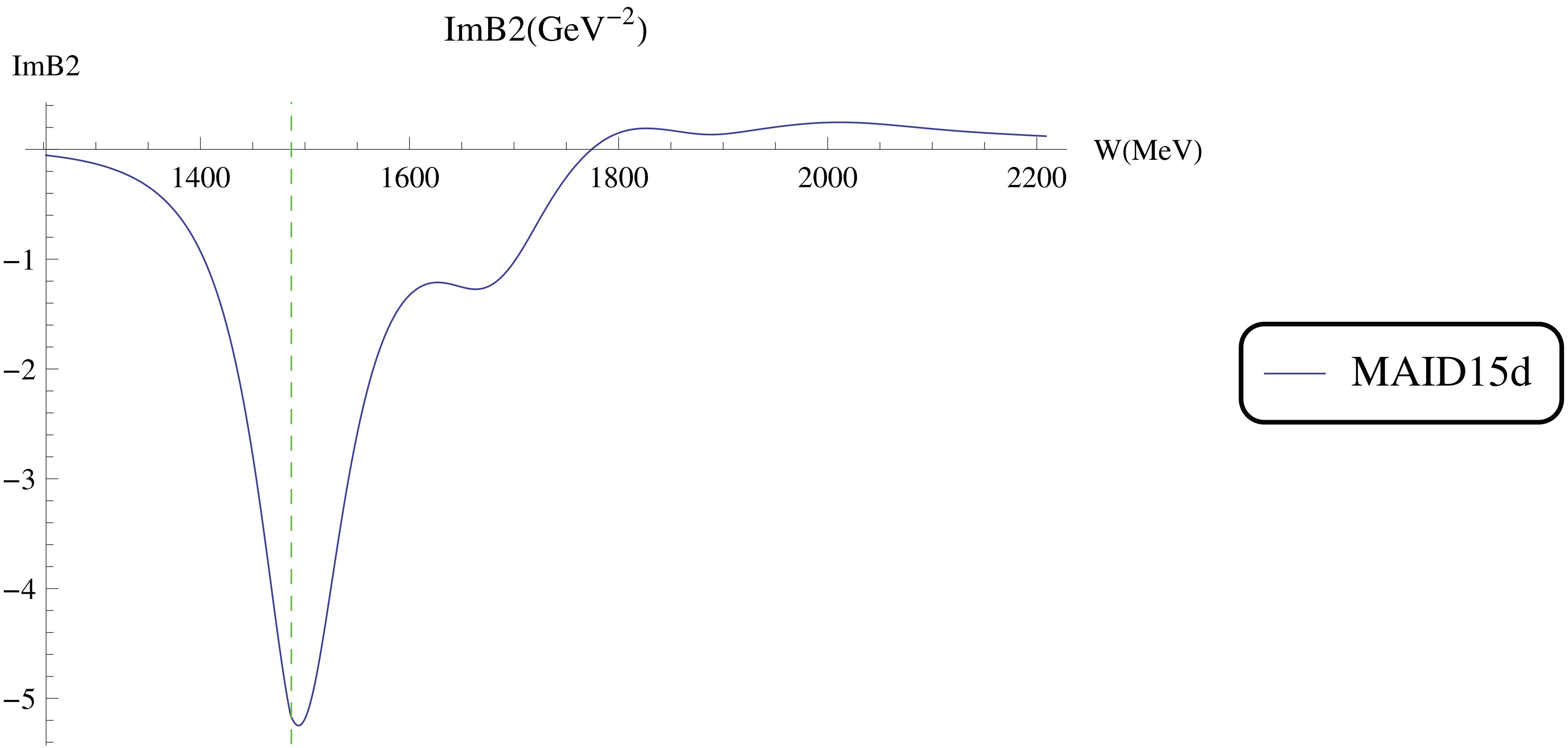

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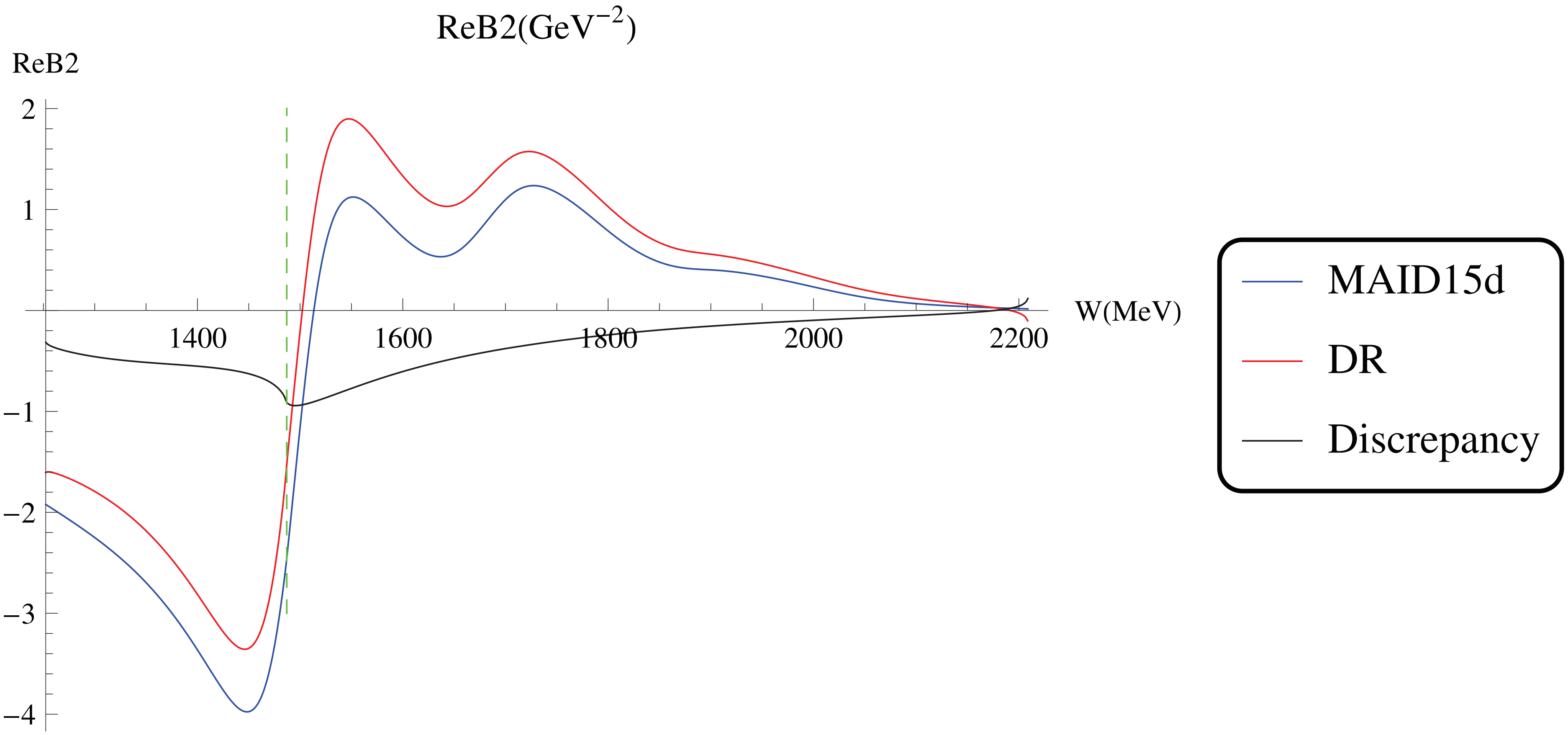

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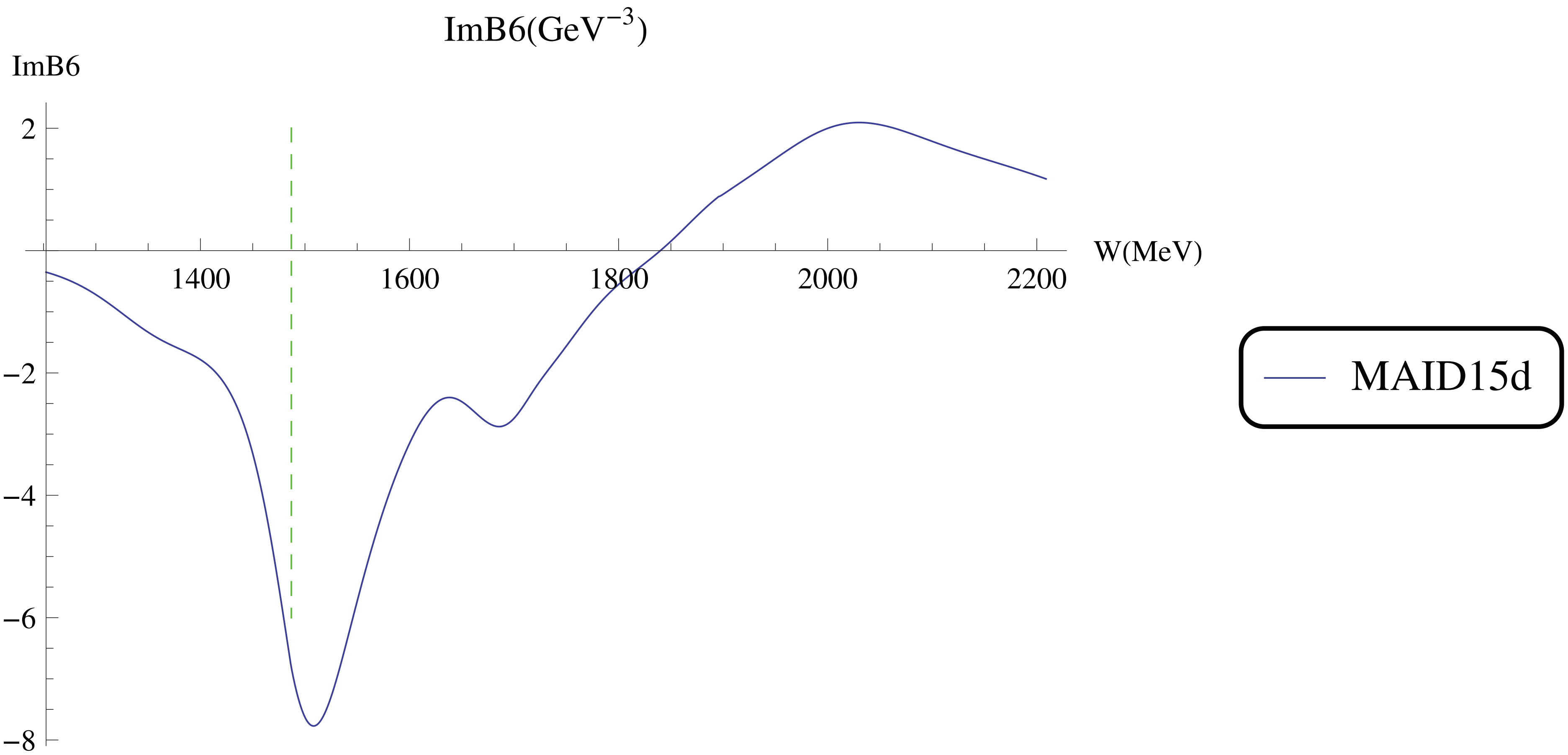

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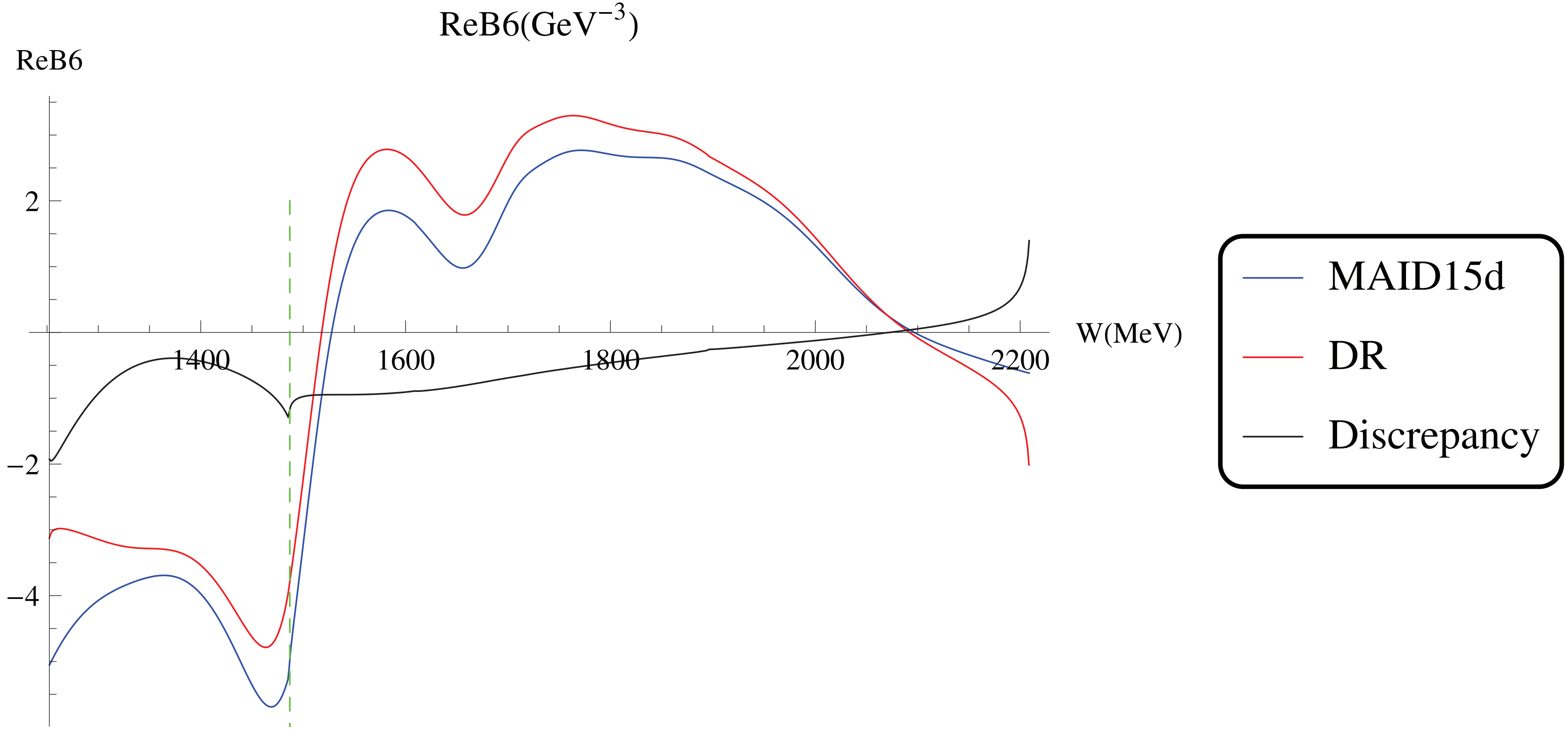

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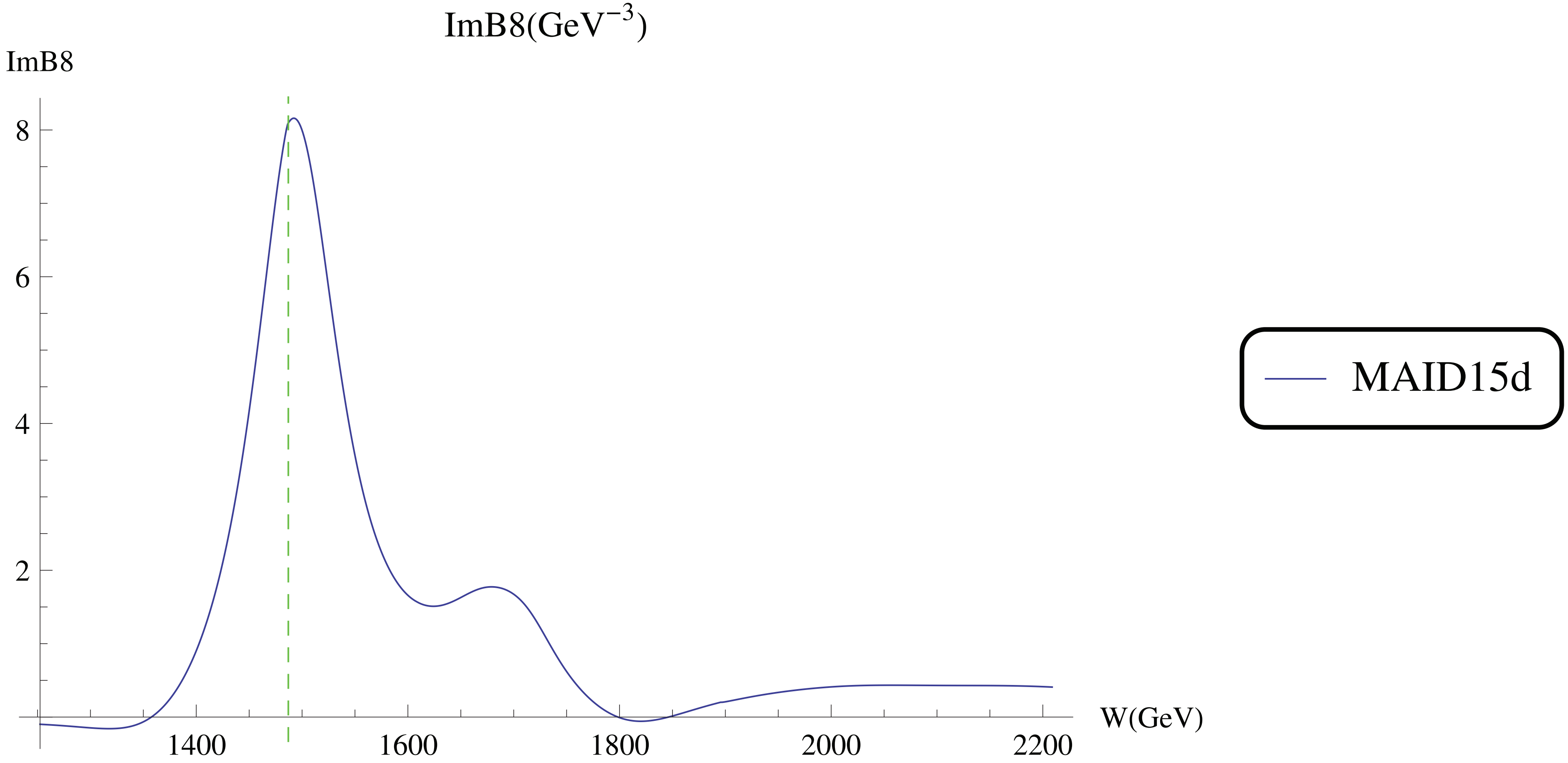

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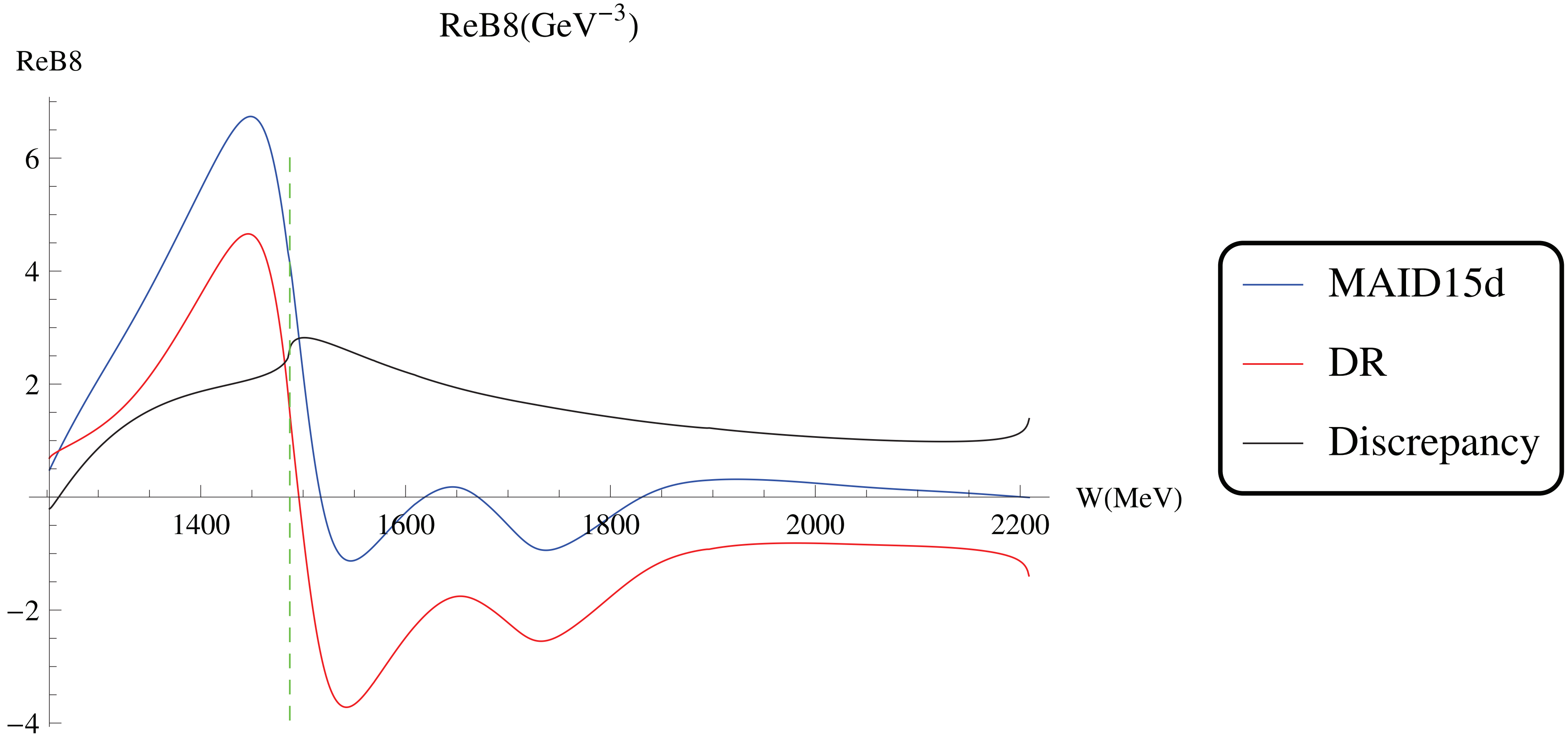

