HW 3: Algebraische Geometrie II

Handing in: Hand in by November 30th 2015. Grades are from 1 to 10. Keep a copy of your work and include an email address + home address to which we can return the graded exercises. You are strongly encouraged to write your solutions in TeX and hand in a printout. If you write your solutions by hand, write legibly! Parts that we cannot easily decipher will simply be ignored.

The rules of the game: Your solutions to these exercises should convince us that you have a good understanding of what has been discussed so far. This involves more than just getting the correct answers, which, anyway, might be obtained from various internet sources or the brain of a fellow student. You may use everything that has been discussed so far in the course. Give full details for your arguments, with precise references for the results you use. What you hand in should be your own, individual work.

Exercise 1. Consider the following operations on a group G:

1. forming the commutator subgroup $[G, G]$ of G;
2. forming the abelianization G^{ab} of G;
3. forming the center $Z(G)$ of G;
4. taking $\mathrm{Hom}(H, G)$ for a fixed group H;
5. forming the automorphism group $\mathrm{Aut}(G)$ of G.

Which of these operations naturally give rise to functors on the category of groups? In case the operation is naturally a functor, explain whether it is a full and/or faithful functor. Motivate your answers.

Exercise 2. Let $\mathcal{C}$ be a category, and let $\mathcal{C}^{op}$ be its opposite category. Show that, if $\mathcal{C}$ is the category of sets, then $\mathcal{C}$ is not equivalent to $\mathcal{C}^{op}$.

Exercise 3. Prove or disprove:

1. A category $\mathcal{C}$ is never equivalent to its opposite category $\mathcal{C}^{op}$.
2. If $\mathcal{C}$ is a category, then there exists an object I of $\mathcal{C}$ such that, for all objects X of $\mathcal{C}$, the set $\mathrm{Hom}_{\mathcal{C}}(I, X)$ is a singleton.