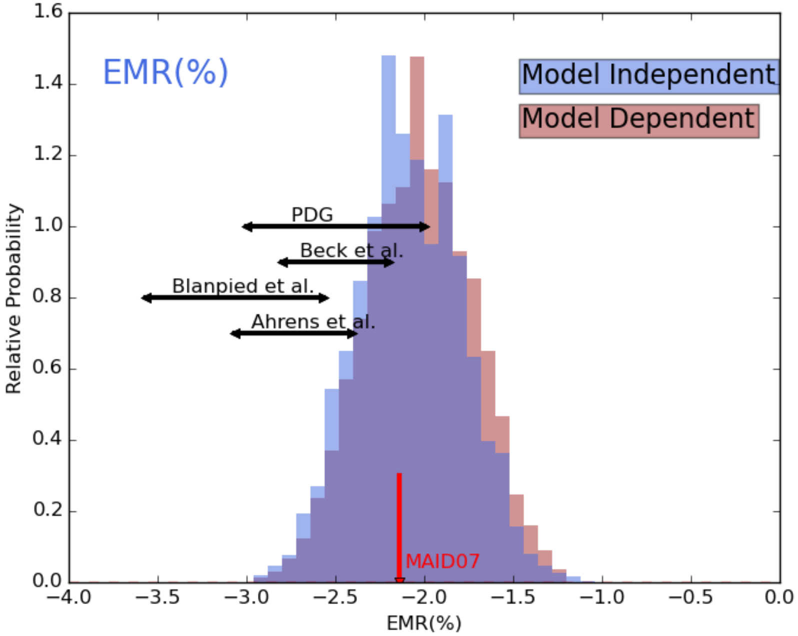


Multipole Amplitude Extraction with the AMIAS

Lefteris Markou

Mainz, 17 Feb 2016

Outline

- Quick reminder what the AMIAS is all about
  - A trivial example of fitting a polynomial
- AMIAS amplitude extraction from MAID07 photoproduction pseudodata
  - A truly model independent analysis
- AMIAS amplitude extraction from MAMI data
  - Handling of double solutions with the AMIAS
  - Results of simultaneous analysis of $p\pi^0$ & $n\pi^+$ data for l = 2, l = 3, and l = 5 ???
  - I(3/2) amplitude extraction from single channel data
- Future Work

Athens Model Independent Analysis Scheme AMIAS

- Based on statistical concepts and relies on Monte Carlo techniques

- Yields the Probability Distributions for parameters
  - Does not assume the shape of a parameter's PDF, e.g. Gaussian Rather it lets the data determine it

- Insensitive parameters are fully accounted and do not bias the result

- All possible correlations are captured due to the randomization process

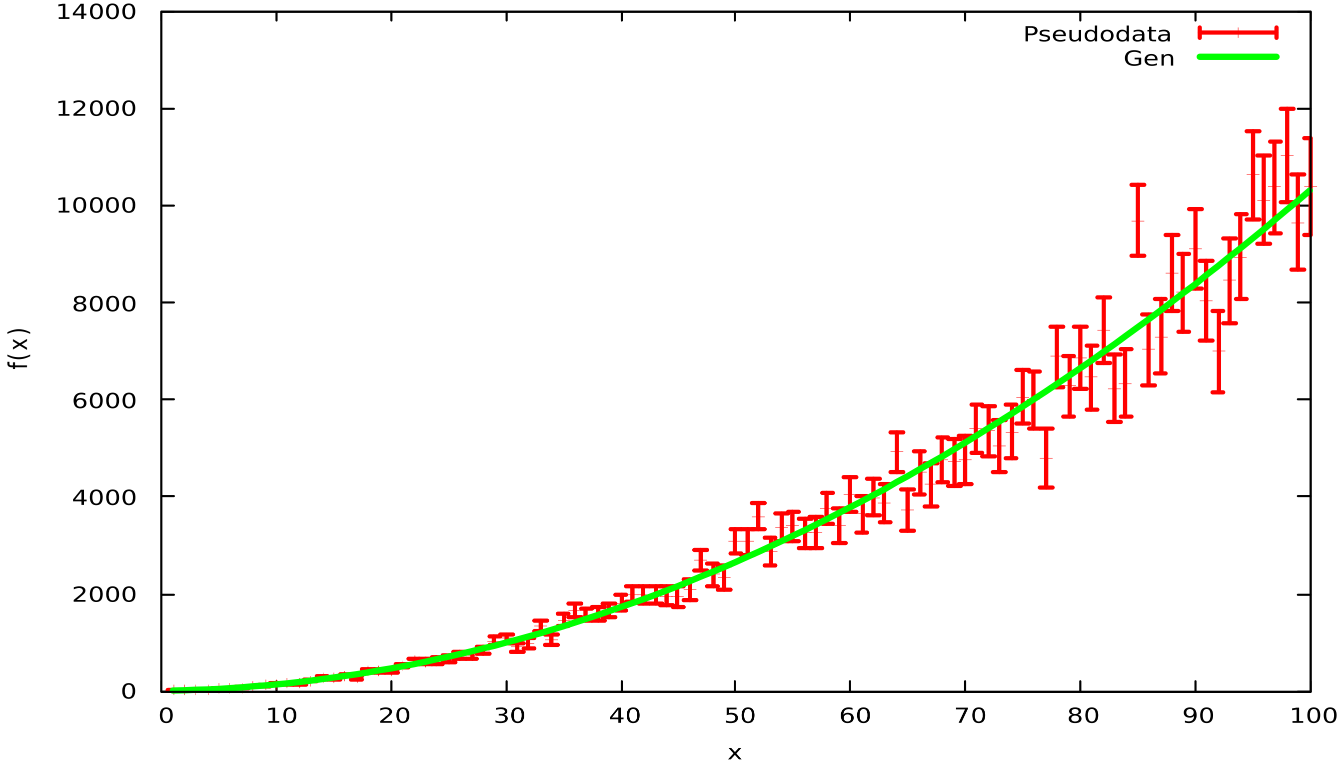
- Does not rely on χ^2-minimization techniques
  - Numerically robust and does not fail for low signal-to-noise-ratio

- Requires High Performance Computing

- Successfully applied in the analysis of experimental data in hadronic physics, of lattice QCD correlators, and in SPECT Image Reconstruction

Athens Model Independent Analysis Scheme
AMIAS

Suppose I would like to fit data with a polynomial model $f(A_n, x) = \sum_{n=0}^{\infty} A_n x^n$

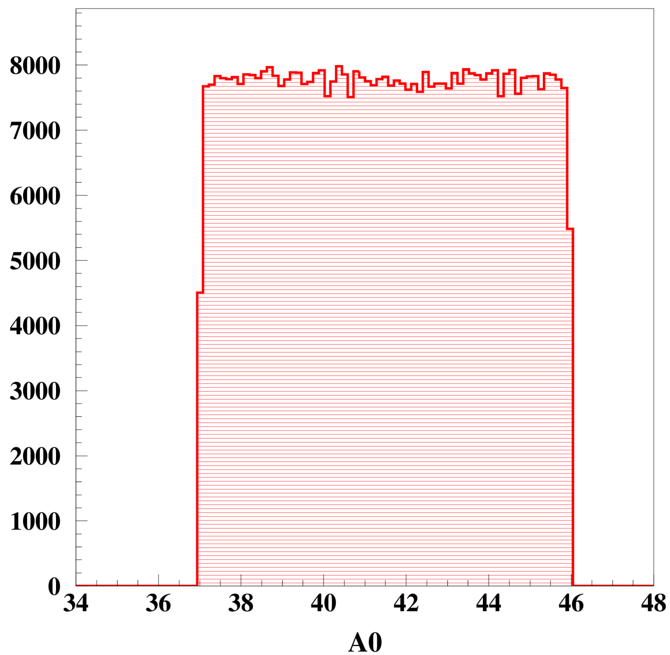


Athens Model Independent Analysis Scheme
AMIAS

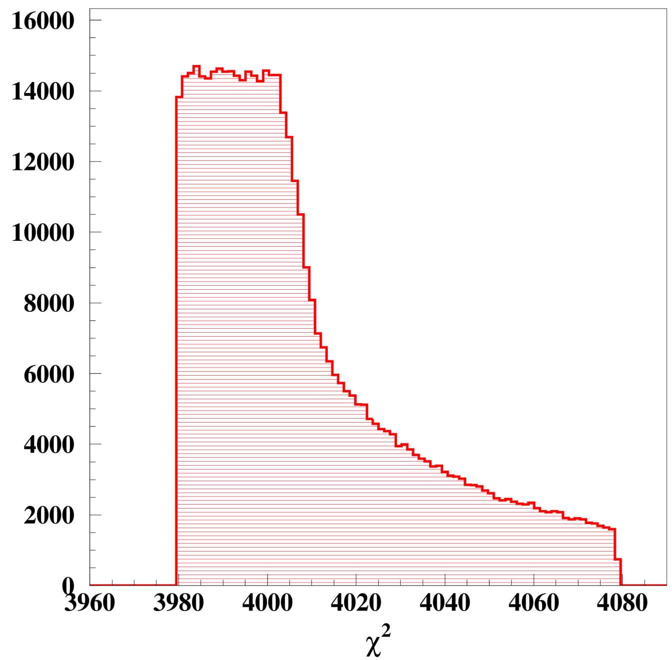
Suppose I would like to fit data with a polynomial model $f(A_n, x) = \sum_{n=0}^{\infty} A_n x^n$

Choose n

Uniformly Sample the parameter space

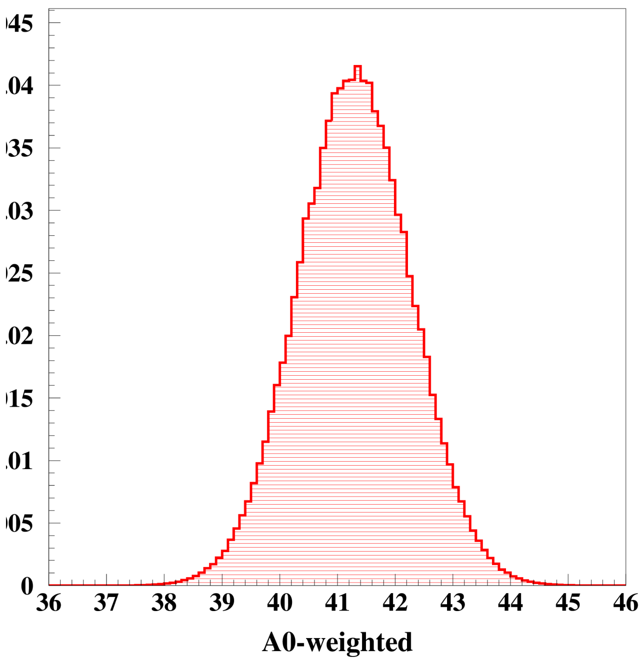


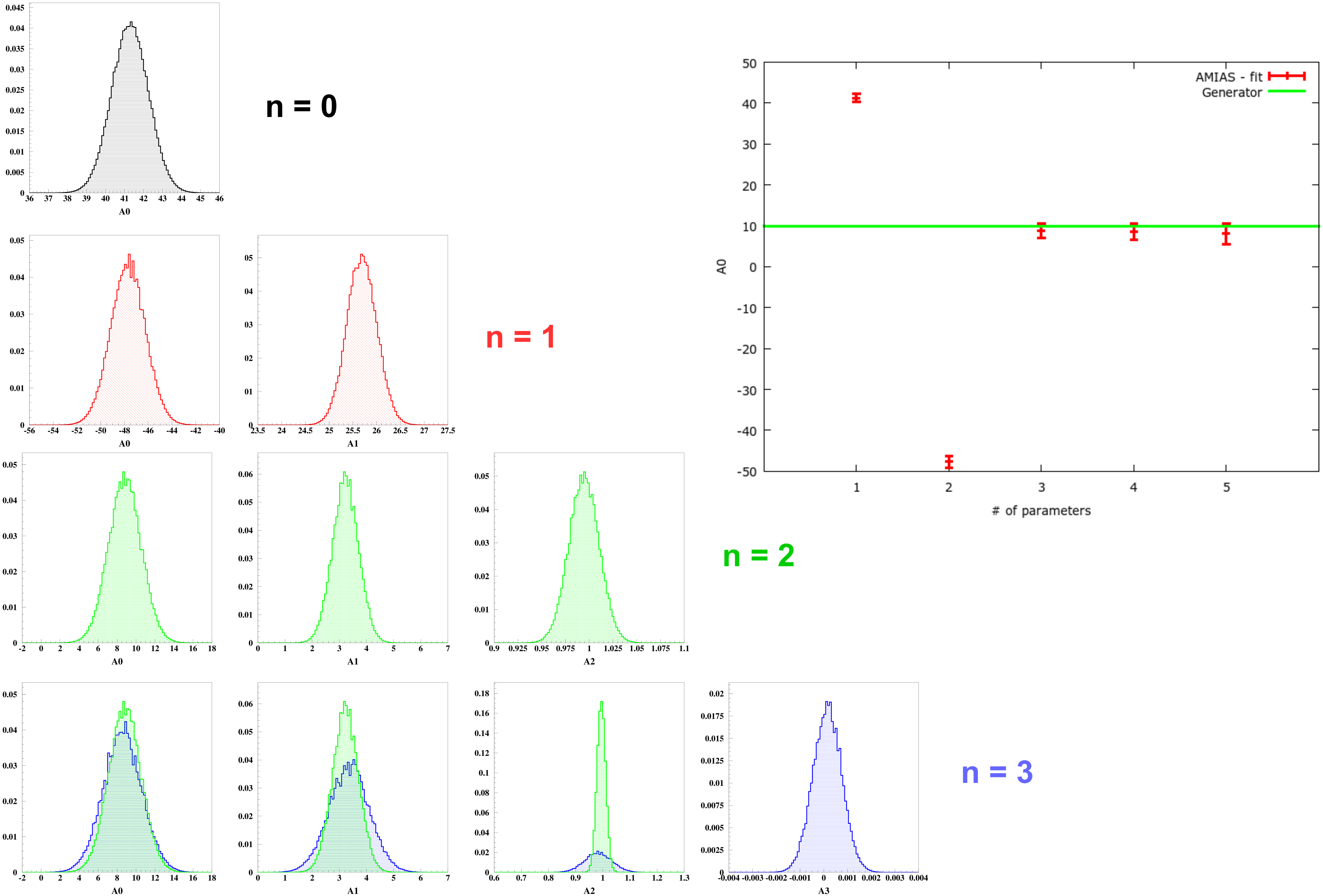
For each point compute χ^2 (or choose another criterion)