Results for $t = -0.5GeV^2$

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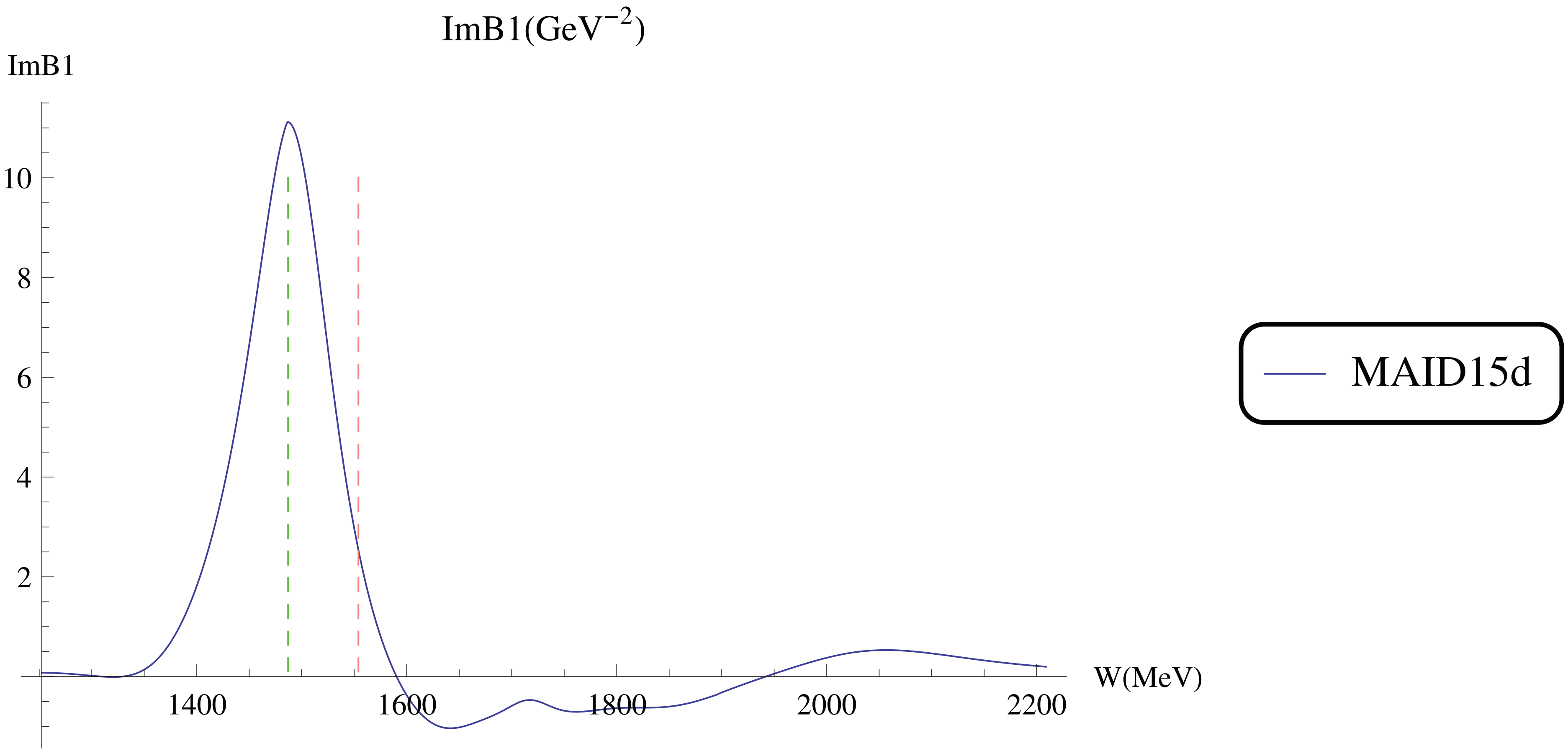

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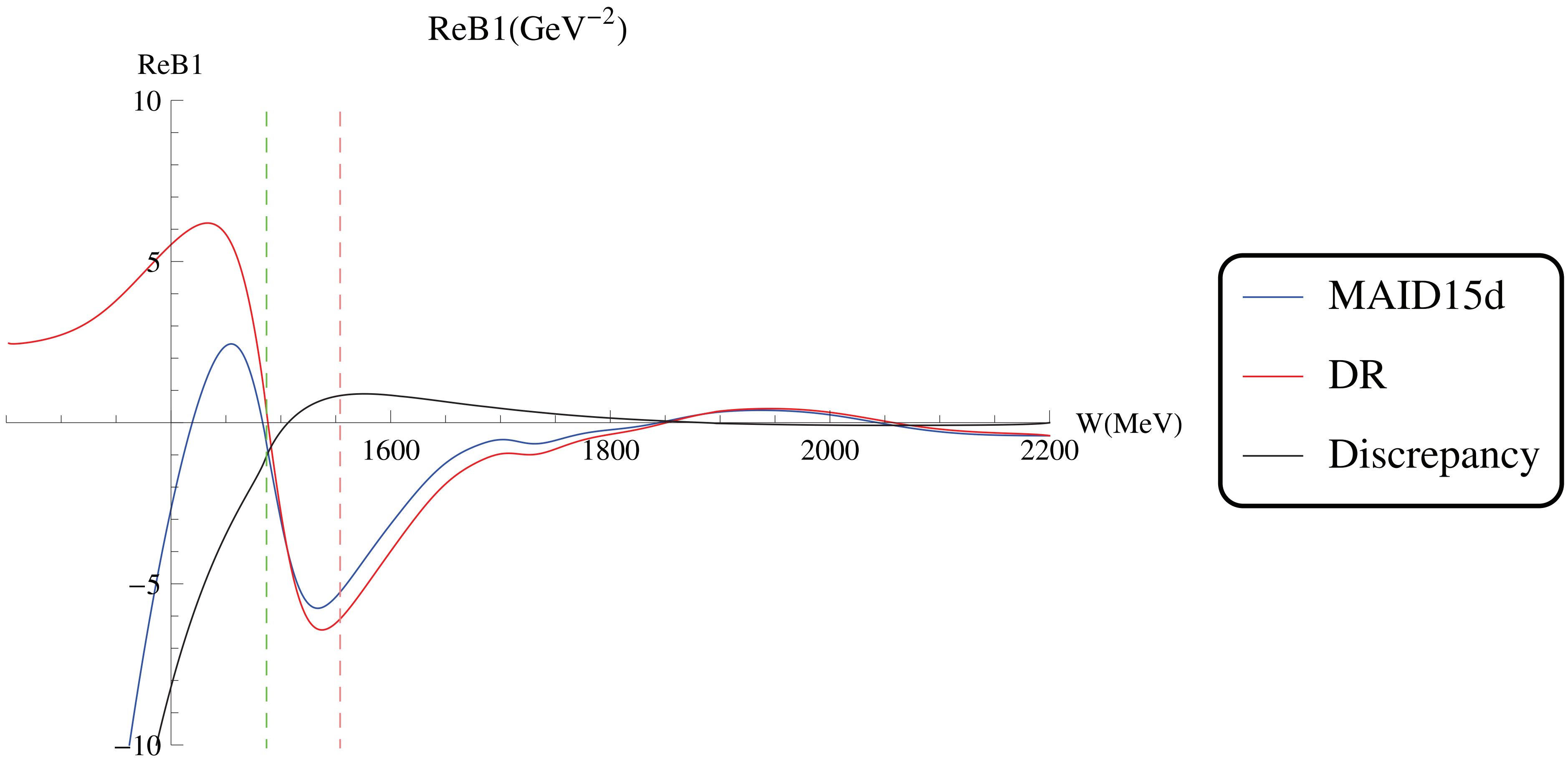

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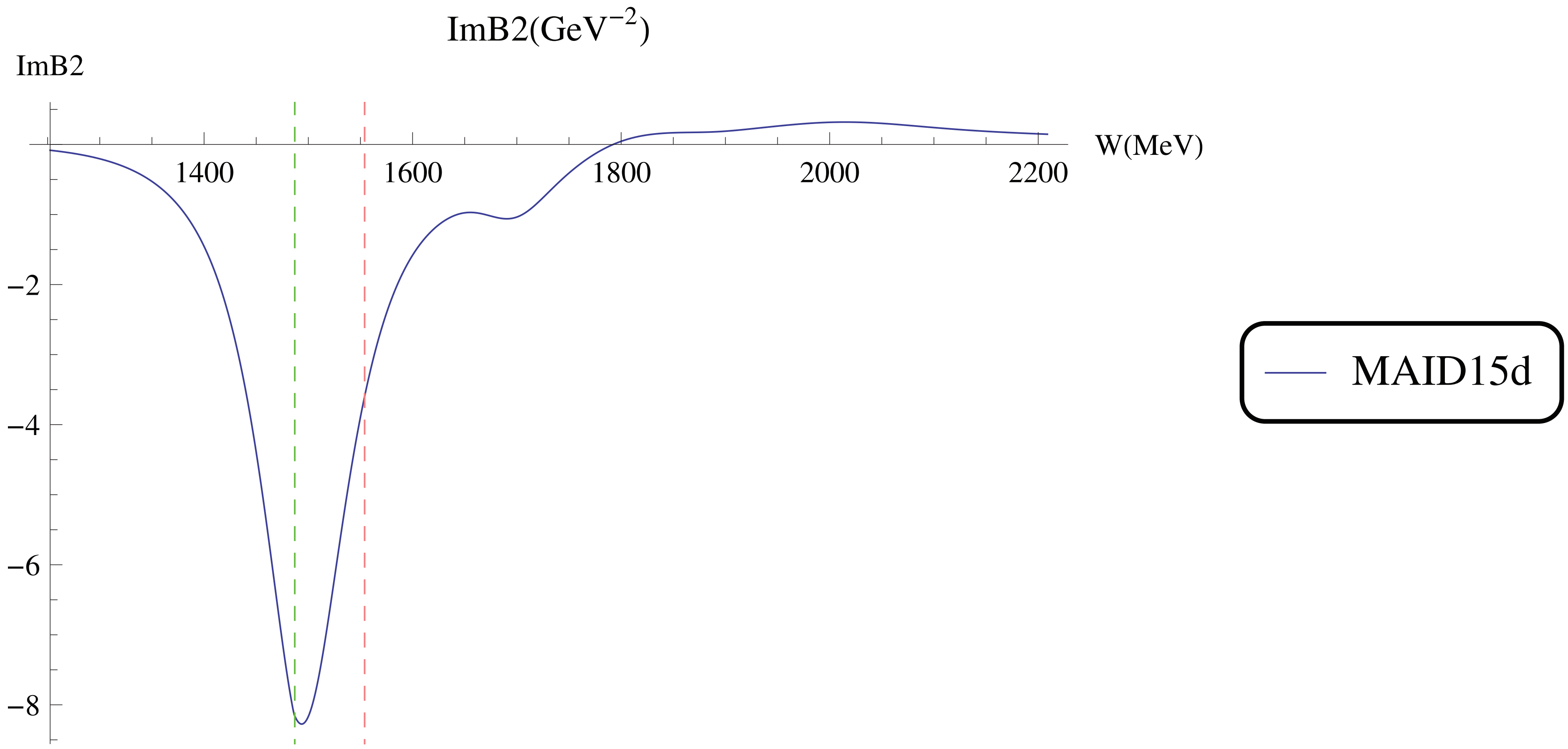

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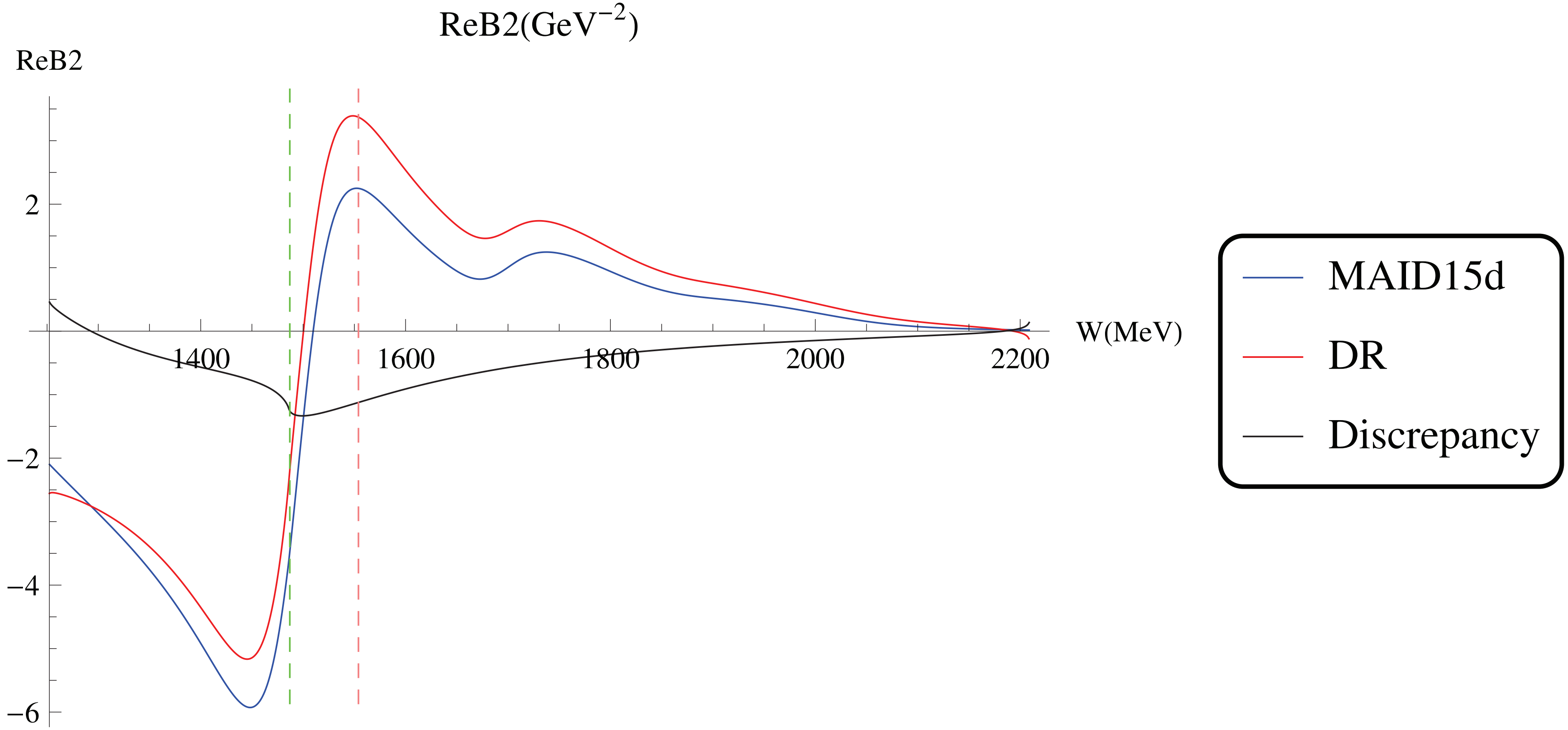

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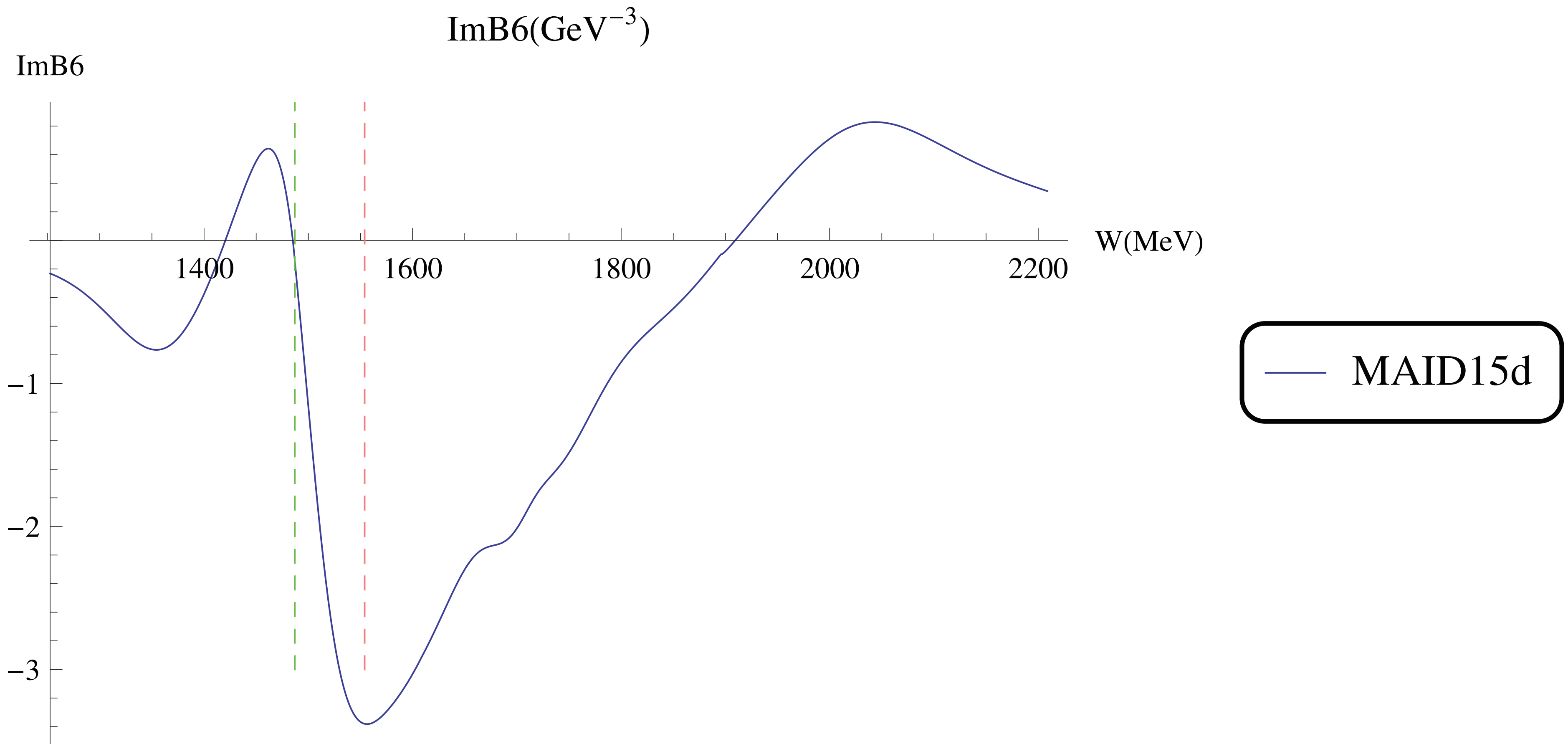

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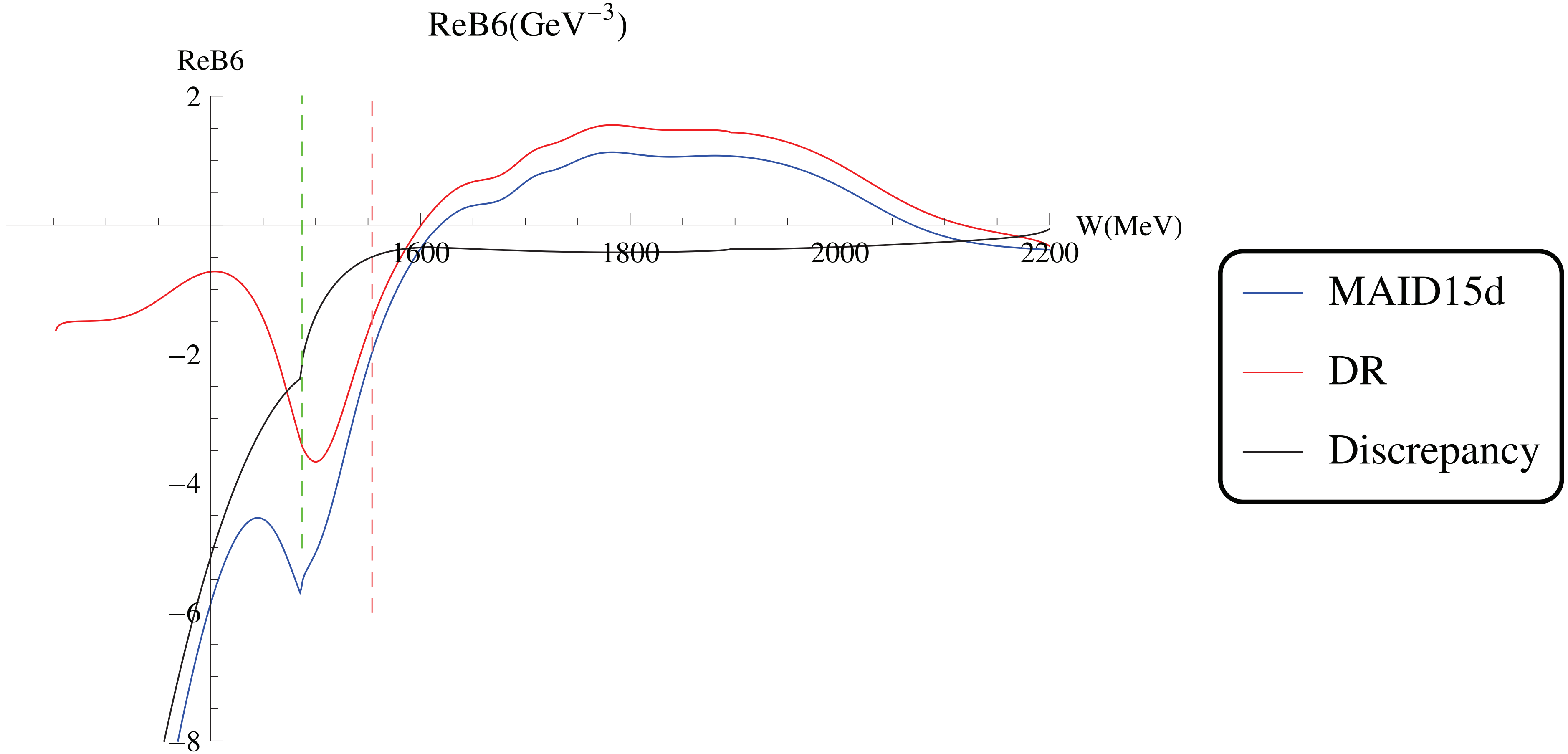

