

STANDARD MODEL OF ELECTROWEAK INTERACTIONS

⇒ GAUGE SYMMETRY GROUP $U(1)$
OF ELECTROMAGNETIC INTERACTION

Ψ : DIRAC SPINOR FIELD

$$\mathcal{L}_0 = \bar{\Psi} (i \gamma^\mu \partial_\mu - m) \Psi$$

* REQUIRE THEORY TO BE INVARIANT
UNDER LOCAL PHASE TRANSFORMATION $U(1)$

$$\Psi(x) \xrightarrow{U(1)} e^{ie\chi(x)} \Psi(x)$$

$\chi(x)$: SCALAR FUNCTION

$$\partial_\mu \Psi \rightarrow e^{ie\chi} \left\{ \partial_\mu \Psi + ie(\partial_\mu \chi) \Psi \right\}$$

$$\bar{\Psi} \gamma^\mu (\partial_\mu \Psi) \rightarrow \bar{\Psi} \gamma^\mu (\partial_\mu \Psi) + \underbrace{ie \bar{\Psi} \gamma^\mu (\partial_\mu \chi) \Psi}_{\text{EXTRA TERM}}$$

$U(1)$ GAUGE INV.

REQUIRES INTRODUCTION OF A VECTOR (GAUGE) FIELD
THAT COMPENSATES EXTRA TERM

REPLACE: $\partial^\mu \Rightarrow D^\mu = \partial^\mu + ie A^\mu$

↑
COVARIANT DERIVATIVE

↑
GAUGE FIELD

$A^\mu \xrightarrow{U(1)} A^\mu - \partial^\mu \chi$

$D^\mu \psi \xrightarrow{U(1)} e^{ie\chi} (D^\mu \psi)$

$\bar{\psi} \gamma_\mu (D^\mu \psi) \rightarrow \bar{\psi} \gamma_\mu (D^\mu \psi)$ INVARIANT

$\mathcal{L}_0 + \mathcal{L}_{INT} = \bar{\psi} (i \gamma^\mu \partial_\mu - m - e \gamma^\mu A_\mu) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

↑
U(1) INVARIANT

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

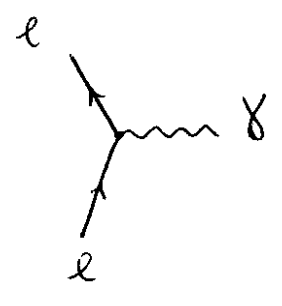
↑
FIELD TENSOR

$F^{\mu\nu} \xrightarrow{U(1)} F^{\mu\nu}$

GAUGE FIELDS (γ)
MASSLESS

$\mathcal{L}_{INT} = -e J_{e.m.}^\mu A_\mu$

↑
E.M. CURRENT



$-ie\gamma^\mu$
FEYNMAN RULE

$J_{e.m.}^\mu = \bar{\psi} \gamma^\mu \psi$

⇒ GAUGE SYMMETRY GROUP $SU(2) \times U(1)$
OF ELECTROWEAK INTERACTIONS

• WEAK ISOSPIN: $SU(2)$

↳ TAKE $\Psi_{\nu_e}^L, \Psi_e^L$ TOGETHER (WEAK INTERACTIONS COUPLE BOTH)

WITH $\Psi^L = \frac{1}{2} (1 - \gamma_5) \Psi$

$$\bar{\Psi}_e^L = \begin{pmatrix} \bar{\Psi}_{\nu_e}^L \\ \bar{\Psi}_e^L \end{pmatrix}$$

↳ TAKE Ψ_e^R SEPARATE (NO Ψ_e^R IN SM)

$$\mathcal{L}_0 = \bar{\Psi}_e^L i \gamma^\mu \partial_\mu \Psi_e^L + \bar{\Psi}_e^R i \gamma^\mu \partial_\mu \Psi_e^R$$

$$= \bar{\Psi}_{\nu_e}^L i \gamma^\mu \partial_\mu \Psi_{\nu_e}^L + \bar{\Psi}_e^L i \gamma^\mu \partial_\mu \Psi_e^L + \bar{\Psi}_e^R i \gamma^\mu \partial_\mu \Psi_e^R$$

↳ GAUGE SYMMETRY: GAUGE GROUP $SU(2)$ WEAK ISOSPIN (t, t_3)

	t	t_3	
$\begin{pmatrix} \nu_e^L \\ e^L \end{pmatrix}$	$1/2$	$+1/2$	} ← WEAK ISOSPIN DOUBLET
	$1/2$	$-1/2$	
e^R	0	0	← WEAK ISOSPIN SINGLET