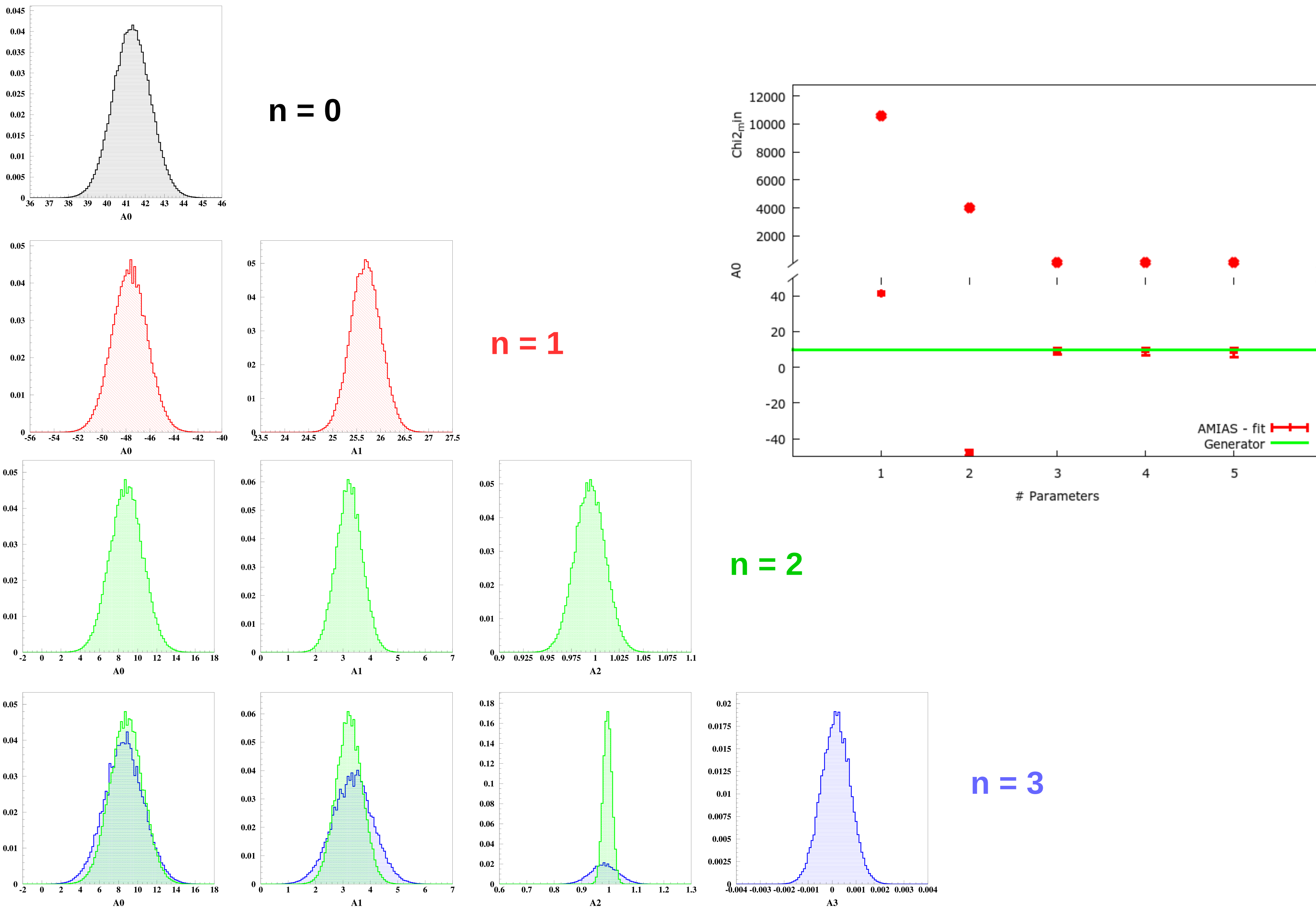
$$e^{\frac{-1}{2}(\chi^2 - \chi^2_{min})}$$

Assign a probability to each point





n = 0
n = 1
n = 2
n = 3
A0
A1
A2
A3
AMIAS - fit
Generator
# of parameters


n = 0
n = 1
n = 2
n = 3
A0
A1
A2
A3
Chi2min
A0
Parameters
AMIAS - fit
Generator

To employ AMIAS we need a model to connect the parameters to be extracted with the observables

CGLN amplitudes to connect multipoles to observables

E_{l+}, E_{l-}, M_{l+}, M_{l-}, L_{l+}, L_{l-}, $0 \leq l \leq l_{cut}$

F_1, F_2, F_3, F_4, F_5, F_6, CGLN

R_T, R_L, R_{TT}, R_{LT}, … Response Functions

$d\sigma$, Σ, T, P, … Spin Asymmetries

AMIAS Flowchart for mutipole extraction (photoproduction)

Define L_{cut}
Total = 4 + (4*(L_{cut}-1)) Complex
multipoles for $L_{cut} > 0$
(times two for Isospin Decompisition)
$\{A_1, A_2, A_3, .., A_n\}$

Random Variation of
all amplitudes A_i
Uniformly
Impose Constraints (e.g. Unitarity)

Build CGLN Amplitudes
(F_i, i=1..4)

Build Cross Sections

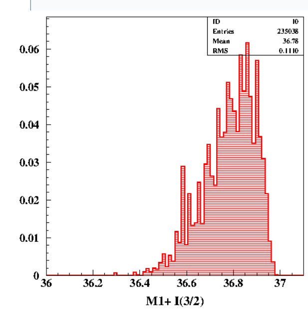
Experimental Data

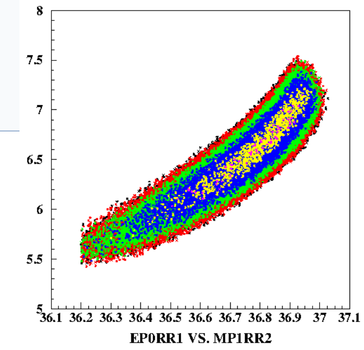
Compute χ^2

Ensemble of Solutions
$\{A_1, A_2, .., A_n, \chi^2\}$

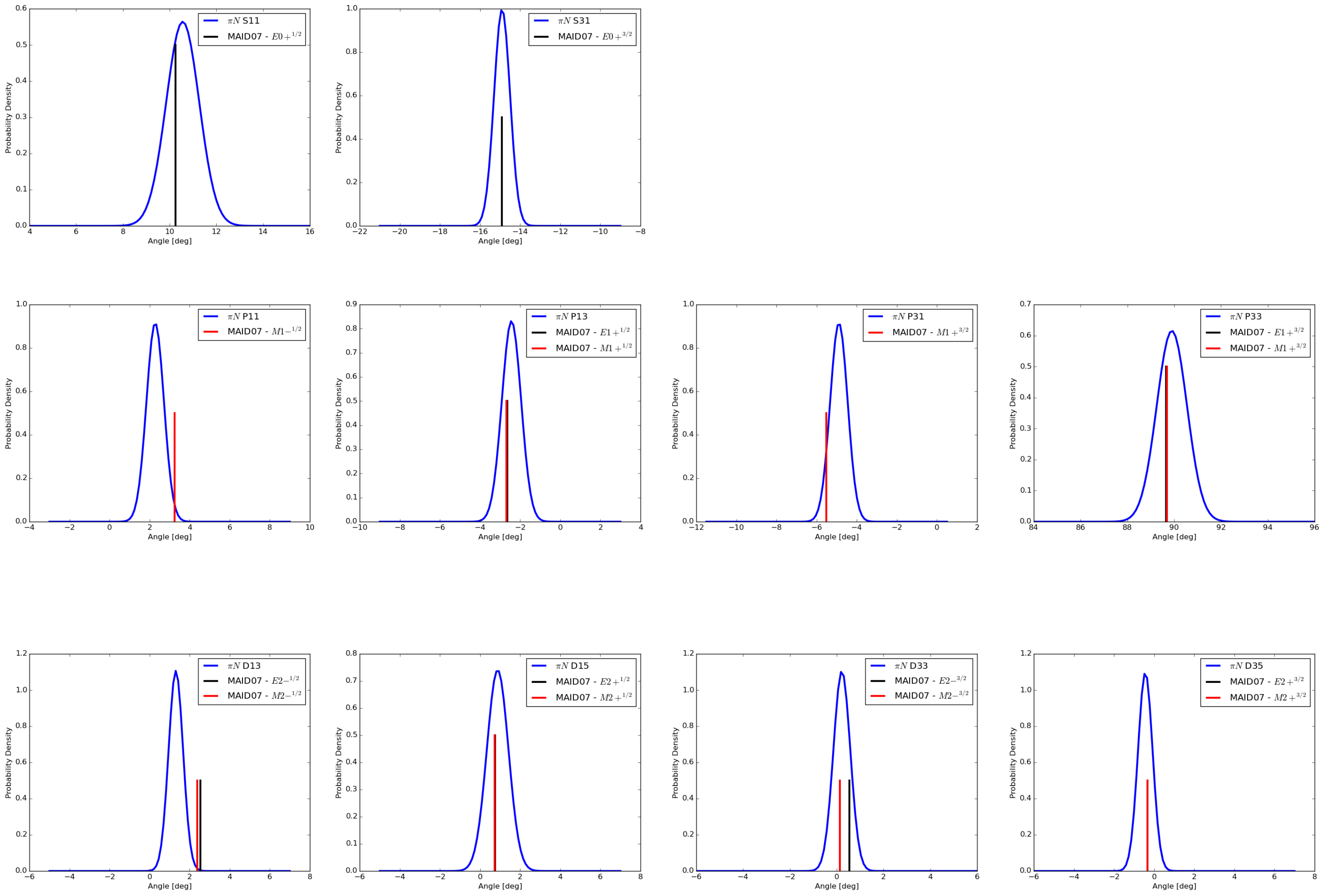
<u>Results</u>
Probability Distribution Functions

<u>Analysis</u>
Correlation plots
Sensitivity



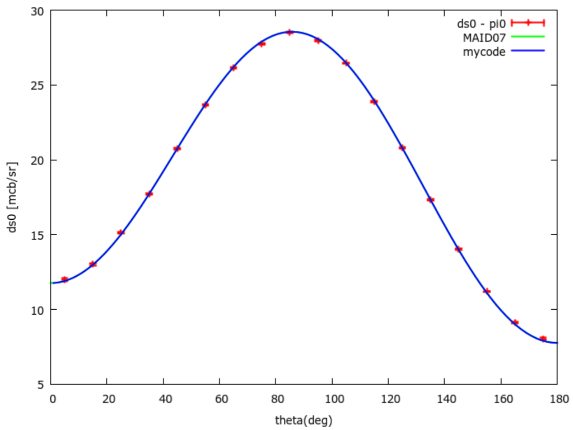


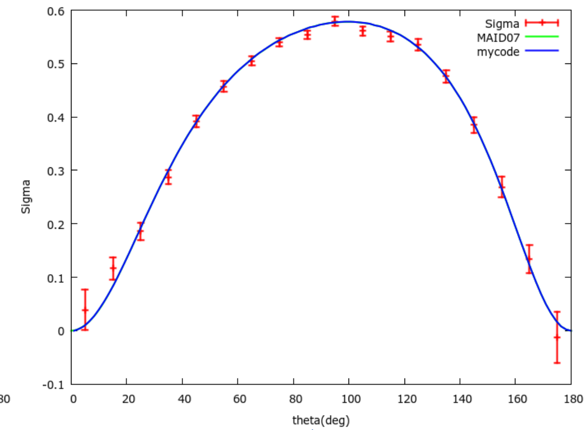
π-N scattering phases values and model predictions @ the Δ

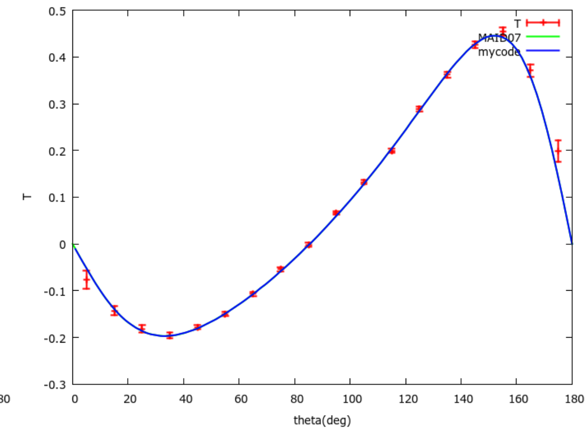


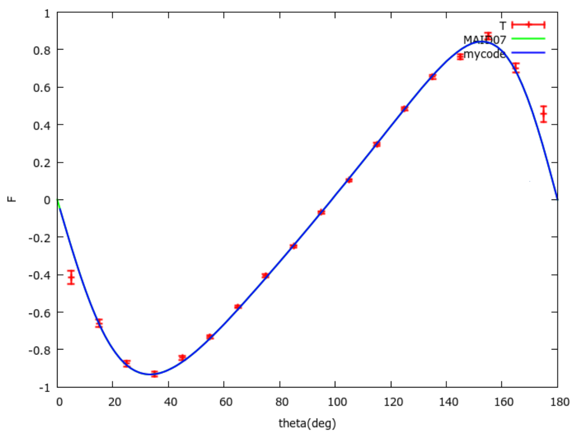
AMIAS amplitude extraction from pseudodata

Wokman et al. arXiv:1102.4897



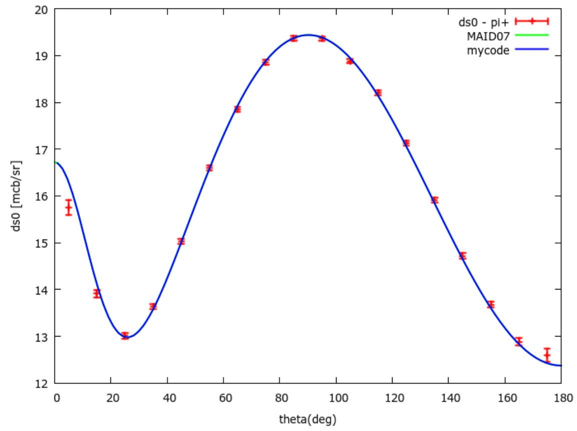


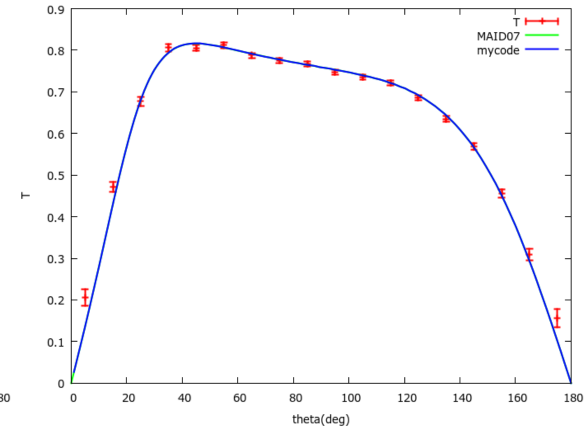


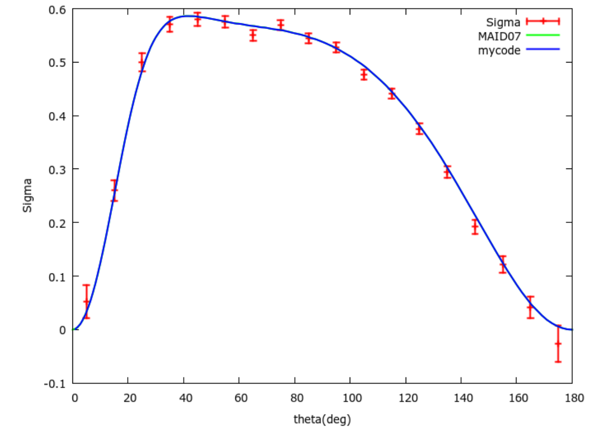


$\gamma p \rightarrow p\pi^0$ data ($d\sigma_0$, Σ, T, F)

