ϕ^4 THEORY (REAL SCALAR FIELD)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4$$ → INTERACTION TERM

LET'S WRITE, FOR COMPLETENESS, EQUATION OF MOTION FOR THIS THEORY

<u>E-L</u> $$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi \qquad \frac{\partial \mathcal{L}}{\partial \phi} = -m^2\phi - \frac{\lambda}{3!}\phi^3$$

$$\partial_\mu \partial^\mu \phi + m^2 \phi + \frac{\lambda}{3!}\phi^3 = 0$$

$$\boxed{(\Box + m^2)\phi = -\frac{\lambda}{3!}\phi^3}$$

LAST TIME WE DISCUSSED TIME ORDERED PRODUCTS OF FIELD

$\langle 0 | T \phi_1 \phi_2 | 0 \rangle$ → FEYNMAN PROPAGATOR

<u>WICK THEOREM</u>:

→ PUTS CREATION OPERATORS TO THE LEFT !! (NORMAL ORDERING)

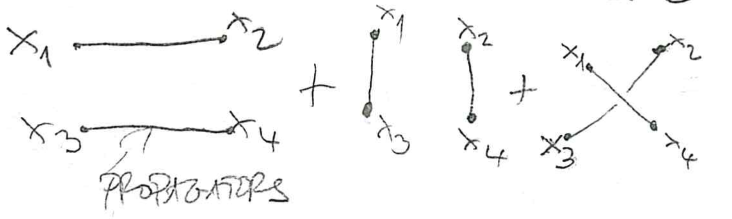
$$T(\phi(x_1)\phi(x_2)\dots\phi(x_n)) = N(\phi(x_1)\phi(x_2)\phi(x_3)\dots\phi(x_n) + \text{CONTRACTIONS})$$

SO $T(\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)) = N(\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) + \overline{\phi(x_1)\phi(x_2)}\phi(x_3)\phi(x_4)$ + MORE TERMS WITH 1 CONTRACTION $+ \overline{\phi(x_1)\phi(x_2)}\,\overline{\phi(x_3)\phi(x_4)} + \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) + \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4))$

$$\overline{\phi\,\phi} = \Delta_F(x-y)$$ → FEYNMAN PROPAGATOR

terms with fields that are not all contracted vanish because $\langle 0 | a^\dagger a | 0 \rangle = 0$ → ON THE LEFT

$\langle 0 | T\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) | 0 \rangle =$



– NO INTERACTION

IF YOU WANT TO CONSIDER INTERACTIONS, INTERACTING HAMILTONIAN NEEDS TO BE ADDED (THIS IS JUST ONE PART OF THE FULL STORY — DYSON FORMULA (DENOMINATOR IS FOR OUR PURPOSES IGNORED))

→ THERE IS $[-i\int d^4z\,\mathcal{H}_I]$ SO $1+\ldots$, THE FIRST TERM BEING INTERACTIONLESS

2nd term in expansion:

$$\langle 0|\, T\big(\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\,[-i\int d^4z\,\mathcal{H}_I]\big)|0\rangle = \Big\{\mathcal{H}_I = \frac{\lambda\phi^4}{4!}$$

$$= \langle 0|\, T\big(\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\,[-i\frac{\lambda}{4!}\int d^4z\,\phi(z)\phi(z)\phi(z)\phi(z)]\big)|0\rangle$$

4 POSSIBILITIES

3 POSSIBILITIES

2 POSSIBILITIES

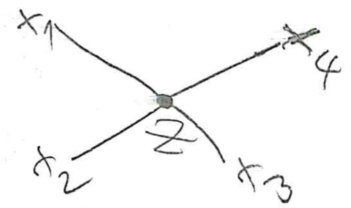
1 POSSIBILITY

$$= \cancel{4!}\left(\frac{-i\lambda}{\cancel{4!}}\right)\int d^4z\,\Delta_F(x_1-z)\,\Delta_F(x_2-z)\,\Delta_F(x_3-z)\,\Delta_F(x_4-z)$$

