Exercise sheet 10
Theoretical physics 1 WS2015/2016
Lecturer: Prof. M. Vanderhaeghen
Assistant: Fabian Ewert

11.01.2016

Exercise 1 (15 points): Tensor of inertia 1: Octahedron

Find the tensor of inertia (with respect to the center of mass) for a rigid body consisting of 6 equal masses that sit on the vertices of an octahedron with edge length a.

Exercise 2 (25 points): Tensor of inertia 2: Paraboloid

We investigate a paraboloid which arises from the rotation of the parabola $z = x^2$ around the z-axis. The volume enclosed by this paraboloid and the plane $z = 4$ is filled with a substance of homogeneous mass distribution. Use the symmetries of the object to find its center of mass and the principal moments of inertia. In which directions do the principal axes point?

Exercise 3 (25 points): Tensor of inertia 3: Hourglass and top toy

Find the center of mass as well as the principal axes and moments of inertia of the following objects. (Their mass distribution is homogeneous.)

a) An hourglass: two identical cones that stand on top of each other.

b) A more realistic model of a top toy: on top of a large cone (height H and radius R) a much thinner one is mounted ($h > H$ and $r < R$). The two axes coincide.

Use the result on the inertia tensor of cones known from the lecture.

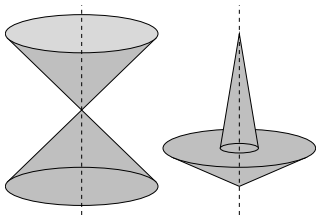

Figure 1: Hourglass and top toy.

Exercise 4 (35 points): Tensor of inertia 4: Prism

Find the center of mass as well as the principal axes and moments of inertia of a right prism of height 2. The base of the prism is the trapezoid defined by the vertices $(0, 0), (2, 0), (1, 1), (0, 1)$. The mass distribution inside the prism is homogeneous.

To calculate the principal axes and moments of inertia first choose a coordinate system, where the required integrations are as simple as possible. Then use Steiner's theorem to calculate the inertia tensor with respect to the center of mass. Finally diagonalize this matrix. With the right choice of the coordinate system the diagonalization can be restricted to two dimensions. (Which symmetry causes this?)