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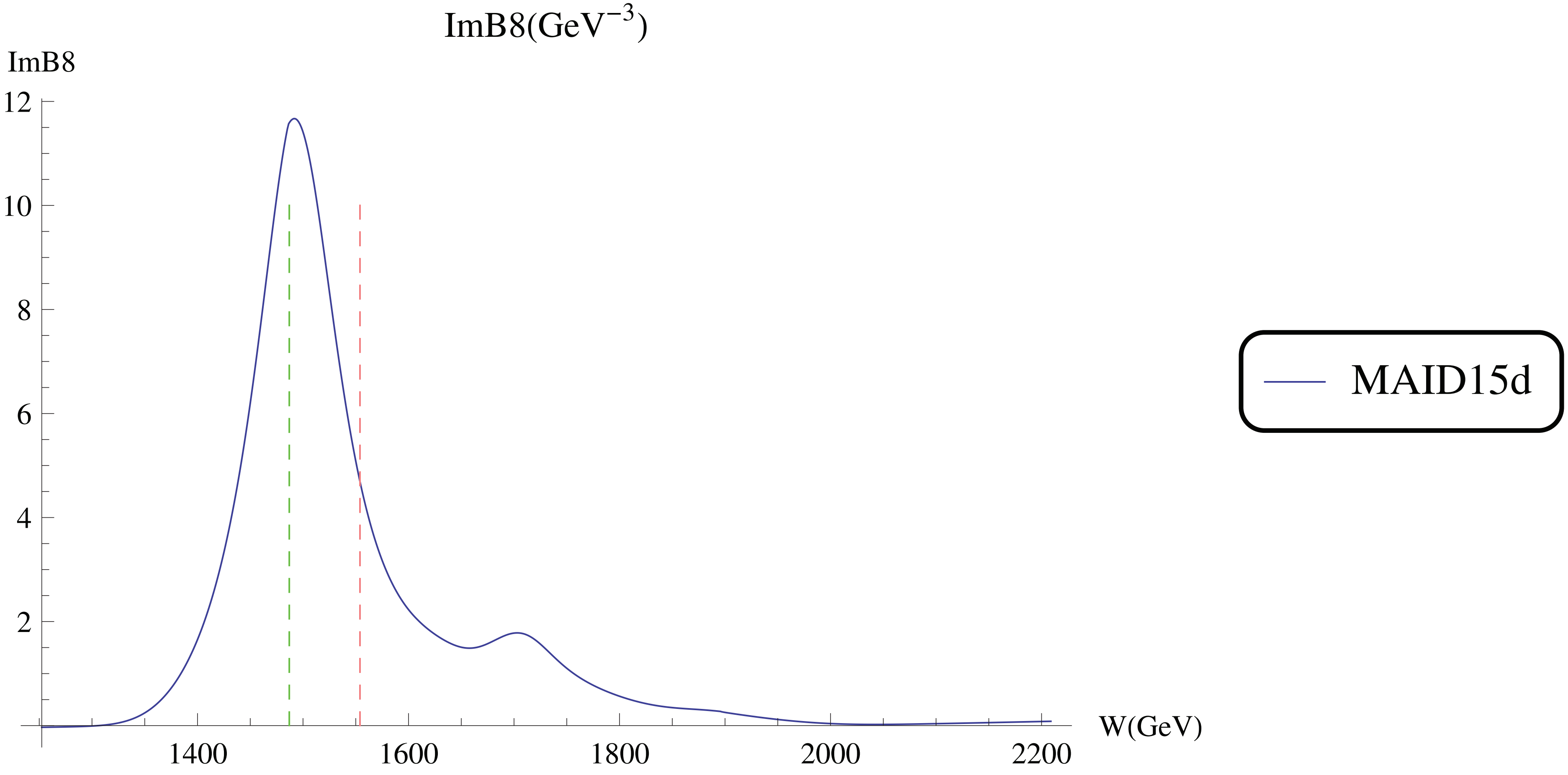

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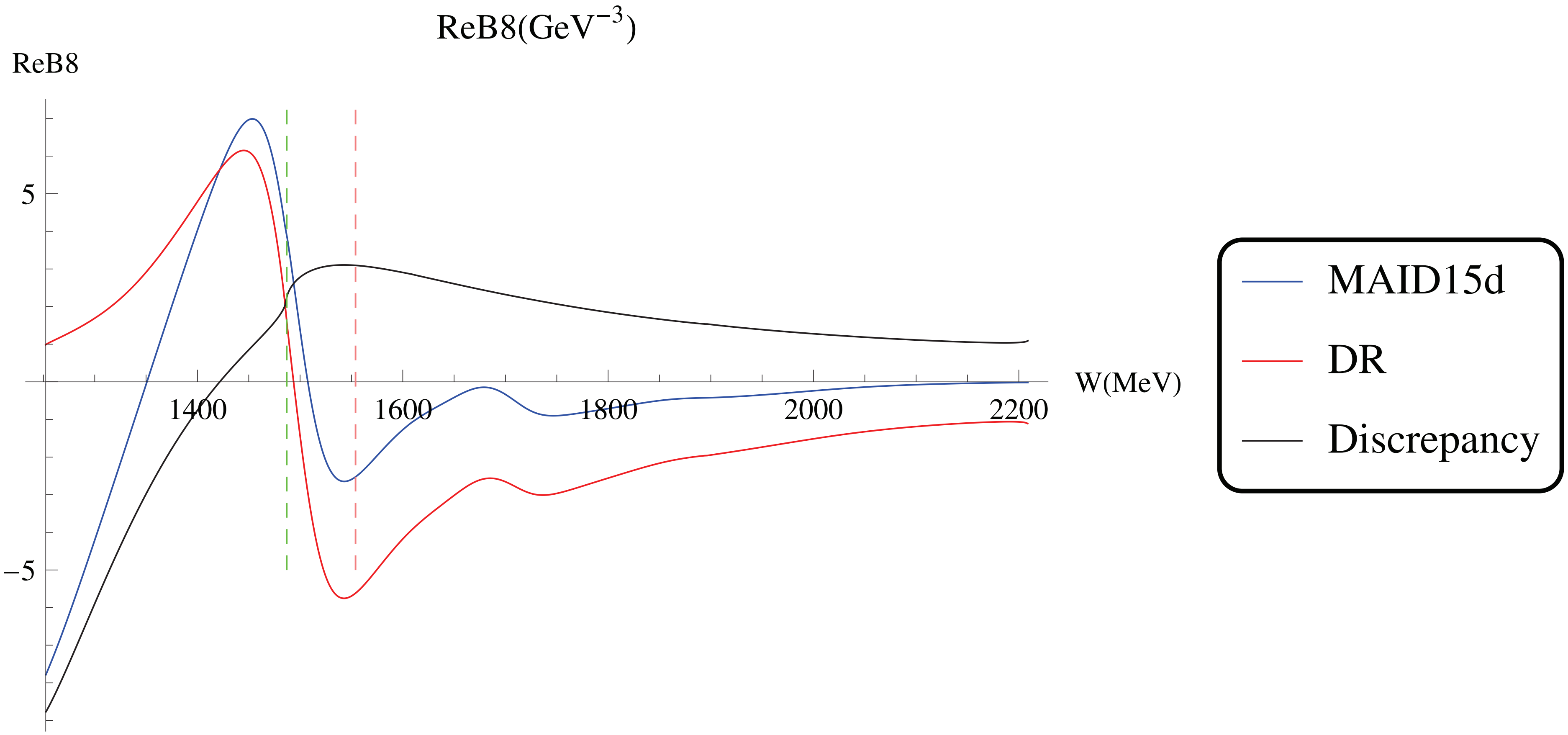

Results for $t = -1.0 GeV^2$

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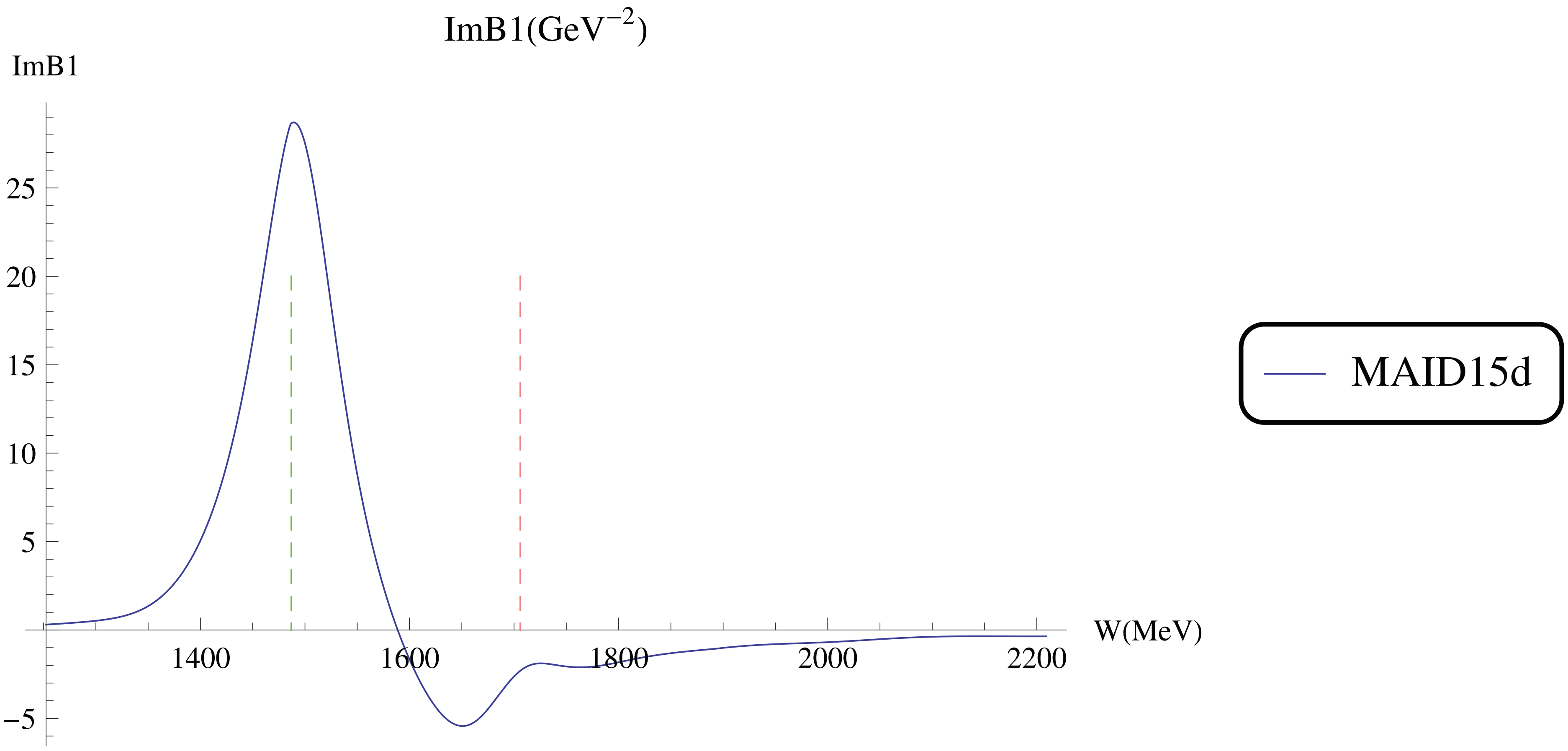

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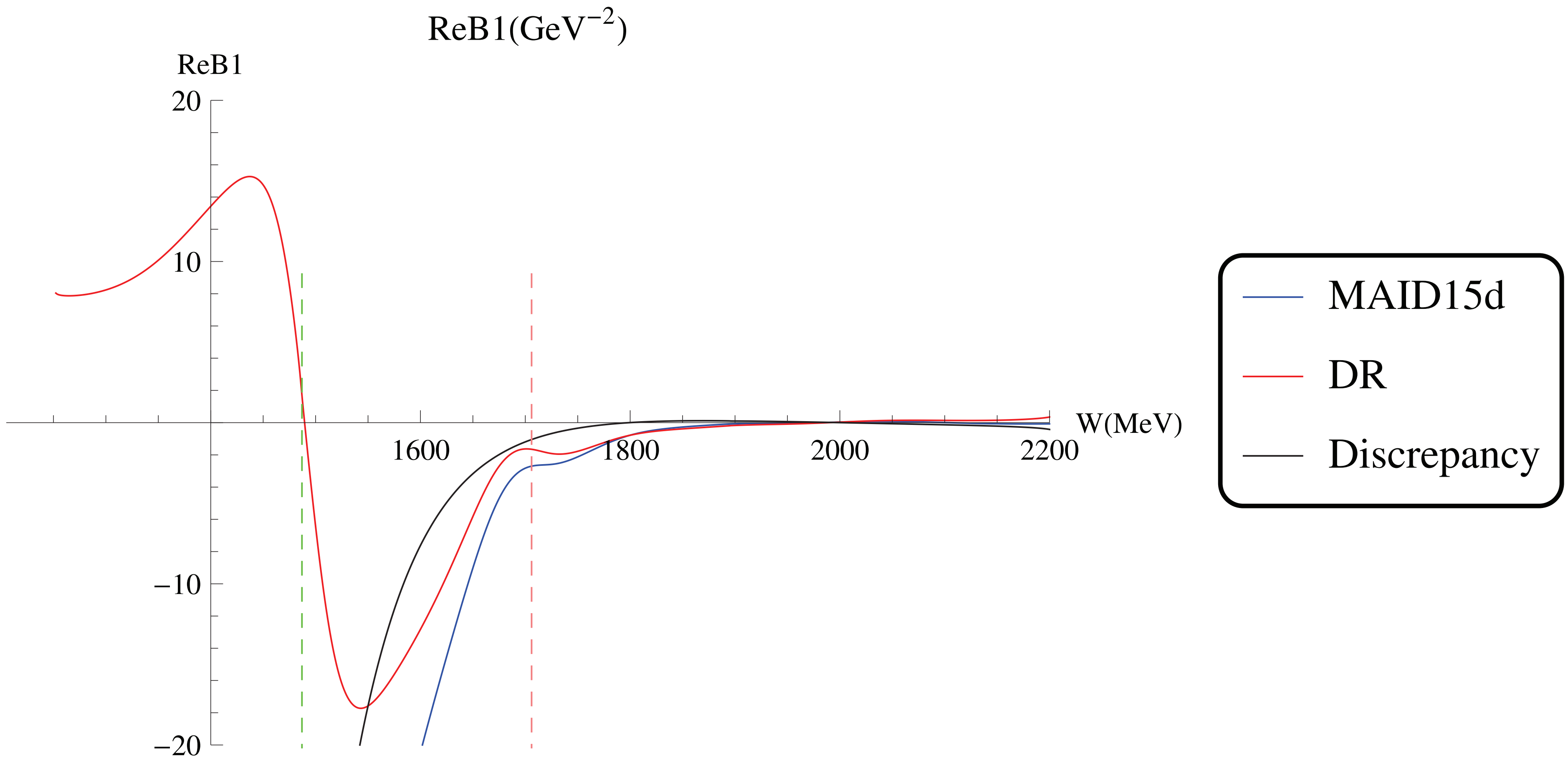

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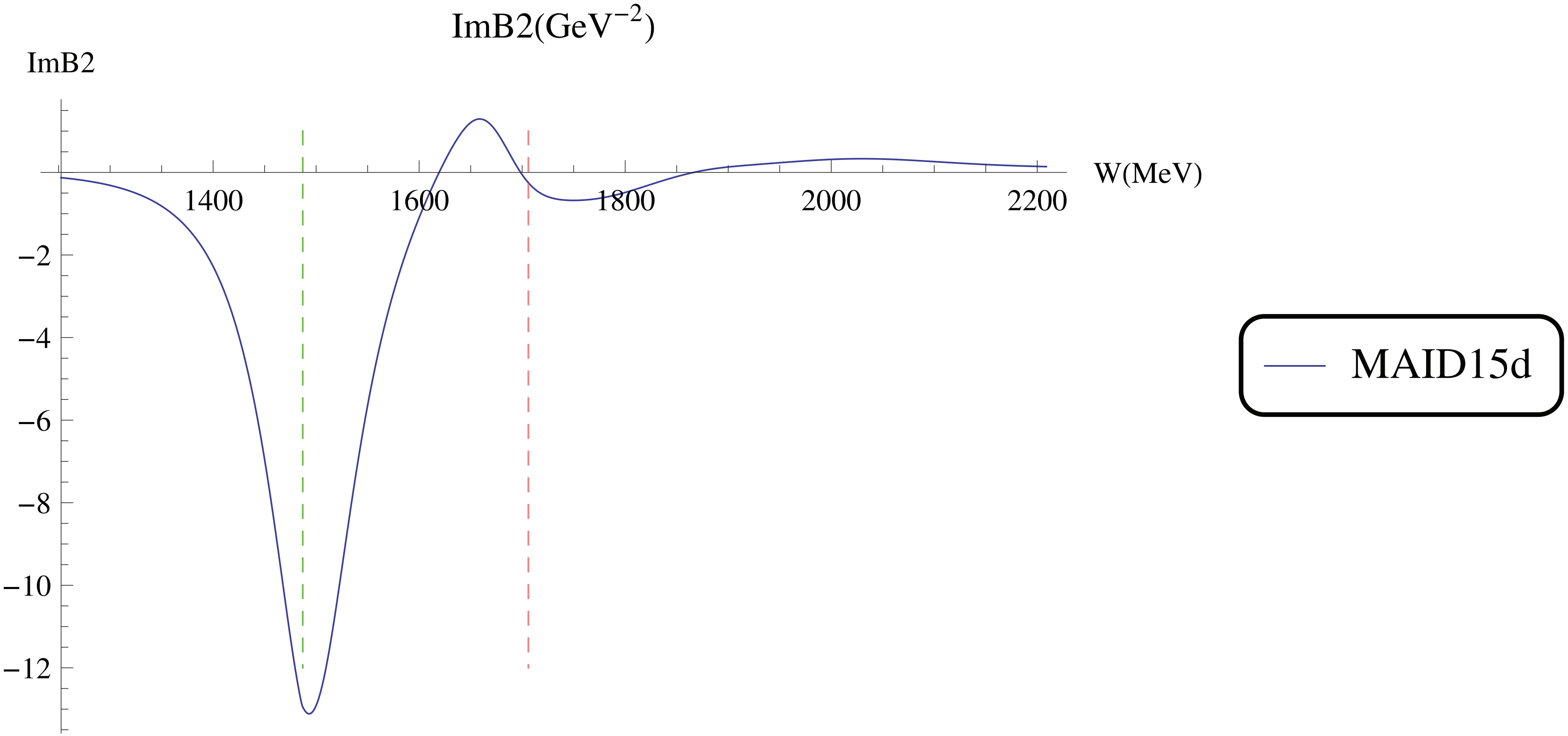

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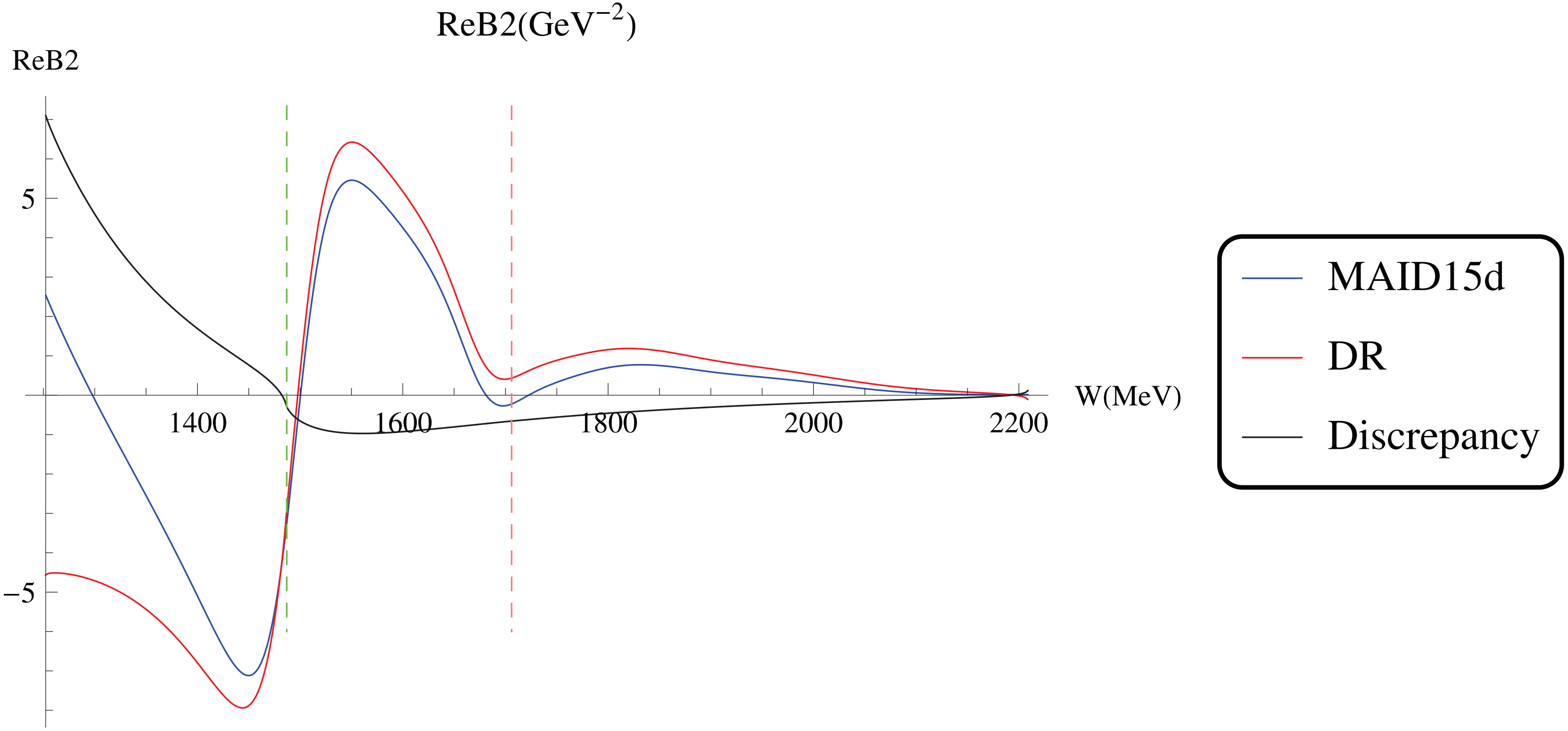

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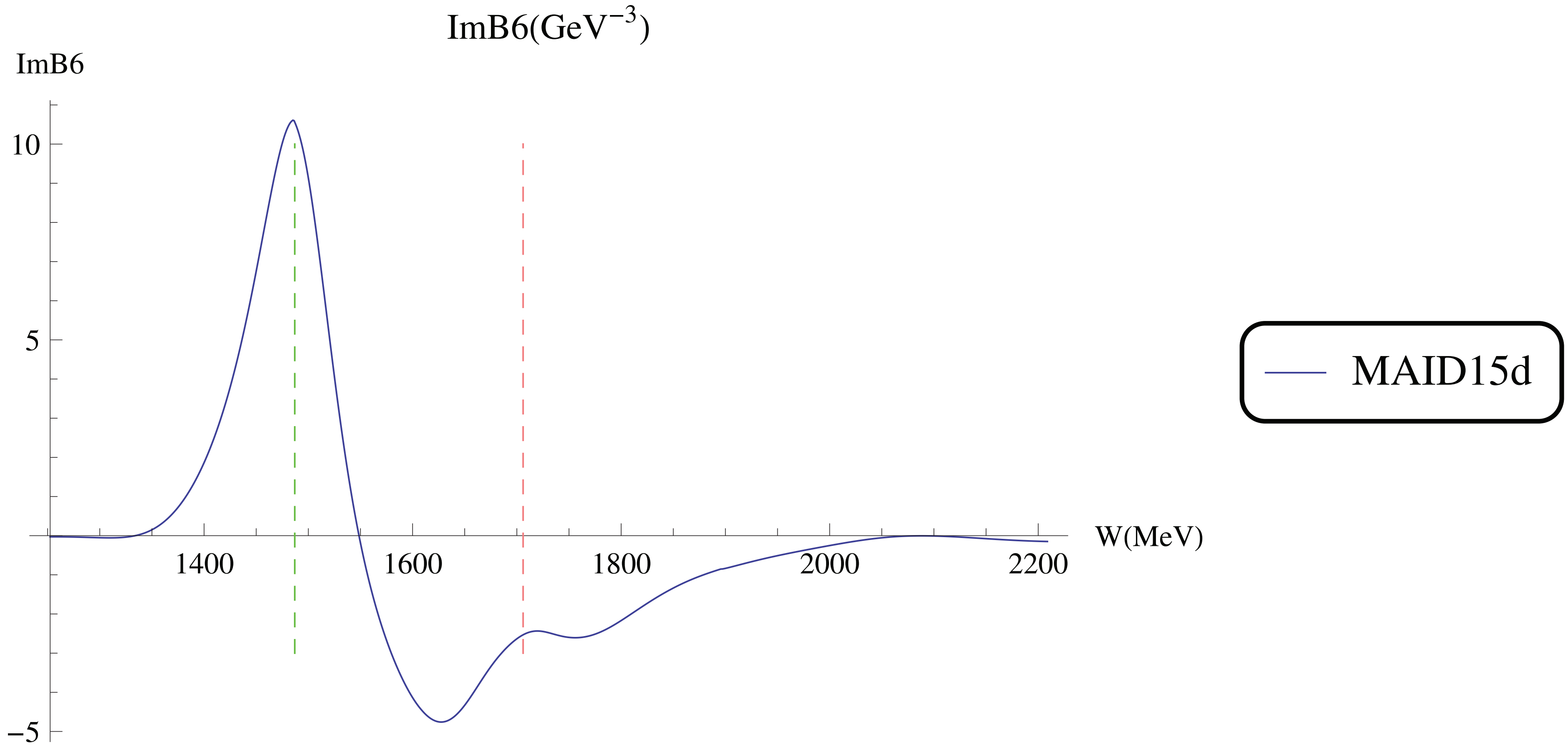

