Exercise sheet 6 (new version)
Theoretical physics 1 WS2015/2016
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Exercise 1 (50 points): Gravitational field of an elliptical galaxy

An elliptical galaxy is an accumulation of relatively old stars. As a good approximation, its mass distribution can be assumed to be continuous. In this exercise we want to find the gravitational potential of a point mass m_0 both inside and outside the galaxy. The mass distribution of the galaxy (given in cylindrical coordinates) is

$$\rho(r,z,\varphi) = \begin{cases} \rho_0 \left(1 - \frac{r^2+\varepsilon^2 z^2}{r_0^2}\right)^2 & \text{for } r^2 + \varepsilon^2 z^2 \leq r_0^2 \\ 0 & \text{else} \end{cases} .$$

a) First, consider the case of a mass point outside the galaxy. Use the formula known from the lecture

$$V(\vec{y}) = -G m_0 \int d^3\vec{x}\, \frac{\rho(\vec{x})}{|\vec{x} - \vec{y}|} \tag{1}$$

and find the Taylor expansion up to second order of $\frac{1}{|\vec{x}-\vec{y}|}$ around $\vec{x} = 0$ to calculate the gravitational potential.

b) For a point mass inside the galaxy the Taylor expansion is no longer appropriate (why?). However, for a spherical galaxy ($\varepsilon = 1$) the integral in (1) can be solved exactly. To this end, introduce appropriate spherical coordinates.

Exercise 2 (20 points): Potential of two balls

Two balls of mass distributions $\rho_1(\vec{x})$ and $\rho_2(\vec{y})$ and radii R_1 and R_2 are separated by the distance $a > R_1 + R_2$. Prove that the potential energy of the two balls is given by

$$V = -G \int\limits_{|\vec{x}|<R_1} d^3\vec{x} \int\limits_{|\vec{y}|<R_2} d^3\vec{y}\, \frac{\rho_1(\vec{x})\rho_2(\vec{y})}{|\vec{x}-\vec{y}|}.$$

Investigate the case of constant mass densities ρ_1 and ρ_2.

Exercise 3 (30 points): Double pendulum

Give the constraints on the coordinates for a plane double pendulum in earth's homogeneous gravity field. Introduce appropriate generalized coordinates. What are their outstanding properties and how many are required?
Draw a sketch of the double pendulum including the coordinates, the possible virtual displacements and the occurring forces (including the constraining forces).
Find the equations of motion for the double pendulum (no solution).

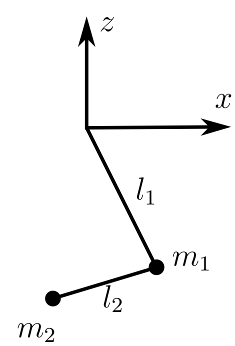

Figure 1: Double pendulum