REQUIRE THEORY TO BE INVARIANT UNDER

$$\boxed{\Psi_e^L \xrightarrow{SU(2)} \exp \{ i g \theta_i(x) t_i \} \Psi_e^L} \equiv U \Psi_e^L$$

$U \in SU(2)$

$$\Downarrow [t_i, t_j] = i \epsilon_{ijk} t_k \quad t_i = \frac{\tau_i}{2}$$

INVARIANCE OF \mathcal{L}_0 REQUIRES INTRODUCTION OF 3 VECTOR (GAUGE) FIELDS W_i^μ ($i=1,2,3$)

$$\boxed{\partial^\mu \Psi_e^L \Rightarrow \left(\partial^\mu + i g t_i W_i^\mu \right) \Psi_e^L}$$

SUCH THAT $\equiv \mathcal{D}^\mu \quad \mathbb{W}^\mu \equiv t_i W_i^\mu$

$$\mathcal{D}^\mu \Psi_e^L \xrightarrow{SU(2)} \exp \{ i g \theta_i t_i \} \mathcal{D}^\mu \Psi_e^L \equiv U (\mathcal{D}^\mu \Psi_e^L)$$

$$\bar{\Psi}_e^L \gamma_\mu (\mathcal{D}^\mu \Psi_e^L) \xrightarrow{SU(2)} \bar{\Psi}_e^L \gamma_\mu (\mathcal{D}^\mu \Psi_e^L)$$

LEADS TO TF. PROPERTY OF GAUGE FIELDS

$$U (\mathcal{D}^\mu \Psi_e^L) = \partial^\mu (U \Psi_e^L) + i g \mathbb{W}^\mu (U \Psi_e^L)$$

||

$$U \left[\partial^\mu \Psi_e^L + i g \mathbb{W}^\mu \Psi_e^L \right]$$

$$U \left[\partial^\mu \underline{\Psi}_e^L + ig \underline{W}^\mu \underline{\Psi}_e^L \right]$$

$$= (\partial^\mu U) \underline{\Psi}_e^L + U (\partial^\mu \underline{\Psi}_e^L) + ig \underline{W}^\mu (U \underline{\Psi}_e^L)$$

$$\Downarrow$$

$$\underline{W}^{\prime\mu} U = U \underline{W}^\mu + \frac{i}{g} (\partial^\mu U)$$

$$\underline{W}^\mu \xrightarrow{SU(2)} \underline{W}^{\prime\mu} = U \underline{W}^\mu U^{-1} + \frac{i}{g} (\partial^\mu U) U^{-1}$$

↓ INFINITESIMAL

$$U \approx 1 + ig \theta_i t_i$$

$$t_i W_i^{\prime\mu} = t_i W_i^\mu + ig \underbrace{[t_j, t_k]}_{i \epsilon_{ijk} t_i} \theta_j W_k^\mu$$

$$+ \frac{i}{g} ig (\partial^\mu \theta_i) t_i$$

$$W_i^{\prime\mu} = W_i^\mu - (\partial^\mu \theta_i) - g \epsilon_{ijk} \theta_j W_k^\mu$$

↑
AS IN
QED

↑
NEW TERM
DUE TO
NON-ABELIAN
GAUGE GROUP
SU(2)

$$\begin{aligned}
 \hookrightarrow \mathcal{L}_0 + \mathcal{L}_{\text{INT}} &= \bar{\Psi}_\ell^L i \gamma^\mu \mathcal{D}_\mu \Psi_\ell^L + \bar{\Psi}_\ell^R i \gamma^\mu \partial_\mu \Psi_\ell^R \\
 &= \mathcal{L}_0 - g \bar{\Psi}_\ell^L \gamma^\mu t_i \Psi_\ell^L W_{i\mu}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}_{\text{INT}} = -g \left\{ \bar{\Psi}_\ell^L \gamma^\mu t_1 \Psi_\ell^L W_{1\mu} + \bar{\Psi}_\ell^L \gamma^\mu t_2 \Psi_\ell^L W_{2\mu} \right. \\
 \left. + \bar{\Psi}_\ell^L \gamma^\mu t_3 \Psi_\ell^L W_{3\mu} \right\}
 \end{aligned}$$

INTRODUCE CHARGED GAUGE FIELDS

$$\begin{aligned}
 \left\| \begin{aligned}
 W^\mu &\equiv \frac{1}{\sqrt{2}} (W_1^\mu - i W_2^\mu) && \text{ANNIHILATES } W^+ \\
 &&& \text{CREATES } W^- \\
 W^{\mu+} &\equiv \frac{1}{\sqrt{2}} (W_1^\mu + i W_2^\mu) && \text{ANNIHILATES } W^- \\
 &&& \text{CREATES } W^+
 \end{aligned} \right.
 \end{aligned}$$