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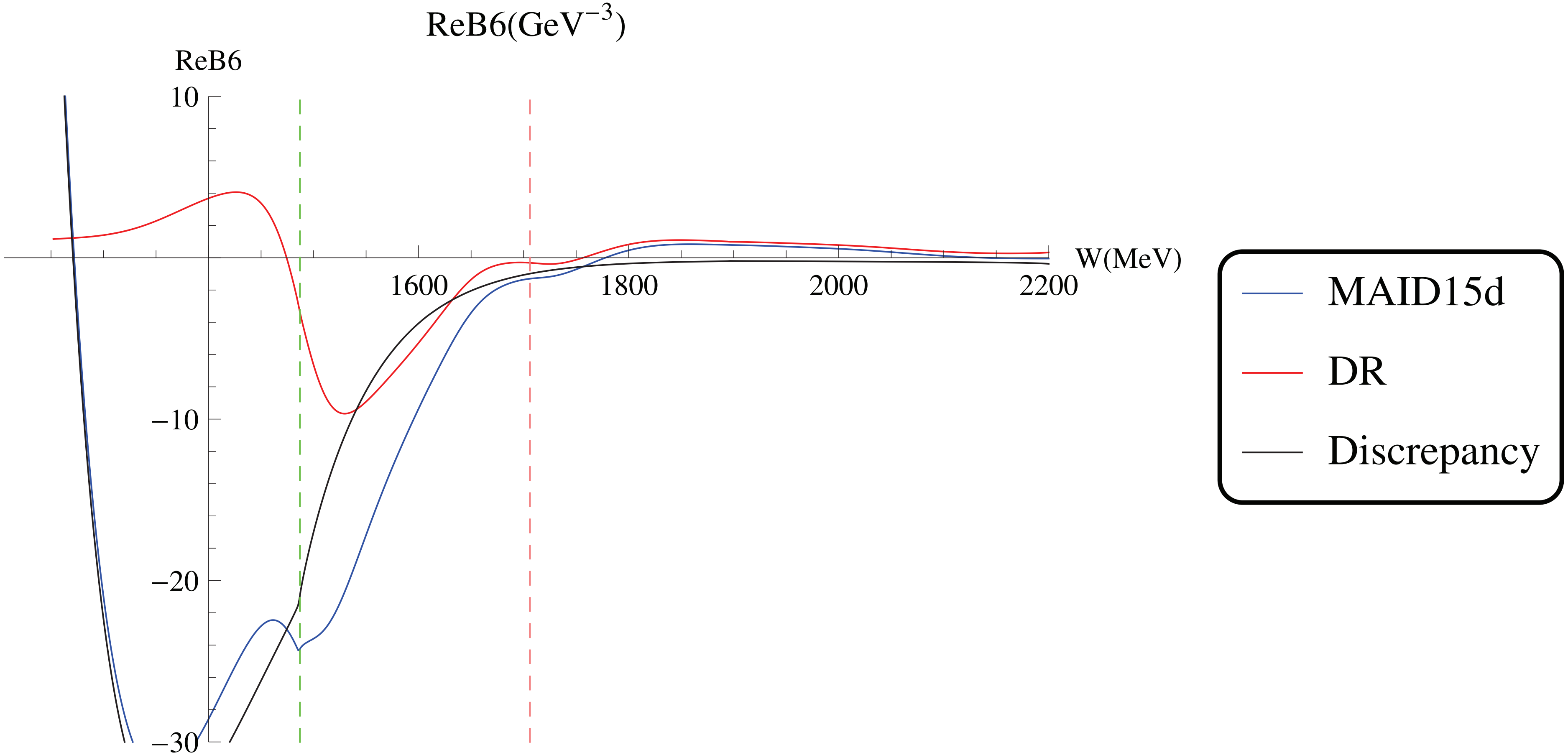

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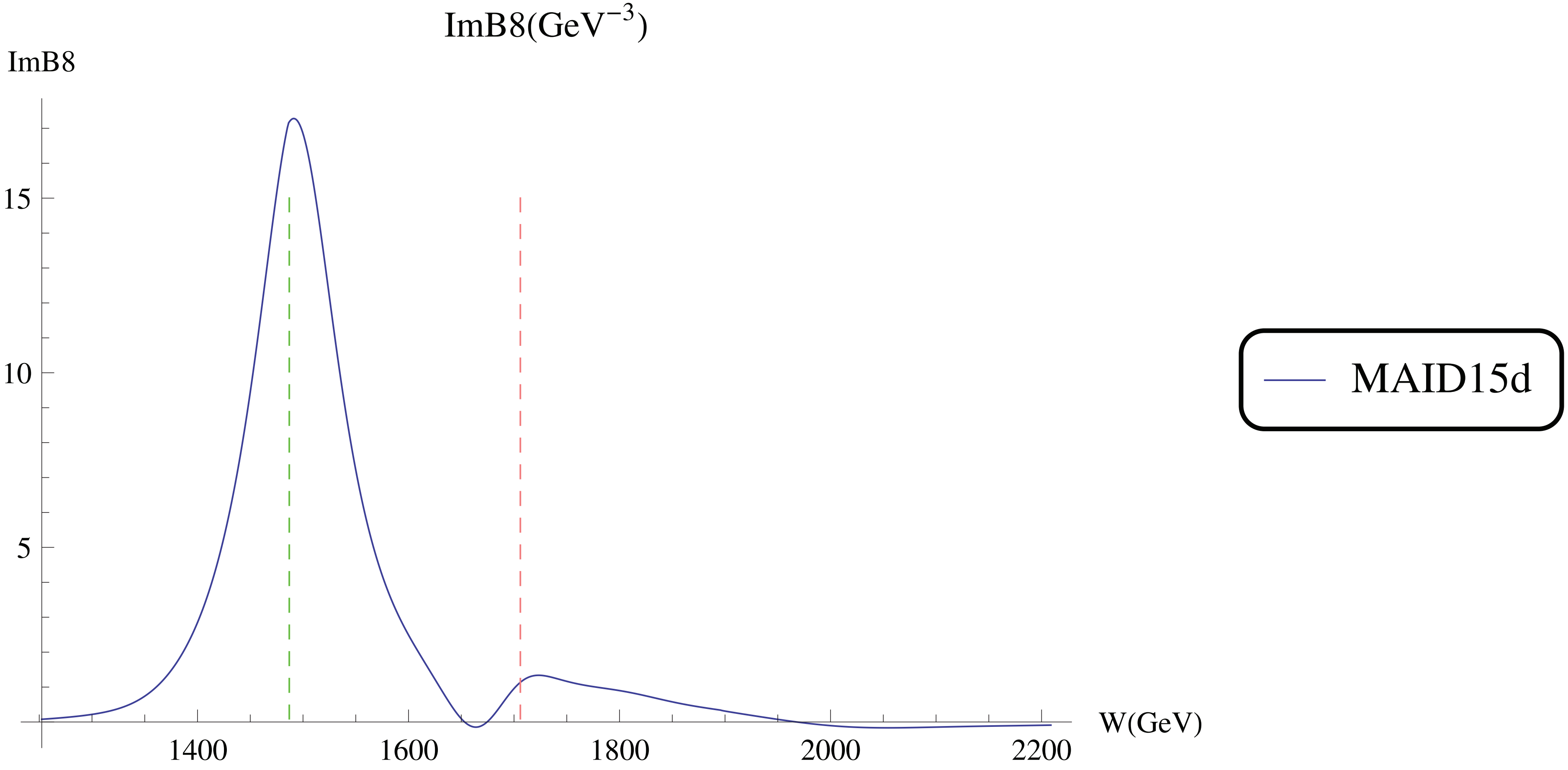

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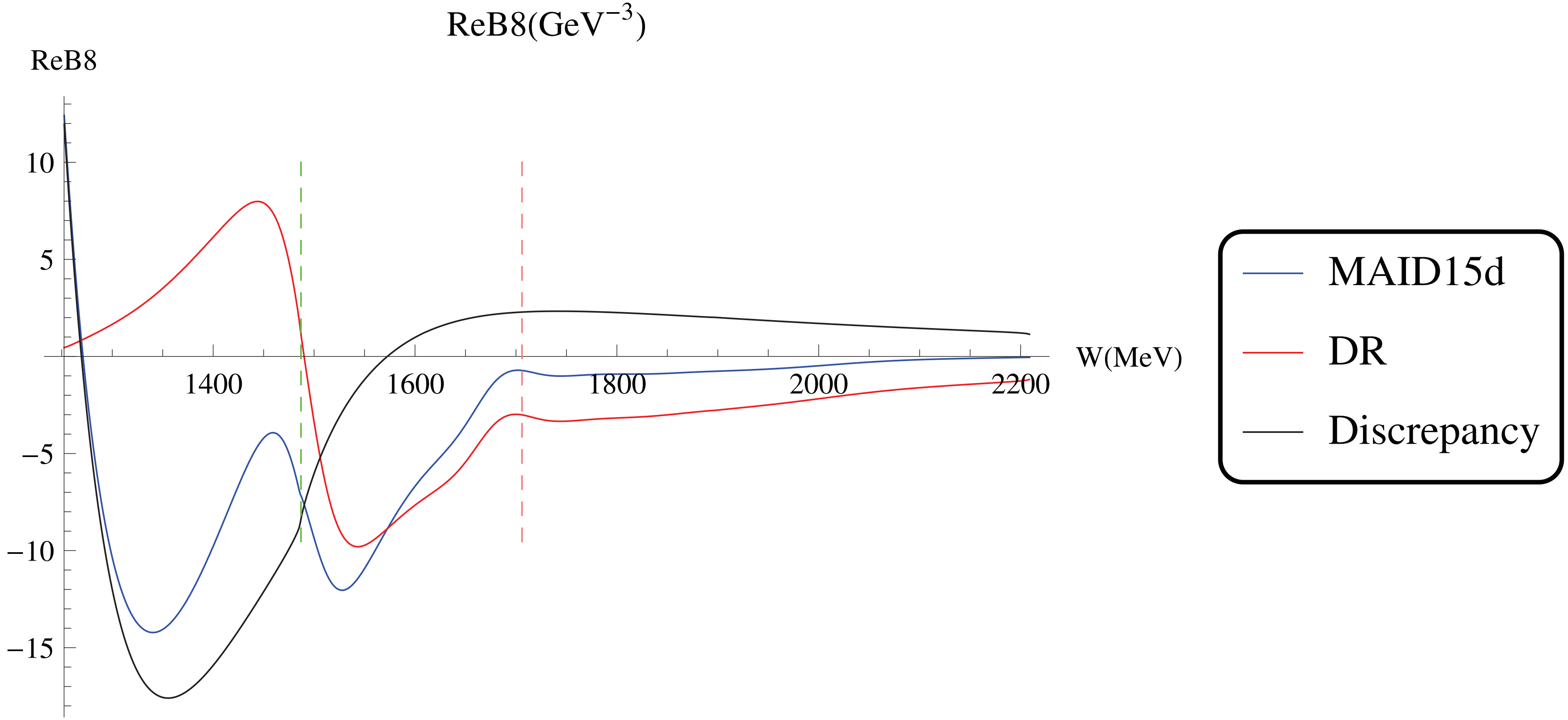

Results for CGLN amplitudes

Results for $W = 1500 MeV$

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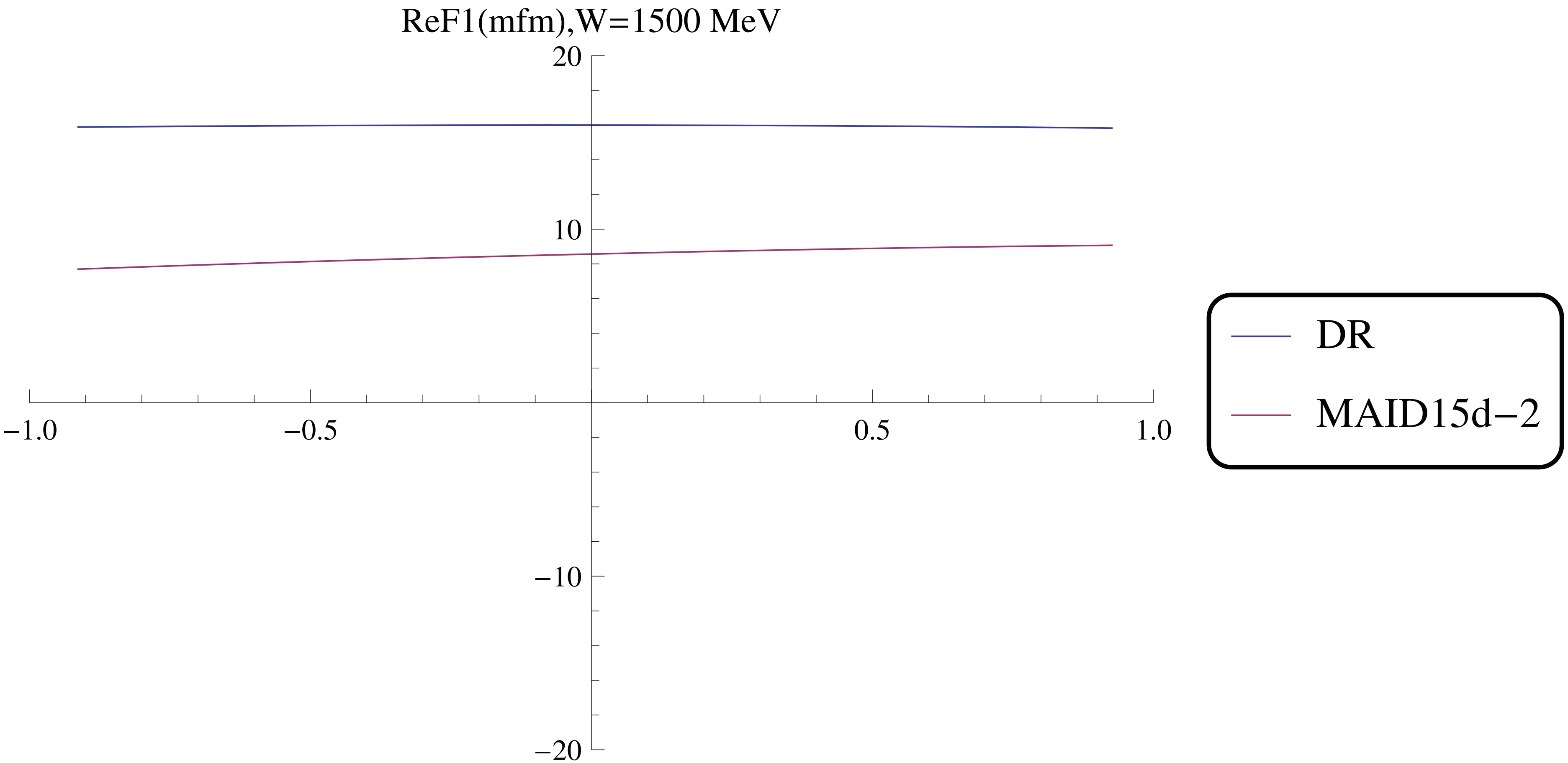
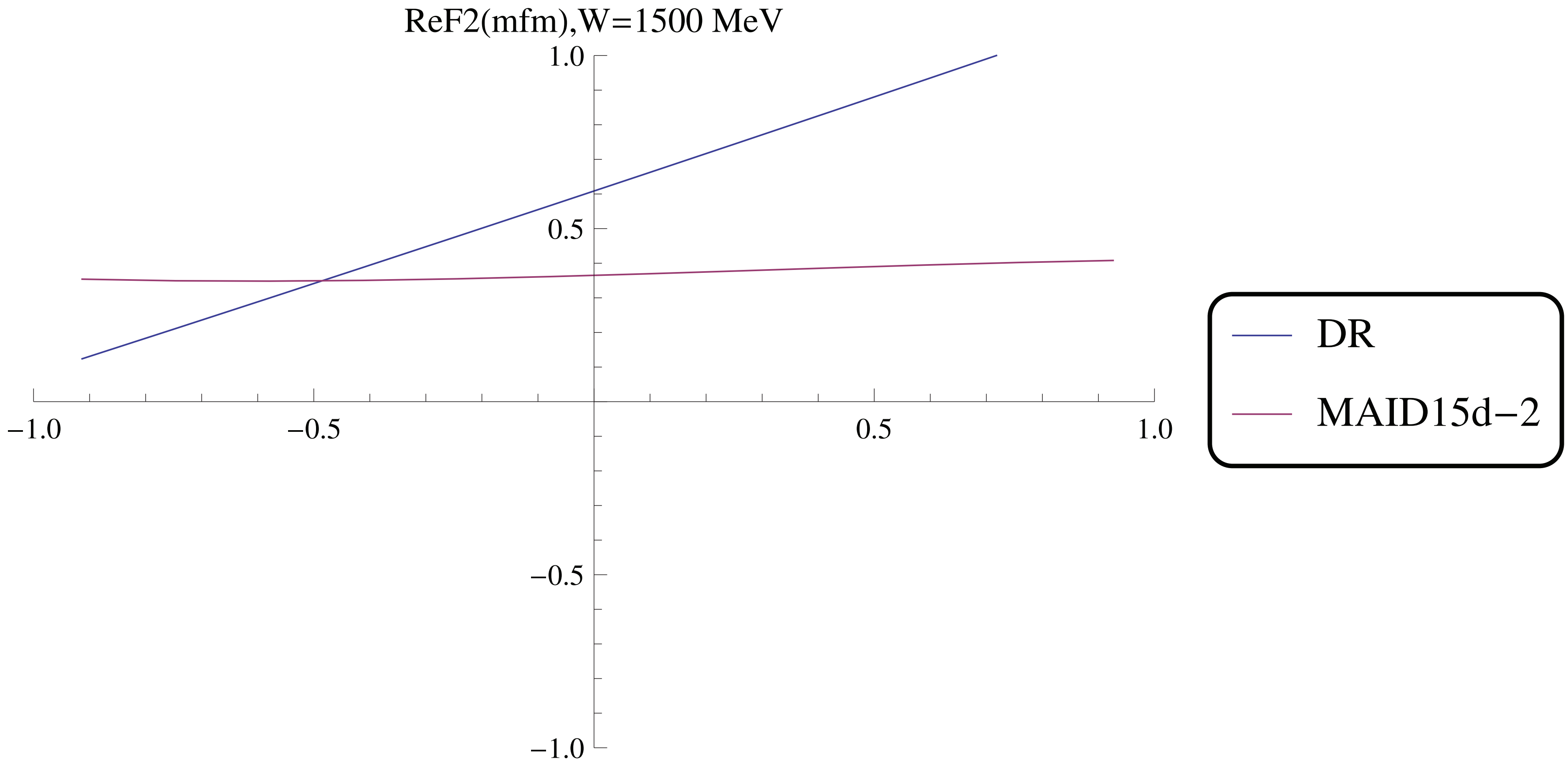
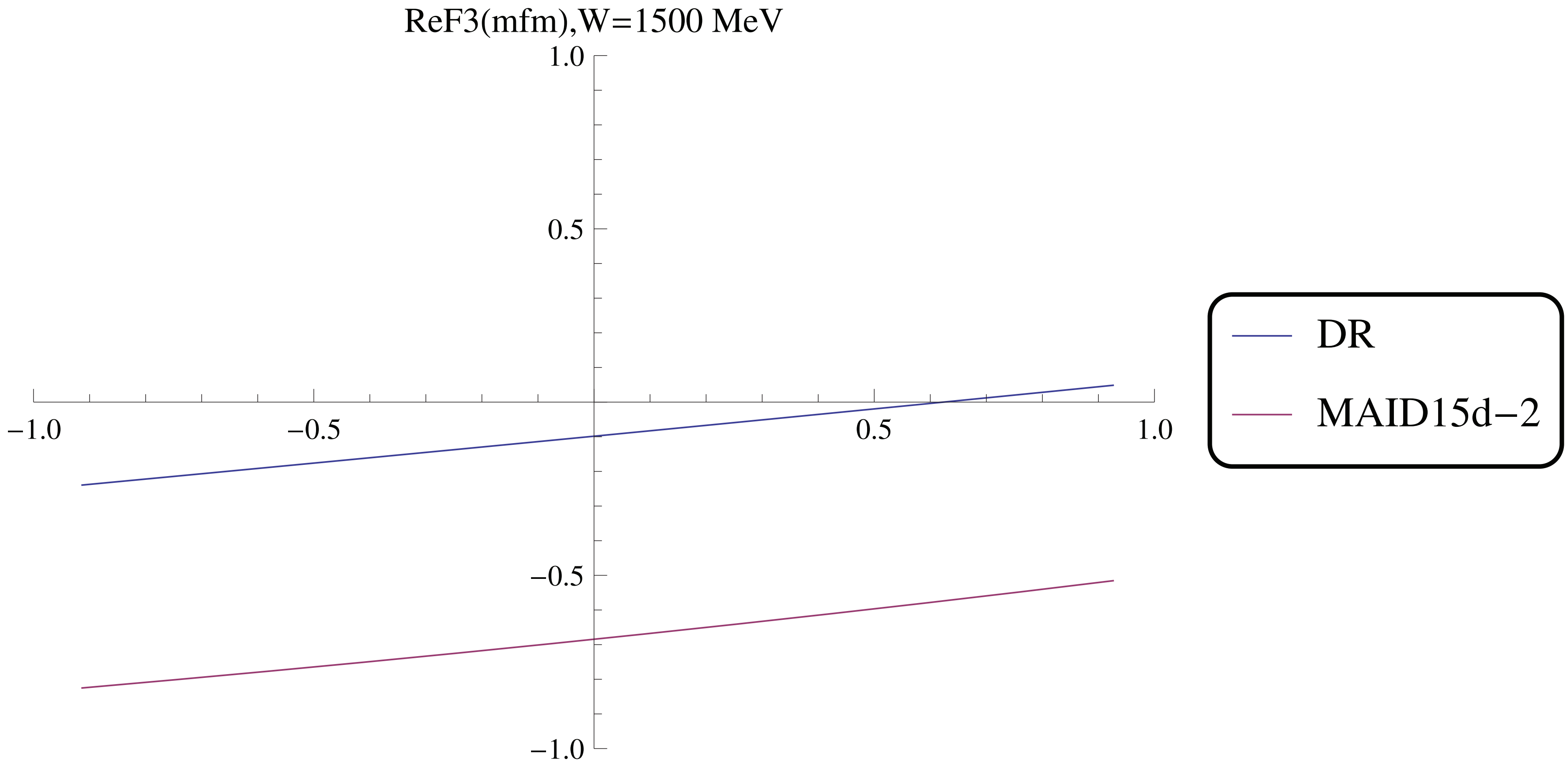