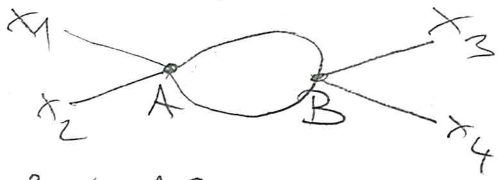
WAYS OF DOING THIS, ALL GIVE IDENTICAL EXPRESSIONS

$-i\lambda\int d^4z$ → FEYNMAN RULE FOR THE VERTEX IN COORDINATE SPACE

DIAGRAMMATICALLY:



CONSIDER 1-LOOP DIAGRAM



THERE IS ADDITIONAL $\frac{1}{2!}$ FROM EXP EXPANSION BUT IS CANCELED BY THE FACTOR FROM INTERCHANGING VERTICES

3rd term in the expansion of exponential

$$\langle 0|\, T\big(\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\,[-i\int d^4A\,\mathcal{H}_I]\,[-i\int d^4B\,\mathcal{H}_I]\big)|0\rangle =$$

$$= \langle 0|\, T\big(\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\left(\frac{-i\lambda}{4!}\right)^2\int d^4A\,\phi(A)\phi(A)\phi(A)\phi(A)\int d^4B\,\phi(B)\phi(B)\phi(B)\phi(B)\big)|0\rangle =$$

4 POSS.

3 POSS.

4 POSS.

3 POSS.

2 POS.

$$= 4\cdot 3\cdot \underbrace{4\cdot 3\cdot 2}_{4!}\,(-i\lambda)^2\,\frac{1}{(4!)^2}\int d^4A\,d^4B\,\Delta_F(x_1-A)\,\Delta_F(x_2-A)\,\Delta_F(x_3-B)\,\Delta_F(x_4-B)\,\Delta_F(A-B)\,\Delta_F(A-B)$$

$$= \frac{12\,\cancel{4!}}{(4!)^{\cancel{2}}}(-i\lambda)^2\ldots = \boxed{\frac{1}{2}(-i\lambda)^2\ldots}$$

SYMMETRY FACTOR

SO, WE GOT SYMMETRY FACTOR = 2 WHICH MEANS THAT THE DIAGRAM IS SYMMETRIC UNDER THE INTERCHANGE OF 2 PROPAGATORS a and b (LINES)

THIS FEYNMAN DIAGRAMS WERE FOR CORRELATION FUNCTIONS, IN PRINCIPLE WE NEED $\underbrace{\text{S-MATRIX ELEMENTS}}$ → AMPLITUDES FOR PROCESSES

$$\langle p_1 p_2 | T\left(-i\frac{\lambda}{4!}\int d^4z\, \phi^4(z)\right) | p_3 p_4 \rangle$$

FEYNMAN RULES FOR ϕ^4 THEORY (IN MOMENTUM SPACE)

1) PROPAGATOR ——— $\dfrac{i}{p^2-m^2}$

2) VERTEX  $-i\lambda$

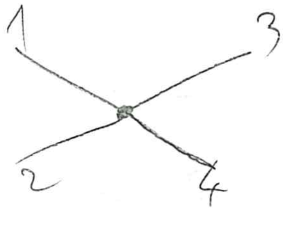
↓ TAKE $\mathcal{L}$, REMOVE FIELDS AND ADD i

3) EXTERNAL LINE 1

4) INTEGRATE OVER EACH UNDETERMINED LOOP MOMENTUM $\int \dfrac{d^4p}{(2\pi)^4}$

5) DIVIDE BY SYMMETRY FACTOR

EXAMPLE: 2 → 2 SCATTERING <u>TREE-LEVEL LEADING CONTRIBUTION</u>



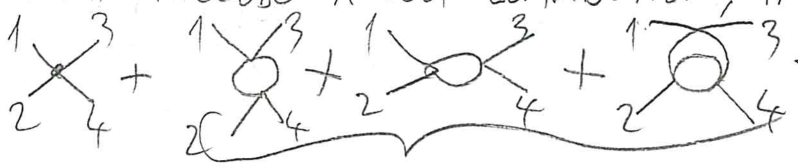
MAYBE NOT USED IN THE QG, PESKIN'S CONVENTION

