Partial wave analysis of $\eta$ photoproduction data

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Outline

- Single energy partial wave analysis - SE PWA-generally
- Imposing the fixed-t analyticity in PWA-generally
- Preliminary results
- Problem: Dependence on constraining PW solution
- Search for unique solution
- Conclusions
- Further research
At a given energy \( W \) minimize a quadratic form:

\[
\chi^2_{\text{data}} = \sum_D \sum_{k=1}^{N_D} \left( \frac{D_{k}^{\text{exp}}(\theta_k) - D_{k}^{\text{fit}}(\theta_k)}{\Delta D_k} \right)^2
\]

- \( D_{k}^{\text{exp}}(\theta_k) \) - values of observable \( D \) measured at angles \( \theta_k \) with errors \( \Delta D_k \).
- \( D_{k}^{\text{fit}}(\theta_k) \) - predictions calculated from partial waves (multipoles) which are parameters in the fit.

Serious problem in SE PWA - ambiguities, no unique solution.

How to resolve it?

First attempt:

Require smoothness of partial waves as a function of energy -

\[ \text{without success.} \]
One must impose more stringent constraints taking into account analyticity of scattering amplitudes.

(J. S. Bowcock, H. Burkhardt, Rep. Prog Phys 38 (1975) 1099)

**Important step forward:**

- E. Pietarinen: Amplitude analysis using fixed-$t$ analyticity of invariant amplitudes
  - E. Pietarinen, Nuovo Cim. 12 (1972) 522
The method consists of two separated analysis:

- Fixed-t amplitude analysis - a method which can determine the scattering amplitudes from exp. data at fixed-t
- Single energy partial wave analysis - SE PWA

Fixed-t AA and SE PWA are coupled. Results from one analysis are used as constraint in another in an iterative procedure.

Method was used in famous KH80 analysis of $\pi N$ scattering data.

In Mainz-Tuzla-Zagreb PWA of $\eta$- photoproduction data we apply the same principles.
Imposing the fixed-t analyticity in PWA of scattering data

Red dashed lines-SE PWA, Green dashed lines - fixed-t amplitude analysis
Imposing the fixed-t analyticity in PWA of scattering data

IA from start solution

At each $t$-value perform FT AA
Minimize:
\[ \chi^2 = \chi^2_{\text{data}} + \chi^2_{\text{IA}} + \Phi \]

At each of $N$ energies perform SE PWA
\[ \chi^2 = \chi^2_{\text{data}} + \chi^2_{\text{FT}} + \Phi_{\text{trunc}} \]

Use results from SE PWA to calculate IA which is used as an constraint in FT amplitude analysis
Pietarinen’s expansion method

The simplest case—$\pi N$ elastic scattering at fixed-$t$.

Apart from nucleon poles, crossing symmetric invariant amplitudes are analytic function in a complex $\nu^2$ plane $\nu_{th}^2 \leq \nu^2 < \infty$, ($\nu_{th} = m_\pi + \frac{t}{4m}$).

Conformal mapping:

$$z = \frac{\alpha - \sqrt{\nu_{th}^2 - \nu^2}}{\alpha + \sqrt{\nu_{th}^2 - \nu^2}}$$

maps a cut $\nu^2$ plane inside and on the circle in a $z$ plane.
Pietarinen’s expansion method

Pietarinen expansion method: Invariant amplitudes $C^\pm, B^\pm$ represented by:

$$C^\pm(\nu^2, t) = C_N^\pm(\nu^2, t) + \hat{C}^\pm(\nu^2, t) \sum_{n=0}^{\infty} c_n^\pm z^n$$

$$B^\pm(\nu^2, t) = B_N^\pm(\nu^2, t) + \hat{B}^\pm(\nu^2, t) \sum_{n=0}^{\infty} b_n^\pm z^n$$

$C_N^\pm, B_N^\pm$ - nucleon pole contributions, $\hat{C}^\pm(\nu^2, t), \hat{B}(\nu^2, t)$ describe high energy behaviour of IA.
Pietarinen’s expansion method

Pietarinen: The best approximants of IA are to be determined by minimizing a quadratic form:

\[ \chi^2 = \chi^2_{data} + \Phi. \]

\(\Phi\) is a convergence test function:

\[ \Phi = \lambda_1 \Phi_1 + \lambda_2 \Phi_2 + \lambda_3 \Phi_3 + \lambda_4 \Phi_4. \]

\[ \Phi_1 = \sum_{n=0}^{N} (n + 1)^3 (c_n^+)^2, \ldots, \Phi_4 = \sum_{n=0}^{N} (n + 1)^3 (b_n^-)^2. \]

For \(N \approx 30\):

\[ \lambda_1 = \frac{N}{\sum_{n=0}^{N} (n + 1)^3 (c_n^+)^2}, \ldots, \lambda_4 = \frac{N}{\sum_{n=0}^{N} (n + 1)^3 (b_n^-)^2}. \]
Our PWA of $\eta$ photoproduction data consists of two analysis:

- Fixed-t amplitude analysis
- SE PWA

Fixed- \( t \) amplitude analysis requires experimental data at a given value of variable \( t \). Experimental data have to be shifted to predefined \( t \)-values using a small steps in \( t \) - \textbf{t-binning}.

SE PWA requires experimental data at a given energy. Experimental data have to be shifted to predefined energies - \textbf{energy binning}.
Fixed-t amplitude analysis

For a given $t$ crossing symmetric invariant amplitudes are represented by two Pietarinen series:

$$B_1 = B_{1N} + \sum_{i=0}^{N_1} b_{1i}^{(1)} z_1^i + \sum_{i=0}^{N_2} b_{1i}^{(2)} z_2^i, \quad B_2 = B_{2N} + \sum_{i=0}^{N_1} b_{2i}^{(1)} z_1^i + \sum_{i=0}^{N_2} b_{2i}^{(2)} z_2^i$$

$$B_6 = B_{6N} + \sum_{i=0}^{N_1} b_{6i}^{(1)} z_1^i + \sum_{i=0}^{N_2} b_{6i}^{(2)} z_2^i, \quad B_8 = \frac{B_{8N}}{\nu} + \sum_{i=0}^{N_1} b_{8i}^{(1)} z_1^i + \sum_{i=0}^{N_2} b_{8i}^{(2)} z_2^i$$

