Exercise sheet 3
Theoretical physics 1 WS2015/2016
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Exercise 1 (30 points): 3-body-problem

In the lecture it was shown that the 2-body-problem can be split into two separate problems, i.e. center of mass motion and relative motion, by introducing appropriate coordinates. This is generalized for the n-body-problem by the Jacobi-coordinates, which separate the center of mass motion. In general, we are left with $n-1$ coupled coordinates. The Jacobi-coordinates are

$$\vec{R}_i = \frac{1}{\sum\limits_{j=1}^{i} m_j} \left(\sum_{k=1}^{i} m_k \vec{r}_k \right) - \vec{r}_{i+1} \quad \forall i < n \quad \text{und} \quad \vec{R}_n = \frac{1}{\sum\limits_{j=1}^{n} m_j} \sum_{k=1}^{n} m_k \vec{r}_k .$$

a) Find the Jacobi-coordinates for the closed 3-body-problem with at least two identical masses ($m_1 = m_2$) and give a sketch including the Jacobi-coordinates. What does each of the Jacobi-coordinates represent?

b) Consider the case of equally strong, harmonic oscillators as interactions between the three particles: $V_{ij} = \frac{1}{2}\kappa|\vec{r}_i - \vec{r}_j|^2$. Express the total energy of the system in terms of the Jacobi-coordinates. What conclusions can you make from this expression?

Exercise 2 (40 points): Ball game on a rotating disk

Four kids (Nils, Erik, Sven and Wiebke) are sitting on the edge of a disk of radius r which rotates with constant angular speed $\omega > 0$ (counterclockwise). At $t = 0$ Sven rolls a ball in the direction of Nils with velocity $v_0 > 0$. Due to the rotation of the disk, Sven sees the ball going straight only in the very beginning.

a) Neglecting the deviation and the slowdown induced by friction, why does the ball follow a curved path?

b) Sketch the path of the ball in the laboratory frame and in the frame rotating with the disk (without any calculations). Especially, mind notable points (like the starting point) the ball must reach as well as the direction of the motion in these points.

c) Sven is not new to the game and by now is able to control the starting velocity and thus the end point of the ball rather precisely. Which of the four kids (including himself) can Sven reach with his technique? (Justify your answer.) What are the required initial velocities? (Find a condition to the velocity. If you cannot solve it, simplify as much as possible and then use Wolfram alpha or another math program.)

d) Calculate the path of the ball in the rotating frame and show explicitly that it solves the equations of motion in the rotating frame.

e) During the entire game Wiebke is focused on the ball. At what angular speed $\tilde{\omega}(t)$ does she turn her head to keep the ball in her line of sight?

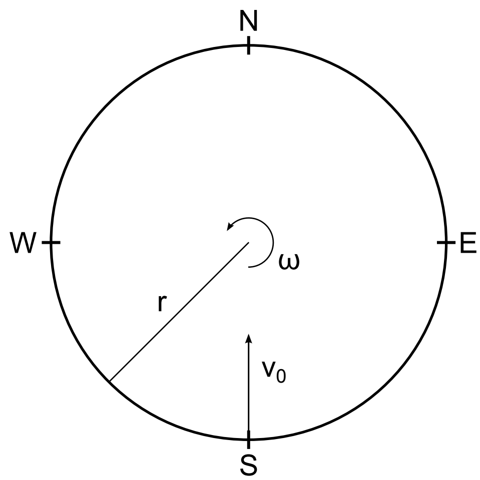

Figure 1: Rotating disk of exercise 2. v_0 is the initial velocity as seen by Sven.

Exercise 3 (30 points): Foucault pendulum

As known from school, a pendulum swings in a constant plane. Only due to the rotation of planet earth the plane of swing seems to rotate for pendulums on earth's surface.

a) First, find the equations of motion of an (ideal, i.e. friction free) pendulum in a coordinate frame in which the plane of swing does not rotate. Simplify these equations with the assumption of small amplitudes and solve them.

b) Now find the equations of motion in a coordinate frame on earth's surface which rotates with the earth. (For a spectator near the pendulum this is the laboratory frame.) Characterize the site of the pendulum on earth's surface by longitude and latitude. The pendulum may swing in north-south-direction in the beginning. Again simplify to obtain a system of equations you can solve. Calculate the path of the pendulum in this frame as well as the angular speed at which the plane of swing rotates.