$$? \quad i\mathcal{M} = -i\lambda$$

$$-i\mathcal{M}^{\dagger} = i\lambda$$

$$\Rightarrow \boxed{|\mathcal{M}|^2 = \lambda^2}$$

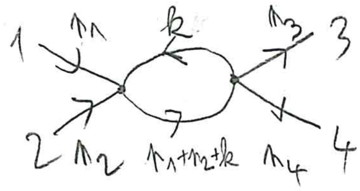
IF YOU WANT TO INCLUDE 1-LOOP CONTRIBUTION, INCLUDE s, t and u DIAGRAMS

$i\mathcal{M} =$  + (COUNTERTERM)*

*MENTION LATER

1-LOOP CONTRIBUTION

LET US CONSIDER s-channel process (1,2 INCOMING, 3,4 OUTGOING PARTICLES)



USING FEYNMAN RULE WE HAVE

$$\underbrace{i\mathcal{M}_s}_{s\text{ channel}} = (-i\lambda)^2 \cdot \underbrace{\frac{1}{2}}_{\text{SYMMETRY FACTOR}} \int \frac{d^4k}{(2\pi)^4}\, \frac{i}{k^2-m^2}\, \frac{i}{(k+p)^2-m^2}$$

$\boxed{\text{WITH ABBREVIATION } p = p_1+p_2}$

TAKE A LOOK AT THE INTEGRAL

$$\sim \int \frac{d^4k}{(2\pi)^4}\, \frac{1}{(k^2-m^2)\left[(k+p)^2-m^2\right]} \xrightarrow[p\to 0]{\text{FOR } k \gg m} \int \frac{\overbrace{d\Omega}^{\text{ANGULAR PART}}\, k^3\, dk}{k^4} =$$

$$\simeq \int \frac{dk}{k} = \ln k \Big| \to \text{POTENTIALLY BIG}$$

$\boxed{\text{LOGARITHMICALLY DIVERGENT}}$

TREATING THE INTEGRAL:

So, we have $i\mathcal{M}_s = +\frac{1}{2}\lambda^2 \underbrace{\int \frac{d^4k}{(2\pi)^4}\, \frac{1}{k^2-m^2}\, \frac{1}{(k+p)^2-m^2}}_{I}$

FIRST STEP : FEYNMAN PARAMETRIZATION

2 DENOMINATORS REWRITE AS ONE RAISED TO THE 2nd POWER

$$\frac{1}{A_1A_2} = \int_0^1 dx\, \frac{1}{\left[xA_1+(1-x)A_2\right]^2} = \int_0^1 dx\, dy\, \delta(x+y-1)\, \frac{1}{(xA_1+yA_2)^2}$$

PROOF: $\int_0^1 dx \frac{1}{\left[xA_1+(1-x)A_2\right]^2} = \int_0^1 dx \frac{1}{(xA_1+A_2-xA_2)^2} = \int_0^1 dx \frac{1}{\left(x(A_1-A_2)+A_2\right)^2}$

$$= \left\{ \begin{matrix} x(A_1-A_2)+A_2 = y \\ (A_1-A_2)\,dx = dy \end{matrix} \right\} = \int_{A_2}^{A_1} dy\, \frac{1}{A_1-A_2}\, \frac{1}{y^2} = \frac{1}{A_1-A_2}\,(-)\,\frac{1}{y}\Big|_{A_2}^{A_1} =$$

$$= -\frac{1}{A_1-A_2}\left(\frac{1}{A_1}-\frac{1}{A_2}\right) = -\frac{1}{A_1-A_2}\,\frac{A_2-A_1}{A_1A_2} = \frac{1}{A_1A_2}$$