$B_{iN}$ are known nucleon pole contributions. Conformal variables $z_1$ and $z_2$ are defined as:

$$z_1 = \frac{\alpha - \sqrt{\nu_{th1}^2 - \nu^2}}{\alpha + \sqrt{\nu_{th1}^2 - \nu^2}}, \quad z_2 = \frac{\beta - \sqrt{\nu_{th2}^2 - \nu^2}}{\beta + \sqrt{\nu_{th2}^2 - \nu^2}}.$$
Coefficients \( \{ b_1^{(k)} \} \) and \( \{ b_2^{(k)} \} \) are obtained by minimizing a quadratic form

\[
\chi^2 = \chi^2_{\text{data}} + \chi^2_{\text{PW}} + \Phi
\]

\[
\chi^2_{\text{data}} = \sum_{i=1}^{N^E} \left( \frac{d\sigma}{d\Omega} (W_i)^{\text{exp}} - \frac{d\sigma}{d\Omega} (W_i)^{\text{fit}} \right)^2 \frac{\Delta d\sigma}{d\Omega} (W_i)^{\text{exp}} \\
+ \sum_{i=1}^{N^E} \left( T(W_i)^{\text{exp}} - T(W_i)^{\text{fit}} \right)^2 \frac{\Delta T(W_i)^{\text{exp}}}{T(W_i)^{\text{exp}}} \\
+ \sum_{i=1}^{N^E} \left( F(W_i)^{\text{exp}} - F(W_i)^{\text{fit}} \right)^2 \frac{\Delta F(W_i)^{\text{exp}}}{F(W_i)^{\text{exp}}} \\
+ \sum_{i=1}^{N^E} \left( \Sigma(W_i)^{\text{exp}} - \Sigma(W_i)^{\text{fit}} \right)^2 \frac{\Delta \Sigma(W_i)^{\text{exp}}}{\Sigma(W_i)^{\text{exp}}}
\]
Fixed-t amplitude analysis

$\chi^2_{PW}$ contains as a “data” the helicity amplitudes calculated from partial wave solution:

$$
\chi^2_{PW} = q \sum_{i=1}^{N_E} \sum_{k=1}^{N^E} \left( \frac{\text{Re } H_k(W_i)^{PW} - \text{Re } H_k(W_i)^{fit}}{(\varphi_R)_{ki}} \right)^2 + q \sum_{k=1}^{N_E} \sum_{i=1}^{N^E} \left( \frac{\text{Im } H_k(W_i)^{PW} - \text{Im } H_k(W_i)^{fit}}{(\varphi_I)_{ki}} \right)^2
$$

$q$ - adjustable weight factor

Errors $\varphi_R$ and $\varphi_I$ are adjusted in such a way to get $\chi^2_{data} \approx \chi^2_{PW}$.

In a first iteration amplitudes $H_k^{PW}$ are calculated from initial, already existing PW solution.

In subsequent iterations $H_k^{PW}$ are calculated from multipoles obtained in SE PWA of the same set of experimental data.
Fixed-t amplitude analysis

Φ is Pietarinen’s convergence test function

\[ \Phi = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 \]

\[ \Phi_k = \lambda_{1k} \sum_{i=0}^{N_1} (b_{1i}^{(k)})^2 (n + 1)^3 + \lambda_{2k} \sum_{i=0}^{N_2} (b_{2i}^{(k)})^2 (i + 1)^3 \]

\[ \lambda_{1k} = \frac{N_1}{\sum_{i=0}^{N_1} (b_{1i}^{(k)})^2 (i + 1)^3}, \quad \lambda_{2k} = \frac{N_2}{\sum_{i=0}^{N_2} (b_{2i}^{(k)})^2 (i + 1)^3} \]

One starts with some initial values of coefficients \( \{b_{1i}^{(k)}\}, \{b_{2i}^{(k)}\} \) and determines \( \lambda_{1k} \) and \( \lambda_{2k} \) in an iterative procedure.
After performing fixed-t amplitude analysis at predetermined t-values, helicity amplitudes may be calculated at any energy $W$ at $N_c$ values of scattering angle

$$
\cos \theta_i = \frac{t_i - m_\eta^2 + 2k\omega}{2kq}
$$

where

$$
|\cos \theta_i| \leq 1, \quad t_i \in [t_{\text{min}}, t_{\text{max}}]
$$

These values of helicity amplitudes are used as constraint in SE PWA.
In a single energy partial wave analysis we minimize a quadratic form:

\[ \chi^2 = \chi^2_{\text{data}} + \chi^2_{\text{FT}} + \Phi_{\text{trunc}} \]

\( \chi^2_{\text{data}} \) contains all experimental data at a given energy \( W \):

\[
\chi^2_{\text{data}} = \sum_{i=1}^{N_1^D} \left( \frac{d\sigma}{d\Omega}(\theta_i)^{\text{exp}} - \frac{d\sigma}{d\Omega}(\theta_i)^{\text{fit}} \right) \frac{\Delta \frac{d\sigma}{d\Omega}(W_i)^{\text{exp}}}{\Delta \frac{d\sigma}{d\Omega}(W_i)^{\text{exp}}} \right)^2
+ \sum_{i=1}^{N_2^D} \left( \frac{T(\theta_i)^{\text{exp}} - T(\theta_i)^{\text{fit}}}{\Delta T(W_i)^{\text{exp}}} \right)^2
+ \sum_{i=1}^{N_3^D} \left( \frac{F(\theta_i)^{\text{exp}} - F(\theta_i)^{\text{fit}}}{\Delta F(W_i)^{\text{exp}}} \right)^2
+ \sum_{i=1}^{N_4^D} \left( \frac{\Sigma(\theta_i)^{\text{exp}} - \Sigma(\theta_i)^{\text{fit}}}{\Delta \Sigma(W_i)^{\text{exp}}} \right)^2
\]
Constrained SE PWA

$\chi^2_{FT}$ contains as the “data” the helicity amplitudes from the fixed-t amplitudes analysis.

$$
\chi^2_{FT} = q \sum_{k=1}^{4} \sum_{i=1}^{N_C} \left( \frac{\text{Re} H_k(\theta_i)^{PW} - \text{Re} H_k(\theta_i)^{fit}}{(\varepsilon_R)_{ki}} \right)^2 \\
+ q \sum_{k=1}^{4} \sum_{i=1}^{N_C} \left( \frac{\text{Im} H_k(\theta_i)^{PW} - \text{Im} H_k(\theta_i)^{fit}}{(\varepsilon_I)_{ki}} \right)^2
$$

$q$ - adjustable weight factor

$N_C$ - number of angles at which constraining amplitudes are determined. Errors $\varepsilon_{Rk}$ and $\varepsilon_{Ik}$ are adjusted in such a way to get

$$
\chi^2_{data} \approx \chi^2_{FT}
$$

Connection between SE PWA and fixed-t AA

Multipoles obtained from SE PWA at $N^E$ energies are used to calculate helicity amplitudes which are used as constraint in the fixed-t amplitude analysis.
\[ \Phi_{\text{trunc}} = \lambda \sum_{\ell=0}^{\ell_{\text{max}}} \left[ |Re T_{\ell\pm}|^2 R_1^{2\ell} + |Im T_{\ell\pm}|^2 R_2^{2\ell} \right]. \quad (1) \]

Expansion in terms of Legendre polynomials converge in an ellipse in \( \cos \theta \) plane having \(-1, 1\) as foci and semi-axis \( y_0(s) \) and \( (y_0^2(s) - 1)^{\frac{1}{2}} \), where \( y_0(s) \) is determined by the edge of the nearest double spectral region.