Results for $W = 1500 MeV$



Results for $W = 1500 MeV$



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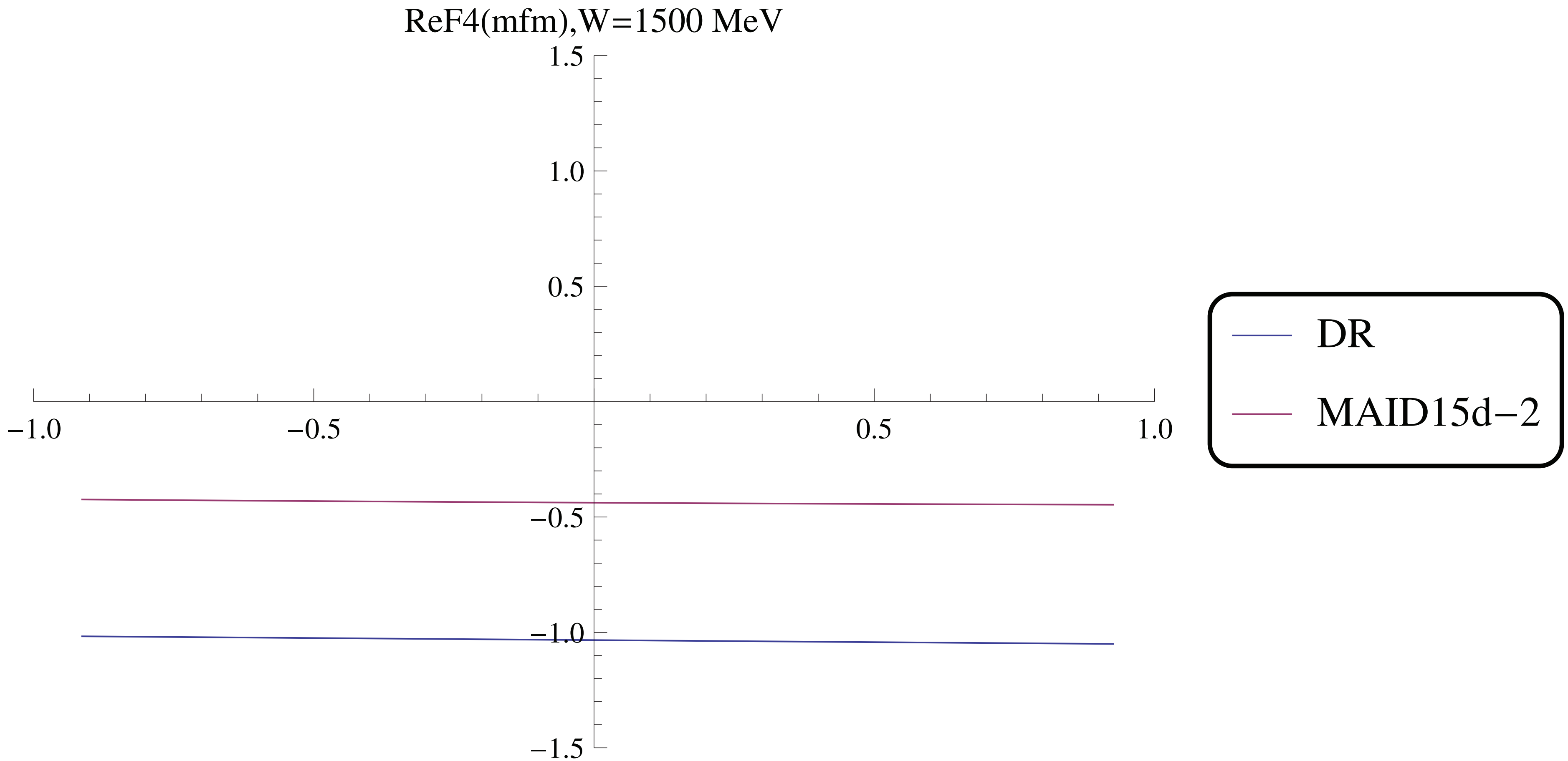

Results for $W = 1600 MeV$

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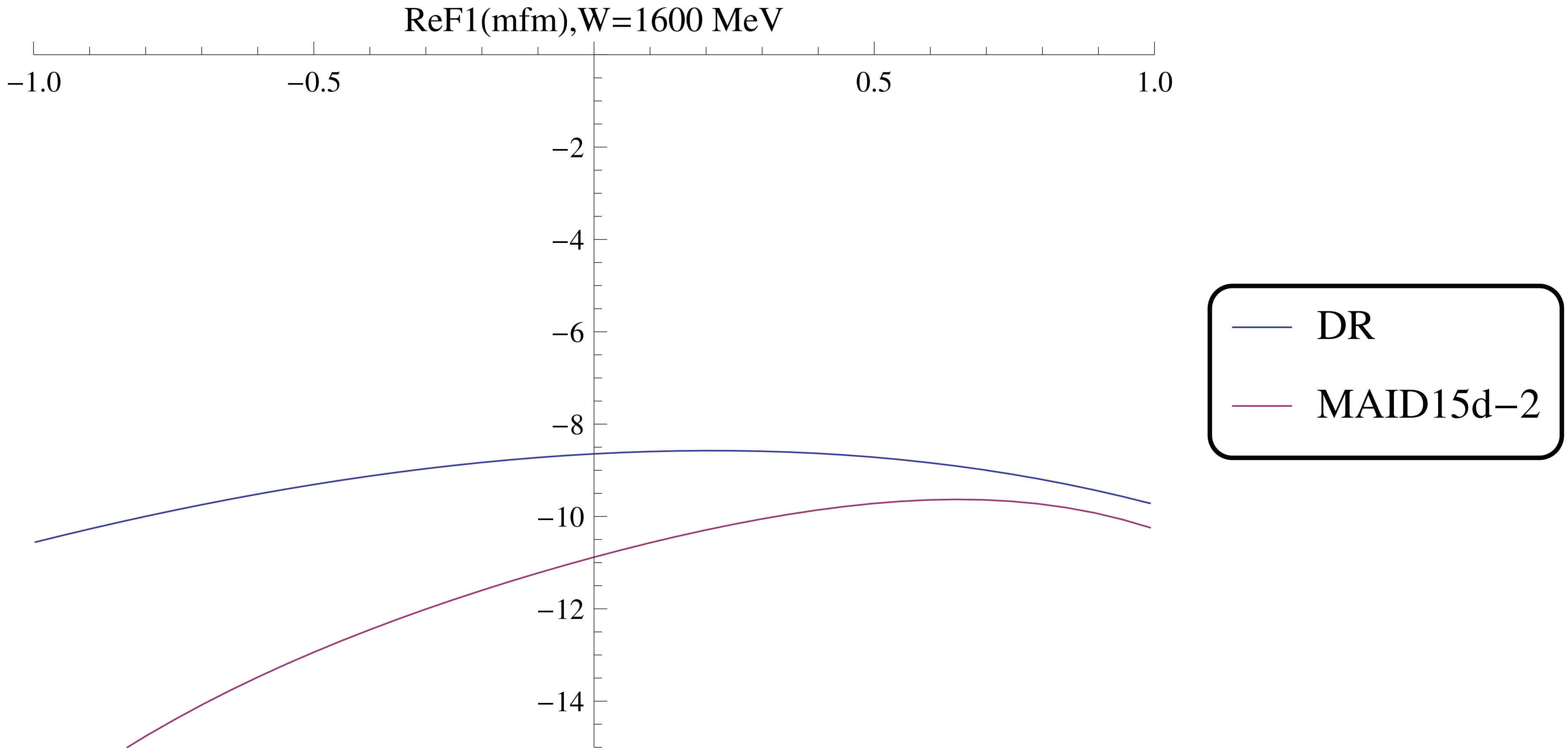
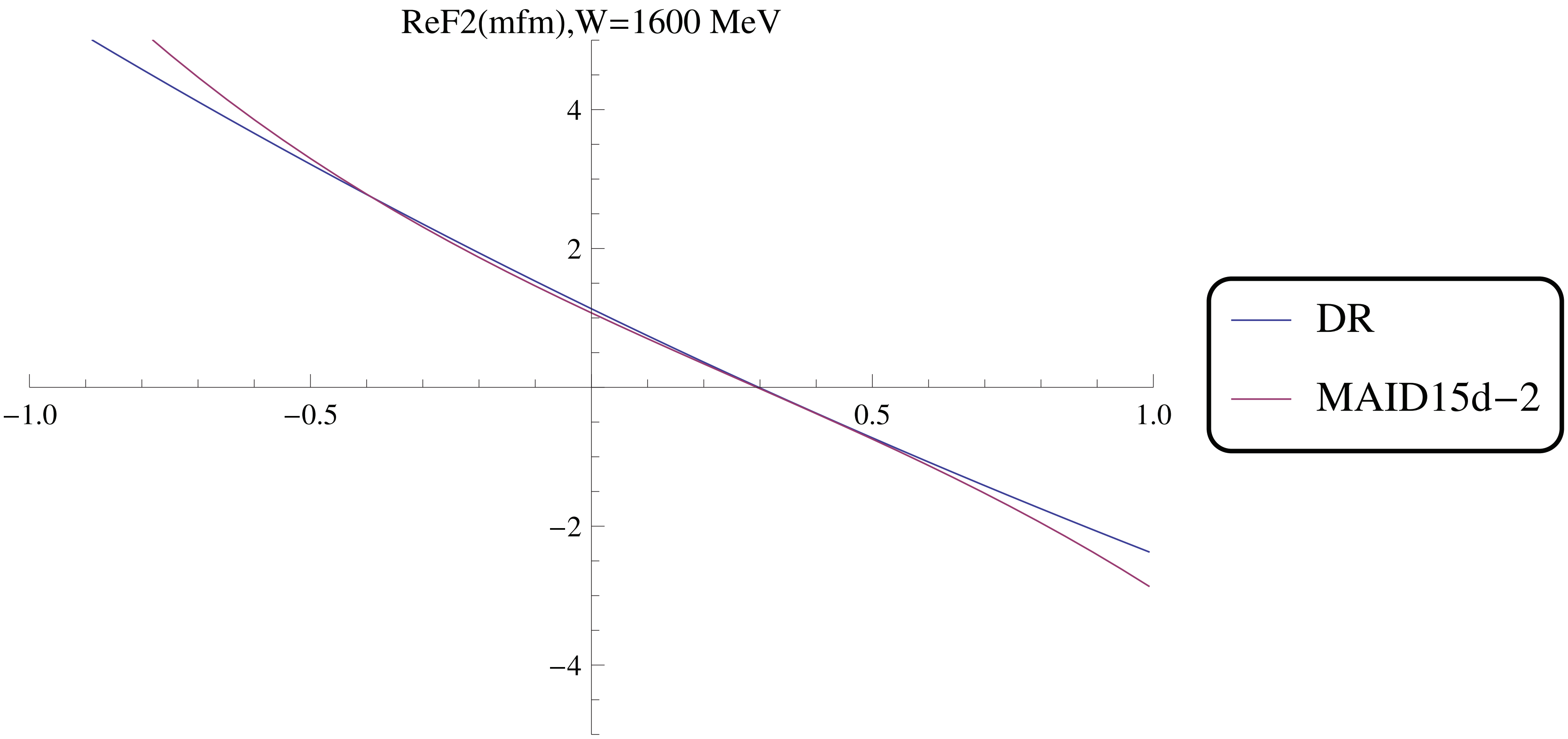
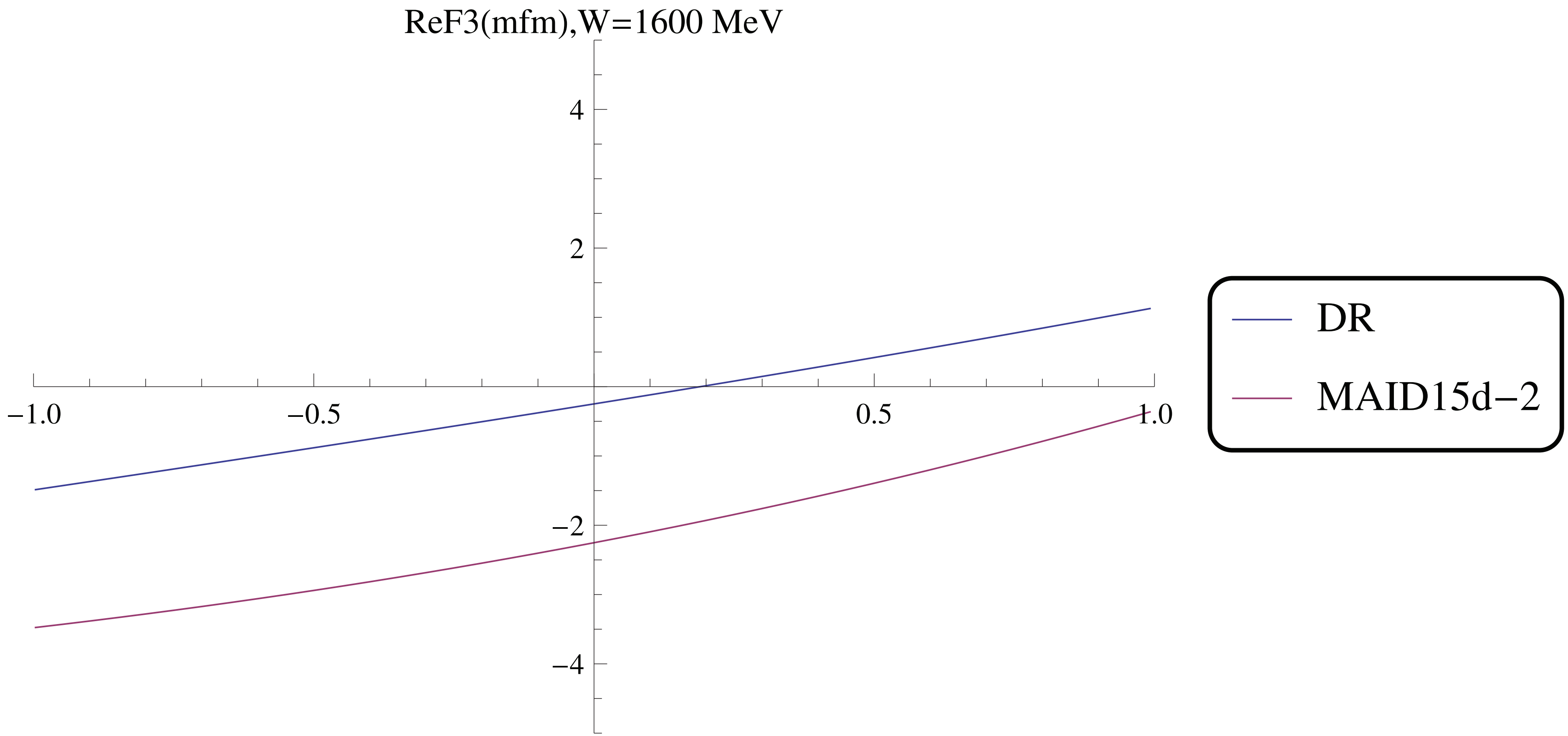


Results for $W = 1600 MeV$



Results for $W = 1600 MeV$



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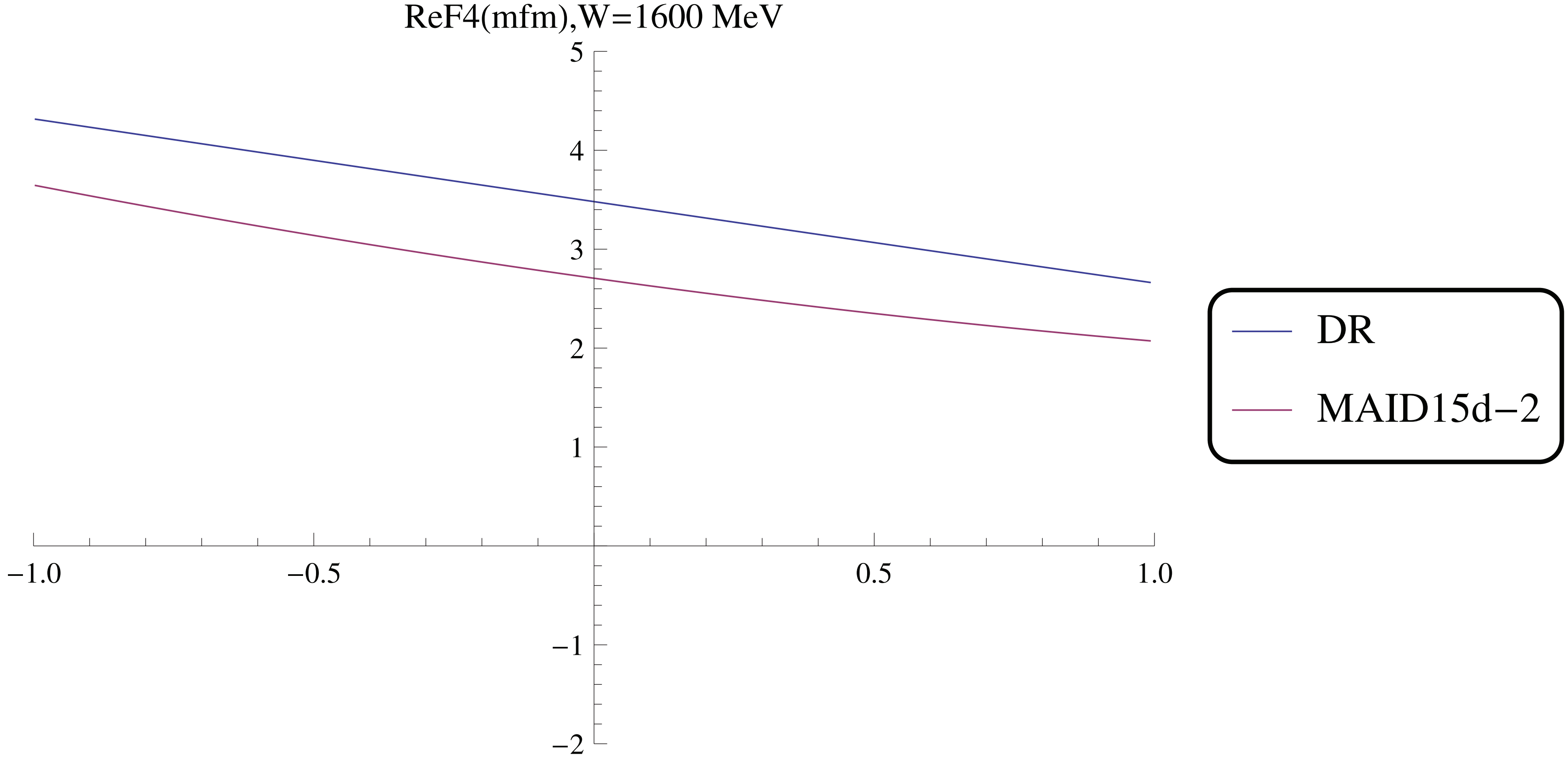