↓

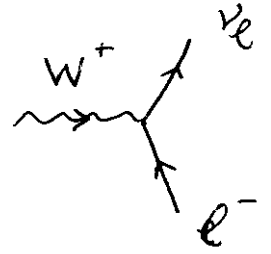
$$W_1^\mu = \frac{1}{\sqrt{2}} (W^\mu + W^{\mu+})$$

$$W_2^\mu = \frac{i}{\sqrt{2}} (W^\mu - W^{\mu+})$$

$$\begin{aligned}
 \mathcal{L}_{\text{INT}} = -g \left\{ \bar{\Psi}_\ell^L \gamma^\mu (t_1 + i t_2) \Psi_\ell^L W_{\mu} \frac{1}{\sqrt{2}} \right. \\
 + \bar{\Psi}_\ell^L \gamma^\mu (t_1 - i t_2) \Psi_\ell^L W_{\mu}^+ \frac{1}{\sqrt{2}} \\
 \left. + \bar{\Psi}_\ell^L \gamma^\mu t_3 \Psi_\ell^L W_{3\mu} \right\}
 \end{aligned}$$

$$t_{\pm} = t_1 \pm i t_2$$

$$\bar{\Psi}_e^L \gamma^\mu t_+ \Psi_e^L = \bar{\Psi}_{\nu_e}^L \gamma^\mu \Psi_e^L$$



$$\bar{\Psi}_e^L \gamma^\mu t_- \Psi_e^L = \bar{\Psi}_e^L \gamma^\mu \Psi_{\nu_e}^L$$

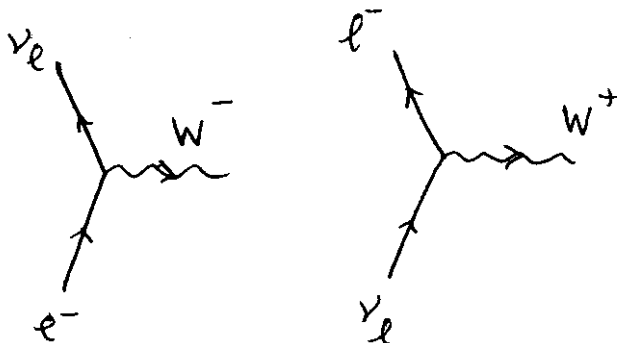
$$= \bar{\Psi}_e \frac{1}{2} (1 + \gamma_5) \gamma^\mu \frac{1}{2} (1 - \gamma_5) \Psi_{\nu_e}$$

$$= \frac{1}{2} \bar{\Psi}_e \gamma^\mu (1 - \gamma_5) \Psi_{\nu_e}$$

∴

$$\begin{aligned} \mathcal{L}_{INT}^{SU(2)} = & - \frac{g}{2\sqrt{2}} \bar{\Psi}_{\nu_e} \gamma^\mu (1 - \gamma_5) \Psi_e W_\mu \\ & - \frac{g}{2\sqrt{2}} \bar{\Psi}_e \gamma^\mu (1 - \gamma_5) \Psi_{\nu_e} W_\mu^+ \\ & - \frac{g}{2(2)} \left\{ \bar{\Psi}_{\nu_e} \gamma^\mu (1 - \gamma_5) \Psi_{\nu_e} - \bar{\Psi}_e \gamma^\mu (1 - \gamma_5) \Psi_e \right\} W_{3\mu} \end{aligned}$$

FIRST 2 TERMS DESCRIBE CHARGED WEAK INTERACTION



FEYNMAN RULE

$$- i \frac{g}{2\sqrt{2}} \gamma^\mu (1 - \gamma_5)$$

• WEAK HYPERCHARGE : $U(1)_Y$

2 NEUTRAL CURRENTS

$$\rightarrow J_3^\mu = \bar{\Psi}_e^L \gamma^\mu t_3 \Psi_e^L \rightsquigarrow \text{COUPLES TO } W_3$$

$$= \frac{1}{2} \bar{\nu}_e^L \gamma^\mu \nu_e^L - \frac{1}{2} \bar{e}^L \gamma^\mu e^L$$

$$\rightarrow J_{em}^\mu = \bar{\nu}_e^L \gamma^\mu \nu_e^L + \bar{e}^R \gamma^\mu e^R \rightsquigarrow \text{COUPLES TO } \gamma$$