In a simplest (spinless) case, pw expansion converges if

\[ (|Im T_{\ell}|^2) \leq [y_0 + \sqrt{y_0^2 - 1}]^{-2\ell} \]

In a first attempt, we take:

\[ R_1 = R_2 = R = x_4 + \sqrt{x_4^2 - 1} \]

when

\[ y_0 = x_4 = \cos \theta (t = 4m^2) \]

\( T_{\ell\pm} \) stands for electric and magnetic multipoles \( E_{\ell\pm} \) and \( M_{\ell\pm} \).

Makes soft cut off of higher partial waves. Effective at low energies.
Constrained PWA of $\eta$ photoproduction data

The whole procedure has to be iterated until reaching reasonable agreement in two subsequent iterations.

**IA from start solution**

At each $t$-value perform FT AA

Minimize:

$$\chi^2 = \chi_{data}^2 + \chi_{IA}^2 + \Phi$$

At each of $N$ energies perform SE PWA

$$\chi^2 = \chi_{data}^2 + \chi_{FT}^2 + \Phi_{trunc}$$

Use results from SE PWA to calculate IA which is used as an constraint in FT amplitude analysis.
Data base consists of following experimental data

- Differential cross section $\frac{d\sigma}{d\Omega}$
  
  CBall/MAMI: E. McNicoll et al., PRC 82(2010) 035208
  
  $E_{lab} = 710, \ldots, 1395\,\text{MeV}$
  
  2400 data points at 120 energies

- Beam asymmetry $\Sigma$
  
  GRAAL: O. Bartalini et al., EPJ A 33 (2007) 169
  
  $E_{lab} = 724, \ldots, 1472\,\text{MeV}$
  
  150 data points at 15 energies

- Target asymmetry $T$
  
  CBall/MAMI: V. Kashevarov (preliminary)
  
  $E_{lab} = 725, \ldots, 1350\,\text{MeV}$
  
  144 data points at 12 energies

- Double-polarisation asymmetry $F$
  
  CBall/MAMI: V. Kashevarov (preliminary)
  
  $E_{lab} = 725, \ldots, 1350\,\text{MeV}$
  
  144 data points at 12 energies
Energy binning

$F, T, \Sigma$

Experimental values of double-polarisation asymmetry $F$, target asymmetry $T$, and beam asymmetry $\Sigma$ for given angles are interpolated to the energies where $\frac{d\sigma}{d\Omega}$ are available. We use a spline fit method. Errors of interpolated data are taken to be equal to errors of nearest measured data points.
Interpolated values of double polarisation $F$
Interpolated values of beam asymmetry $\Sigma$

- $\theta = 38.99^\circ$
- $\theta = 53.60^\circ$
- $\theta = 66.28^\circ$
- $\theta = 78.40^\circ$
- $\theta = 90^\circ$
- $\theta = 101.36^\circ$
- $\theta = 113.72^\circ$
- $\theta = 127.37^\circ$
- $\theta = 143.31^\circ$
- $\theta = 160.53^\circ$
Input data $\frac{d\sigma}{d\Omega}$, $T$, $F$ and $\Sigma$ for t-binning are obtained from energy binning procedure (113 energies).

- Observables $\frac{d\sigma}{d\Omega}$, $T$, $F$ and $\Sigma$ are available at different t-values (different $\cos \theta$).
- Fixed-t amplitude analysis is performed at t-values in the range $-0.05 \text{GeV}^2 < t < -1.00 \text{GeV}^2$.
- Using spline fit, experimental data ($\frac{d\sigma}{d\Omega}$, $T$, $F$ and $\Sigma$) are shifted to the predetermined t-values from above interval.
Interpolated values of measurable quantities at $t = -0.15\text{GeV}^2$
Interpolated values of measurable quantities at $t = -0.30 \text{GeV}^2$
MAID15 solutions-comparison of invariant amplitudes

B1 [GeV$^2$] at t=-0.2 GeV$^2$

- Real B
- Imag B

B1 (Maid2015a)
B1 (Maid2015b)
B1 (Maid2015c)
B1 (Maid2015d2)
MAID15 solutions-comparison of invariant amplitudes

B2 [GeV$^{-2}$] at t=-0.2 GeV$^2$

Real B

Imag B

B2 (Maid2015a)

B2 (Maid2015b)

B2 (Maid2015c)

B2 (Maid2015d2)

PW A meeting in Mainz, February, 2016.
MAID15 solutions-comparison of invariant amplitudes

B6 [GeV$^{-3}$] at $t=-0.2$ GeV

![Graphs showing B6 solutions for different comparisons of invariant amplitudes.](image)

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PWA of eta photoproduction data
MAID15 solutions—comparison of invariant amplitudes

B8 [GeV$^3$] at t=-0.2 GeV$^2$

Real B

Imag B

B8 (Maid2015a)

W[MeV]

B8 (Maid2015b)

W[MeV]

B8 (Maid2015c)

W[MeV]

B8 (Maid2015d2)

W[MeV]

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MAID15 solutions-comparison of invariant amplitudes

B2 [GeV$^{-2}$] at $t=-0.5$ GeV$^2$

Real $B$

Imag $B$

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MAID15 solutions—comparison of invariant amplitudes

B6 [GeV⁻³] at t = -0.5 GeV²

Real B

Imag B

B6 (Maid2015b)

B6 (Maid2015c)

B6 (Maid2015d2)
MAID15 solutions-comparison of invariant amplitudes

B8 [GeV$^{-3}$] at $t=-0.5$ GeV$^2$

Real B

Imag B

PW A meeting in Mainz, February, 2016.

PWA of eta photoproduction data
MAID15 solutions-comparison of helicity amplitudes

H1 [mfm] at $t=-0.2 \text{ GeV}^2$

Real H

Imag H

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PWA of eta photoproduction data
MAID15 solutions-comparison of helicity amplitudes

PWA meeting in Mainz, February, 2016.

PWA of eta photoproduction data
MAID15 solutions-comparison of helicity amplitudes

H3 [mfm] at t=−0.2 GeV²

Real H

Imag H

PW A meeting in Mainz, February, 2016.

PWA of eta photoproduction data
MAID15 solutions-comparison of helicity amplitudes

H4 [mfm] at $t=-0.2$ GeV$^2$

Real H

Imag H

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MAID15 solutions-comparison of helicity amplitudes

H1 [mfm] at t=-0.5 GeV²

Real H

H1 [mfm] at t=-0.5 GeV²

Imag H

PW A meeting in Mainz, February, 2016.
MAID15 solutions-comparison of helicity amplitudes

H2 [mfm] at $t=-0.5 \text{ GeV}^2$

Real H

Imag H

PW A meeting in Mainz, February, 2016.