Results for $W = 1700 MeV$

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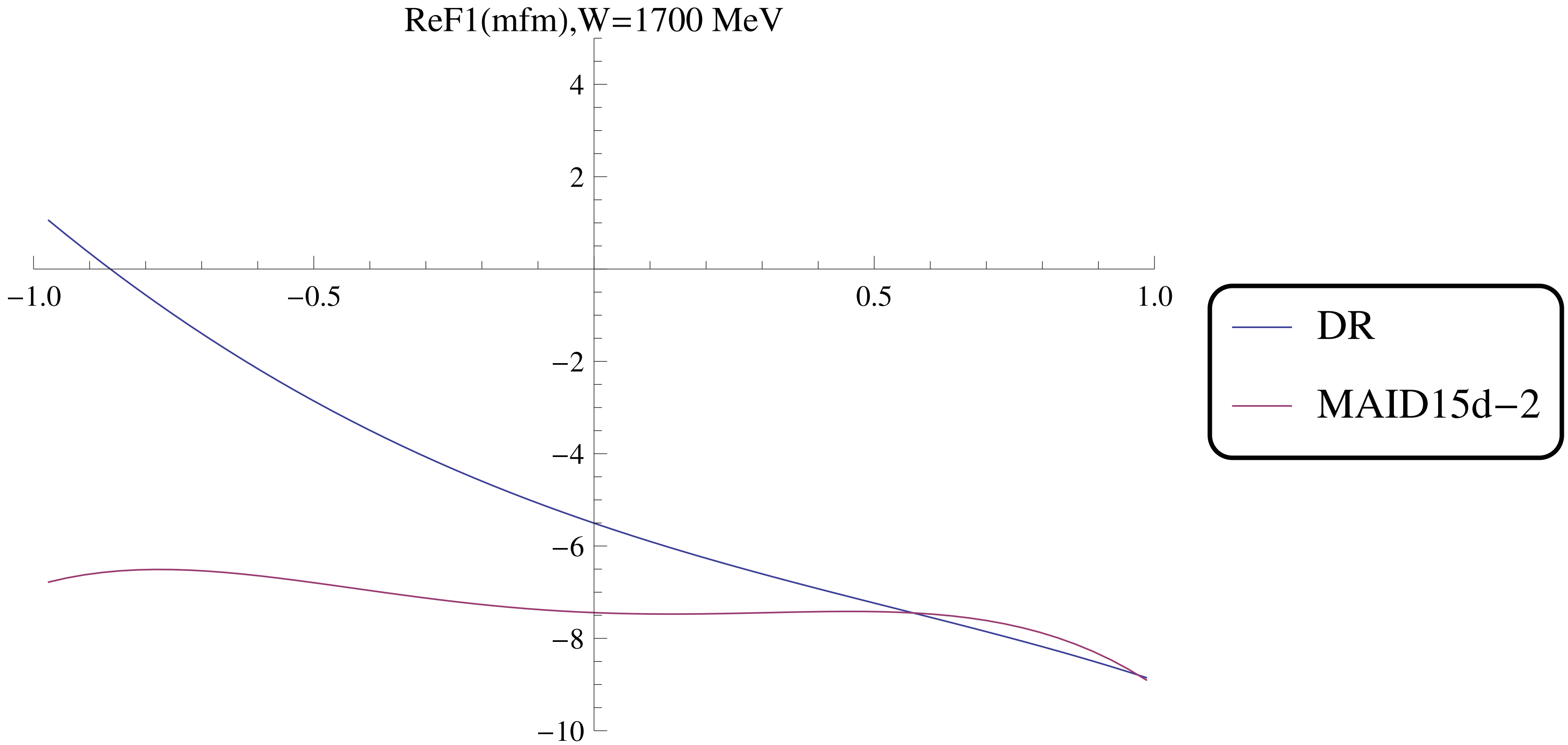
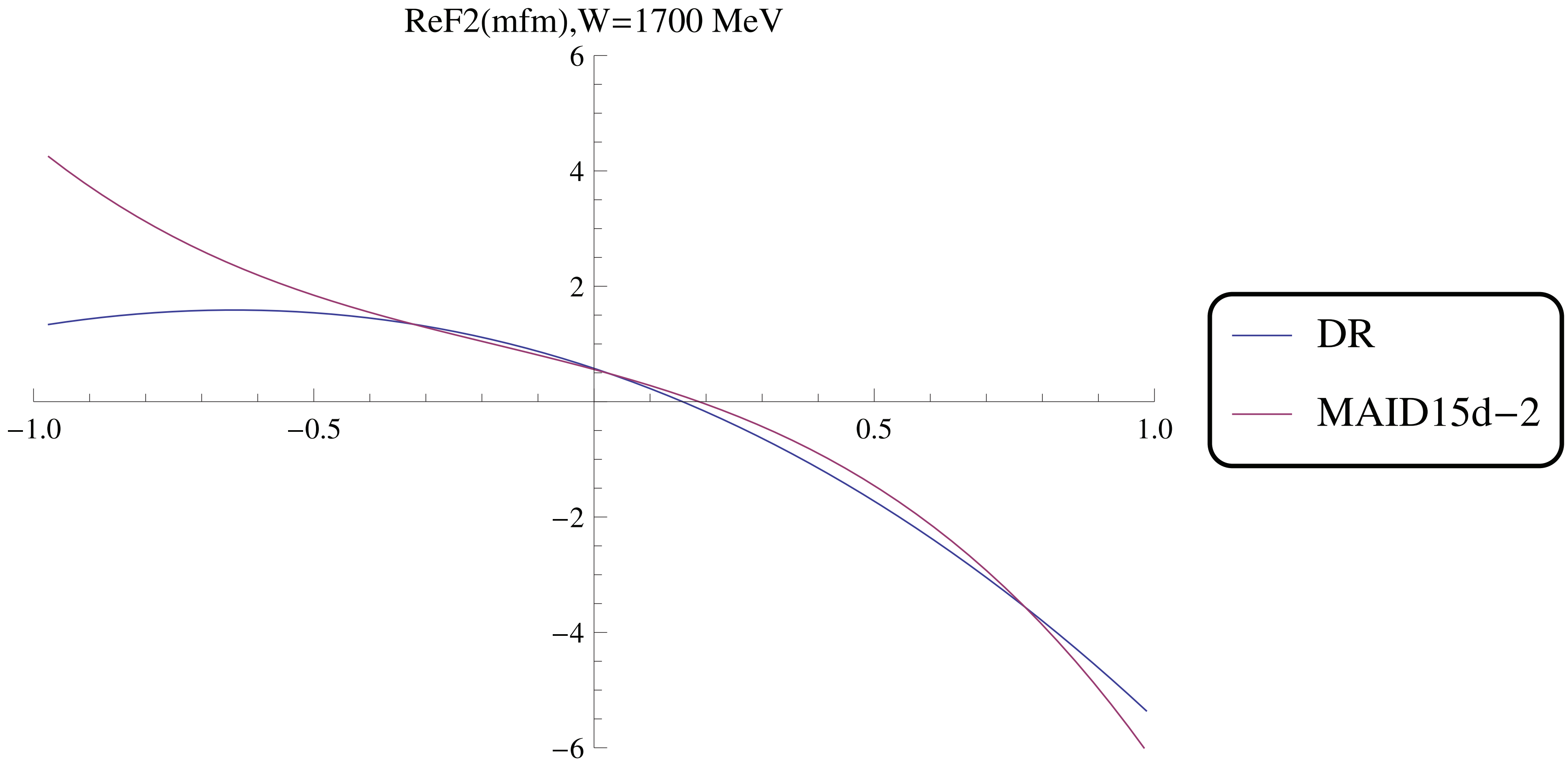
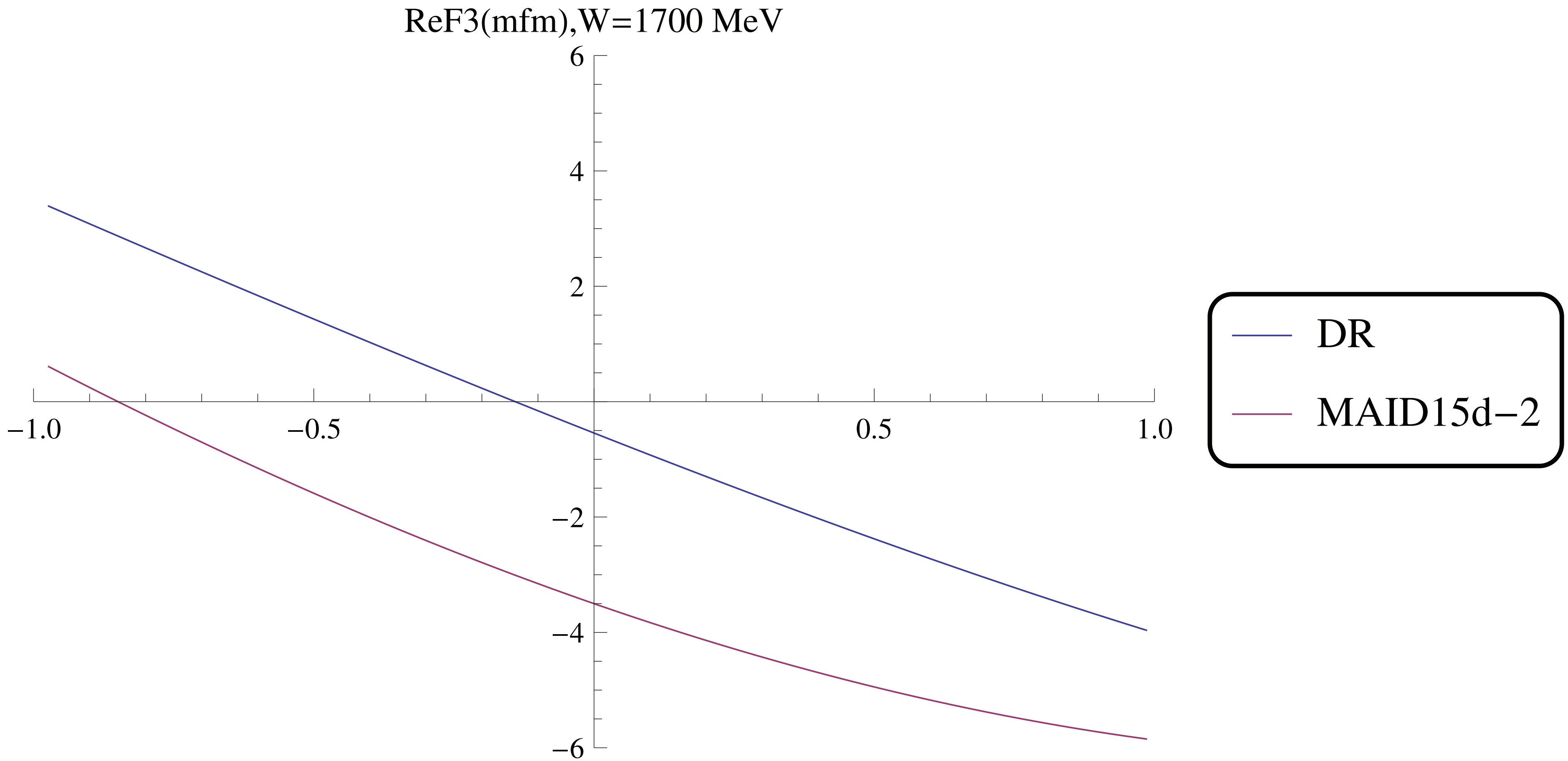


Results for $W = 1700 MeV$



Results for $W = 1700 MeV$



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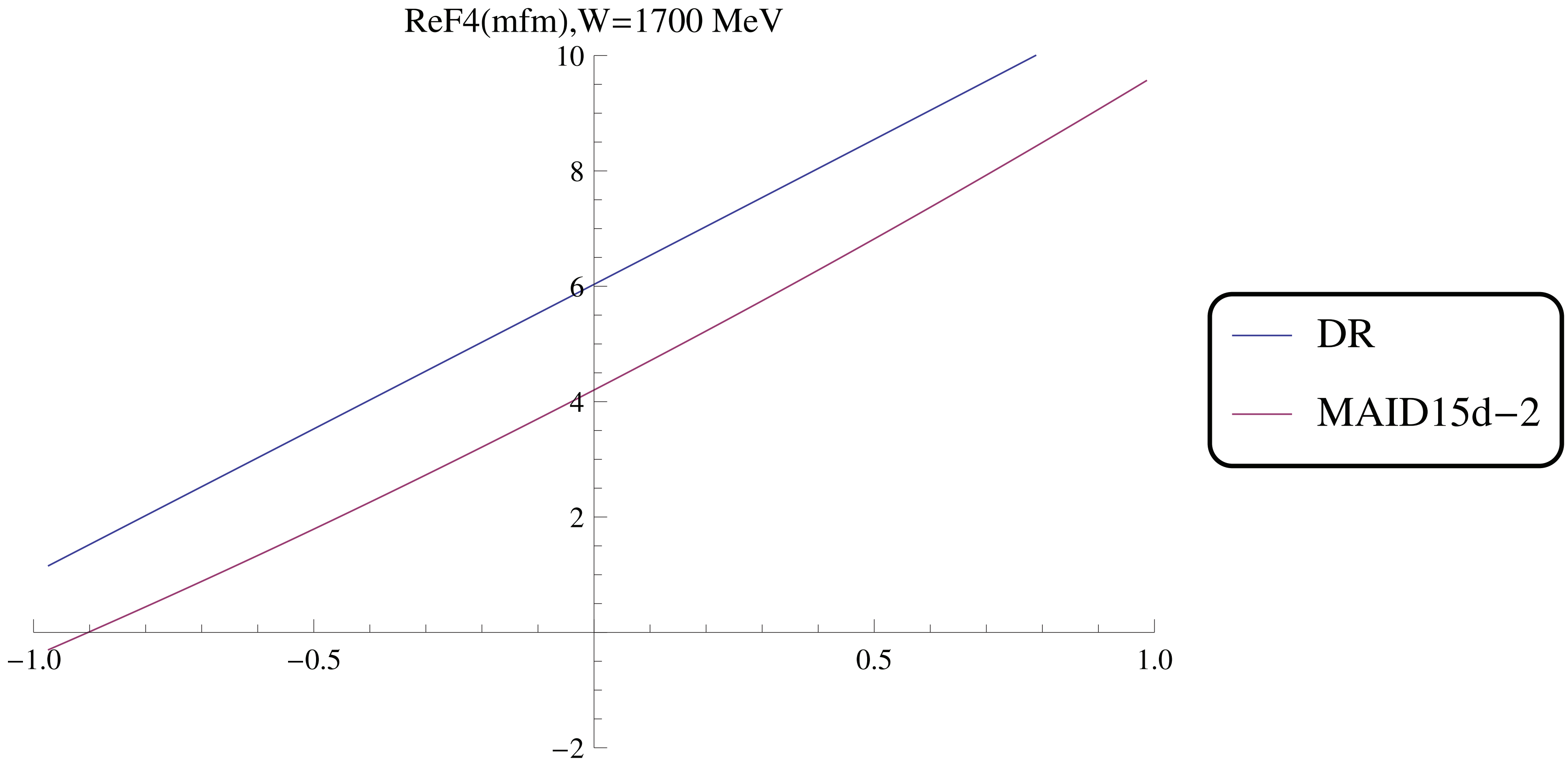

Results for $W = 1800 MeV$

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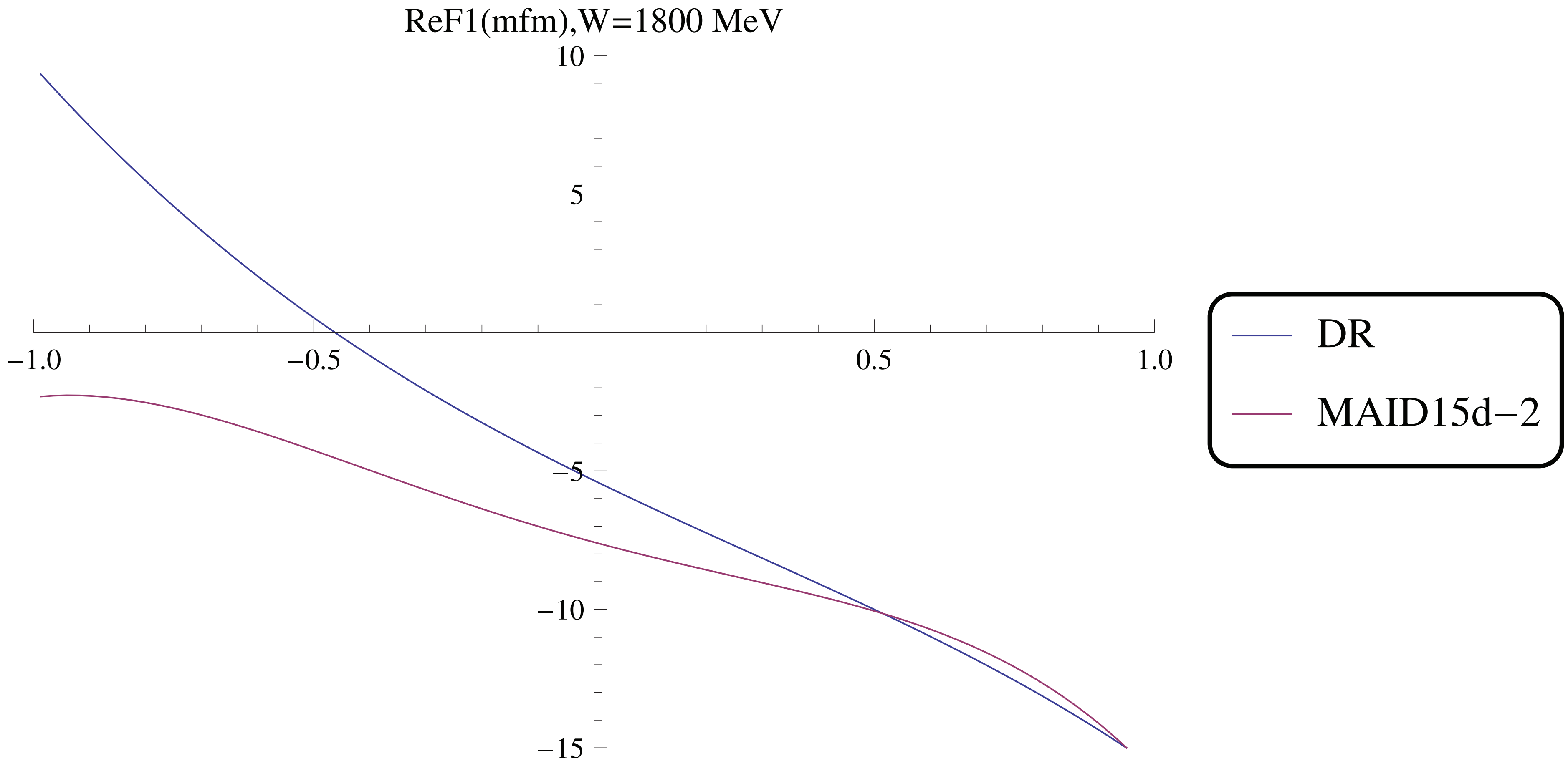
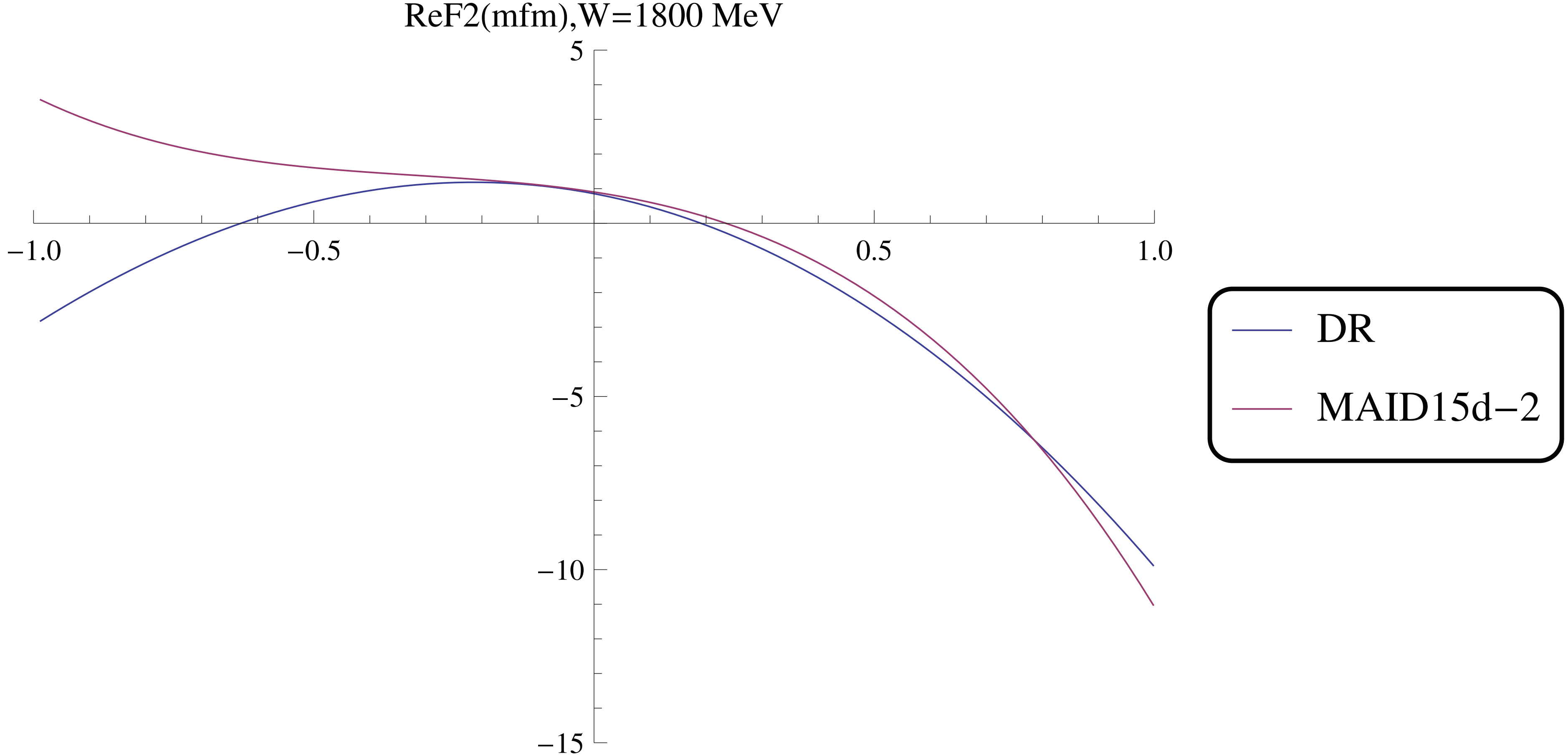
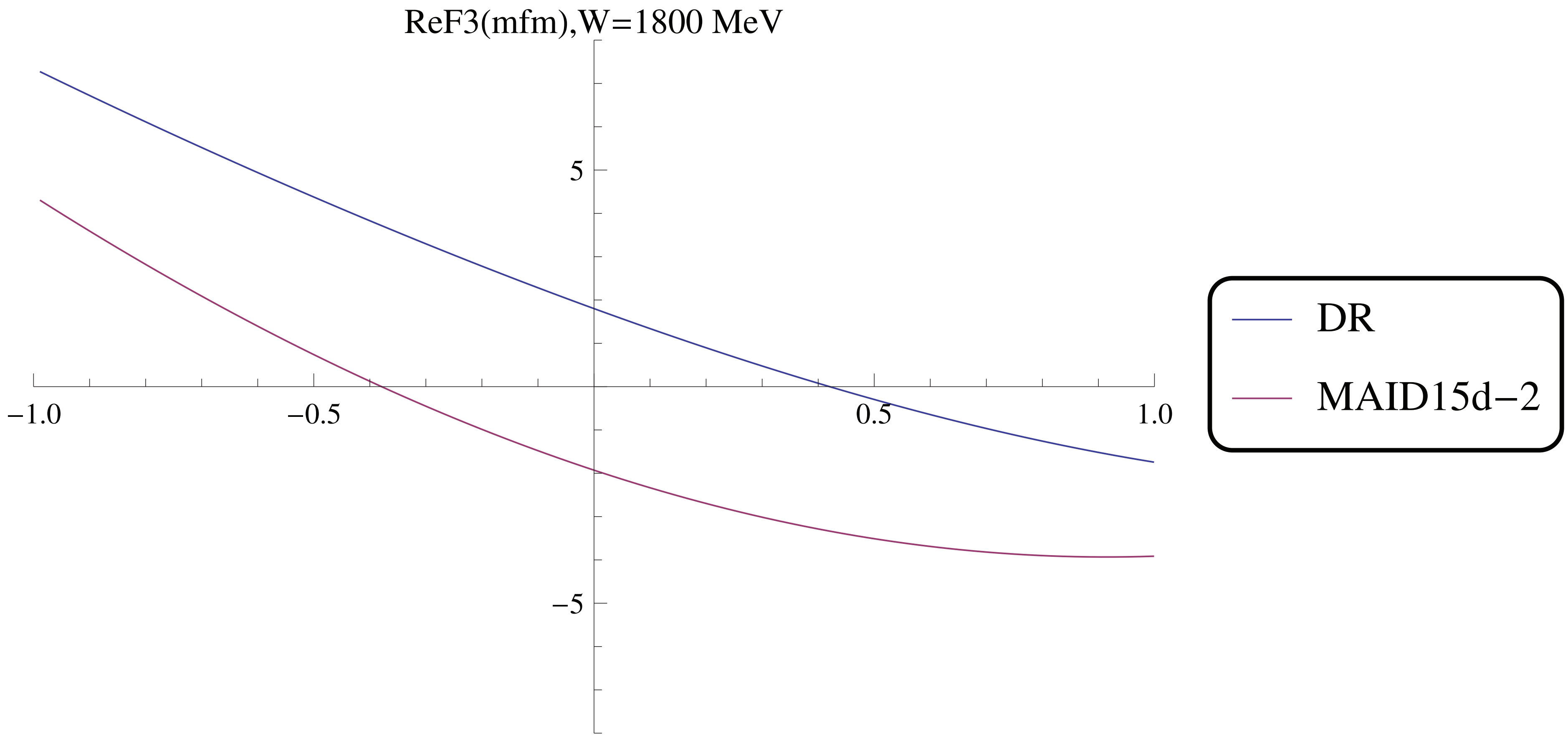