IN OUR CASE

$$\frac{1}{k^2-m^2}\,\frac{1}{(k+n)^2-m^2} = \int_0^1 dx \frac{1}{\left[\left[(k+n)^2-m^2\right]x+(1-x)(k^2-m^2)\right]^2} =$$

$$= \int_0^1 dx \frac{1}{\left(\cancel{xk^2}+x2kn+xn^2-\cancel{xm^2}+k^2-m^2-\cancel{xk^2}+\cancel{xm^2}\right)^2} =$$

$$= \int_0^1 dx \frac{1}{\left(k^2+2knx+xn^2-m^2\right)^2} = \left\{ \begin{array}{l} \text{COMPLETE THE SQUARE} \\ \text{ADD } x^2n^2 - x^2n^2 \\ l = k + xn \\ \quad dl = dk \end{array} \right\} =$$

$$= \int_0^1 dx \frac{1}{\left(l^2+xn^2(1-x)-m^2\right)^2}$$

SO INTEGRAL NOW LOOKS LIKE

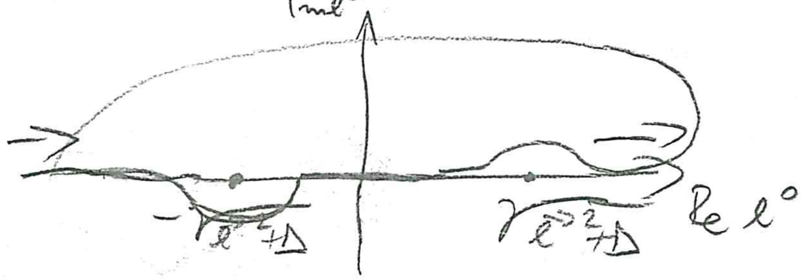
$$I = \int_0^1 dx \int \frac{d^4 l}{(2\pi)^4} \frac{1}{\left(l^2+\underbrace{xn^2(1-x)-m^2}_{-\Delta}\right)^2}$$

SECOND STEP: WICK ROTATION

→ BRUTE FORCE METHOD: SEPARATELY INTEGRATE OVER l^0 AND $\vec{l}$

MORE ELEGANT → WICK ROTATION

WE ARE IN PRINCIPLE "REMOVING" −SIGNS IN METRIC



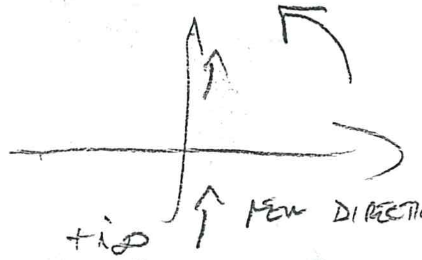
POLES: $l^2-\Delta=0$

$l^{0\,2}-\vec{l}^2-\Delta=0$

$l^{0\,2}=\vec{l}^2+\Delta$

$l_0=\pm\sqrt{\vec{l}^2+\Delta}$

ROTATE CONTOUR COUNTERCLOCKWISE BY $90°$



$$\int_{-\infty}^{+\infty} dl^0 \rightarrow \int_{-i\infty}^{+i\infty} dl^0 = i\int_{-\infty}^{+\infty} d\underset{\substack{\uparrow\\ \text{EUCLIDEAN}}}{l_E^0}$$

THEREFORE, DEFINE $\boxed{dl^0 = i\,dl_E^0}$

$l = l_E$ SPATIAL PART THE SAME

$$l^2 = l^{0\,2} - \vec{l}^{\,2} = -l_E^{0\,2} - \vec{l}_E^{\,2} = -\underbrace{\left(l_E^{0\,2} + \vec{l}_E^{\,2}\right)}_{l_E^2} = -l_E^2$$

OUR INTEGRAL CANNOT BE WICK ROTATED YET $\quad \Delta = m^2 - x\Lambda^2(1-x)$

$$I = \int_0^1 dx \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2-\Delta)^m} = \boxed{(-1)^m i \int_0^1 dx \int \frac{d^4 l_E}{(2\pi)^4} \frac{1}{(l_E^2+\Delta)^m}}$$

for $m=2$ (OUR CASE) WICK ROTATION DOESN'T WORK SINCE INTEGRAL DOESN'T DECREASE SUFFICIENTLY FAST (BLOWS UP AT INFINITY)