$\gamma p \rightarrow n\pi^+$ data ($d\sigma_0$, Σ, T)







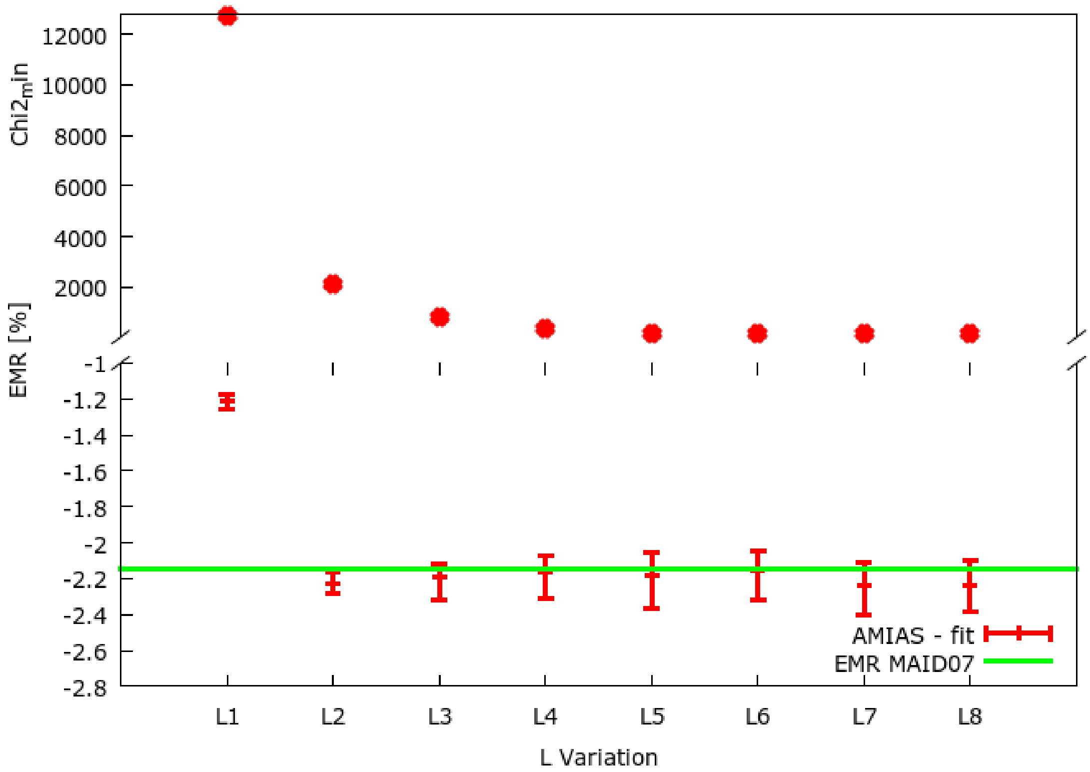
AMIAS amplitude extraction from pseudodata

We analyze the data each time allowing more multipole amplitudes to vary

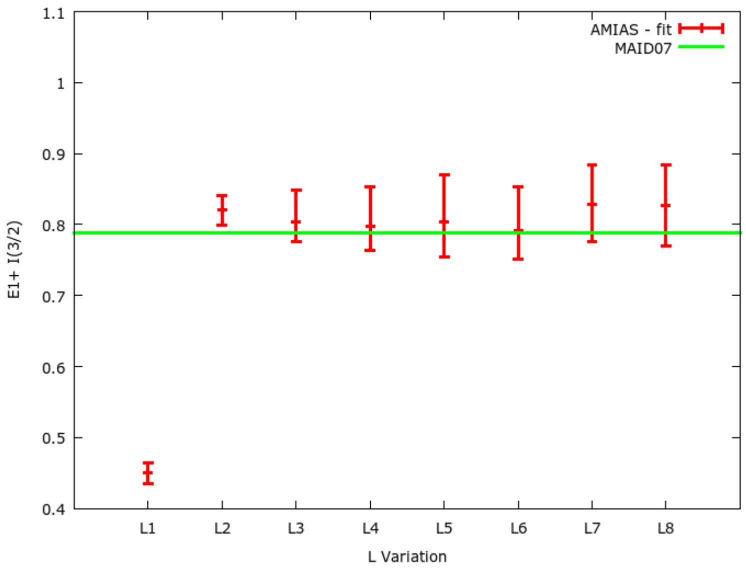
NO amplitudes are fixed to a model

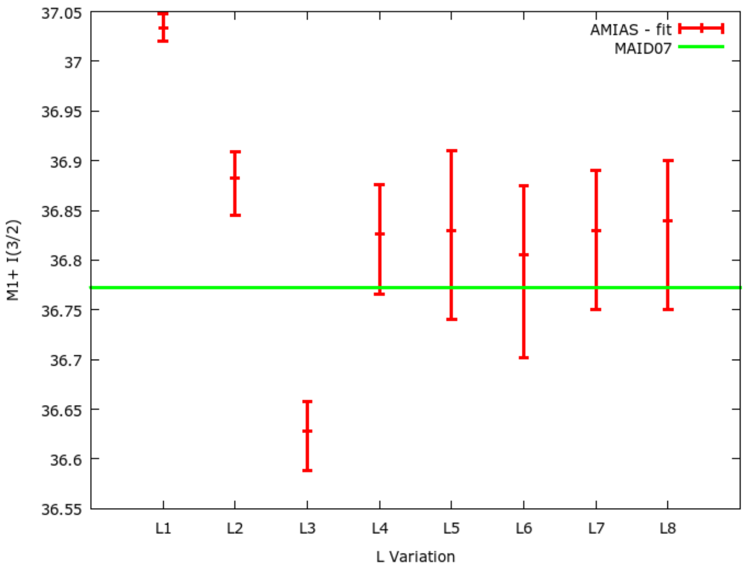
Below 2-pion threshold, phases fixed according to **F-W**

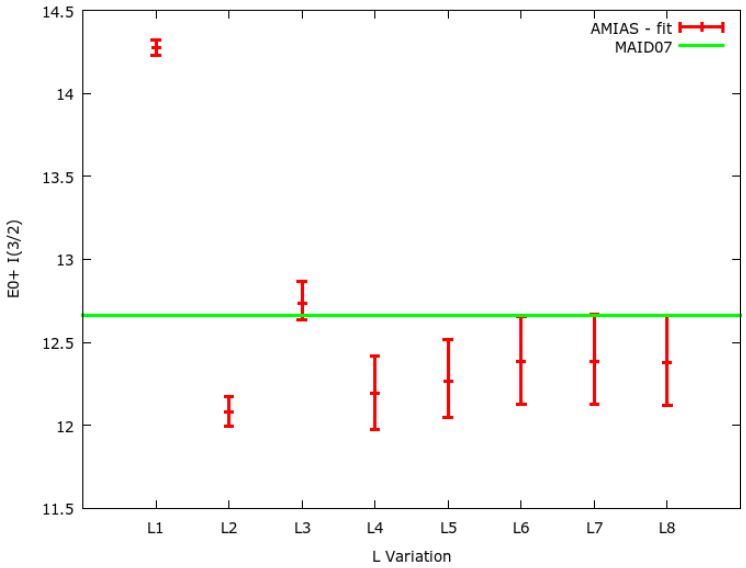
The analysis is complete once solutions have converged, χ^2_{min} reaches a minimum and adding more parameters to the variation does not affect the derived values

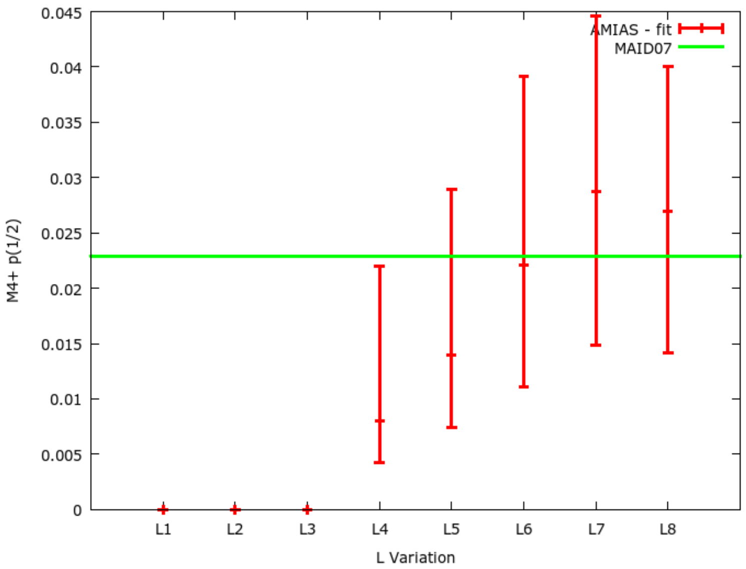


AMIAS amplitude extraction from photoproduction pseudodata









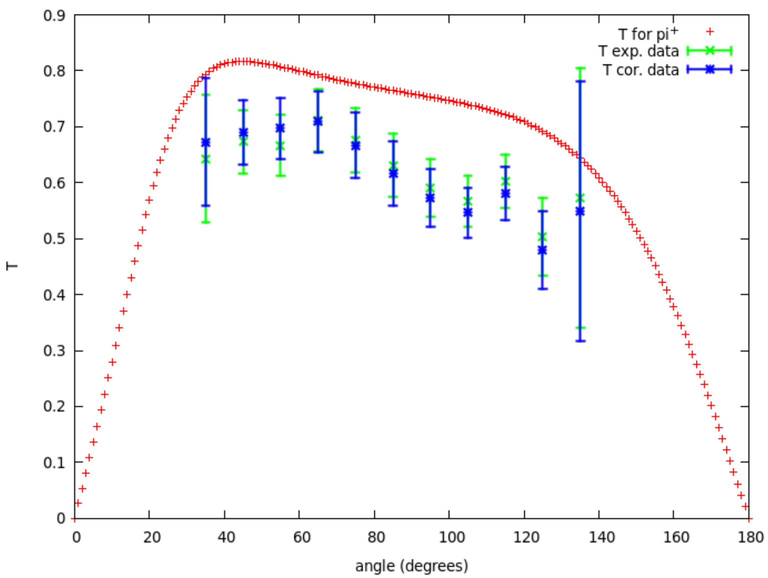
Experimental photoproduction data analysed in this work

		W (MeV)					
		1201	1209	1217	1225	1232	1239
$\gamma p \rightarrow p\pi^0$							
$d\sigma_0/d\Omega$	MAMI	√	√	√	√	√	√
Σ	MAMI	√	√	√	√	√	√
$T\sigma_0$	MAMI	√	√	√	√	√	√
$F\sigma_0$	MAMI	√	√	√	√	√	√
$\gamma p \rightarrow n\pi^+$							
$d\sigma_0/d\Omega$	MAMI	√	√	√	√	√	√
Σ	MAMI	√	√	√	√	√	√
T	GWU	√	X	√	X	√	X

Energy Correction

To bring an observable from the experimentally measured energy (w) to the desired energy (w') we use the formula

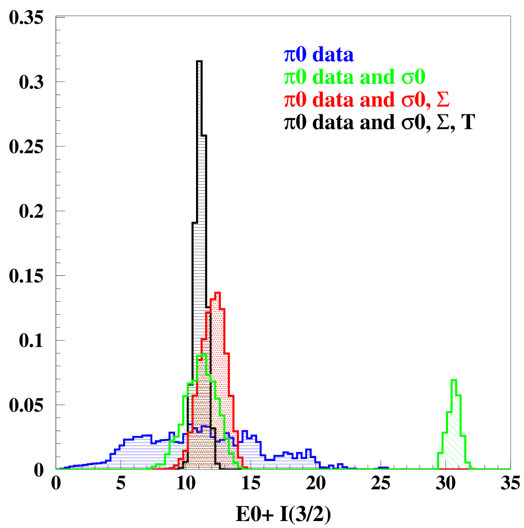
$$O(w') = O(w) + \frac{\partial O(w)}{\partial w}\Delta(w'-w) + \frac{\partial^2 O(w)}{\partial w^2}\Delta(w'-w)^2 + \ldots$$

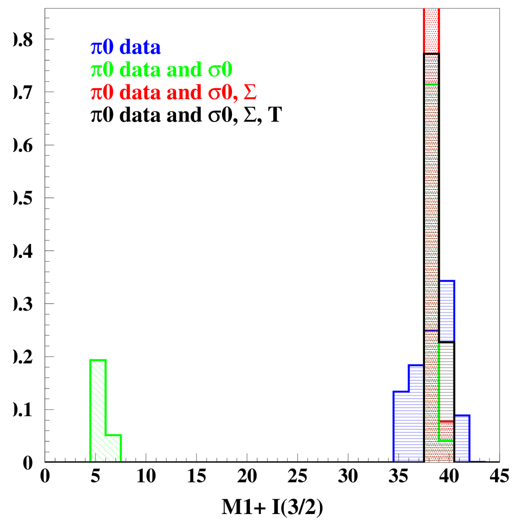


	Photon		
Target	Unp.	Circular	Linear
Unp.	dσ/dΩ		Σ
Long		E	G
Trans	T	F	H

where the partial derivative of O in respect to the energy w can be computed through a model, e.g. MAID07

Full isospin decomposition and double solutions

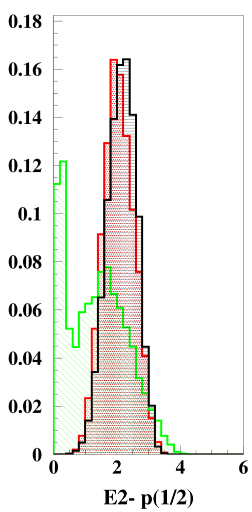


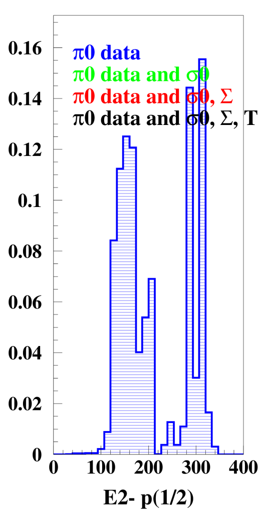


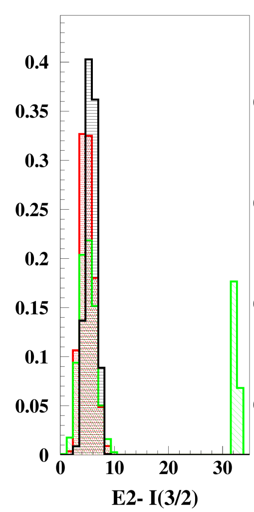
- Need combined data for iospin decomposition