Results for $W = 1800 MeV$



Results for $W = 1800 MeV$



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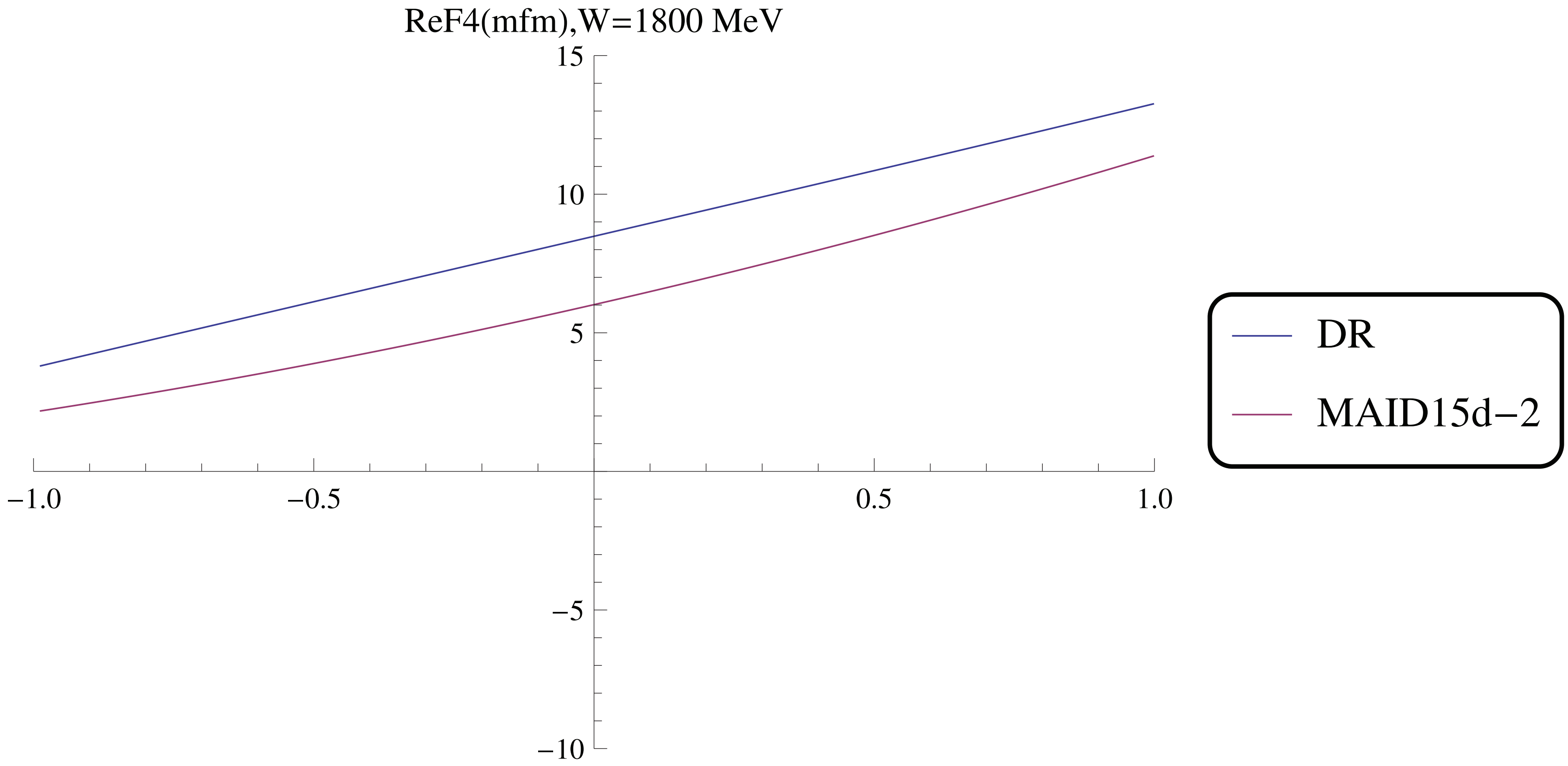
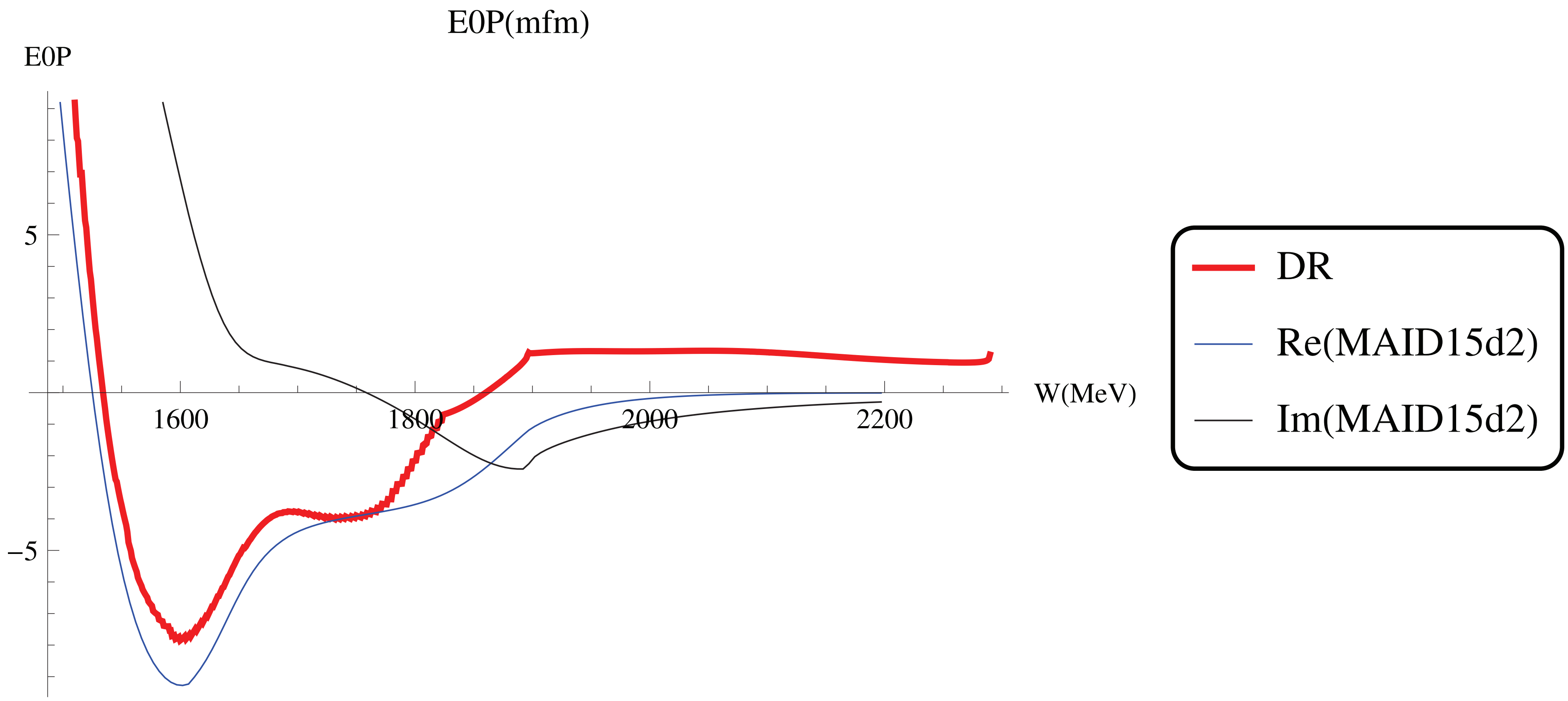
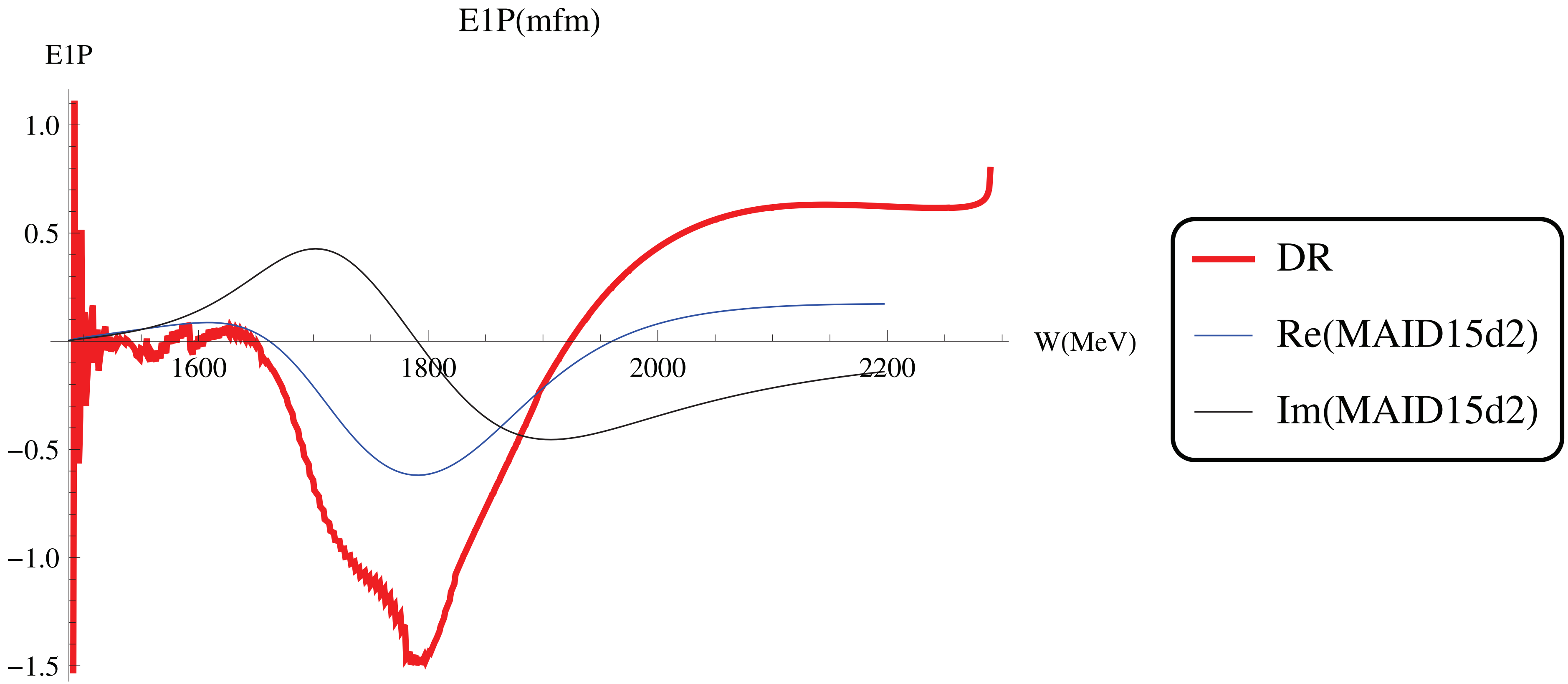