THIRD STEP : DIMENSIONAL REGULARIZATION

– let's consider general dimension d, and at the very end go back to $d \to 4$

$$\int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2+\Delta)^2} \to \int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2+\Delta)^2} = i \int \frac{d^d l_E}{(2\pi)^d} \frac{1}{(l_E^2+\Delta)^2}$$

$$= i \int \frac{d\Omega_d}{(2\pi)^d} \int_0^\infty dl_E \frac{l_E^{d-1}}{(l_E^2+\Delta)^2}$$

$d\Omega_d \to$ AREA OF SPHERE IN d DIMENSIONS

LIKE $\int_{-\infty}^{+\infty} dx\,dy\, e^{-x^2-y^2} = \int d\phi \int_0^\infty \rho\, d\rho\, e^{-\rho^2}$

ANGULAR PART

START FROM

label each individual x with $1,2,\dots n$

$$\left(\sqrt{\pi}\right)^d = \left(\int_{-\infty}^{+\infty} dx\, e^{-x^2}\right)^d = \int d^d x \exp\left(-\sum_{i=1}^{d} x_i^2\right) =$$

$$= \int d\Omega_d \int_0^\infty dx\, x^{d-1} e^{-x^2} = \int d\Omega_d \frac{1}{2} \int_0^\infty d(x^2) (x^2)^{\frac{d}{2}-1} e^{-(x^2)} =$$

$$= \left\{ \Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx \right\} = \int d\Omega_d \cdot \frac{1}{2} \Gamma\left(\frac{d}{2}\right)$$

So $\displaystyle\int d\Omega_d = \frac{2\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)}$

$$3D: \frac{2\pi^{\frac{3}{2}}}{\Gamma\left(\frac{3}{2}\right)} = \left\{ \Gamma(t+1) = t\,\Gamma(t) \right\} = \frac{2\pi^{\frac{3}{2}}}{\frac{1}{2}\Gamma\left(\frac{1}{2}\right)} = \frac{2\pi^{\frac{3}{2}}}{\frac{1}{2}\pi^{\frac{1}{2}}} = 4\pi$$

$$4D: \frac{2\pi^2}{\Gamma(2)} = 2\pi^2$$

SPATIAL PART:

$$\int_0^\infty dl_E \frac{l_E^{d-1}}{(l_E^2+\Delta)^2} = \frac{1}{2}\int_0^\infty d(l_E^2)\frac{(l_E^2)^{\frac{d}{2}-1}}{(l_E^2+\Delta)^2} =$$

$$= \left\{ \begin{array}{c|c} \text{SUBSTITUTION} & dx = \frac{-\Delta}{(l_E^2+\Delta)^2}\, d(l_E^2) \\ x = \frac{\Delta}{l_E^2+\Delta} & \hline \\ & l_E^2 = \Delta\,\frac{1-x}{x} \end{array} \right\} =$$

$$= \frac{1}{2}\int_1^0 (-1)\,\frac{\cancel{(l_E^2+\Delta)^2}}{\Delta}\,dx\,\left(\Delta\,\frac{1-x}{x}\right)^{\frac{d}{2}-1}\frac{1}{\cancel{(l_E^2+\Delta)^2}} =$$

$$= \frac{1}{2}(\Delta)^{\frac{d}{2}-2}\underbrace{\int_0^1 dx\,(1-x)^{\frac{d}{2}-1}\,x^{1-\frac{d}{2}}}_{\frac{\Gamma\left(2-\frac{d}{2}\right)\Gamma\left(\frac{d}{2}\right)}{\Gamma(2)}} =$$

$$\int_0^1 dx\, x^{\alpha-1}(1-x)^{\beta-1} = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$= \boxed{\frac{1}{2}(\Delta)^{\frac{d}{2}-2}\,\frac{\Gamma\left(2-\frac{d}{2}\right)\Gamma\left(\frac{d}{2}\right)}{\Gamma(2)}}$$

