Theoretical Elementary Particle Physics
Exercise 3

June 13, 2015

-to be handed in on Monday, 29th of June

1. **Gluon self-energy (100 points)**

   One-loop contributions of the gluon self energy (2 gluon, 1 ghost and 1 fermion loop) are shown in fig. 1.

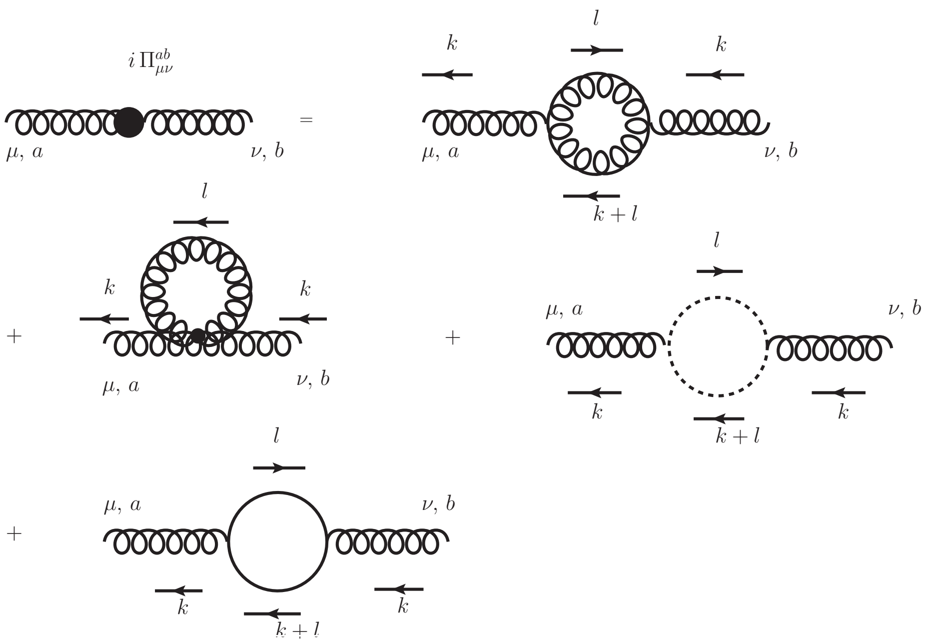

Figure 1: One-loop contributions of the gluon self energy. The first two diagrams are due to the triple and quartic gluon coupling, respectively. The third diagram is a ghost loop and the last one represents a fermion loop. The 4-momenta are labelled on each diagram. Color indices a and b are also shown.

Because of the gauge invariance

$$k_\mu \Pi^{\mu\nu}_{ab}(k) = 0 \ , \tag{1}$$

$\Pi^{\mu\nu}_{ab}(k)$ can be written in the following form

$$\Pi^{\mu\nu}_{ab}(k) = \delta_{ab}\left(k^\mu k^\nu - k^2 g^{\mu\nu}\right)\Pi(k^2) \ . \tag{2}$$

It is convenient to directly compute $\Pi(k^2)$ which is in 4 space-time dimensions expressed as :

$$i\Pi(k^2) = -\frac{1}{3k^2}\, g_{\mu\nu}\, i\Pi^{\mu\nu}_{aa}\,(k) \tag{3}$$

where k stands for external 4-momentum. The idea is to use the Feynman rules to obtain 1-loop contribution for each diagram in fig. 1. By doing so, one gets expressions $i\Pi^{j;\mu\nu}_{aa}\,(k)$ where the color index b is replaced by the index a because of the trivial δ_{ab}, and j takes values from 1 to 4 since there are 4 different 1-loop contributions. The total one-loop correction to the gluon propagator is obtained by adding all 4 contributions :

$$i\,\Pi^{\mu\nu}_{aa}\,(k) = \sum_{j=1}^{4} i\,\Pi^{j;\mu\nu}_{aa}\,(k) \tag{4}$$

(a)(20 points) Write down one-loop contributions of the gluon self energy (gluon loops, ghost loop and quark loop contributions) in Feynman gauge $\xi = 1$. The Feynman rules for triple and quartic gluon couplings may be found in the "Skript 1" whereas the Feynman rules for the ghost field are written in the "Skript 3". In the diagram with the ghost loop additional $(-)$ sign should be included. In the case of a fermion loop in addition to the $(-)$ sign, trace of the integrand should be applied (for example see problem 3. in Exercise 2). Also use N_f as multiplicative factor because there is in principle N_f different diagrams with different quark flavors propagating in the loop. In the Standard model, $N_f = 6$ corresponding to u, d, c, s, b, and t quarks. Notice that even though all these particles have different masses that enter into propagators, the divergent part is mass independent and N_f can be added as multiplicative factor for the purpose of renormalization.

Note also that the two diagrams with the gluon loop have a symmetry factor equal to 2.

(b)(70 points) Using dimensional regularization, extract the divergent part (in $1/\epsilon$) for the gluon loop contribution, the ghost loop contribution and the quark loop contribution separately. (**hint** : Start by applying eq. (3).)
After employing Feynman parametrization bypass the Wick rotation by directly applying formulae :

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2 - \Delta)^n} = \frac{(-1)^n i}{(4\pi)^{d/2}} \frac{\Gamma(n - \frac{d}{2})}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}} \tag{5}$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^2}{(l^2 - \Delta)^n} = \frac{(-1)^{n-1} i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n - \frac{d}{2} - 1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1} \tag{6}$$

Use $d = 4 - 2\epsilon$ which implies the following expansion of the Γ function around pole $z = -1$

$$\Gamma(-1 + \epsilon) \approx -\frac{1}{\epsilon} + \gamma - 1\ , \tag{7}$$

and $z = 0$

$$\Gamma(\epsilon) \approx \frac{1}{\epsilon} - \gamma \ . \tag{8}$$

(**Solution**:
diagram 1. $\rightarrow i\Pi_1(k^2) = -i\frac{9}{2}\frac{g^2}{(4\pi)^2}\frac{1}{\epsilon}$
diagram 2. $\rightarrow i\Pi_2(k^2) = 0$
diagram 3. $\rightarrow i\Pi_3(k^2) = -i\frac{1}{2}\frac{g^2}{(4\pi)^2}\frac{1}{\epsilon}$
diagram 4. $\rightarrow i\Pi_4(k^2) = i\frac{2}{3}N_F\frac{g^2}{(4\pi)^2}\frac{1}{\epsilon}$)

(c)(10 points) Absorbing this divergence into the gluon field renormalization (see fig. 2) constant Z_3, show that the one-loop contribution to the gluon renormalization constant is given by

$$Z_3 = 1 - \frac{g^2}{(4\pi)^2}\frac{1}{\epsilon}\left[-5 + \frac{2}{3}N_f\right] \ . \tag{9}$$

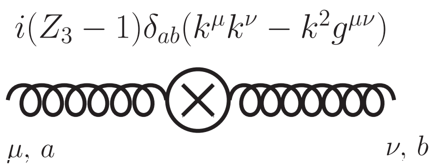

Figure 2: Counterterm that absorbs divergence of the diagrams in fig. 1 together with the corresponding Feynman rule.

Additional useful formulae

$$\sum_{b,c} f_{abc} f_{abc} = 3 \tag{10}$$

$$\mathrm{Tr}\left[\frac{\lambda_a}{2}\frac{\lambda_b}{2}\right] = \frac{1}{2}\delta_{ab} \tag{11}$$

where f_{abc} are antisymmetric SU(3) structure constants and λ_x stands for Gell-Mann matrices.