Multipoles

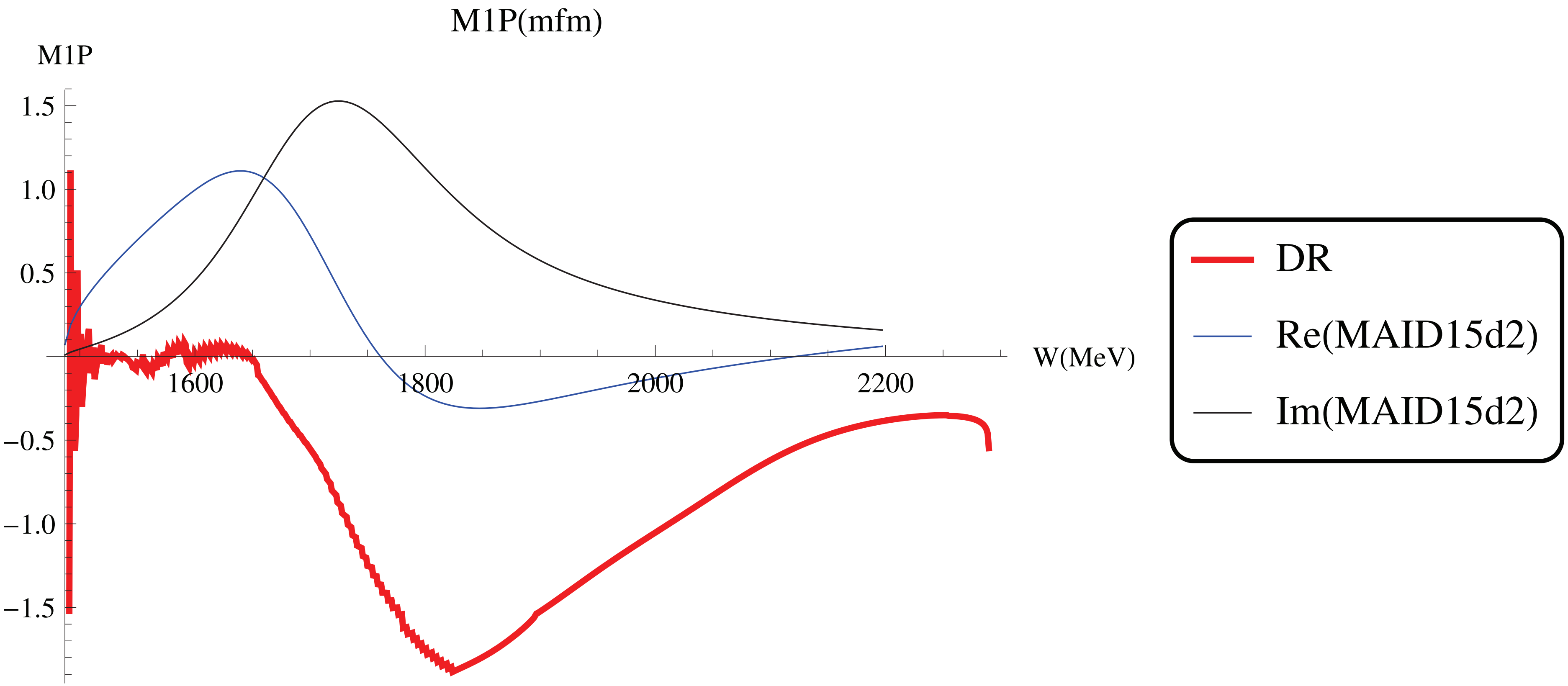
Results for E0+



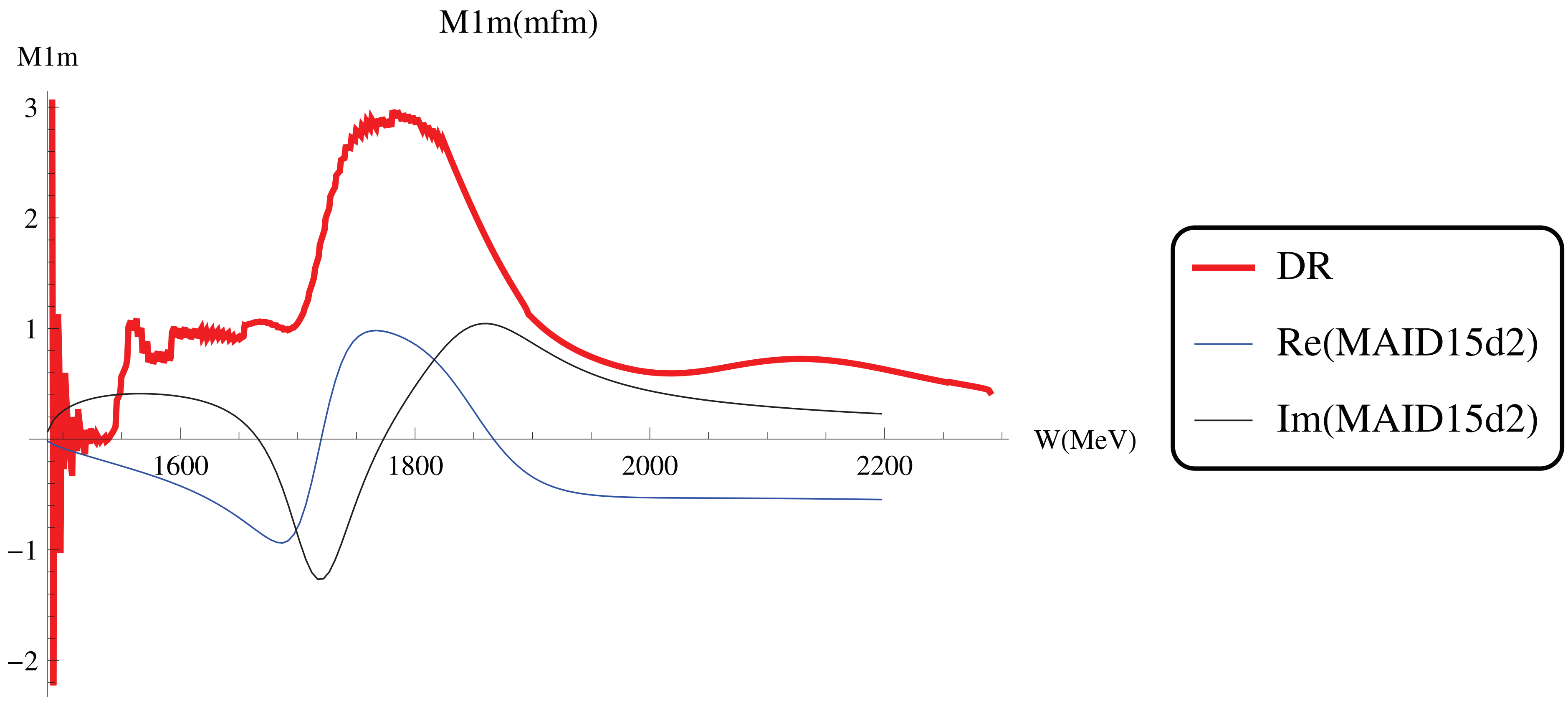
Results for E1+



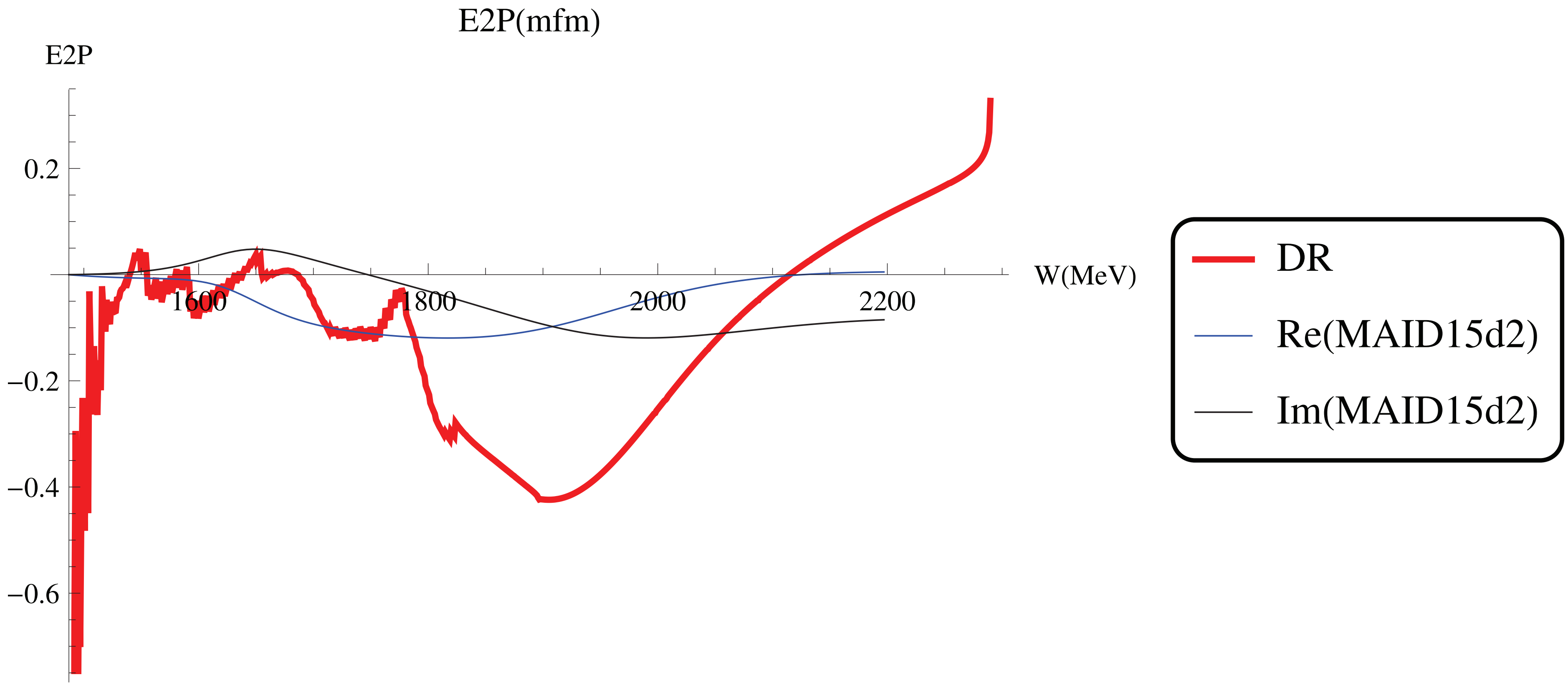
Results for M1+



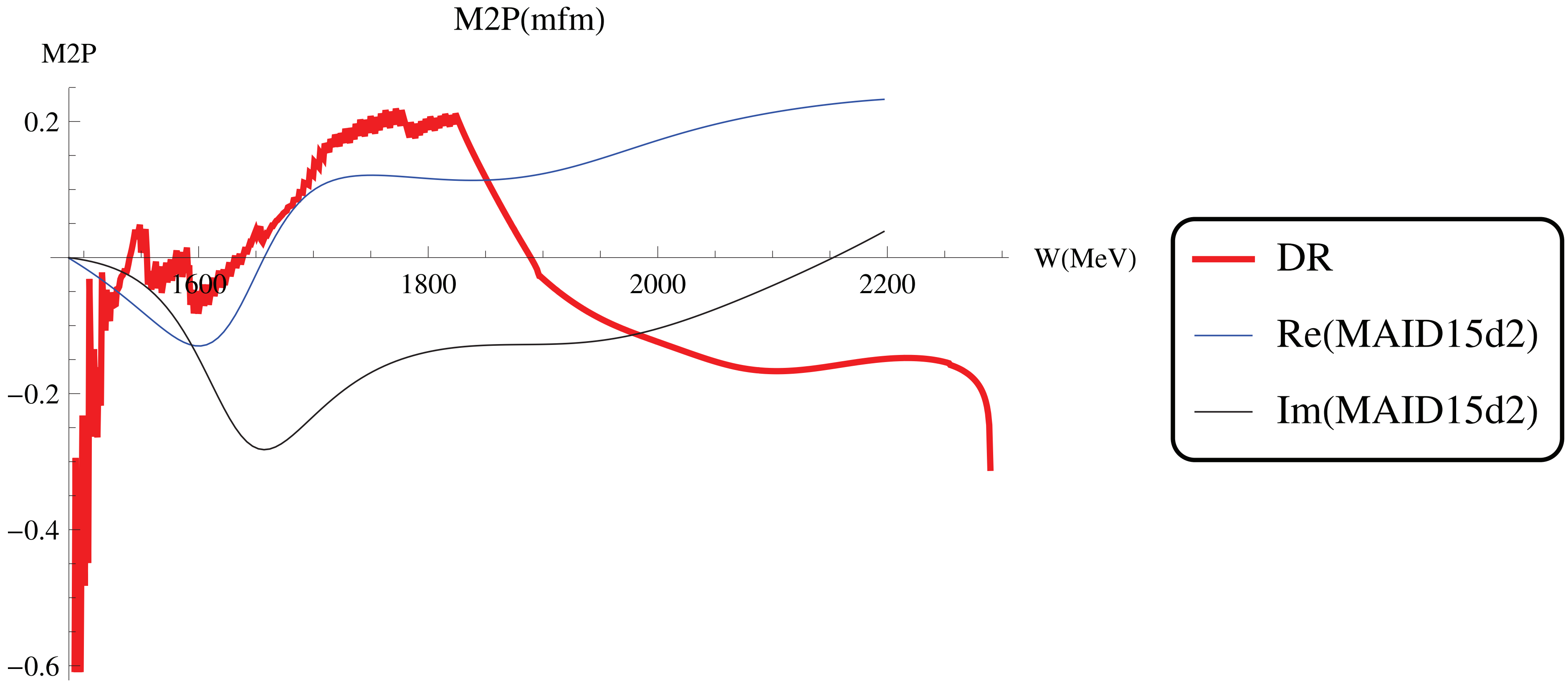
Results for M1-



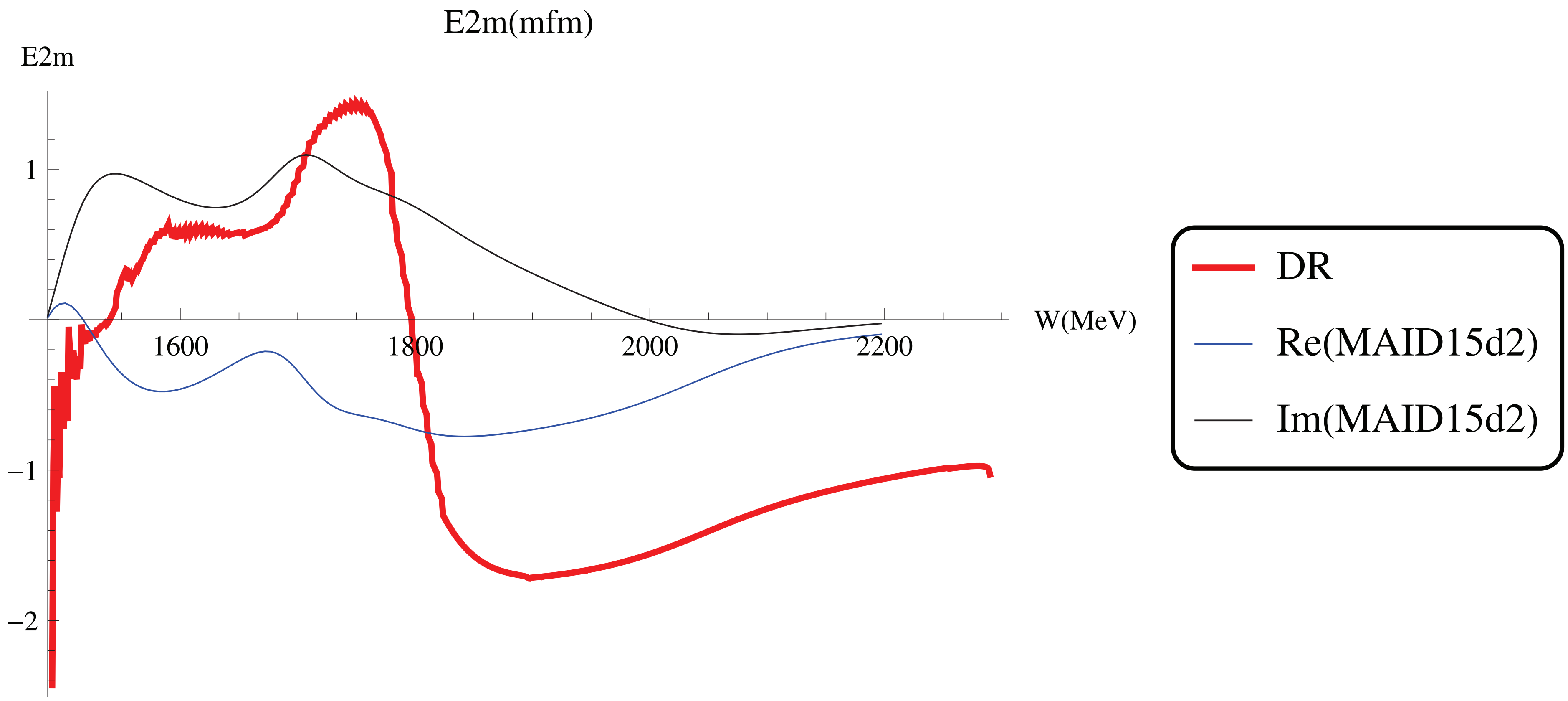
Results for E2+



Results for M2+



Results for E2-



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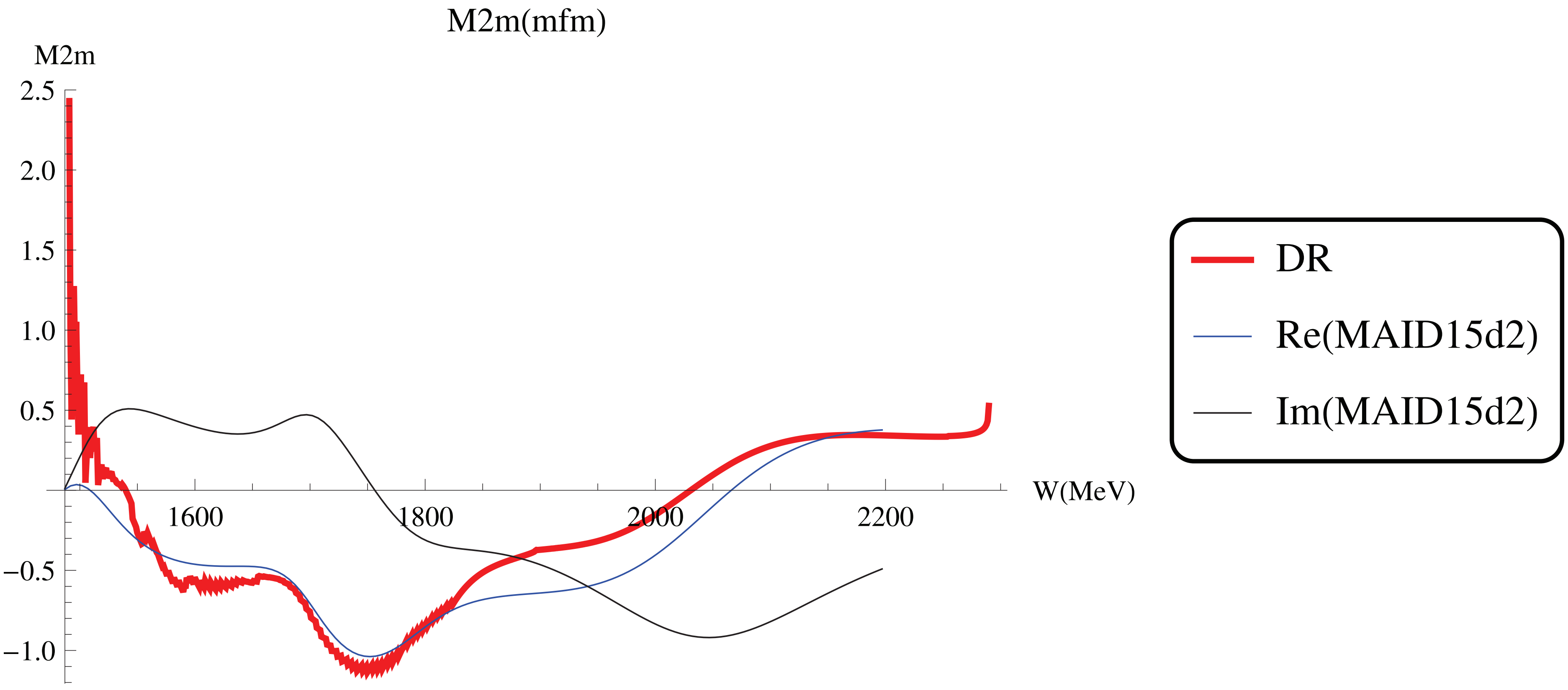
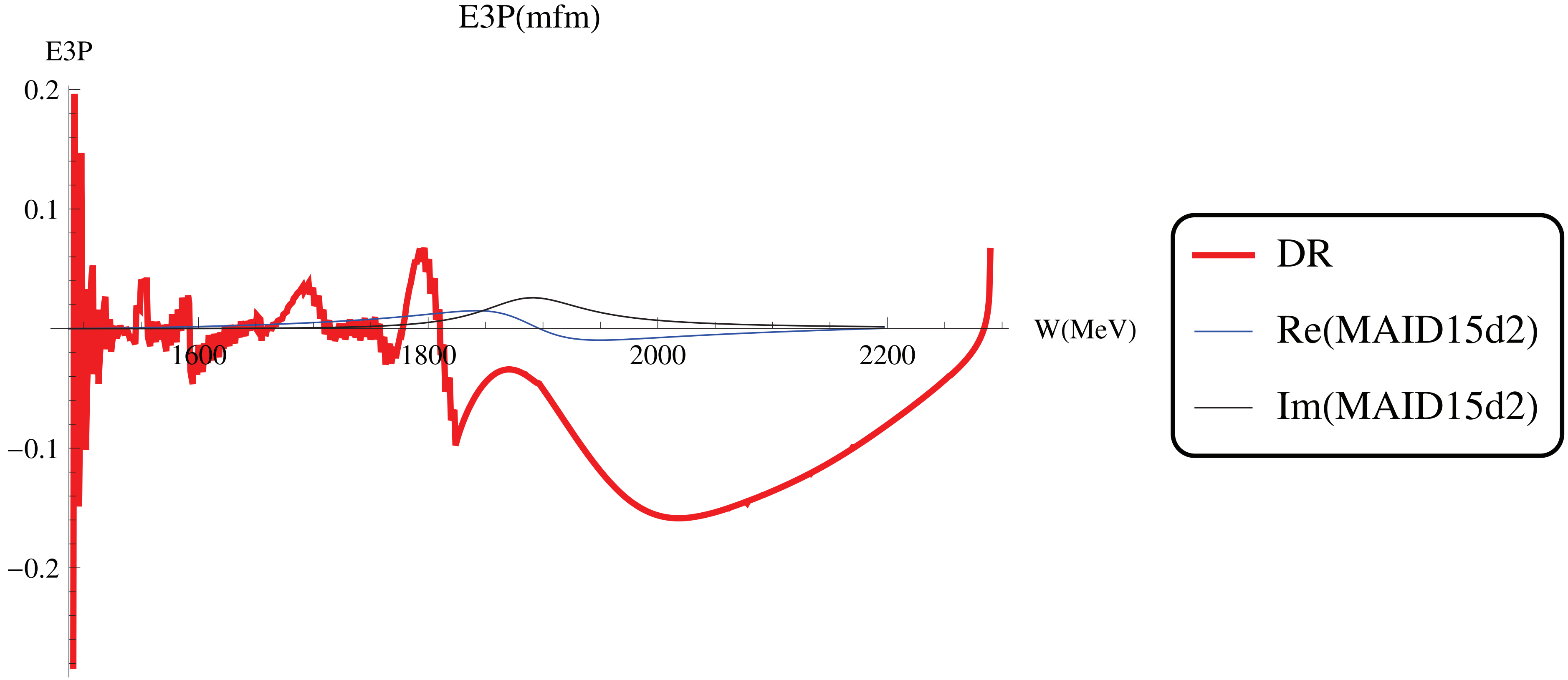
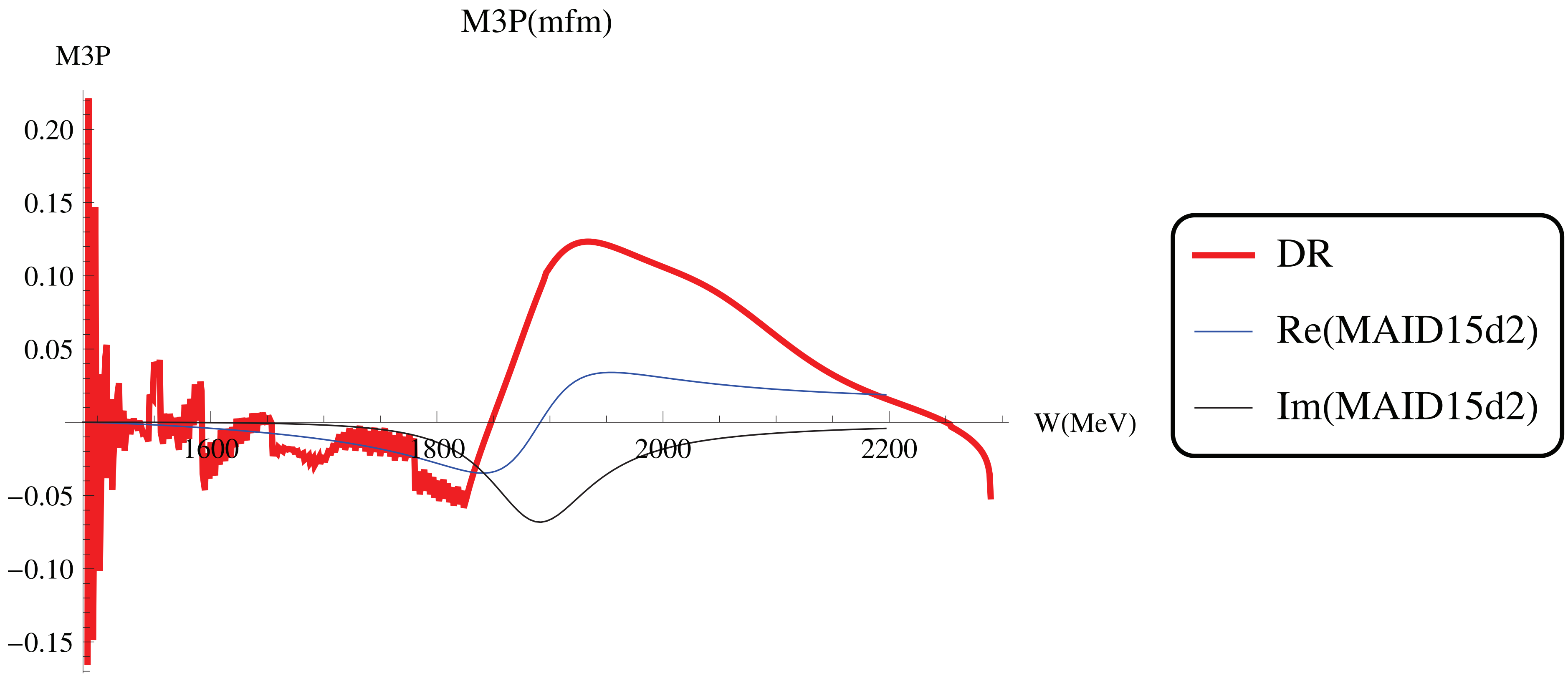


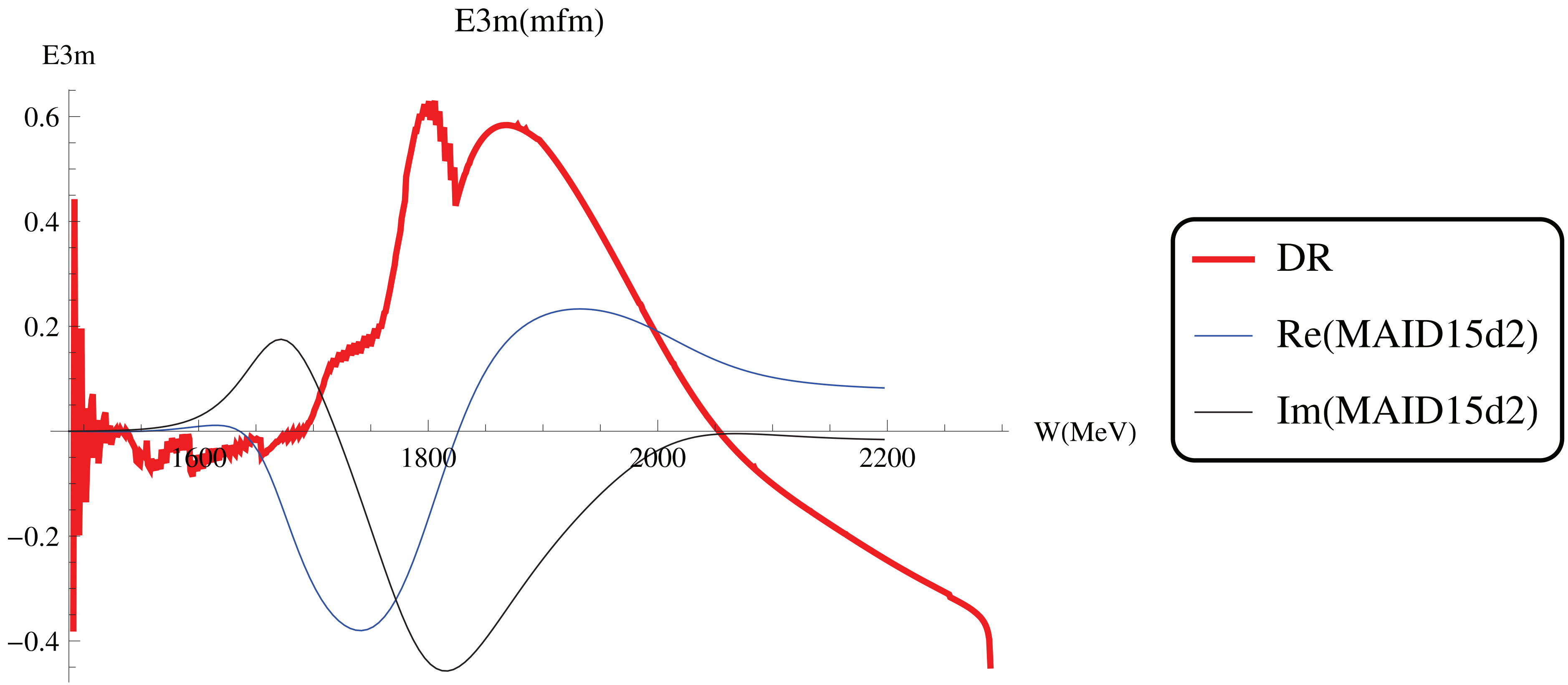
Results for E3+



Results for M3+



Results for E3-



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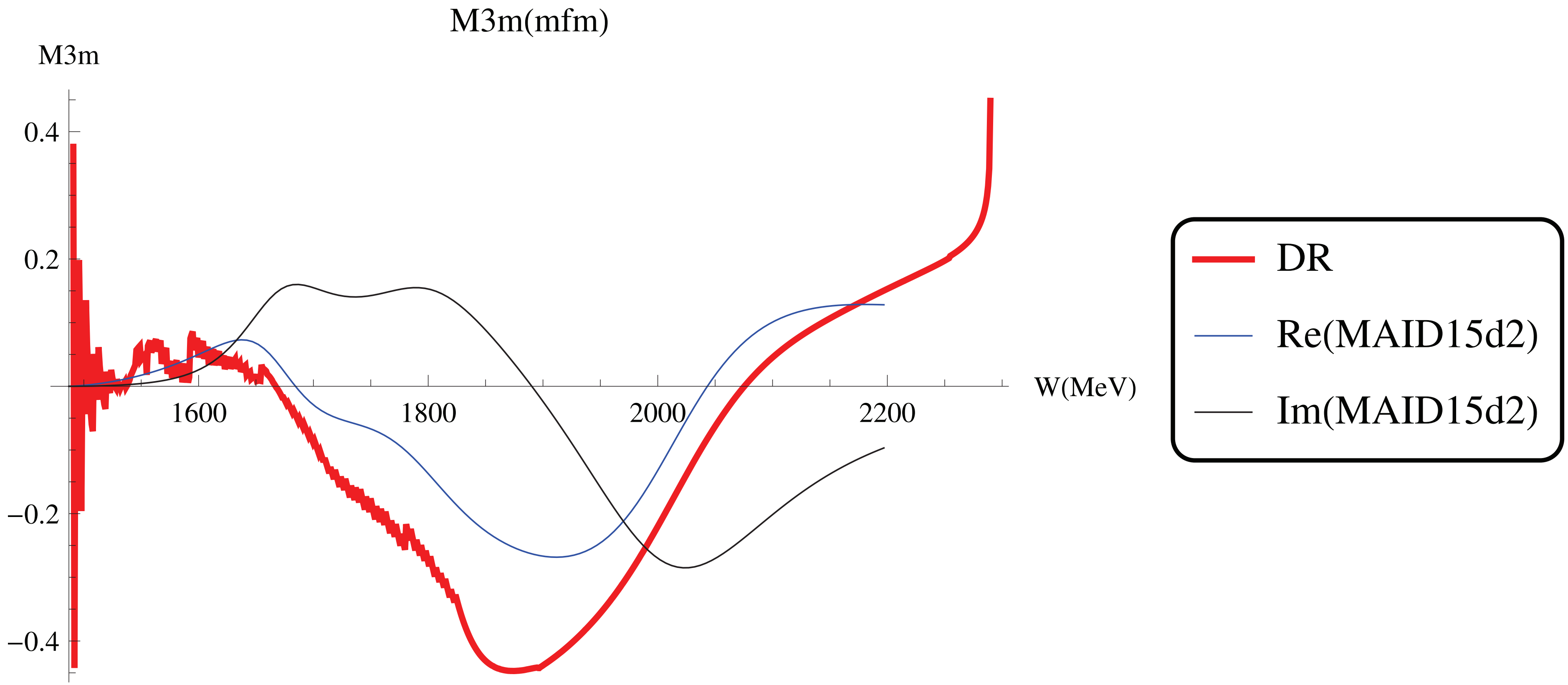

Conclusions

- We have multipoles!!! :)
- Spikes :(
- Things such as number of points, accuracy, bugs need to be studied :]