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MAID15 solutions-comparison of helicity amplitudes

H4 [mfm] at t=-0.5 GeV^2

Real H

Imag H

W[MeV]

H4 (Maid2015a)

H4 (Maid2015b)

H4 (Maid2015c)

H4 (Maid2015d2)
A quick check of consistency of MAID15 solutions with fixed-t analyticity.

\[ \text{Re} \bar{B}_i(\nu^2, t) = \frac{1}{\pi} \int_{\nu_{th1}^2}^{\nu_{th2}^2} \frac{\text{Im} B_i(\nu'^2, t)}{\nu'^2 - \nu^2} d\nu'^2 + \frac{1}{\pi} \int_{\nu_{th2}^2}^{\infty} \frac{\text{Im} B_i(\nu'^2, t)}{\nu'^2 - \nu^2} d\nu'^2 \]

\[ \text{Re} \bar{B}_i(\nu^2, t) = \frac{1}{\pi} \int_{\nu_{th2}^2}^{\nu_{th1}^2} \frac{\text{Im} B_i(\nu'^2, t)}{\nu'^2 - \nu^2} d\nu'^2 + \text{Dis} = \text{PVI} + \text{Dis} \]

\[ \nu_{th1} = \frac{2(m + m_\pi)^2 - \Sigma - t}{4m}, \nu_{th2} = \frac{2(m + m_\eta)^2 - \Sigma - t}{4m}, \Sigma = 2m^2 + m_\eta^2 \]

\[ \text{Dis} = \text{Re} \bar{B}_i(\nu^2, t) - \frac{1}{\pi} \int_{\nu_{th1}^2}^{\nu_{th2}^2} \frac{\text{Im} B_i(\nu'^2, t)}{\nu'^2 - \nu^2} d\nu'^2 - \frac{1}{\pi} \int_{\nu_{th2}^2}^{\nu_{max}^2} \frac{\text{Im} B_i(\nu'^2, t)}{\nu'^2 - \nu^2} d\nu'^2 \]

PVI - principal value integral. Dis should be smooth function without

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Check of fixed-\( t \) analyticity - FTDR

**\( B_1 \)**

![Graphs showing analyticity check for \( B_1 \) and \( W \) vs. \( W \) for \( t = -0.2 \) GeV\(^2\) and \( t = -0.5 \) GeV\(^2\).](image)

**EtaMaid2015b**
- Real \( \tilde{B} \)
- PVI
- Discrepancy

**EtaMaid2015c**
- Real \( \tilde{B} \)
- PVI
- Discrepancy

**Note:** Figures show data points and curves for different GeV\(^2\) values.
Check of fixed-\(t\) analiticity - FTDR

\(B_2\)

\[\begin{array}{ccc}
\text{EtaMaid2015b} & \text{PVI} & \text{EtaMaid2015c} \\
\text{Real } B & \text{Discrepancy} & \\
\end{array}\]

\[\begin{array}{c}
\text{t=-0.2 GeV}^2 \\
\text{t=-0.5 GeV}^2 \\
\end{array}\]

\[\begin{array}{c}
B_2 [\text{GeV}^{-2}] \\
W[\text{GeV}] \\
\end{array}\]

\[\begin{array}{c}
B_2 [\text{GeV}^{-2}] \\
W[\text{GeV}] \\
\end{array}\]

\[\begin{array}{c}
B_2 [\text{GeV}^{-2}] \\
W[\text{GeV}] \\
\end{array}\]

\[\begin{array}{c}
B_2 [\text{GeV}^{-2}] \\
W[\text{GeV}] \\
\end{array}\]
Check of fixed-\(t\) analyticity - FTDR

\(B_6\)

![Graphs showing the comparison of EtaMaid2015b and EtaMaid2015c models for real and PVI cases at \(t = -0.2\) GeV\(^2\) and \(t = -0.5\) GeV\(^2\).]
Check of fixed-\( t \) analyticity -FTDR

\( B_8 \)

\begin{align*}
\text{EtaMaid2015b} & \quad \text{Real } B \\
\text{PVI} & \quad \text{Discrepancy}
\end{align*}

\begin{align*}
t = -0.2 \text{GeV}^2 & \\
t = -0.5 \text{GeV}^2
\end{align*}

\[ B_8 \text{ [GeV]} \]

\[ W \text{ [GeV]} \]

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Check of fixed-t analyticity - FTDR

\[ B_1 \]

\begin{align*}
\text{Maid 2015b} & \quad \text{Real } B \\
\text{Maid 2015d2} & \quad \text{PVI} \quad \text{Discrepancy}
\end{align*}

\begin{align*}
t = -0.2 \text{ GeV}^2 \\
t = -0.5 \text{ GeV}^2
\end{align*}

\begin{align*}
B_1 \text{ [GeV}^{-2}] \\
W \text{ [GeV]}
\end{align*}
Check of fixed-\(t\) analyticity - FTDR

\[ B_2 \]

**Maid 2015b**

- Real \( \tilde{B} \)

**Maid 2015d2**

- PVI
- Discrepancy

\( t = -0.2 \text{ GeV}^2 \)

\( t = -0.5 \text{ GeV}^2 \)
Check of fixed-$t$ analyticity - FTDR

$B_6$

**Maid 2015b**

Real $B$

**Maid 2015d2**

PVI

Discrepancy

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$B_6$ vs $W$ for $t=-0.2$ GeV$^2$

$B_6$ vs $W$ for $t=-0.5$ GeV$^2$
Check of fixed-t analyticity - FTDR

\[ B_8 \]

\begin{align*}
\text{Maid 2015b} & \quad \text{Real } \bar{B} & \quad \text{PVI} & \quad \text{Discrepancy} \\
\text{Maid 2015d2} & \quad \text{Real } \bar{B} & \quad \text{PVI} & \quad \text{Discrepancy}
\end{align*}

\begin{align*}
\text{t} = -0.2 \text{ GeV}^2 & \quad \text{t} = -0.2 \text{ GeV}^2 \\
\text{t} = -0.5 \text{ GeV}^2 & \quad \text{t} = -0.5 \text{ GeV}^2
\end{align*}

PWA meeting in Mainz, February, 2016.
Important contributions are missing.
In present calculations 4 observables were fitted: $d\sigma/d\Omega$, F, T and $\Sigma$.

\[
\chi^2 = \chi^2_{\text{data}} + \chi^2_{PW} + \Phi
\]

\[
\chi^2_{PW} = q \sum_{k=1}^{4} \sum_{n=1}^{N_{th}} \left[ \left( \frac{\text{Re}H_k(\omega, x_n)^{\text{fit}} - \text{Re}H_k(\omega, x_n)^{\text{start}}}{\epsilon_{Re}^{k,n}} \right)^2 + \left( \frac{\text{Im}H_k(\omega, x_n)^{\text{fit}} - \text{Im}H_k(\omega, x_n)^{\text{start}}}{\epsilon^{Im}^{k,n}} \right)^2 \right]
\]

$H_k$ -helicity amplitudes from SE ($-0.09 GeV^2 > t > -1.00 GeV^2$)
$q$ - adjustable weight factor. ($q = 1.0$).
$\epsilon_{Re}^{k,n}$ and $\epsilon^{Im}^{k,n}$ are errors. In this case $\epsilon = \epsilon^{Im}^{k,n} = 1.0$. 

