Exercise sheet 9
Theoretical physics 1 WS2015/2016
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Exercise 1 (40 points): Poisson brackets and angular momentum

a) For an arbitrary function $f(q_i, p_i)$ of a three-dimensional system in Cartesian coordinates ($i = 1, 2, 3$) show that

$$\{l_i, f\} = \sum_{k,l=1}^{3} \varepsilon_{ikl} \left[p_l \frac{\partial f}{\partial p_k} - q_k \frac{\partial f}{\partial q_l} \right]$$

where l_i is the i-component of the angular momentum $\vec{r} \times \vec{p}$.

b) Use this to prove that for any vector $\vec{w} = c_1\vec{r} + c_2\vec{p} + c_3\vec{l}$ with arbitrary constants c_1, c_2, c_3

$$\{l_i, w_j\} = \sum_{k=1}^{3} \varepsilon_{ijk} w_k$$

and therefore $\{l_i, l_j\} = \sum_k \varepsilon_{ijk} l_k$ holds.

Exercise 2 (30 points): Jacobi identity

Prove the Jacobi identity for Poisson brackets:

$$\{A, \{B, C\}\} + \{B, \{C, A\}\} + \{C, \{A, B\}\} = 0$$

Exercise 3 (30 points): Liouville's theorem: Vertical throw

In the lecture the harmonic oscillator was shown as an example for Liouville's theorem: a circular area in phase space propagates on a circular path around the origin without being deformed.

In this exercise we investigate the phase space of the vertical throw. For simplicity we set $m = g = 1$ which gives the Hamiltonian function

$$H(q, p) = \frac{p^2}{2} + q.$$

Find the trajectory in phase space for arbitrary initial values q_0, p_0 using the canonical equations.

How does the circle of radius 0.5 around $(\tilde{q}, \tilde{p}) = (0.5, 3)$ evolve in time? Follow both the center of this circle and multiple points on the circle (e.g., the cardinal points). Sketch the resulting area in phase space for the times $t = 0, 1, 2.5, 5$.