Exercise sheet 3
Theoretical physics 1: WS2015/2016
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Exercise 1 (50 points): Virial theorem

In a closed system the total energy is conserved. In general, however, it is unclear how it is distributed on kinetic and potential energy. The virial theorem gives this distribution when averaging the energy contributions over time. The average over time of a physical quantity is defined as

$$\langle A \rangle = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau A(t) dt.$$

Assuming that the particles of a closed n-particle system are confined in a finite area and that their velocities are bounded as well, prove the virial theorem

$$2 \langle T \rangle = \left\langle \sum_{i=1}^{n} \vec{r}_i \cdot \vec{\nabla}_i V \right\rangle.$$

Here the potential V is the sum of the two-particle interaction energies as given in the lecture

$$V = \sum_{i=1}^{n} \sum_{j=i+1}^{n} V_{ij}(r_{ij}).$$

Start your proof similar to the proof of energy conservation with the equations of motion and multiply these with an appropriate quantity.

What does the virial theorem state for the (rather common) case of interaction potentials of the form $V_{ij}(r_{ij}) = \alpha_{ij} r_{ij}^{\beta}$ with $\beta \in \mathbb{Z}$?

Exercise 2 (25 points): Conservative forces

We examine the three force fields

$$\vec{F}_1(\vec{r}) = -e^{-r}\vec{r}, \quad \vec{F}_2(\vec{r}) = \begin{pmatrix} zy \\ -zx \\ 0 \end{pmatrix}, \quad \vec{F}_2(\vec{r}) = \begin{pmatrix} -2xz \\ -2yz \\ 2z^2 - x^2 - y^2 \end{pmatrix}.$$

a) For each of these fields calculate the work required to move a mass point on the two different paths C_1 and C_2 from point $P_1 = a\vec{e}_x$ to point $P_2 = -a\vec{e}_y + \frac{3\pi}{2}\vec{e}_z$. The path C_1 is part of the spiral $(a\cos(t), a\sin(t), t)$ and C_2 is the polygonal chain connecting the points P_1, $(a, 0, \frac{3\pi}{2})$, $(0, 0, \frac{3\pi}{2})$ and P_2.

b) Which of the force fields are conservative? What are their potentials?

Exercise 3 (25 points): Rotations

a) Let the coordinate system K' be rotated with respect to the coordinate system K in a way that the x-axis coincides with the y'-axis, the y-axis with the z'-axis and the z-axis with the x'-axis. Find the rotation matrix R of the corresponding Galilean transformation and the rotation axis $\vec{n}$ as well as the rotation angle φ.

b) In general, the rotation matrix can be written as

$$[R(\vec{n}, \varphi)]_{ij} = (1 - \cos\varphi) n_i n_j + \cos\varphi \delta_{ij} + \sin\varphi \sum_{k=1}^{3} \varepsilon_{ijk} n_k,$$

where ε_{ijk} is the Levi-Civita symbol. Use this to check your results of part a).