Exercise sheet 8
Theoretical physics 1 WS2015/2016
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On this sheet you can reach a total of 200 points. Turn in by 04.01.2016.

Exercise 1 (50 points): Three-dimensional HO

The interaction of two particles of masses m_1 and m_2 in three-dimensional space is given by $V(\vec{x}, \vec{y}) = \alpha|\vec{x} - \vec{y}|^2$ with $\alpha > 0$.

a) Use the techniques of chapter I 2) to calculate the path $r(\varphi)$ of the relative coordinate. Result:

$$r(\varphi) = \frac{r_0}{\sqrt{1 + \varepsilon \cos(2\varphi)}}$$

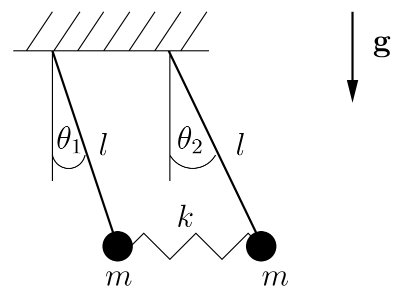
b) Show that both particles follow elliptic paths. (Note: Other than for the Kepler problem, the center of mass is not in one of the focal points.)

c) Are there also circular, parabolic and hyperbolic paths, as is the case for the Kepler problem?

Hint: $\displaystyle\int \frac{dx}{x\sqrt{ax^2 + bx + c}} = -\frac{1}{\sqrt{c}} \operatorname{arsinh}\left(\frac{bx + 2c}{|x|\sqrt{4ac - b^2}}\right)$

Exercise 2 (50 points): Coupled pendulums

Two plane pendulums of mass m and length l are coupled by a spring with spring constant k. The masses of the pendulum rods and of the spring are negligible. The length of the spring in rest is exactly the distance between the suspension points of the pendulums.



a) Find the Lagrange function of this system as a function of the angles θ_1 and θ_2. Simplify this for the case of small amplitudes.

b) What are the equations of motion in the case of small amplitudes?

c) Solve the equations obtained in b) by finding the eigenfrequencies and corresponding normal modes. (The matrix-approach of exercise 1 on sheet 5 might be helpful.)

d) Draw a sketch of the normal modes and describe them briefly.

Exercise 3 (30 points): Hamiltonian function

A system is described by the Lagrangian function

$$L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\dot{x}\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y}^2 - k\sqrt{x^2 + y^2}$$

with constants a, b, c, f, g, k. What is the Hamiltonian function of the system? What quantities are conserved?

Exercise 4 (30 points): Canonical equations

A system's Lagrangian function is given by

$$L = \dot{q}_1^2 + \frac{\dot{q}_2^2}{a + b q_1^2} + k_1 q_1^2 + k_2 \dot{q}_1 \dot{q}_2,$$

with constants a, b, k_1, k_2. Find the canonical equations.

Exercise 5 (40 points): Noether-theorem – particle in the homogeneous electric field

A particle with charge q in the constant, homogeneous electric field $\vec{E}$ can be described by the Lagrangian function

$$L(\vec{r}, \dot{\vec{r}}) = \frac{m}{2}\dot{\vec{r}}^2 + q\vec{E} \cdot \vec{r}.$$

a) The system is invariant with respect to translations in space. Derive the conserved quantity corresponding to this symmetry.

b) Find the coordinate transformation corresponding to the conservation of the component of angular momentum parallel to the electric field.