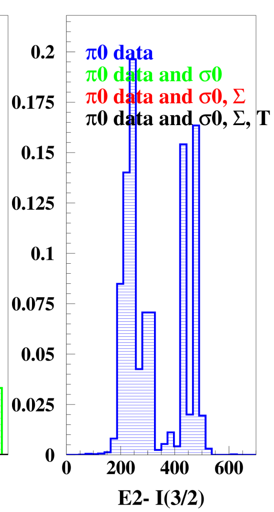
$$A_{\pi 0} = A_p^{1/2} + 2/3\, A^{3/2}$$
$$A_{\pi+} = A_p^{1/2} - 1/3\, A^{3/2}$$

- AMIAS explores the whole parameter space so any possible solution is captured. When faced with double solutions I choose the one which provides continuity







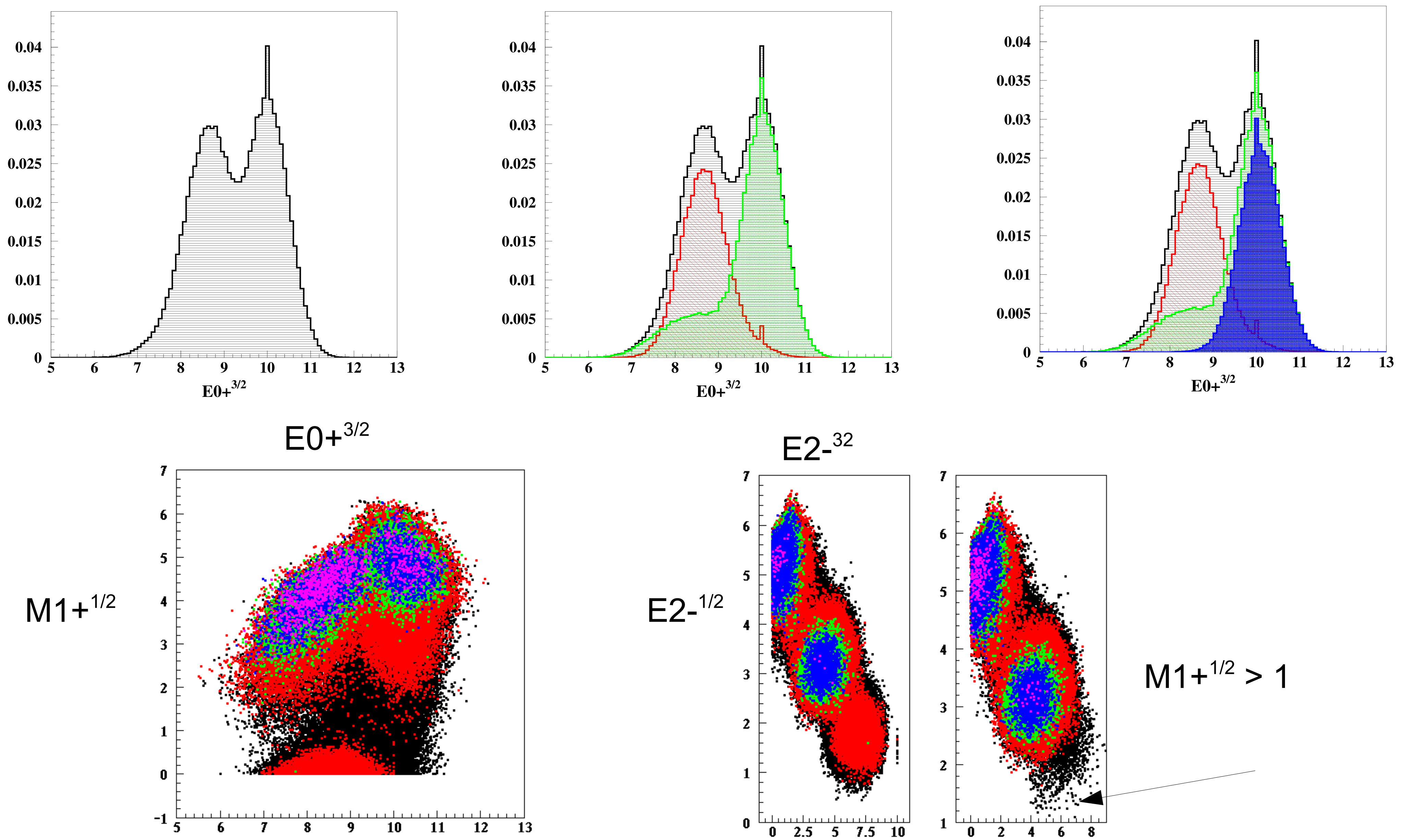


- **$T_{\pi+}$ is not as precise as the recent measurements, yet it helps reduce the determined parameter uncertainty (compare with red)**

- ***for $I_{var} < 3$**

The “hard” Double solutions – Graphic Analysis needed!

Example, W = 1232 MeV, Observables: $d\sigma_0/d\Omega$, Σ, $T\sigma_0$, $F\sigma_0$, $d\sigma_0/d\Omega$, Σ, T

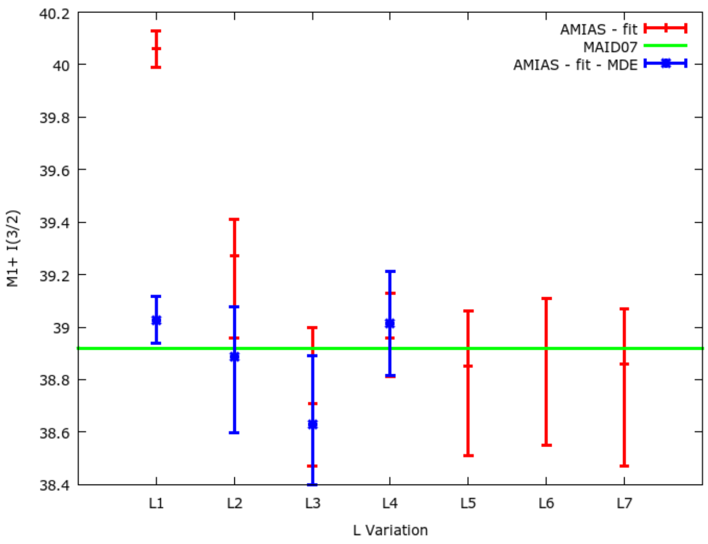


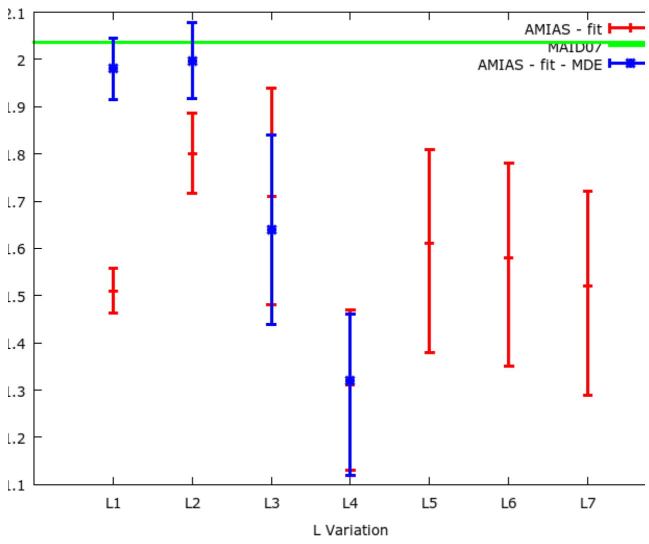
AMIAS Model Independent Analysis of experimental photoproduction data

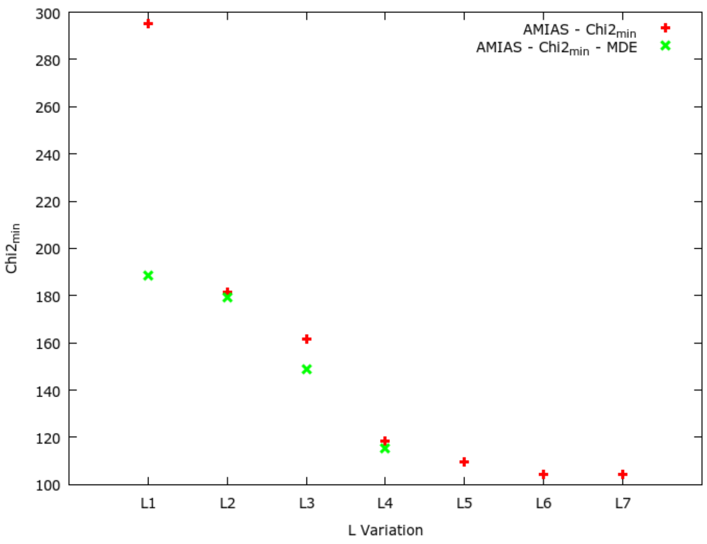
We follow the same methodology as with the pseudodata example

Uniformly and Randomly Vary multipoles until convergeance is reached

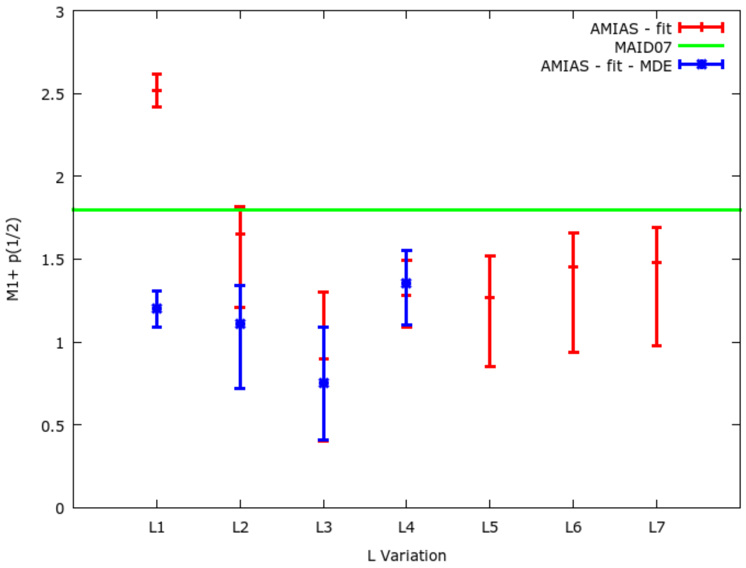
Example, W = 1201 MeV, Observables: $d\sigma_0/d\Omega$, Σ, $T\sigma_0$, $F\sigma_0$, $d\sigma_0/d\Omega$, Σ, T