INTRODUCE 'WEAK HYPERCHARGE' Y SUCH THAT

$$Q = t_3 + \frac{1}{2} Y$$

\uparrow \uparrow \uparrow
 ELECTRIC WEAK WEAK
 CHARGE ISOSPIN HYPERCHARGE

	t	t_3	Y	Q
$\begin{pmatrix} \nu_e^L \\ e^L \end{pmatrix}$	$1/2$	$+1/2$	-1	0
e^R	0	0	-2	-1

$$\frac{1}{2} J_Y^\mu = J_{em}^\mu - J_3^\mu$$

↳ INVARIANCE OF \mathcal{L} UNDER $U(1)_Y$

$$\Psi_e \xrightarrow{U(1)_Y} \exp \left\{ i g' \chi(x) \frac{Y}{2} \right\} \Psi_e$$

⇓

$$\partial^\mu \Psi_e \Rightarrow \left(\partial^\mu + i g' \frac{Y}{2} B^\mu \right) \Psi_e$$

↑
GAUGE FIELD

$$B^\mu \xrightarrow{U(1)} B^\mu - \partial^\mu \chi$$

$$\mathcal{L}_{\text{INT}} \xrightarrow{U(1)_Y} - g' \left\{ \bar{\Psi}_e^L \gamma^\mu \left(\frac{-1}{2} \right) \Psi_e^L + \bar{\Psi}_e^L \gamma^\mu \left(-\frac{1}{2} \right) \Psi_e^L + \bar{\Psi}_e^R \gamma^\mu \left(-\frac{2}{2} \right) \Psi_e^R \right\} B_\mu$$

↳ NEUTRAL CURRENT

$$\mathcal{L}_{\text{INT}}^{\text{NC}} = - g \bar{\Psi}_e^L \gamma^\mu t_3 \Psi_e^L W_{3\mu} - g' \bar{\Psi}_e^L \gamma^\mu \left(\frac{Y}{2} \right) \Psi_e^L B_\mu - g' \bar{\Psi}_e^R \gamma^\mu \left(\frac{Y}{2} \right) \Psi_e^R B_\mu$$

↓

REPLACE $\frac{1}{2} Y = Q - t_3$

$$\mathcal{L}_{\text{INT}}^{\text{NC}} = - \bar{\Psi}_e^L \gamma^\mu t_3 \Psi_e^L (g W_{3\mu} - g' B_\mu) - g' B_\mu \underbrace{(\bar{\Psi}_e^L \gamma^\mu Q \Psi_e^L + \bar{\Psi}_e^R \gamma^\mu Q \Psi_e^R)}$$

$$\bar{\Psi}_e \gamma^\mu Q \Psi_e \equiv J_{em}^\mu$$

↑
E.M. CURRENT

$$\mathcal{L}_{\text{INT}}^{\text{NC}} = - \bar{\Psi}_e^L \gamma^\mu t_3 \Psi_e^L (g W_{3\mu} - g' B_\mu) - J_{em}^\mu g' B_\mu$$

PHYSICAL GAUGE BOSON FIELDS (γ, Z^0)

ARE LINEAR COMBINATIONS OF (W_3, B)

SUCH THAT $g W_{3\mu} - g' B_\mu$ DOES NOT CONTAIN γ

(BECAUSE IT COUPLES TO NEUTRINO)

→ ELECTROWEAK UNIFICATION

• **ELECTROWEAK UNIFICATION.**

GLASHOW-WEINBERG-SALAM
MODEL (1967, 1968)

↳ PHYSICAL GAUGE BOSON FIELDS ARE
MIXTURE OF W_3, B

↓
MIXING ANGLE : WEINBERG ANGLE θ_W

$$\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

→ PHYSICAL GAUGE BOSON FIELDS

↳ $g W_{3\mu} - g' B_\mu$ SHOULD NOT CONTAIN A_μ

$$= (g \cos \theta_W + g' \sin \theta_W) Z_\mu + \underbrace{(g \sin \theta_W - g' \cos \theta_W)}_{= 0} A_\mu$$

$$g \sin \theta_W = g' \cos \theta_W$$

$$\tan \theta_W = \frac{g'}{g}$$

↳ $g' B_\mu$

$$= -g' \sin \theta_W Z_\mu + \underbrace{g' \cos \theta_W}_{\text{wavy}} A_\mu$$

$g' B_\mu$ COUPLES TO E.M. CURRENT \Rightarrow

$$g' \cos \theta_W = e$$

↳ \mathcal{L}_{INT}^{NC} IN TERMS OF A^μ & Z^μ

$$\mathcal{L}_{INT}^{NC} = - \bar{\Psi}_e^L \gamma^\mu t_3 \Psi_e^L \left(\frac{g}{\cos \theta_W} \right) Z_\mu$$

$$- J_{em}^\mu \left(- \frac{g}{\cos \theta_W} \sin^2 \theta_W \right) Z_\mu - e J_{em}^\mu A_\mu$$