TOGETHER WITH ANGULAR PART: $\rightarrow$ FRIDAY

$$\int \frac{d^d l_E}{(2\pi)^d}\frac{1}{(l_E^2+\Delta)^2} = \underbrace{\frac{\cancel{2}\pi^{\frac{d}{2}}}{\cancel{\Gamma\left(\frac{d}{2}\right)}}}_{\int d\Omega_d}\cdot\frac{1}{(2\pi)^d}\,\frac{1}{\cancel{2}}(\Delta)^{\frac{d}{2}-2}\,\frac{\Gamma\left(2-\frac{d}{2}\right)\cancel{\Gamma\left(\frac{d}{2}\right)}}{\underbrace{\Gamma(2)}_{1}}$$

$$= \frac{\pi^{\frac{d}{2}}}{\underbrace{(2\pi)^d}_{(4\pi^2)^{\frac{d}{2}}}}(\Delta)^{\frac{d}{2}-2}\,\Gamma\left(2-\frac{d}{2}\right) = \boxed{\frac{1}{(4\pi)^{\frac{d}{2}}}(\Delta)^{\frac{d}{2}-2}\,\Gamma\left(2-\frac{d}{2}\right)}$$

POLES AT $d=4,6,8\ldots$

FOR Γ FUNCTION :

$$\Gamma(z)^{-1} = z e^{\gamma z} \prod_{n=1}^{\infty} \left(1+\frac{z}{n}\right) e^{-z/n} =$$

$$= z e^{\gamma z} (1+z) e^{-z} + \text{HIGHER ORDER IN } z =$$

$$\simeq z(1+\gamma z)(1+z)(1-z) + \dots =$$

$$= z(1+\gamma z)(1-z^2) = z(1-z^2+\gamma z - \cancel{\gamma z^3})$$

$$\simeq z(1+\gamma z)$$

$$\frac{1}{1+x} \simeq 1-x \quad x \text{ SMALL}$$

$$\Gamma(z) \simeq \frac{1}{z(1+\gamma z)} = \frac{1}{z}(1-\gamma z) = \frac{1}{z} - \gamma$$

$z \approx 0$ $\quad \gamma = \lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4} \dots - \ln n\right)$ EULER-MASCHERONI $\simeq 0,5771$

$$\text{SO} \quad \boxed{\Gamma(z) \simeq \frac{1}{z} - \gamma + O(z)}$$

NOW WHEN WE KNOW THIS, LET'S SIT AT $d = 4 - \varepsilon$ WHERE ε IS TINY AND EVALUATE THE PREVIOUS EXPRESSION

$$\int \frac{d^d l_E}{(2\pi)^d} \frac{1}{(l_E^2+\Delta)^2} = \left\{ \begin{array}{l} \text{ALSO } a^x = e^{x \ln a} \simeq \underline{1 + x \ln a} \text{ FOR SMALL } x \\ \text{THEREFORE } (\Delta)^{\frac{d}{2}-2} = \Delta^{-\frac{\varepsilon}{2}} = 1 - \frac{\varepsilon}{2}\ln\Delta \\ (4\pi)^{\frac{d}{2}} = (4\pi)^2 (4\pi)^{-\frac{\varepsilon}{2}} = (4\pi)^2\left(1-\frac{\varepsilon}{2}\ln 4\pi\right) \end{array} \right\} =$$

$$= \frac{1}{(4\pi)^2\left(1-\frac{\varepsilon}{2}\ln 4\pi\right)}\left(1-\frac{\varepsilon}{2}\ln\Delta\right)\Gamma\left(\frac{\varepsilon}{2}\right) =$$