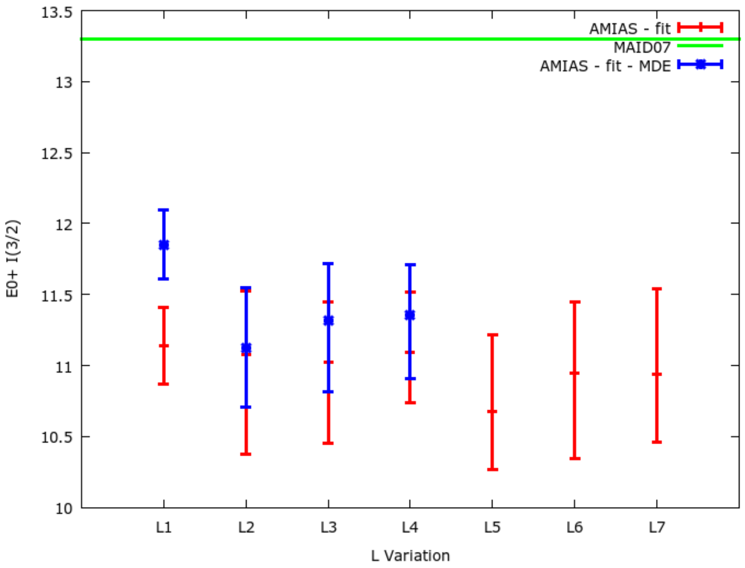


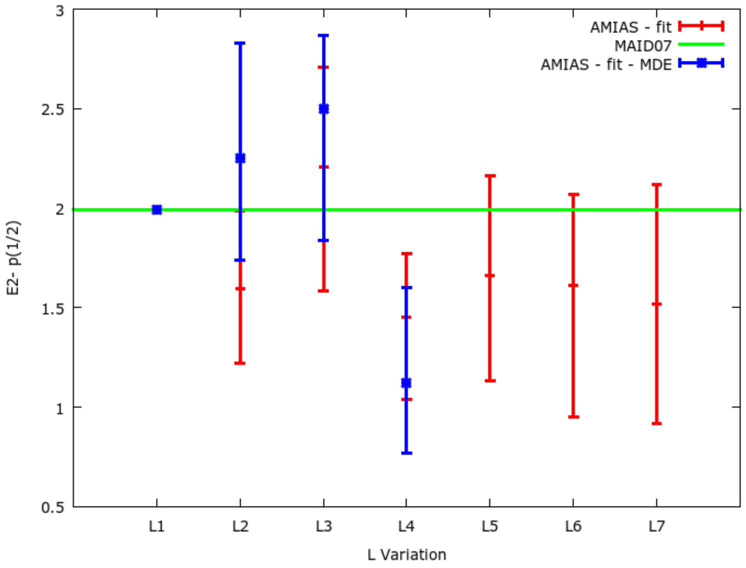


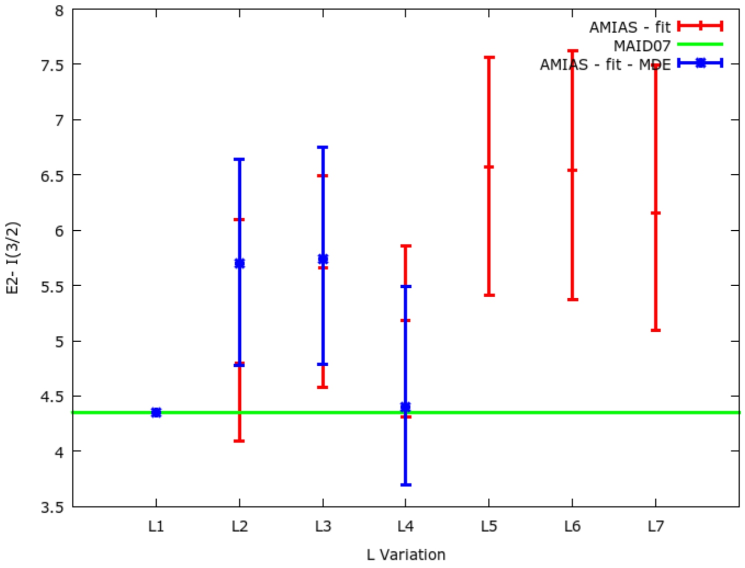


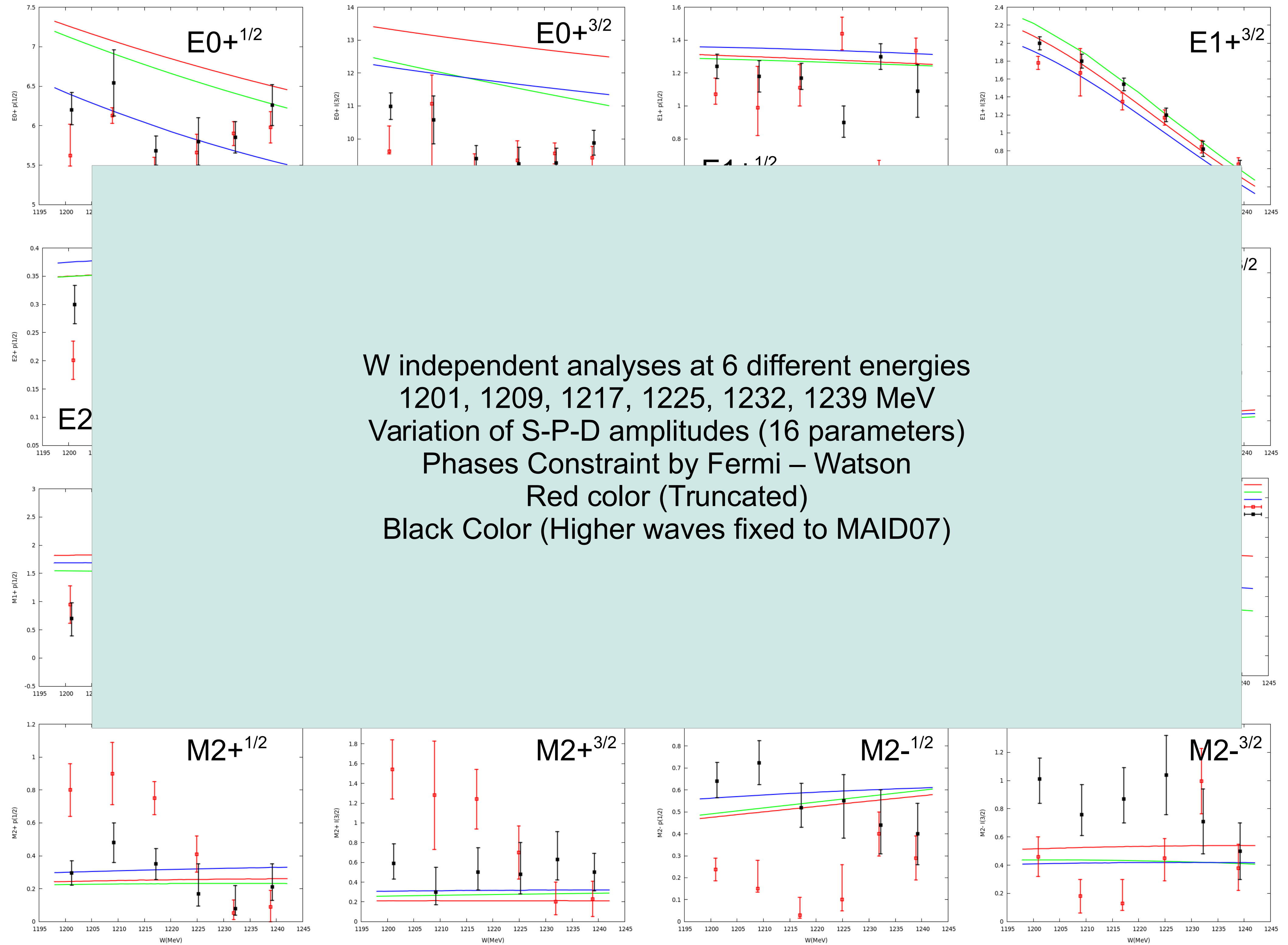
AMIAS Model Independent Analysis of experimental photoproduction data



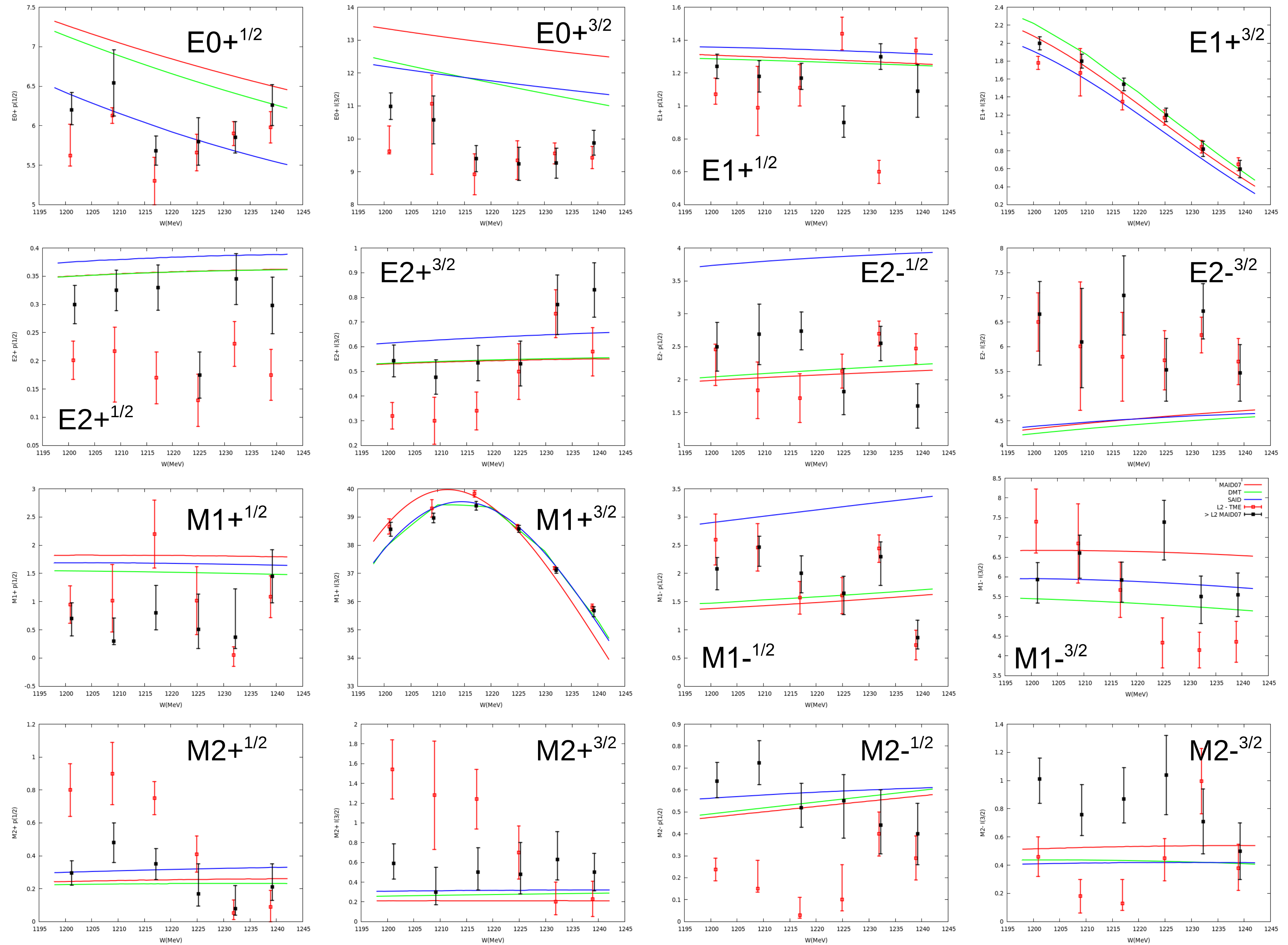




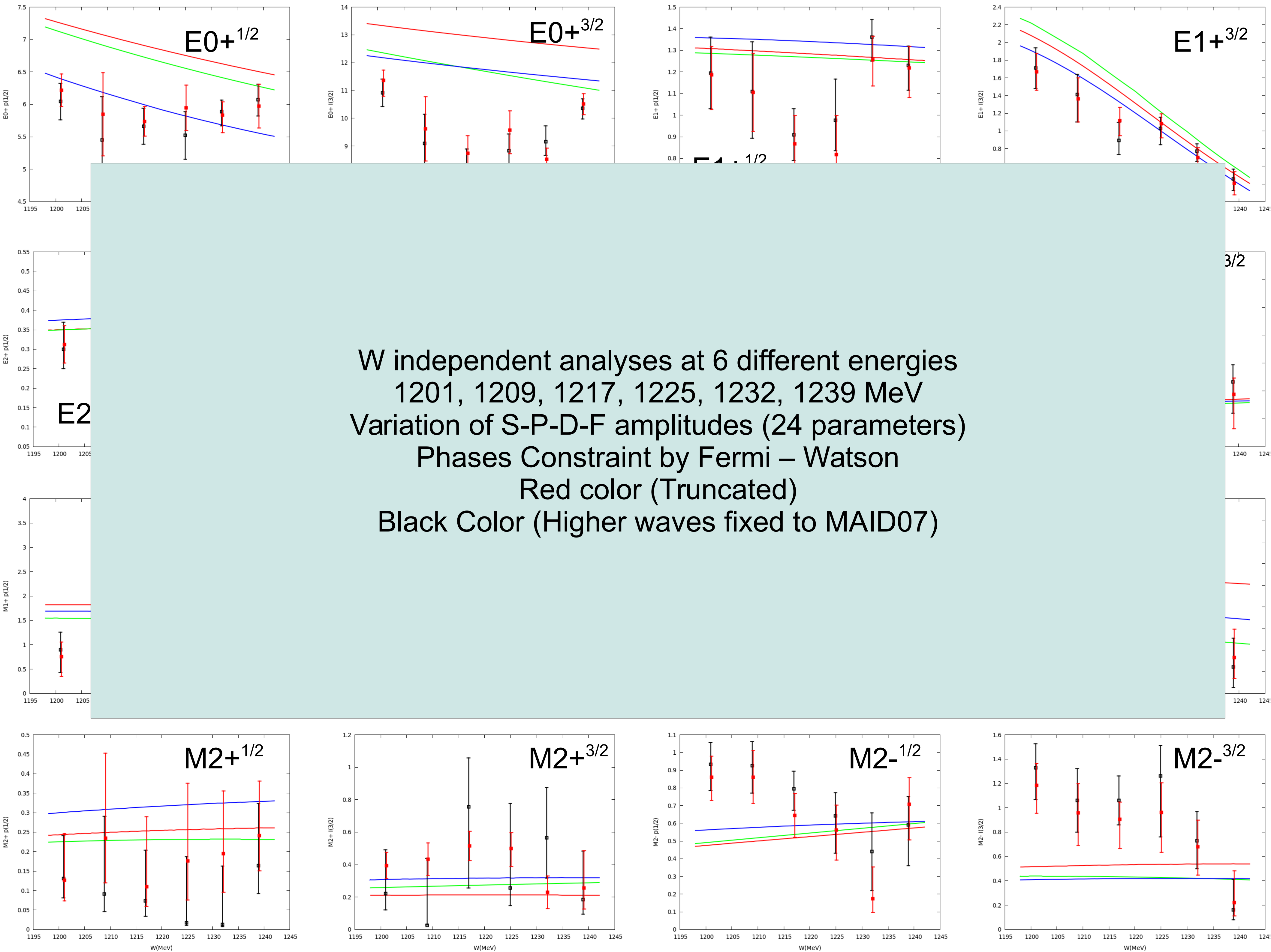




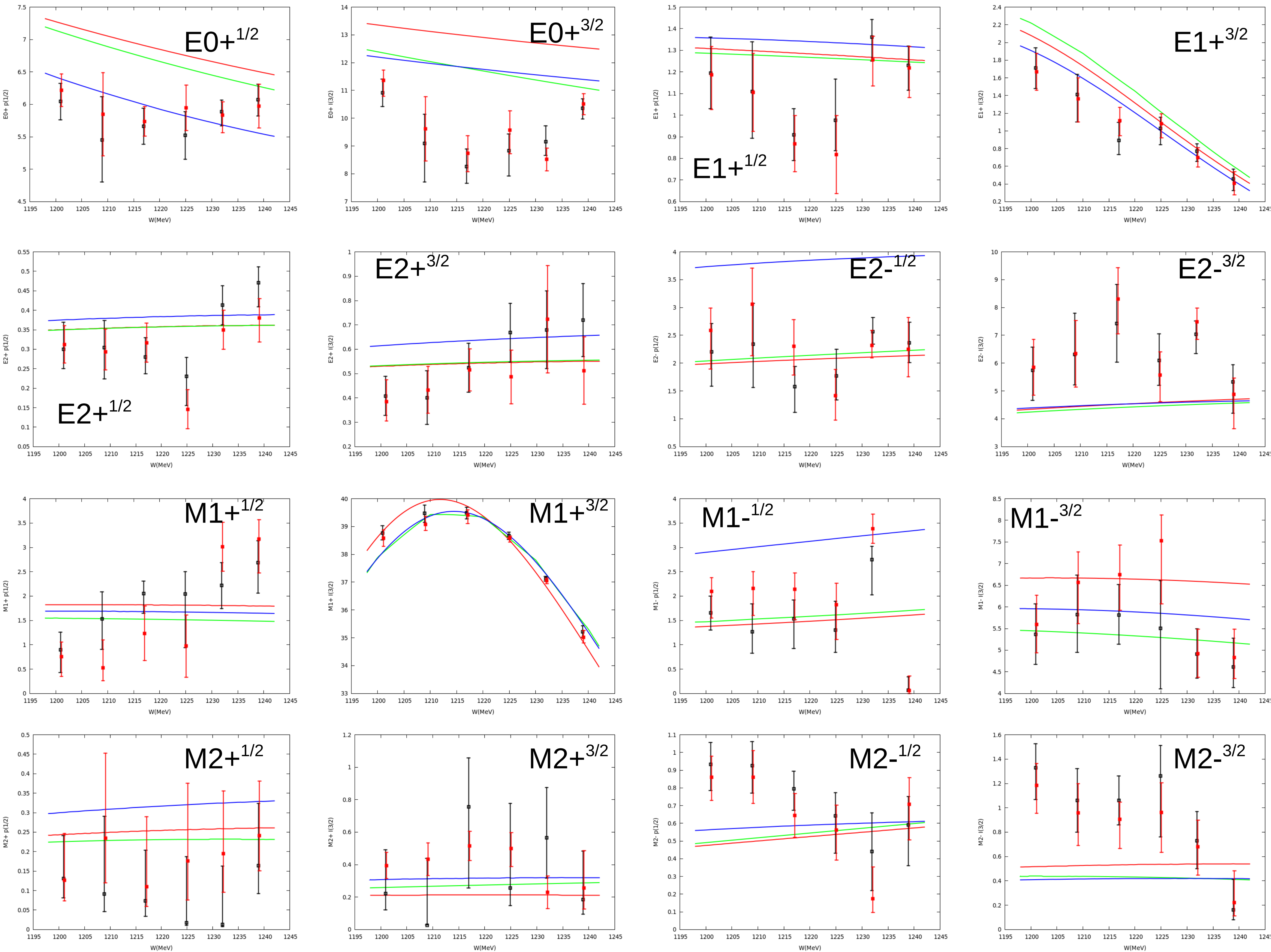
W independent analyses at 6 different energies
1201, 1209, 1217, 1225, 1232, 1239 MeV
Variation of S-P-D amplitudes (16 parameters)
Phases Constraint by Fermi – Watson
Red color (Truncated)
Black Color (Higher waves fixed to MAID07)