↓

$$\mathcal{L}_{INT}^{NC} = - e J_{em}^\mu A_\mu$$

$$- \frac{g}{\cos \theta_W} \left(J_3^\mu - \sin^2 \theta_W J_{em}^\mu \right) Z_\mu$$

$$J_{NC}^\mu$$

↑

NEUTRAL WEAK CURRENT

1^o TERM : E.M. INTERACTION

2^o TERM : NEUTRAL WEAK INTERACTION

↓

NEUTRAL WEAK CURRENT J_{NC}^μ

$$J_{NC}^\mu = \frac{1}{4} \bar{\nu}_{\nu_e} \gamma^\mu (1 - \gamma_5) \nu_{\nu_e}$$

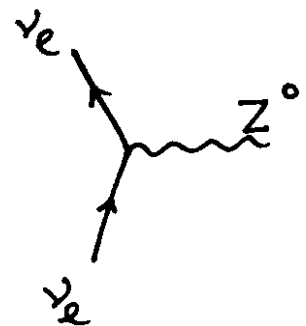
$$- \frac{1}{4} \bar{\nu}_e \gamma^\mu (1 - \gamma_5) \nu_e + \sin^2 \theta_W \bar{\nu}_e \gamma^\mu \nu_e$$

↓

(LEPTON HAS NEGATIVE CHARGE)

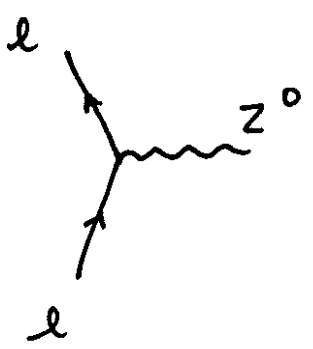
$$J_{NC}^{\mu} = \frac{1}{4} \bar{\Psi}_{\nu_e} \gamma^{\mu} (1 - \gamma_5) \Psi_{\nu_e} - \frac{1}{4} \bar{\Psi}_e \gamma^{\mu} \left([1 - 4 \sin^2 \theta_W] - \gamma_5 \right) \Psi_e$$

↳ FEYNMAN RULES : NEUTRAL WEAK INTERACTION



$$- i \frac{g}{4 \cos \theta_W} \gamma^{\mu} (1 - \gamma_5)$$

(V-A)



$$+ i \frac{g}{4 \cos \theta_W} \gamma^{\mu} \left([1 - 4 \sin^2 \theta_W] - \gamma_5 \right)$$

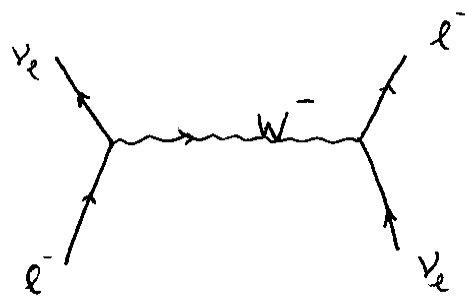
ELECTROWEAK INTERACTIONS

DESCRIBED BY 1 COUPLING : e
 & 1 MIXING ANGLE $\sin \theta_W \Rightarrow$ TO BE DETERMINED BY EXPERIMENT

g & g' FOLLOW FROM

$$g \sin \theta_W = g' \cos \theta_W = e$$

⇒ GAUGE BOSON MASSES



$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{2\sqrt{2}} \right)^2 \cdot \frac{1}{M_W^2} = \frac{g^2}{8M_W^2}$$

$$\downarrow \quad g = \frac{e}{\sin \theta_W}$$

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 \sin^2 \theta_W} \cdot \frac{1}{M_W^2}$$

↓ G_F KNOWN FROM m β -DECAY

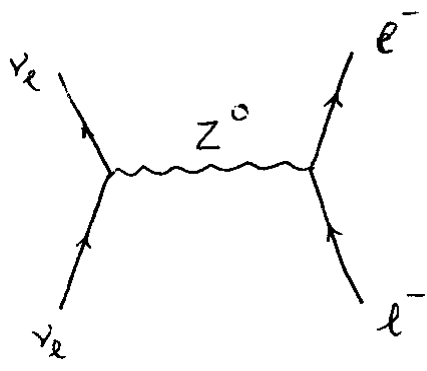
$$M_{W^\pm}^2 = \frac{e^2 \sqrt{2}}{8 G_F \sin^2 \theta_W}$$

PREDICTED ONCE $\sin^2 \theta_W$
IS KNOWN!