Helmholtz equation: $\partial_t H_k = \alpha \left( H_k - H^{\text{fit}} \right)$
We use Pietarinen’s expansion with two thresholds ($\pi N$ and $\eta N$) and two conformal variables

$$z_1 = \frac{\alpha - \sqrt{\nu_{th1}^2 - \nu^2 - i \cdot \text{eps}}}{\alpha + \sqrt{\nu_{th1}^2 - \nu^2 - i \cdot \text{eps}}}$$

$$z_2 = \frac{\beta - \sqrt{\nu_{th2}^2 - \nu^2 - i \cdot \text{eps}}}{\beta + \sqrt{\nu_{th2}^2 - \nu^2 - i \cdot \text{eps}}}.$$ 

$\alpha = \beta = 0.9$, $Th(\pi N) = 1.07325 \text{GeV}$, $Th(\eta N) = 1.486 \text{GeV}$, $\nu = \frac{s-u}{4m}$.

$$B_1 = B_{1N} + P_R(z_1) \cdot (1 + z_1) \cdot \sum_i b^{(1)}_{1i} z_1^i + (1 + z_2) \cdot \sum_i b^{(2)}_{1i} z_2^i$$

$$B_2 = B_{2N} + P_R(z_1) \cdot (1 + z_1) \cdot \sum_i b^{(1)}_{2i} z_1^i + (1 + z_2) \cdot \sum_i b^{(2)}_{2i} z_2^i$$

$$B_6 = B_{6N} + P_R(z_1) \cdot (1 + z_1) \cdot \sum_i b^{(1)}_{6i} z_1^i + (1 + z_2) \cdot \sum_i b^{(2)}_{6i} z_2^i$$

$$\frac{B_8}{\nu} = \frac{B_{8N}}{\nu} + P_R(z_1) \cdot (1 + z_1) \cdot \sum_i b^{(1)}_{8i} z_1^i + (1 + z_2) \cdot \sum_i b^{(2)}_{8i} z_2^i.$$
A factor

\[ P_R(z_1) = \frac{(1 + z_R)(1 + z_R^*)}{(z_1 - z_R)(z_1 - z_R^*)}, \]

was introduced to take into account contribution from the Roper resonance.

\[ z_R = \frac{\alpha - \sqrt{\nu_{th1}^2 - \nu_R^2}}{\alpha + \sqrt{\nu_{th1}^2 - \nu_R^2}}. \]

\( z_R \) is a value of conformal variable \( z \) at \( P_{11} \) pole

\( W_R = (1.365 - 0.095i) \) GeV.
Dependence of amplitude solution on initial PW solution and PW constraint

Fixed-t invariant amplitudes $t = -0.20 GeV^2$ (EtaMAID15b)

**Figure**: Corresponding fixed-t invariant amplitudes are obtained using initial solution etaMAID2015b (red diamonds and blue circles). Red and blue solid lines are fits of invariant amplitudes $B_i$. 

PW meeting in Mainz, February, 2016.
Fixed-t invariant amplitudes \( t = -0.20\text{GeV}^2 \) (EtaMAID15d2)

Figure: Corresponding fixed-t invariant amplitudes are obtained using initial solution etaMAID2015d2 (red diamonds and blue circles). Red and blue solid lines are fits of invariant amplitudes \( B_i \).
Figure: Corresponding fixed-t invariant amplitudes are obtained using initial solution etaMAID2015b (red diamonds and blue circles). Red and blue solid lines are fits of invariant amplitudes $B_i$. 
Fixed-t invariant amplitudes

\[ t = -0.50 GeV^2 (\text{EtaMAID15d2}) \]

**Figure**: Corresponding fixed-t invariant amplitudes are obtained using initial solution etaMAID2015d2 (red diamonds and blue circles). Red and blue solid lines are fits of invariant amplitudes \( B_i \).
Fixed-t invariant amplitudes $t = -1.00\, GeV^2$ (EtaMAID15b)

Figure: Corresponding fixed-t invariant amplitudes are obtained using initial solution etaMAID2015b (red diamonds and blue circles). Red and blue solid lines are fits of invariant amplitudes $B_i$. 
Fixed-t invariant amplitudes $t = -1.00 \, GeV^2$ (EtaMAID15d2)

**Figure:** Corresponding fixed-t invariant amplitudes are obtained using initial solution etaMAID2015d2 (red diamonds and blue circles). Red and blue solid lines are fits of invariant amplitudes $B_i$.  

PWA meeting in Mainz, February, 2016.
Fit of experimental data $t = -0.20 GeV^2$ (EtaMAID15b)
Fit of experimental data $t = -0.50 \text{GeV}^2$ (EtaMAID15b)

Data

Fit

$W$ [MeV]

$W$ [MeV]

$W$ [MeV]

$W$ [MeV]

$W$ [MeV]

$W$ [MeV]

PWA meeting in Mainz, February, 2016.

PWA of eta photoproduction data
Fit of experimental data $t = -1.0 \text{GeV}^2$ (EtaMAID15b)

Data

Fit

PWA meeting in Mainz, February, 2016.
We present SE fits to the real data. In present calculations 4 observables were fitted: $d\sigma/d\Omega$, $\Sigma d\sigma/d\Omega$, $Td\sigma/d\Omega$ and $Fd\sigma/d\Omega$. Multipoles up to $H$-waves ($l = 5$) were fitted.

$$\chi^2 = \chi^2_{data} + \chi^2_{PW} + \Phi_{trunc}$$

$$\chi^2_{PW} = q \sum_{k=1}^{4} \sum_{n=1}^{N_{th}} \left[ \left( \frac{ReH_k(\omega,x_n)^{fit} - ReH_k(\omega,x_n)^{start}}{\varepsilon_{k,n}^{Re}} \right)^2 + \left( \frac{ImH_k(\omega,x_n)^{fit} - ImH_k(\omega,x_n)^{start}}{\varepsilon_{k,n}^{Im}} \right)^2 \right]$$

$H_k$ -helicity amplitudes from FT ($-0.09 \text{GeV}^2 > t > -1.00 \text{GeV}^2$). As a constraint we used etaMAID2015a. $q$ - adjustable weight factor. ($q = 1.5$). $\varepsilon_{k,n}^{Re}$ and $\varepsilon_{k,n}^{Im}$ are errors. In this case $\varepsilon_{k,n}^{Re} = \varepsilon_{k,n}^{Im} = 1$. 
\[ \Phi_{\text{trunc}} = \lambda \sum_{\ell=0}^{\ell_{\text{max}}} [\left| \text{Re} T_{\ell \pm} \right|^2 R^{2\ell} + \left| \text{Im} T_{\ell \pm} \right|^2 R^{2\ell}] . \]

\( \lambda \) is adjustable weight factor (\( \lambda = 0.3 \) in present calculations.).

In a first attempt, we take \( R = x_4 + \sqrt{x_4^2 - 1} \), where

\[ x_4 = \cos \theta(t = 4m_{\pi}^2) = \frac{4m_{\pi}^2 - m_\eta^2 + 2k(s)\omega(s)}{2k(s)q(s)}. \]

\( T_{\ell \pm} \) stands for electric and magnetic multipoles \( E_{\ell \pm} \) and \( M_{\ell \pm} \).
Smooth truncation—an example

As example we show results obtained using etaMAID2015a pseudo data and etaMAID2015b as a constraint.