$$\simeq \frac{1}{(4\pi)^2}\left(1+\frac{\varepsilon}{2}\ln 4\pi\right)\left(1-\frac{\varepsilon}{2}\ln\Delta\right)\left(\frac{2}{\varepsilon}-\gamma\right) = \left\{\begin{array}{l}\text{KEEP} \\ \text{JUST} \\ \frac{1}{\varepsilon}, 1\end{array}\right.$$

$$= \frac{1}{(4\pi)^2}\left(1+\frac{\varepsilon}{2}\ln 4\pi - \frac{\varepsilon}{2}\ln\Delta + \cancel{\frac{\varepsilon^2}{2^2}\ln 4\pi \ln\Delta}\right)\left(\frac{2}{\varepsilon}-\gamma\right) =$$

$$\boxed{= \frac{1}{(4\pi)^2}\left(\frac{2}{\varepsilon} + \ln 4\pi - \ln\Delta - \gamma + O(\varepsilon)\right)}$$

PUTTING EVERYTHING TOGETHER

AFTER WICK ROTATION

$$I = i\int_0^1 dx \int \frac{d^4 l_E}{(2\pi)^4} \frac{1}{(l_E^2+\Delta)^2} = \left\{ \begin{array}{l} \text{NOW PUT} \\ \varepsilon \to 0 \end{array} \right\}$$

$$\frac{i}{(4\pi)^2}\int_0^1 dx \left(\frac{2}{\varepsilon} - \gamma + \log 4\pi - \log \Delta \right)$$

$m^2 - x\lambda^2(1-x)$

SO WE JUST HAVE REWRITTEN EVERYTHING IN MORE CONVENIENT FORM BUT IN PRINCIPLE DIDN'T GET RID OF INFINITIES

SOLUTION → RENORMALIZATION

AGAIN LET'S START FROM LAGRANGIAN

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_B)(\partial^\mu \phi_B) - \frac{1}{2} m_B^2 \phi_B^2 - \frac{\lambda_B}{4!}\phi_B^4$$

B → BARE MEANS NOT MEASURABLE QUANTITIES IN EXPERIMENTS (DIVERGENT)

RESCALING : $\phi_B = Z_\phi^{\frac{1}{2}}\phi$

$$\Rightarrow \mathcal{L} = \frac{1}{2} Z_\phi (\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2} m_B^2 Z_\phi \phi^2 - \frac{\lambda_B Z_\phi^2}{4!}\phi^4$$

NOW DEFINE : $\delta Z_\phi = Z_\phi - 1$ $\quad \delta_\lambda = \lambda_B Z_\phi^2 - \lambda$

$\delta_m = m_B^2 Z_\phi - m^2$

$\mathcal{L}$ IS NOW

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + \frac{1}{2}\delta Z_\phi (\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}\delta_m \phi^2 - \frac{1}{2} m^2\phi^2 - \frac{\delta_\lambda}{4!}\phi^4 - \frac{\lambda}{4!}\phi^4 =$$

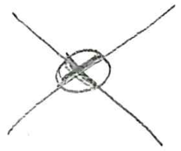
$$= \underbrace{\left(\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2} m^2\phi^2 - \frac{\lambda\phi^4}{4!} \right)}_{\text{renormalized part}} + \left(\frac{1}{2}\delta_{Z_\phi}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}\delta_m\phi^2 - \frac{\delta_\lambda}{4!}\phi^4 \right)$$

→ COUNTER TERMS

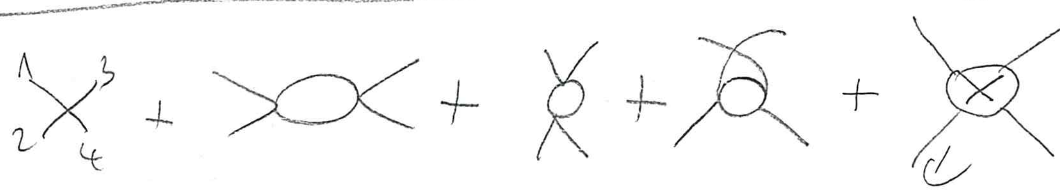
$\Rightarrow$ WE ABSORB INFINITIES INTO COUPLINGS