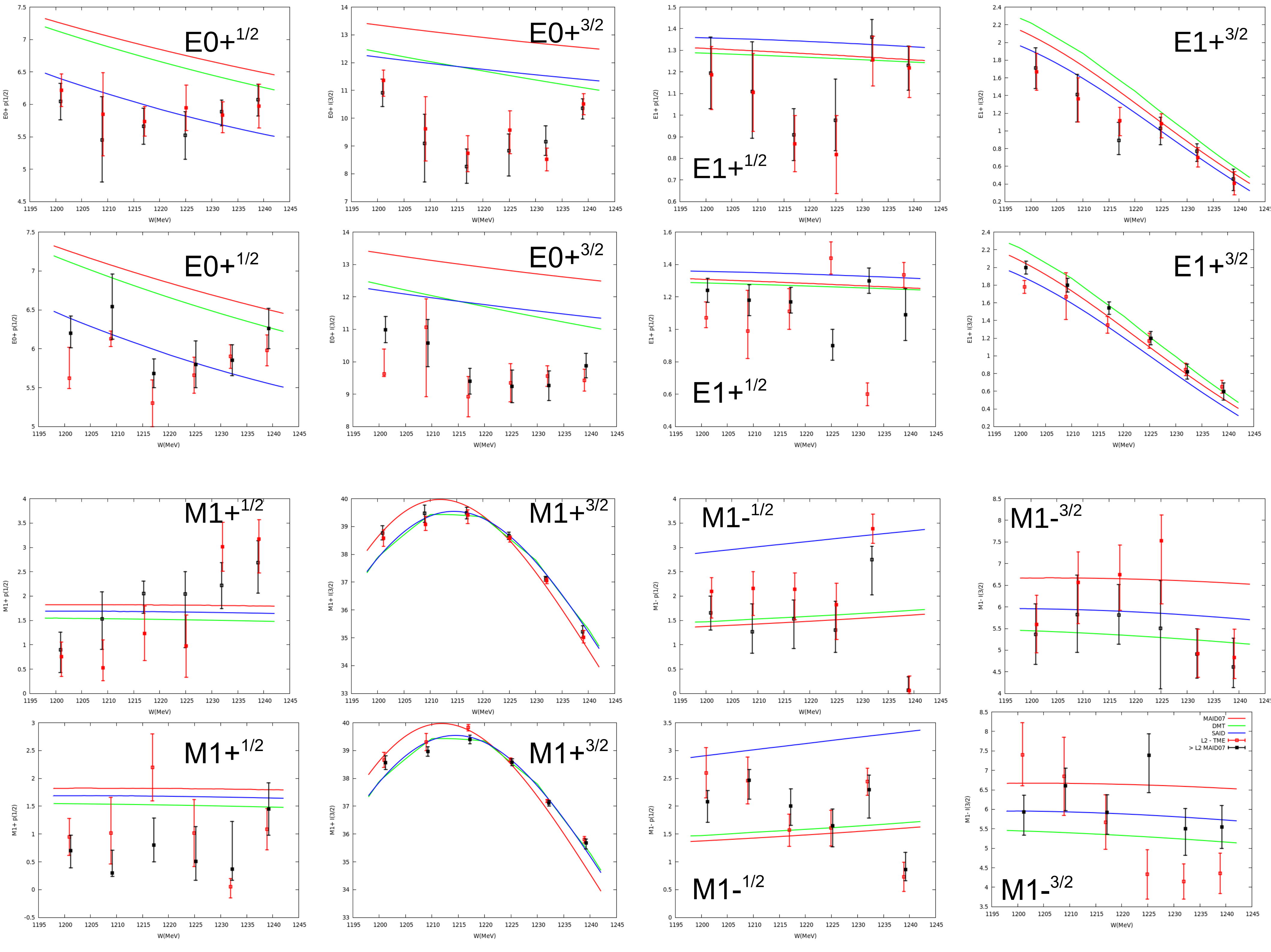


E0+$^{1/2}$
E0+ p(1/2)
E0+$^{3/2}$
E0+ I(3/2)
E1+$^{1/2}$
E1+ p(1/2)
E1+$^{3/2}$
E1+ I(3/2)
E2+$^{1/2}$
E2+ p(1/2)
E2+$^{3/2}$
E2+ I(3/2)
E2-$^{1/2}$
E2- p(1/2)
E2-$^{3/2}$
E2- I(3/2)
M1+$^{1/2}$
M1+ p(1/2)
M1+$^{3/2}$
M1+ I(3/2)
M1-$^{1/2}$
M1- p(1/2)
M1-$^{3/2}$
M1- I(3/2)
MAID07
DMT
SAID
L2 - TME
> L2 MAID07
M2+$^{1/2}$
M2+ p(1/2)
M2+$^{3/2}$
M2+ I(3/2)
M2-$^{1/2}$
M2- p(1/2)
M2-$^{3/2}$
M2- I(3/2)
W(MeV)

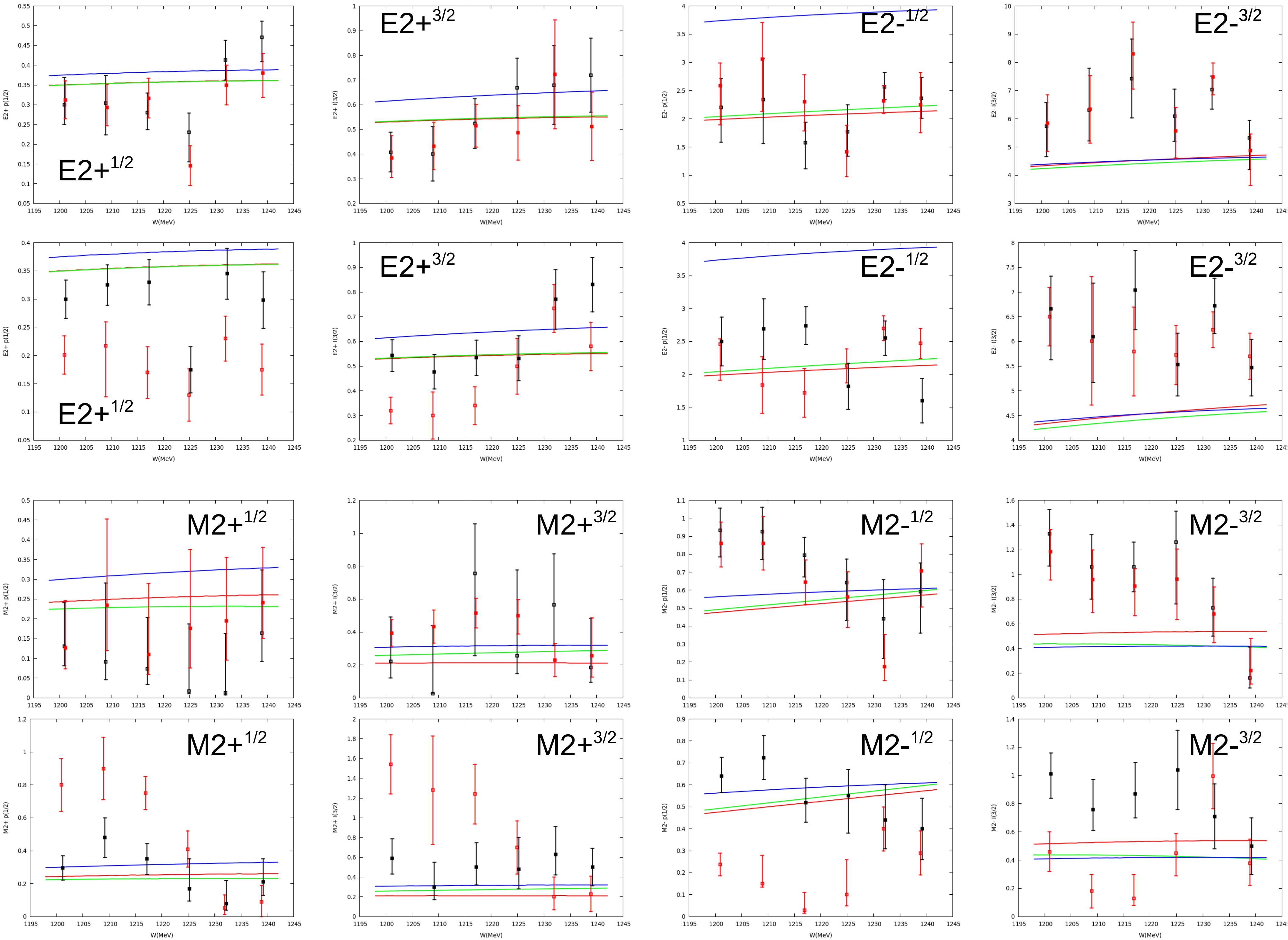


W independent analyses at 6 different energies
1201, 1209, 1217, 1225, 1232, 1239 MeV
Variation of S-P-D-F amplitudes (24 parameters)
Phases Constraint by Fermi – Watson
Red color (Truncated)
Black Color (Higher waves fixed to MAID07)

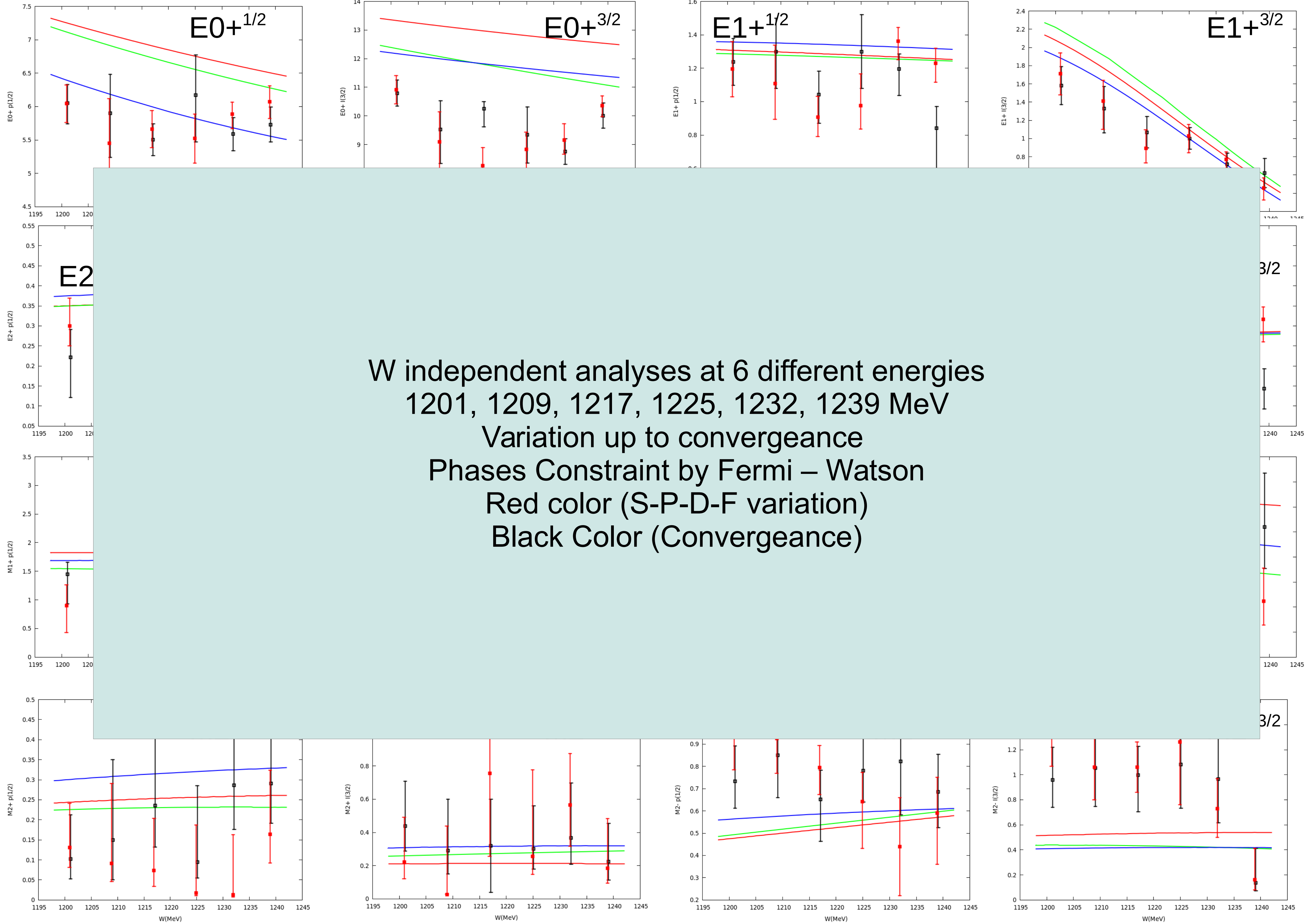

E0+1/2
E0+ p(1/2)
W(MeV)
E0+3/2
E0+ I(3/2)
W(MeV)
E1+1/2
E1+ p(1/2)
W(MeV)
E1+3/2
E1+ I(3/2)
W(MeV)
E2+1/2
E2+ p(1/2)
W(MeV)
E2+3/2
E2+ I(3/2)
W(MeV)
E2-1/2
E2- p(1/2)
W(MeV)
E2-3/2
E2- I(3/2)
W(MeV)
M1+1/2
M1+ p(1/2)
W(MeV)
M1+3/2
M1+ I(3/2)
W(MeV)
M1-1/2
M1- p(1/2)
W(MeV)
M1-3/2
M1- I(3/2)
W(MeV)
M2+1/2
M2+ p(1/2)
W(MeV)
M2+3/2
M2+ I(3/2)
W(MeV)
M2-1/2
M2- p(1/2)
W(MeV)
M2-3/2
M2- I(3/2)
W(MeV)