PRESENT DAY VALUE

$$M_{W^\pm} \approx 80.423 \pm 0.039 \text{ GeV}$$

OBSERVED CERN (1983)



$$\begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & -\sin \theta_W \\ \sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix}$$

$$\begin{cases} Z^\mu = \cos \theta_W W_3^\mu - \sin \theta_W B^\mu \\ A^\mu = \sin \theta_W W_3^\mu + \cos \theta_W B^\mu \end{cases}$$

BOSONS : FIELD EQ. INVOLVE SQUARE OF MASSES

INTERFERENCE
↓

$$M_Z^2 = \cos^2 \theta_W M_W^2 + \sin^2 \theta_W M_B^2 - 2 \sin \theta_W \cos \theta_W M_{BW}^2$$

$$0 = M_Y^2 = \sin^2 \theta_W M_W^2 + \cos^2 \theta_W M_B^2 + 2 \sin \theta_W \cos \theta_W M_{BW}^2$$

BOTH STATES ARE ORTHOGONAL

$$\rightarrow 0 = \sin \theta_W \cos \theta_W (M_W^2 - M_B^2) + M_{BW}^2 (\cos^2 \theta_W - \sin^2 \theta_W)$$

↓ ELIMINATE M_{BW}²

$$\bullet M_{BW}^2 = (M_B^2 - M_W^2) \frac{\sin \theta_W \cos \theta_W}{\cos^2 \theta_W - \sin^2 \theta_W}$$

$$\bullet (1) + (2) \quad M_Z^2 = M_W^2 + M_B^2$$

$$\cos^2 \theta_W M_Z^2 = (\cos^2 \theta_W - \sin^2 \theta_W) M_W^2 - \frac{2 \sin^2 \theta_W \cos^2 \theta_W}{(\cos^2 \theta_W - \sin^2 \theta_W)} (M_Z^2 - 2 M_W^2)$$

↓

$$\underbrace{(\cos^4 \theta_W + \sin^2 \theta_W \cos^2 \theta_W)}_{\cos^2 \theta_W} M_Z^2 = (\cos^4 \theta_W + \sin^4 \theta_W + 2 \sin^2 \theta_W \cos^2 \theta_W) M_W^2$$

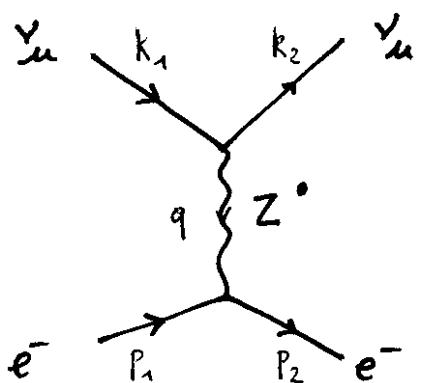
$$M_Z^2 = \frac{M_W^2}{\cos^2 \theta_W}$$

$$\underline{\underline{M_Z^0 \approx 91.1876 \pm 0.0021 \text{ GeV}}}$$

⇒ DETERMINATION OF $\sin^2 \theta_W$

- $\nu_\mu e^- \rightarrow \nu_\mu e^-$ ELASTIC SCATTERING

↳ CAN ONLY PROCEED VIA NEUTRAL CURRENT WEAK INTERACTION



$$\rightarrow -\frac{i g}{4 \cos \theta_W} \gamma^\mu (1 - \gamma_5)$$

$$\rightarrow \frac{i g}{4 \cos \theta_W} \gamma^\mu (c_V - \gamma_5)$$

$$s \equiv (k_1 + p_1)^2$$

$$t \equiv (k_1 - k_2)^2 \ll M_Z^2$$

+ NEGLECT e^- MASS

$$\underline{c_V \equiv 1 - 4 \sin^2 \theta_W}$$

$$\Rightarrow \mathcal{M} \approx \frac{i g^2}{16 \cos^2 \theta_W} \frac{1}{M_Z^2} \cdot \bar{U}(k_2, s'_\nu) \gamma^\mu (1 - \gamma_5) U(k_1, s_\nu) \cdot \bar{U}(p_2, s'_e) \gamma_\mu (c_V - \gamma_5) U(p_1, s_e)$$

$$\downarrow M_Z^2 \cos^2 \theta_W = M_W^2$$

$$\frac{g^2}{8 M_W^2} = \frac{G_F}{\sqrt{2}}$$

$$\mathcal{M} = i \frac{G_F}{2\sqrt{2}} \cdot \bar{U}(k_2, s'_\nu) \gamma^\mu (1 - \gamma_5) U(k_1, s_\nu) \cdot \bar{U}(p_2, s'_e) \gamma_\mu (c_V - \gamma_5) U(p_1, s_e)$$