(a) \( \chi^2 = \chi^2_{data} + \chi^2_{FT} \)  \hspace{1cm}  (b) \( \chi^2 = \chi^2_{data} + \chi^2_{FT} + \Phi_{trunc} \)
Smooth truncation—an example

\[(c) \chi^2 = \chi^2_{data} + \chi^2_{FT}\]

\[(d) \chi^2 = \chi^2_{data} + \chi^2_{FT} + \Phi_{trunc}\]
Smooth truncation—an example

\[(e) \chi^2 = \chi^2_{data} + \chi^2_{FT} \]

\[(f) \chi^2 = \chi^2_{data} + \chi^2_{FT} + \Phi_{trunc} \]
Smooth truncation - an example

\( g) \chi^2 = \chi^2_{\text{data}} + \chi^2_{\text{FT}} \)

\( h) \chi^2 = \chi^2_{\text{data}} + \chi^2_{\text{FT}} + \Phi_{\text{trunc}} \)
Smooth truncation—an example

\[(i) \chi^2 = \chi^2_{\text{data}} + \chi^2_{\text{FT}} \]

\[(j) \chi^2 = \chi^2_{\text{data}} + \chi^2_{\text{FT}} + \Phi_{\text{trunc}}\]
Figure: Red and blue solid lines-initial solution $\eta$MAID2015a
Figure: Red and blue solid lines—initial solution $\text{etaMAID2015a}$
Figure: Red and blue solid lines—initial solution etaMAID2015a
Figure: Red and blue solid lines—initial solution \textit{etaMAID2015a}
Figure: Red and blue solid lines—initial solution \textit{etaMAID2015a}
Elab = 739.70 MeV, Wcm = 1506.13 MeV; Chi2/Ndata = 0.86

Data

Fit

Tdσ/dΩ cos θ

Sdσ/dΩ cos θ

PWA meeting in Mainz, February, 2016.
Elab = 756.40 MeV, Wcm = 1516.50 MeV; Chi²/Ndata = 0.72

PWA meeting in Mainz, February, 2016.
Elab = 772.90 MeV, Wcm = 1526.67 MeV; Chi2/Ndata = 0.56
Elab = 902.50 MeV, Wcm = 1604.35 MeV; Chi2/Ndata = 0.74

Data

Fit

Tdσ/dΩ

Sdσ/dΩ

PW A meeting in Mainz, February, 2016.
Data and Fit curves for $d\sigma/d\Omega \cos \theta$ with $\theta$ values:

- $0.05$
- $0.1$
- $0.15$
- $0.2$
- $0.25$
- $0.3$
- $0.35$
- $0.4$

$Elab = 1002.60$ MeV, $Wcm = 1661.86$ MeV; $\chi^2/N_{data} = 0.63$
Elab = 1125.90 MeV, Wcm = 1730.07 MeV; \( \chi^2/\text{Ndata} = 0.56 \)

Data

Fit

\( \frac{d\sigma}{d\Omega} \cos \theta \)

\( \frac{d\sigma}{d\Omega} \cos \theta \)

\( \frac{d\sigma}{d\Omega} \cos \theta \)

\( \frac{d\sigma}{d\Omega} \cos \theta \)
Elab = 1204.20 MeV, Wcm = 1772.03 MeV; Chi²/ndata = 0.54

Data

Elab = 1204.20 MeV, Wcm = 1772.03 MeV; Chi²/ndata = 0.54

Fit
Elab = 1287.40 MeV, Wcm = 1815.55 MeV; \chi^2/N_{data} = 1.01

Data

Fit

PWA meeting in Mainz, February, 2016.

PWA of eta photoproduction data
Helicity amplitudes

Elab = 830.40 MeV, Wcm = 1561.61 MeV; Chi2/Ndata = 1.01

Re FT  Re SE  Im FT  Im SE

$H_1$ [mfm]  $H_2$ [mfm]  $H_3$ [mfm]  $H_4$ [mfm]

PW A meeting in Mainz, February, 2016.  PWA of eta photoproduction data
Helicity amplitudes

Elab = 930.00 MeV, Wcm = 1620.35 MeV; \( \chi^2/N_{\text{data}} = 0.58 \)

- \( H_1 \) [mfm] \( \cos \theta \)
- \( H_2 \) [mfm] \( \cos \theta \)
- \( H_3 \) [mfm] \( \cos \theta \)
- \( H_4 \) [mfm] \( \cos \theta \)

PWA meeting in Mainz, February, 2016.
Helicity amplitudes

Elab = 1070.20 MeV, Wcm = 1699.60 MeV; Chi²/Ndata = 0.60

PWA meeting in Mainz, February, 2016.  

PWA of eta photoproduction data
Helicity amplitudes

Elab = 1172.40 MeV, Wcm = 1755.11 MeV; \( \text{Chi}_2/\text{Ndata} = 0.92 \)
Helicity amplitudes

Elab = 1260.20 MeV, Wcm = 1801.44 MeV; \( \chi^2/\text{Ndata} = 0.66 \)

\( H_1 \) [mfm] \( \cos \theta \)

\( H_2 \) [mfm] \( \cos \theta \)

\( H_3 \) [mfm] \( \cos \theta \)

\( H_4 \) [mfm] \( \cos \theta \)
Helicity amplitudes

Elab = 1335.20 MeV, Wcm = 1840.09 MeV; Chi2/Ndata = 0.67
Problem: Dependence on constraining PW solution
Dependance of SE PWA solution on constraint from FT AA

Real part

Imag part

Maid2015 model

Maid15a

Maid15b

Maid15c

Maid15d

PW A meeting in Mainz, February, 2016.

PWA of eta photoproduction data
Dependence of SE PWA solution on constraint from FT AA

Real part  

Imag part  

Maid15 model  

E1+ (Maid15a)  

W[MeV]  

E1+ (Maid15c)  

W[MeV]  

E1+ (Maid15b)  

W[MeV]  

E1+ (Maid15d2)  

W[MeV]  

PW A meeting in Mainz, February, 2016.
Dependance of SE PWA solution on constraint from FT AA

Real part

Imag part

Maid15 model

Maid15 model

1500 1550 1600 1650 1700 1750 1800 1850

E2+ (Maid15a)  W[MeV]

E2+ (Maid15c)  W[MeV]

E2+ (Maid15d)  W[MeV]
Dependance of SE PWA solution on constraint from FT AA

PWA meeting in Mainz, February, 2016.
Dependence of SE PWA solution on constraint from FT AA

Real part
Imag part
Maid15 model

\[
\begin{align*}
E_{3\text{-}(\text{Maid15a})} & \quad \text{W[MeV]} \\
E_{3\text{-}(\text{Maid15b})} & \quad \text{W[MeV]} \\
E_{3\text{-}(\text{Maid15c})} & \quad \text{W[MeV]} \\
E_{3\text{-}(\text{Maid15d2})} & \quad \text{W[MeV]}
\end{align*}
\]
Dependance of SE PWA solution on constraint from FT AA

![Graphs showing the dependance of SE PWA solution on constraint from FT AA.](image)

E3+ (Maid15a) in red, E3+ (Maid15b) in blue, E3+ (Maid15c) in red, and E3+ (Maid15d2) in blue. The graphs show the real part and imaginary part for different energies.