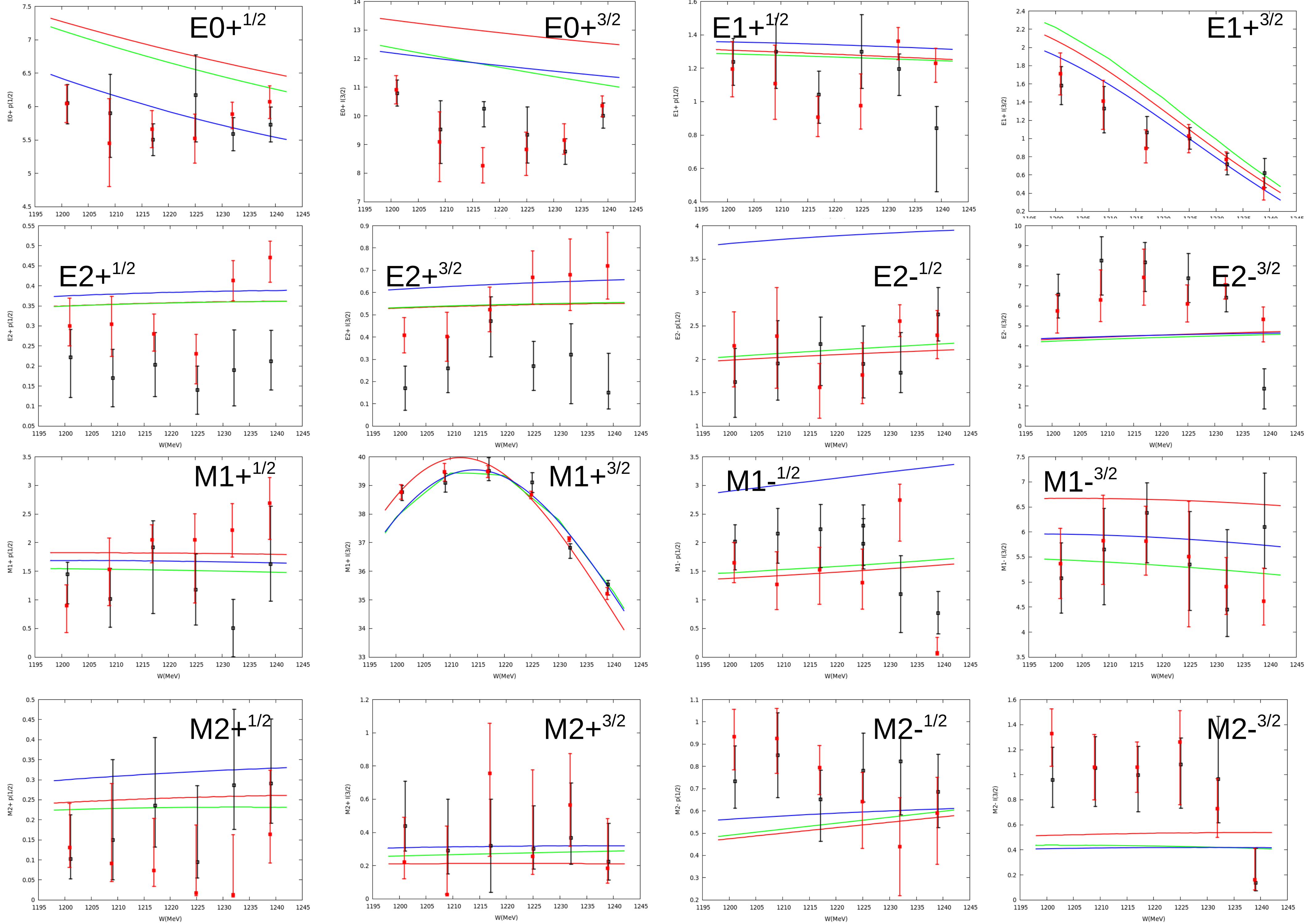
E0+1/2
E0+ p(1/2)
E0+3/2
E0+ I(3/2)
E1+1/2
E1+ p(1/2)
E1+3/2
E1+ I(3/2)
M1+1/2
M1+ p(1/2)
M1+3/2
M1+ I(3/2)
M1-1/2
M1- p(1/2)
M1-3/2
M1- I(3/2)
W(MeV)
MAID07
DMT
SAID
L2 - TME
> L2 MAID07

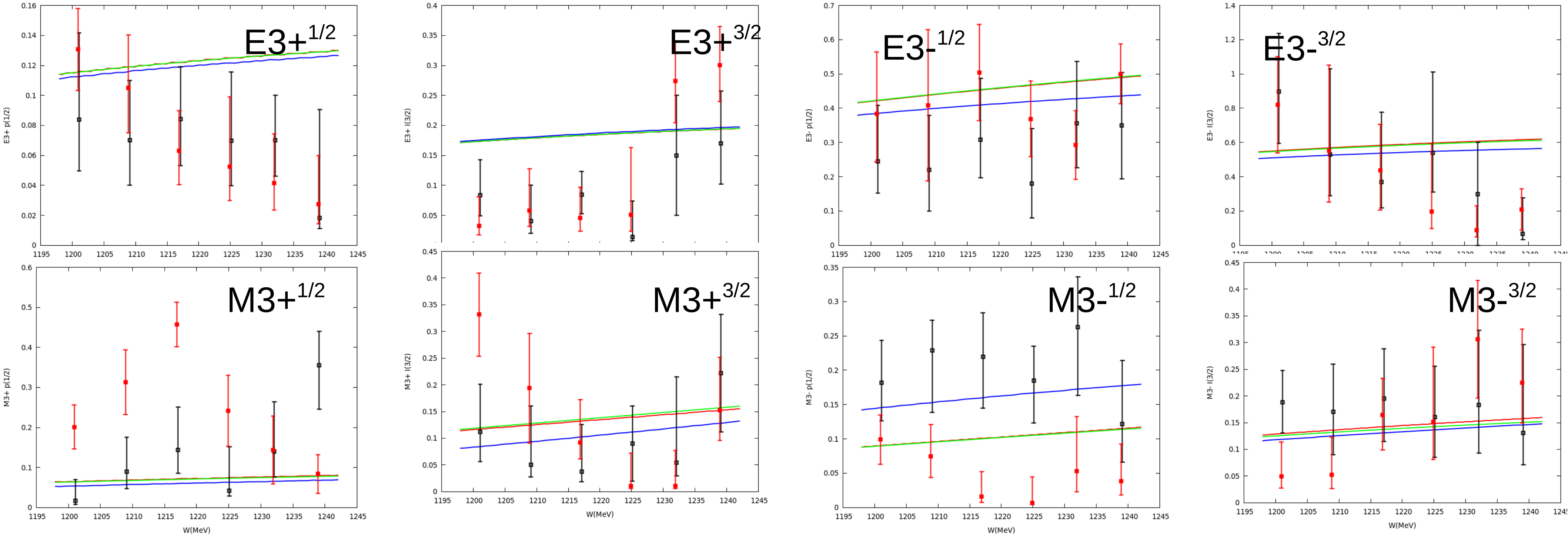


E2+^1/2
E2+^3/2
E2-^1/2
E2-^3/2
E2+^1/2
E2+^3/2
E2-^1/2
E2-^3/2
M2+^1/2
M2+^3/2
M2-^1/2
M2-^3/2
M2+^1/2
M2+^3/2
M2-^1/2
M2-^3/2
W(MeV)



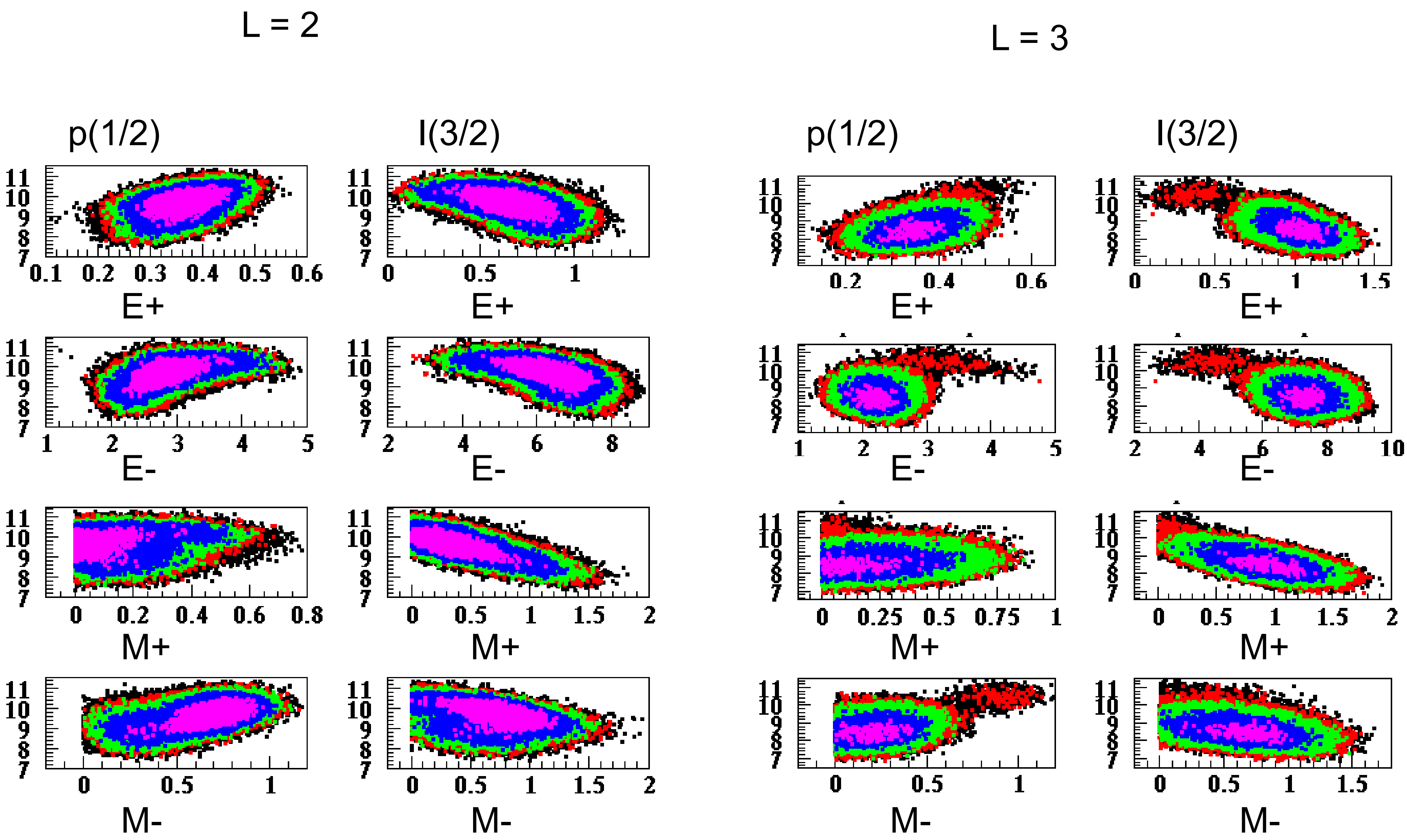
W independent analyses at 6 different energies
1201, 1209, 1217, 1225, 1232, 1239 MeV
Variation up to convergeance
Phases Constraint by Fermi – Watson
Red color (S-P-D-F variation)
Black Color (Convergeance)


E0+1/2
E0+3/2
E1+1/2
E1+3/2
E2+1/2
E2+3/2
E2-1/2
E2-3/2
M1+1/2
M1+3/2
M1-1/2
M1-3/2
M2+1/2
M2+3/2
M2-1/2
M2-3/2
E0+ p(1/2)
E0+ I(3/2)
E1+ p(1/2)
E1+ I(3/2)
E2+ p(1/2)
E2+ I(3/2)
E2- p(1/2)
E2- I(3/2)
M1+ p(1/2)
M1+ I(3/2)
M1- p(1/2)
M1- I(3/2)
M2+ p(1/2)
M2+ I(3/2)
M2- p(1/2)
M2- I(3/2)
W(MeV)



Black: convergeance
Red: S-P-D-F extraction

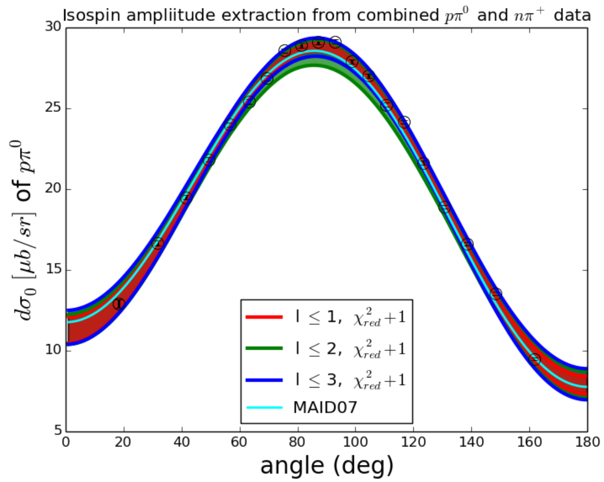
Correlation Plots of E0+$^{3/2}$ – D-waves

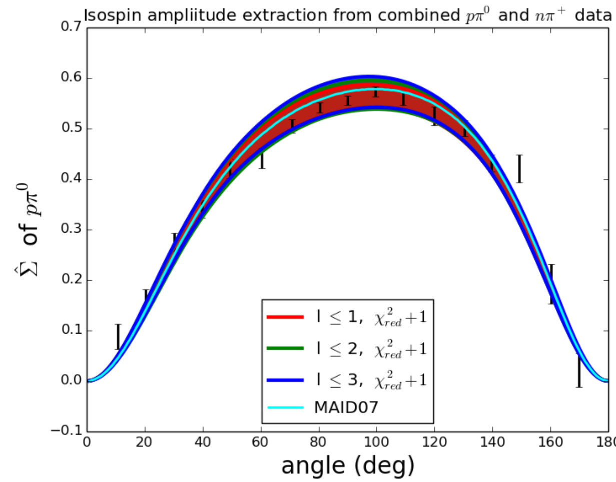


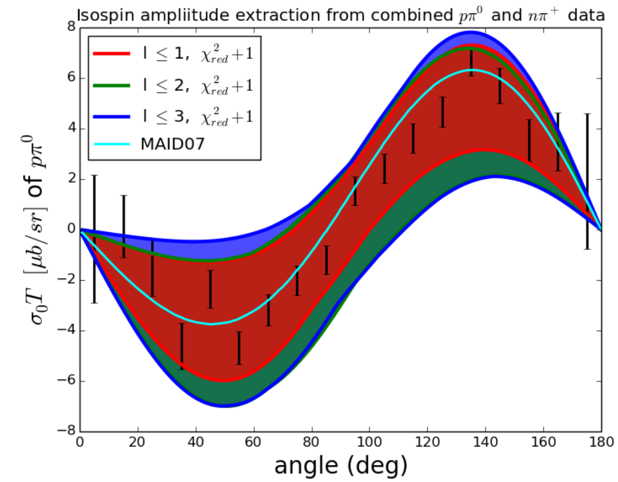
We need D-waves to extract E0+
F waves help extract D waves