$$\Rightarrow \frac{1}{2} \sum_{s_\nu s_e} \sum_{s'_\nu s'_e} |\mathcal{M}|^2$$

$$\stackrel{\substack{\text{ONLY} \\ 1\nu \\ \text{SPIN STATE}}}{=} \frac{G_F^2}{4} \cdot \frac{1}{4} \text{Tr} \left\{ \gamma^\mu (1 - \gamma_5) \not{k}_1 \gamma^\nu (1 - \gamma_5) \not{k}_2 \right\}$$

$$\cdot \text{Tr} \left\{ \gamma_\mu (c_V - \gamma_5) \not{P}_1 \gamma_\nu (c_V - \gamma_5) \not{P}_2 \right\}$$

$$= \frac{G_F^2}{4} \cdot \text{Tr} \left\{ (1 + \gamma_5) \gamma^\mu \not{k}_1 \gamma^\nu \not{k}_2 \right\}$$

$$\cdot \text{Tr} \left\{ \left[\frac{1}{2} (c_V^2 + 1) + \gamma_5 c_V \right] \gamma_\mu \not{P}_1 \gamma_\nu \not{P}_2 \right\}$$

$$= 4 G_F^2 \left\{ k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - (k_1 \cdot k_2) g^{\mu\nu} + i \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \right\}$$

$$\cdot \left\{ \frac{1}{2} (1 + c_V^2) \left[P_{1\mu} P_{2\nu} + P_{1\nu} P_{2\mu} - P_1 \cdot P_2 g_{\mu\nu} \right] \right.$$

$$\left. + c_V i \epsilon_{\mu\nu\gamma\delta} P_1^\gamma P_2^\delta \right\}$$

$$= 4 G_F^2 \left\{ \frac{1}{2} (1 + c_V^2) 2 \left[(k_1 \cdot P_1) (k_2 \cdot P_2) + (k_1 \cdot P_2) (k_2 \cdot P_1) \right] \right.$$

$$\left. + c_V 2 \left[(k_1 \cdot P_1) (k_2 \cdot P_2) - (k_1 \cdot P_2) (k_2 \cdot P_1) \right] \right\}$$

$$\downarrow$$

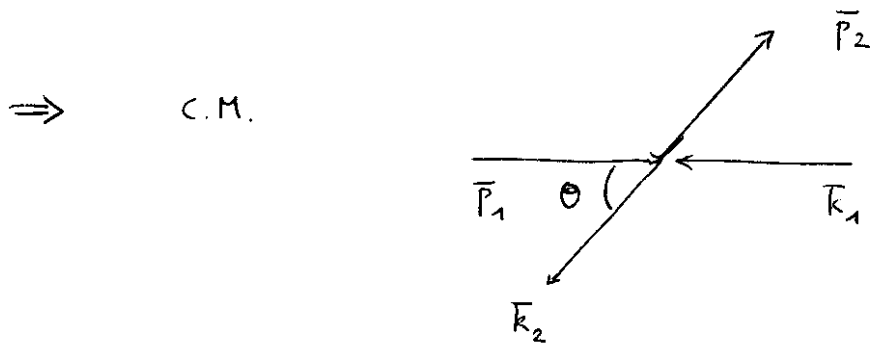
$$\Rightarrow = 2 k_1 \cdot P_1 = 2 k_2 \cdot P_2.$$

$$\frac{P_1 \cdot k_2}{P_1 \cdot k_1} = \frac{P_2 \cdot k_1}{P_2 \cdot k_2} \equiv 1 - Y$$

\hookrightarrow DIMENSIONLESS

$$= 2 G_F^2 s^2 \left\{ \frac{1}{2} (1 + c_V^2) [1 + (1 - y)^2] + c_V [1 - (1 - y)^2] \right\}$$

$$= G_F^2 s^2 \left\{ (1 + c_V)^2 + (1 - c_V)^2 (1 - y)^2 \right\}$$



$$|\bar{p}_1| = |\bar{p}_2| = |\bar{k}_1| = |\bar{k}_2| = \frac{\sqrt{s}}{2}$$

$$\frac{p_1 \cdot k_2}{p_1 \cdot k_1} = \frac{1}{2} (1 + \cos \theta) \Rightarrow \frac{1}{2} d \cos \theta = dy$$

$$d\sigma = \frac{1}{2 |\bar{p}_1| 2 |\bar{k}_1| \cdot 2} \cdot \frac{d^3 \bar{k}_2}{(2\pi)^3 2 |\bar{k}_2|} \cdot \frac{d^3 \bar{p}_2}{(2\pi)^3 2 |\bar{p}_2|} (2\pi)^4 \delta^4(p_1 + k_1 - p_2 - k_2)$$