PW A meeting in Mainz, February, 2016.
Dependance of SE PWA solution on constraint from FT AA

Real part

Imag part

Maid15 model

E4+ (Maid15a)

W [MeV]

E4+ (Maid15b)

W [MeV]

E4+ (Maid15c)

W [MeV]

E4+ (Maid15d2)

W [MeV]
Dependance of SE PWA solution on constraint from FT AA

![Graphs showing real and imaginary parts of E5+ (Maid15a) and Maid15 model over different energy ranges.](image)

PWA meeting in Mainz, February, 2016.
Dependance of SE PWA solution on constraint from FT AA

PWA meeting in Mainz, February, 2016.  
PWA of eta photoproduction data
Dependance of SE PWA solution on constraint from FT AA

![Graphs showing real and imaginary parts of M1- for Maid15 model](image)

-3
-2
-1
0
1
2
1500 1550 1600 1650 1700 1750 1800 1850
W[MeV]

M1- (Maid15a)
M1- (Maid15b)
M1- (Maid15c)
M1- (Maid15d2)

PW A meeting in Mainz, February, 2016.

PWA of eta photoproduction data

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Dependence of SE PWA solution on constraint from FT AA

M1+ (Maid15a)

M1+ (Maid15b)

M1+ (Maid15c)

Maid2015 model

Real part

Imag part

Real part

Imag part

M1+ (maid15d2)

M1+ (Maid15d2)

1500 1550 1600 1650 1700 1750 1800 1850
W[MeV]

1500 1550 1600 1650 1700 1750 1800 1850
W[MeV]

1500 1550 1600 1650 1700 1750 1800 1850
W[MeV]

1500 1550 1600 1650 1700 1750 1800 1850
W[MeV]
Dependance of SE PWA solution on constraint from FT AA

PWA meeting in Mainz, February, 2016.

PWA of eta photoproduction data 107 / 146
Dependence of SE PWA solution on constraint from FT AA

Maid15 model

Real part

Imag part

W [MeV]

1500 1550 1600 1650 1700 1750 1800 1850

M3- (Maid15a)

M3- (Maid15b)

M3- (Maid15c)

M3- (Maid15d2)

PWA meeting in Mainz, February, 2016.

PWA of eta photoproduction data
Dependence of SE PWA solution on constraint from FT AA

Maid15 model

Real part

Imag part

M3+ (Maid15a)

W[MeV]

M3+ (Maid15b)

W[MeV]

M3+ (Maid15c)

W[MeV]

M3+ (Maid15d2)

W[MeV]
Dependance of SE PWA solution on constraint from FT AA

Maid15 model

Real part

Imag part

M4- (Maid15a)

W[MeV]

M4- (Maid15b)

W[MeV]

M4- (Maid15c)

W[MeV]

M4- (Maid15d2)

W[MeV]
Dependance of SE PWA solution on constraint from FT AA

PWA meeting in Mainz, February, 2016.
Dependence of SE PWA solution on constraint from FT AA

Real part

Imag part

Maid15 model

M5- (Maid15a)

M5- (Maid15b)

M5- (Maid15c)

M5- (Maid15d2)

PWA meeting in Mainz, February, 2016.

PWA of eta photoproduction data
Dependance of SE PWA solution on constraint from FT AA

Real part

Imag part

Maid15 model

Maid15 model

W[MeV]

W[MeV]

M5+ (Maid15a)

M5+ (Maid15b)

M5+ (Maid15c)

M5+ (Maid15d2)

PWA meeting in Mainz, February, 2016. PWA of eta photoproduction data

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Search for unique solution
Reduced multipoles

\[ E_{0+}(W) = |E_{0+}(W)|e^{i\phi_0} \]

\[ E_{\ell\pm}(W) = |E_{\ell\pm}(W)|e^{i\phi_{\ell}} \]

\[ M_{\ell\pm}(W) = |M_{\ell\pm}(W)|e^{i\phi_{\ell}} \]

Reduced multipoles are

\[ \tilde{E}_{\ell\pm}(W) = |E_{\ell\pm}(W)|e^{i(\phi_{\ell}-\phi_0)}, \quad \ell = 0, \ldots, 5 \]

\[ \tilde{M}_{\ell\pm}(W) = |M_{\ell\pm}(W)|e^{i(\phi_{\ell}-\phi_0)}, \quad \ell = 1, \ldots, 5 \]
Reduced multipoles

Figure: Red and blue-reduced multipoles obtained using 15a as constraint in FT. Green and magenta-reduced multipoles obtained using 15c as constraint in FT. Cyan and yellow-reduced multipoles obtained using 15d2 as constraint in FT.
Figure: Red and blue-reduced multipoles obtained using 15a as constraint in FT. Green and magenta-reduced multipoles obtained using 15c as constraint in FT. Cyan and yellow-reduced multipoles obtained using 15d2 as constraint in FT.
Figure: Red and blue-reduced multipoles obtained using 15a as constraint in FT. Green and magenta-reduced multipoles obtained using 15c as constraint in FT. Cyan and yellow-reduced multipoles obtained using 15d2 as constraint in FT.
Reduced helicity amplitudes

\[ H_1^{a,c,d_2}(x, W) = |H_1^{a,c,d_2}(x, W)| e^{i\phi_1^{a,c,d_2}(x, W)} \]

Reduced helicity amplitudes

\[ \tilde{H}_k^{a,c,d_2}(x, W) = |H_k^{a,c,d_2}| e^{i(\phi_k^{a,c,d_2}(x, W) - \phi_1^{a,c,d_2}(x, W))}, \quad k = 1, 2, 3, 4 \]
Figure: Red and blue-reduced helicity amplitudes obtained using 15a as constraint in FT. Green and magenta-reduced helicity amplitudes obtained using 15c as constraint in FT. Cyan and yellow-reduced helicity amplitudes obtained using 15d2 as constraint in FT.
Reduced helicity amplitudes-$W=1602\text{MeV}$

Figure: Red and blue-reduced helicity amplitudes obtained using 15a as constraint in FT. Green and magenta-reduced helicity amplitudes obtained using 15c as constraint in FT. Cyan and yellow-reduced helicity amplitudes obtained using 15d2 as constraint in FT
Reduced helicity amplitudes-$W=1699\text{MeV}$

**Figure:** Red and blue-reduced helicity amplitudes obtained using 15a as constraint in FT. Green and magenta-reduced helicity amplitudes obtained using 15c as constraint in FT. Cyan and yellow-reduced helicity amplitudes obtained using 15d2 as constraint in FT.
Reduced helicity amplitudes- $W=1801\,\text{MeV}$