NEW FEYNMAN RULES:

 $i\left(p^2\delta_Z - \delta_m\right)$

 $-i\delta_\lambda$

"NEW TERM"

WE HAD $iM =$ 

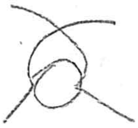
AND HERE YOU PUT ∞

OUR  IS FINALLY

$$iM_s = +\frac{\lambda^2}{2}\frac{1}{(4\pi)^2}\, i\int_0^1 dx\left(\frac{2}{\varepsilon} - \gamma + \log 4\pi - \log\Delta\right)$$

there is $\Delta = (p_1+p_2)^2$ in Δ

IN  IT WILL BE THE SAME, ONLY $\Delta(t) \to (p_1-p_3)^2$

 $\Delta(u) \to (p_1-p_4)^2$

REQUIRE THAT δ_λ CANCELS INFINITIES:

$$-i\lambda + 3\frac{\lambda^2}{2}\frac{1}{(4\pi)^2}\, i\int_0^1 dx\left(\frac{2}{\varepsilon} - \gamma + \log 4\pi\right)$$

$$-\frac{\lambda^2}{2}\frac{1}{(4\pi)^2}\, i\int_0^1 dx\left[\log\left(m^2 - x(1-x)\Delta\right) + \log\left(m^2 - x(1-x)t\right) + \log\left(m^2 - x(1-x)u\right)\right]$$

$$-i\delta_\lambda = \text{FINITE}$$

$$\Rightarrow \boxed{\delta_\lambda = +\frac{3\lambda^2}{32\pi^2}\underbrace{\int_0^1 dx\,\frac{2}{\varepsilon}}_{\frac{2}{\varepsilon}}}$$

$\to$ MS SCHEME JUST ABSORB INFINITY

$\overline{MS}$ SCHEME ABSORBS ALSO γ AND $\ln 4\pi$, AND INTRODUCES RENORMALIZATION SCALE μ (UNPHYSICAL)

$$\delta_{\lambda\,\overline{MS}} = + \frac{3\lambda^2}{32\pi^2} \int_0^1 dx \left(\frac{2}{\varepsilon} - \gamma + \log 4\pi - \log \mu^2 \right)$$

MOTIVATION: $\displaystyle\int \frac{d^4k}{(2\pi)^4 (k^2-m^2)((k+n)^2-m^2)} \to \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2-m^2)((k+n)^2-m^2)}$

INTEGRAL HAS DIM $= 0$ DIM $d-4$

INTRODUCE $\displaystyle\int \frac{d^dl\,\mu^{4-d}}{(2\pi)^d} \frac{1}{(l_E^2+\Delta)^2} = \ldots$

$$\sim \int_0^1 dx \left(\frac{2}{\varepsilon} - \gamma + \log 4\pi - \ln\left(\frac{\Delta}{\mu^2}\right) \right)$$

DIMENSIONLESS

LOOK JUST THAT PART:

$$- \frac{\lambda^2}{32\pi^2} \int_0^1 dx \left(\log\left(m^2 - x(1-x)\Delta\right) - \log \mu^2 \right) \ldots$$

~~ADDENDUM~~:

IF WE HAD SPINORS AND GAMMA MATRICES FROM GAUGE BOSON COUPLINGS THERE ARE IDENTITIES IN $\underbrace{4-\varepsilon}_{=d}$ DIMENSIONS

$$\gamma^\mu \gamma^\nu \gamma_\mu = -(2-\varepsilon)\gamma^\nu$$
$$\gamma^\mu \gamma^\nu \gamma^\rho \gamma_\mu = 4g^{\nu\rho} - \varepsilon \gamma^\nu \gamma^\rho$$
$$\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_\mu = -2\gamma^\sigma \gamma^\rho \gamma^\nu + \varepsilon \gamma^\nu \gamma^\rho \gamma^\sigma$$
$$\gamma_\mu \gamma^\mu = d = 4 - \varepsilon$$