Exercise sheet 11
Theoretical physics 1 WS2015/2016
Lecturer: Prof. M. Vanderhaeghen
Assistant: Fabian Ewert

18.01.2016

Exercise 1 (40 points): Rolling cone

A homogeneous cone of height h, opening angle 2α and density ρ is rolling on a plane surface without slipping. After the time T it again reaches its initial position. Find the kinetic energy and the angular momentum of the cone.

Hint: Express the rolling of the cone as a superposition of the motion of the center of mass and a rotation. What is the instantaneous rotational axis?

Exercise 2 (25 points): Polar motion

Earth can be described , approximately, as a rotational ellipsoid with principal moments of inertia $I_1 = I_2 < I_3$, where $I_3 \approx 8.04 \cdot 10^{37}\,\mathrm{kg\,m^2}$ and $I_3 - I_1 \approx 2.61 \cdot 10^{35}\,\mathrm{kg\,m^2}$. Since earth's rotational axis does not coincide with the principal axis exactly, the kinematic north pole circles around the geographical north pole. Use the Euler equations (see lecture on Friday 22nd) to find the period of one such revolution. In which direction does the kinematic north pole circle around the geographical?

Exercise 3 (35 points): Finite rotation

Show that the rotation matrix

$$R(\vec{\varphi}) = \exp\left[|\vec{\varphi}|(\hat{\varphi}_1 j_1 + \hat{\varphi}_2 j_2)\right]$$

is

$$\mathbb{1} + (\cos|\vec{\varphi}| - 1)\begin{pmatrix} \hat{\varphi}_2^2 & -\hat{\varphi}_1\hat{\varphi}_2 & 0 \\ -\hat{\varphi}_1\hat{\varphi}_2 & \hat{\varphi}_1^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \sin|\vec{\varphi}|\begin{pmatrix} 0 & 0 & \hat{\varphi}_2 \\ 0 & 0 & -\hat{\varphi}_1 \\ -\hat{\varphi}_2 & \hat{\varphi}_1 & 0 \end{pmatrix}.$$