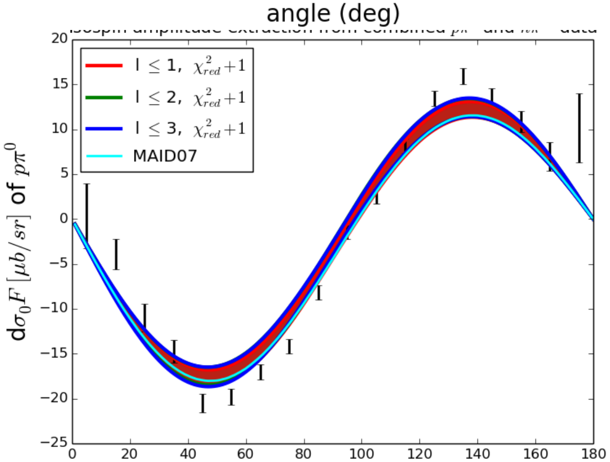
Focus on the resonant $\Delta(1232)$

Bands of allowed solutions (1-sigma) @ W1232 MeV



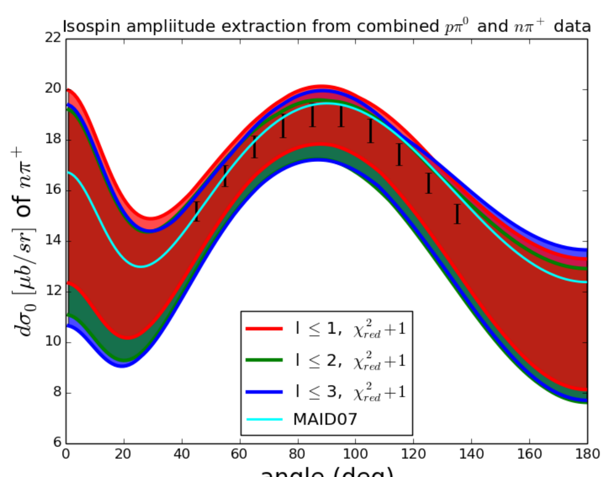


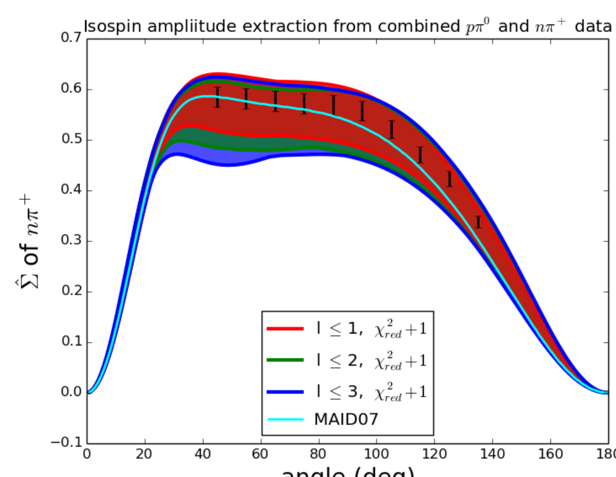


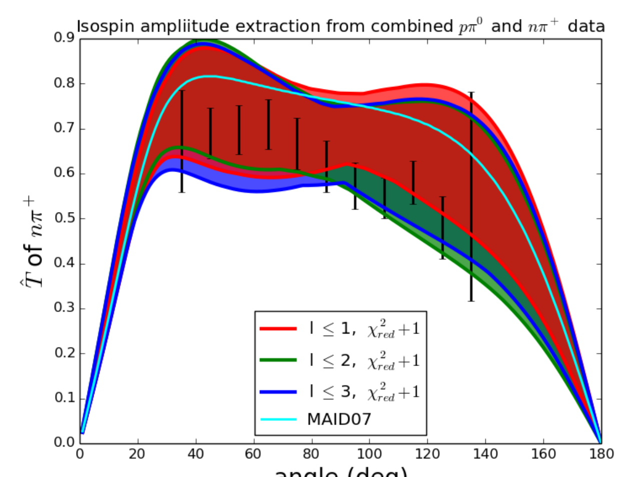


Solutions with $\chi^2 < \chi^2_{red} + 1$

Angular coverage in one region does not confine solutions in another

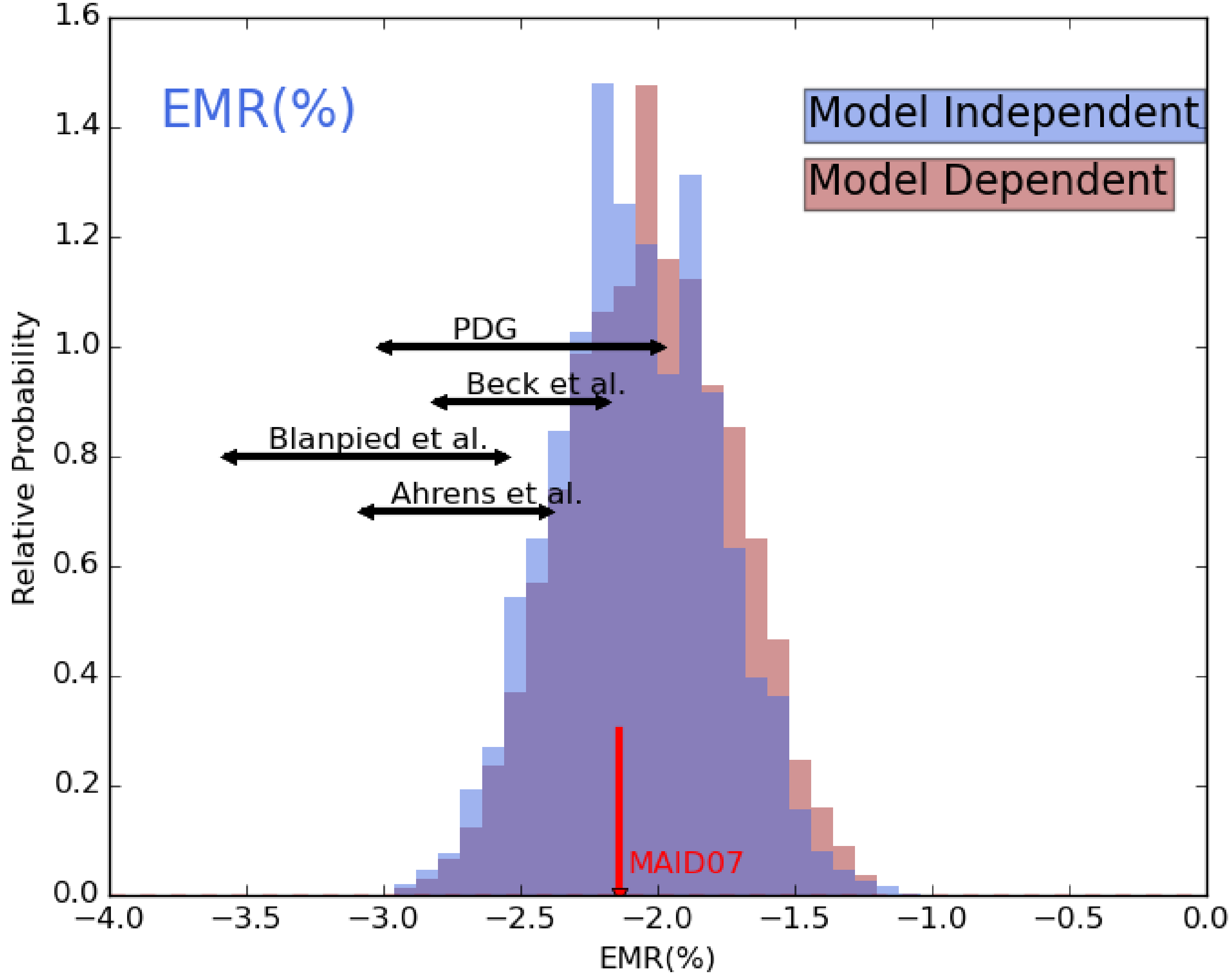




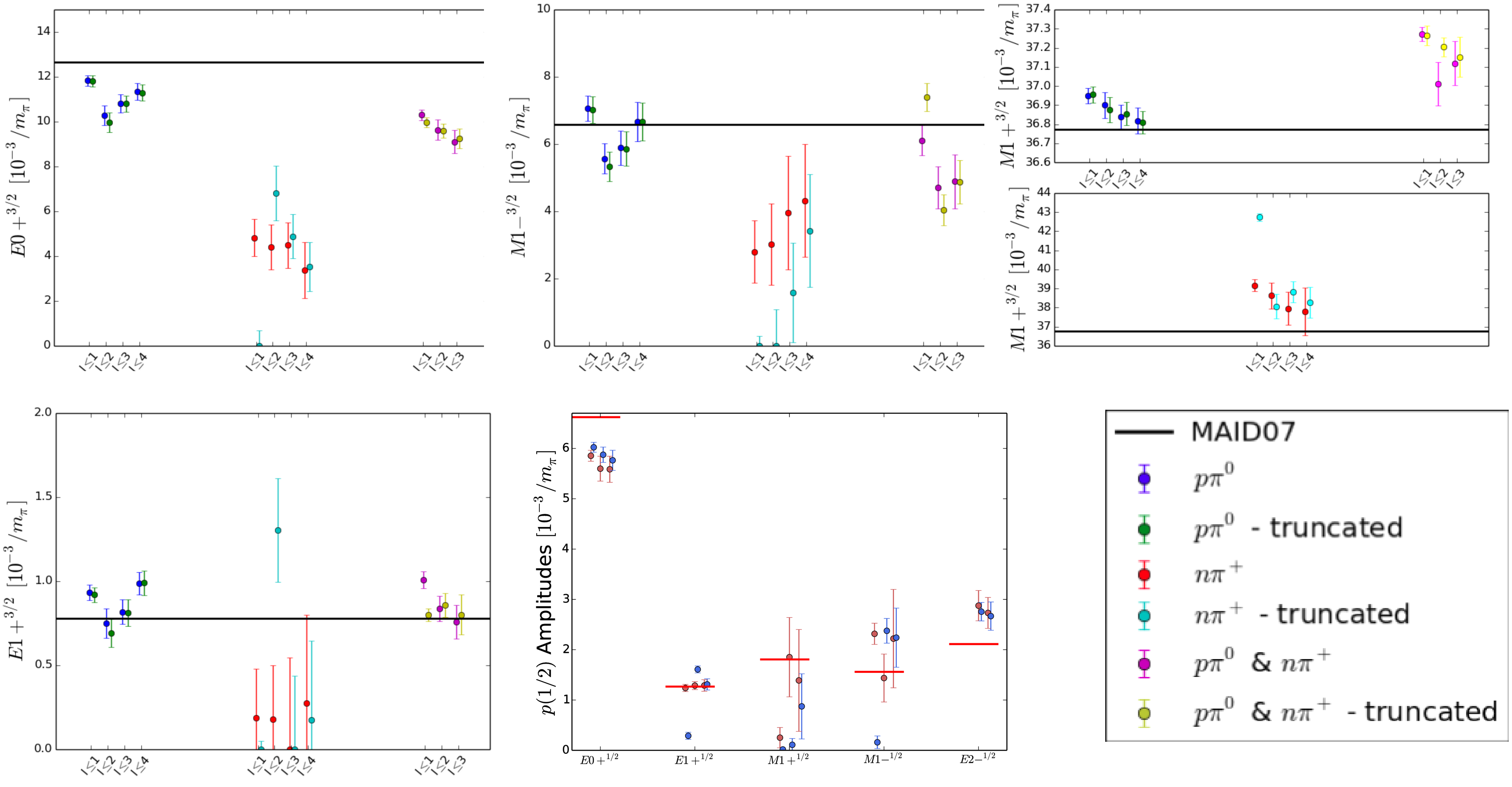


Extracted Electric to Magnetic Ratio (EMR) @ W1232 MeV

$$EMR_{p\pi^0 \,\&\, n\pi^+} = -\left(2.09 \pm {}^{0.29}_{0.26}\right)\%$$



Amplitude extraction from single channel data @ W1232 MeV



- $E0+^{3/2}$ drasstically changes with the incluion of D-waves
- Higher L_{cut} needed to describe the $n\pi^+$ data
- The values of E0+1/2 and E2-1/2 as determined by the data signifficantly differ from the MAID07 prediction which was used as model input for the single channel analyses

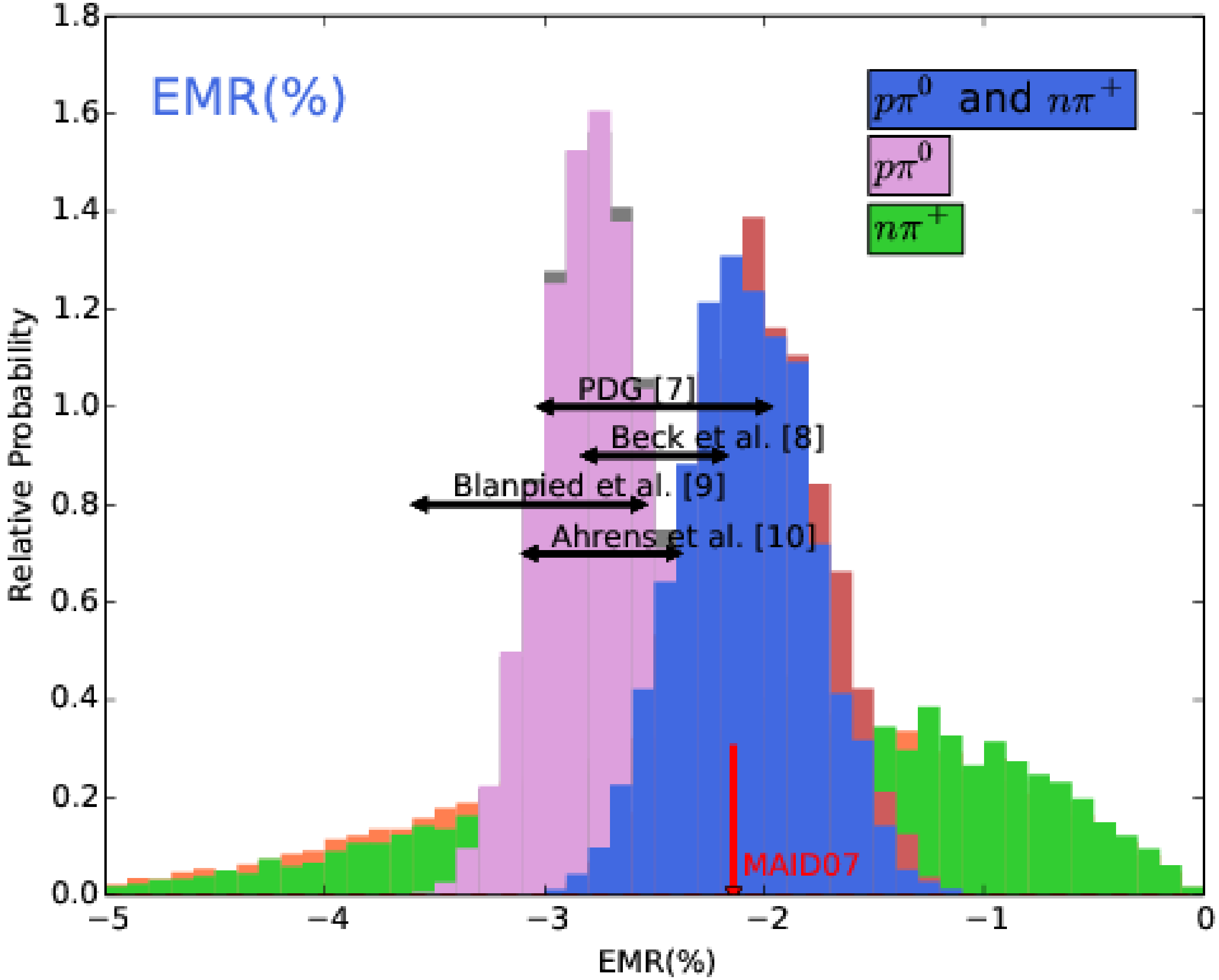
Extracted Electric to Magnetic Ratio (EMR) @ W1232 MeV

At the Δ(1232) we have also extracted I(3/2) amplitudes by fixing p(1/2) amplitudes to a model

$$EMR_{p\pi^0} = -(2.8 \pm 0.3)\,\%$$

$$EMR_{p\pi^0\,\&\,n\pi^+} = -\left(2.09 \pm {}^{0.29}_{0.26}\right)\%$$

$$EMR_{n\pi^+} = -\left(1.4 \pm {}^{0.7}_{1.4}\right)\%$$



Future Work (with real data)

- Analysis of $p\pi^0$ data in a wider energy range
  - Extraction of resonant amplitudes with p(1/2) amplitudes fixed
  - Extraction of Real and Imaginary parts
- Include some of the world data to my analyses

Thank You !