Exercise sheet 7
Theoretical physics 1 WS2015/2016
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Exercise 1 (20 points): Conservation laws

a) For a Lagrangian function that does not depend explicitly on q_{i_0}, show that $p_{i_0} = \frac{\partial L}{\partial \dot{q}_{i_0}}$ is conserved.

b) For a Lagrangian function that does not depend explicitly on the time, show that

$$\mathcal{E} = \sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L$$

is conserved.

Exercise 2 (15 points): Ambiguity of the Lagrangian

Two Lagrangian functions differ by the time derivative of a function $f(q,t)$ with continuous second partial derivatives:

$$L'(q,\dot{q},t) = L(q,\dot{q},t) + \frac{df(q,t)}{dt}.$$

Show that both Lagrangian functions – L and L' – lead to the same equation of motion.

Exercise 3 (40 points): Soap tube

When pulling two parallel rings of thin wire, separated by the distance d, and each with radius R out of soapy water a thin tube of soap forms between the rings. Due to the surface tension, the tube is not merely a cylinder but shaped to minimize its surface. The goal is to determine the shape of the tube.

a) What is the functional you are minimizing and what functions are used?

b) From the functional find the Lagrangian function and derive the Euler-Lagrange-equation.

c) Find the shape of the tube. If you are unable to solve the Euler-Lagrange-equation: there is another differential equation (of first order) the solution must obey.

d) Will there be a tube for every combination of d and R? State both a mathematical and a physical reason for your answer.

Exercise 4 (25 points): Known systems

Use the variational principle to derive the equations of motion for

a) the Kepler problem,

b) a three dimensional harmonic oscillator.

Which of the conserved quantities of Exercise 1 do you find? What physical quantities do they correspond to?