**Figure**: Red and blue-reduced helicity amplitudes obtained using 15a as constraint in FT. Green and magenta-reduced helicity amplitudes obtained using 15c as constraint in FT. Cyan and yellow-reduced helicity amplitudes obtained using 15d2 as constraint in FT.
Figure: Red and blue-reduced helicity amplitudes obtained using 15a as constraint in FT. Green and magenta-reduced helicity amplitudes obtained using 15c as constraint in FT. Cyan and yellow-reduced helicity amplitudes obtained using 15d2 as constraint in FT.
We present fixed-t fits to the MAID2015a pseudodata. In present calculations 8 observables were fitted: $d\sigma/d\Omega$, $\Sigma d\sigma/d\Omega$, $T d\sigma/d\Omega$, $F d\sigma/d\Omega$, $Ed\sigma/d\Omega$, $Gd\sigma/d\Omega$, $Hd\sigma/d\Omega$, and $Pd\sigma/d\Omega$.

$$\chi^2 = \chi^2_{data} + \chi^2_{FT} + \Phi$$

$$\chi^2_{FT} = q \sum_{k=1}^{4} \sum_{n=1}^{N_{th}} \left[ \left( \frac{ReH_k(\omega, x_n)^{fit} - ReH_k(\omega, x_n)^{start}}{\epsilon_k^{Re}} \right)^2 + \left( \frac{ImH_k(\omega, x_n)^{fit} - ImH_k(\omega, x_n)^{start}}{\epsilon_k^{Im}} \right)^2 \right]$$

$H_k$-helicity amplitudes from SE-1st ($-0.075\text{GeV}^2 > t > -2.00\text{GeV}^2$) $q$ - adjustable weight factor. ($q = 1$).

$\epsilon_k^{Re}$ and $\epsilon_k^{Im}$ are errors. In this case $\epsilon_k^{Re} = \epsilon_k^{Im} = 1$. As a constraint we used etaMAID2015b. Red diamonds and blue circles shown etaMAID2015b initial solution. Red and blue solid lines are FT fit of IA.
FT invariant amplitudes

Real part

Imag part

B1 \([1/GeV^2]\)

B2 \([1/GeV^2]\)

B6 \([1/GeV^2]\)

B8 \([1/GeV^2]\)

W[MeV]
Fit of pseudo data etaMAID2015a

Data vs Fit at $t=-0.200\text{GeV}^2$

- Data
- Fit

<table>
<thead>
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<th>$W\text{[MeV]}$</th>
<th>$d\sigma/d\Omega$</th>
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</table>

PWA meeting in Mainz, February, 2016.
FT invariant amplitudes

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**Real part**

- $B_1$ vs. $W$ [MeV]
- $B_2$ vs. $W$ [MeV]

**Imag part**

- $B_6$ vs. $W$ [MeV]
- $B_8$ vs. $W$ [MeV]

---

PWA meeting in Mainz, February, 2016.
Fit of pseudo data etMAID2015a

Data $t=-0.800\text{GeV}^2$

Fit

PWA meeting in Mainz, February, 2016.

PWA of eta photoproduction data
We present SE fits to the MAID2015a pseudodata. In present calculations 12 observables were fitted: \(d\sigma/d\Omega\), \(\Sigma d\sigma/d\Omega\), \(Td\sigma/d\Omega\), \(Fd\sigma/d\Omega\), \(Ed\sigma/d\Omega\), \(Gd\sigma/d\Omega\), \(Hd\sigma/d\Omega\), and \(Pd\sigma/d\Omega\). Multipoles up to H-waves \((l = 5)\) were fitted.

\[
\chi^2 = \chi^2_{data} + \chi^2_{FT} + \Phi_{trunc}
\]

\(H_k\) -helicity amplitudes from FT \((-0.075 GeV^2 > t > -2.00 GeV^2\)). As a constraint we used etaMAID2015b.

\(q\) - adjustable weight factor. \((q = 1.)\).

\(\varepsilon^{Re}_{k,n}\) and \(\varepsilon^{Im}_{k,n}\) are errors. In this case \(\varepsilon^{Re}_{k,n} = \varepsilon^{Im}_{k,n} = 1\).

Red and blue solid lines are multipoles from etaMAID2015a.

Magenta and green solid lines are multipoles from etaMAID2015b.
SE PWA pseudo etaMAID2015a

PWA meeting in Mainz, February, 2016.

PWA of eta photoproduction data
PW A meeting in Mainz, February, 2016.

PWA of eta photoproduction data
The graphs show the real and imaginary parts of the functions $E_5^+$, $E_5^-$, $M_5^+$, and $M_5^-$ as functions of $W$ (in MeV). The data points are shown with error bars, and the fitted curves are represented by solid lines. The graphs cover the energy range from 1400 to 2200 MeV.
Elab= 1000.00 MeV, Wcm= 1660.40 MeV; Chi²/Ndata= 0.40

Chi²/Ndata= 0.40

Data

Fit

Chi²/Ndata= 0.40

Data

Fit
SE PWA pseudo helicity amplitudes

Elab= 800.00 MeV, Wcm= 1543.25 MeV; Chi2/Ndata= 0.56

Elab= 1000.00 MeV, Wcm= 1660.40 MeV; Chi2/Ndata= 0.40

PWA meeting in Mainz, February, 2016.
SE PWA pseudo helicity amplitudes

Elab = 1300.00 MeV, Wcm = 1822.06 MeV; $\chi^2/N_{data} = 0.48$

Elab = 1500.00 MeV, Wcm = 1922.29 MeV; $\chi^2/N_{data} = 0.49$

PWA meeting in Mainz, February, 2016.
SE PWA pseudo helicity amplitudes

![Graphs showing pseudo helicity amplitudes](image)

Elab = 1800.00 MeV, Wcm = 2063.54 MeV; Chi2/Ndata = 0.94

Elab = 2000.00 MeV, Wcm = 2152.55 MeV; Chi2/Ndata = 0.73

PWA meeting in Mainz, February, 2016.
Conclusions

- Applied method of PWA is based only on Mandelstam hypothesis and fixed-t and fixed-s analyticity, and, as such is model independent.
- PWA with constraint from fixed-t amplitude analysis produce multipoles which are consistent with crossing symmetry and fixed-t analyticity.
- Invariant amplitudes (Helicity amplitudes) obtained in fixed-t AA show a good consistency with fixed-s analyticity. It implies that our amplitudes are consistent with both fixed-t and fixed-s analyticity.
- Weak point and the main problem is strong dependance of our results on constraining solution.
Further research

- Include input from “red” region taking results from Aznauryan’s work - it will make our analysis (slightly?) model dependent
- Include results on imaginary parts of IA from “orange” region
- Spread constraining PWA solution by randomizing constraining solution changing it randomly (let say 10%)

PWA meeting in Mainz, February, 2016.
Further research

- Include input from “red” region taking results from Aznauryan’s work - it will make our analysis (slightly?) model dependent
- Include results on imaginary parts of IA from “orange” region
- Spread constraining PWA solution by randomizing constraining solution changing it randomly (let say 10%)
Further research

- Include input from “red” region taking results from Aznauryan’s work - it will make our analysis (slightly?) model dependent.
- Include results on imaginary parts of IA from “orange” region.
- Spread constraining PWA solution by randomizing constraining solution changing it randomly (let say 10%).