$$\cdot \frac{1}{2} \sum |\mathcal{M}|^2$$

$$= \frac{1}{2s} \frac{1}{(2\pi)^2} \frac{d^3 \bar{k}_2}{s} \underbrace{\delta(\sqrt{s} - 2|\bar{k}_2|)}_{\frac{1}{2} \delta(|\bar{k}_2| - \frac{\sqrt{s}}{2})} \cdot \frac{1}{2} \sum |\mathcal{M}|^2$$

$$d\sigma = \frac{1}{2s} \frac{1}{(2\pi)^2} \frac{1}{8} d\cos\theta d\phi$$

$$\cdot G_F^2 s^2 \left\{ (1+c_V)^2 + (1-c_V)^2 (1-Y)^2 \right\}$$

$$\Downarrow$$

$$e^- \bar{\nu}_\mu \rightarrow e^- \nu_\mu$$

$$\frac{d\sigma}{dy} = \frac{G_F^2 s}{16\pi} \left\{ (1+c_V)^2 + (1-c_V)^2 (1-Y)^2 \right\}$$

$$\text{FOR } e^- \bar{\nu}_\mu \rightarrow e^- \bar{\nu}_\mu : c_V \rightarrow -c_V$$

$$\Rightarrow \sin^2 \theta_W \approx 0.23113 \pm 6.5 \times 10^{-5}$$

⇒ QUARK ELECTROWEAK CURRENTS : $SU(2) \times U(1)$

• MULTIPLETS FOR $SU(2) \times U(1)$

$$\hookrightarrow \Psi_{q_1}^L = \begin{pmatrix} u^L \\ d_c^L \end{pmatrix} \quad \text{WEAK ISOSPIN DOUBLET}$$

↑
1^o FAMILY

$$\Psi_{q_2}^L = \begin{pmatrix} c^L \\ s_c^L \end{pmatrix}$$

↑
2^o FAMILY

$$\text{WITH } \begin{pmatrix} d_c^L \\ s_c^L \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d^L \\ s^L \end{pmatrix}$$

↑ WEAK INTERACTION EIGENSTATES ↑ STRONG INTERACTION (MASS) EIGENSTATES

$$\rightarrow \theta_c : \text{CABIBBO ANGLE} \quad \cos \theta_c \approx 0.975$$

$$\sin \theta_c \approx 0.221$$

→ FOR 3 FAMILIES OF QUARKS : CKM MATRIX

↳ RIGHT HANDED QUARKS ARE SINGLET

u^R

d^R

• QUANTUM NUMBERS

$$Q = t_3 + \frac{1}{2} Y \quad \left(\frac{1}{2} Y \equiv Q - t_3 \right)$$

\uparrow \uparrow \uparrow
 ELECTRIC WEAK WEAK
 CHARGE ISOSPIN HYPERCHARGE

	t	t_3	Y	Q
$\begin{pmatrix} u^L \\ d^L \end{pmatrix}$	$1/2$	$+1/2$	$+1/3$	$+2/3$
u^R	0	0	$+4/3$	$+2/3$
d^R	0	0	$-2/3$	$-1/3$

• WEAK ISOSPIN SU(2)

$$\mathcal{L}_0 = \bar{\Psi}_q^L i \gamma^\mu \partial_\mu \Psi_q^L + \bar{U}^R i \gamma^\mu \partial_\mu U^R + \bar{d}^R i \gamma^\mu \partial_\mu d^R$$

MINIMAL SUBSTITUTION

$$\Downarrow \quad \partial_\mu \Rightarrow \underline{\underline{\mathbb{D}_\mu \equiv \partial_\mu + i g t_i W_{i\mu}}}$$

$$\mathcal{L}_{INT}^{SU(2)} = -g \bar{\Psi}_q^L \gamma^\mu t_i \Psi_q^L W_{i\mu}$$

INTRODUCE W^μ & $W^{\mu\dagger}$ FIELDS
 THAT ANNIHILATE / CREATE W^\pm

$$W^\mu \equiv \frac{1}{\sqrt{2}} (W_1^\mu - i W_2^\mu)$$

$$W^{\mu\dagger} \equiv \frac{1}{\sqrt{2}} (W_1^\mu + i W_2^\mu)$$

↓

$$W_1^\mu = \frac{1}{\sqrt{2}} (W^\mu + W^{\mu\dagger})$$

$$W_2^\mu = \frac{i}{\sqrt{2}} (W^\mu - W^{\mu\dagger})$$

$$\begin{aligned} \hookrightarrow \mathcal{L}_{\text{INT}}^{SU(2)} &= -\frac{g}{\sqrt{2}} \bar{\Psi}_q^L \gamma^\mu (t_1 + i t_2) \Psi_q^L W_\mu \\ &\quad - \frac{g}{\sqrt{2}} \bar{\Psi}_q^L \gamma^\mu (t_1 - i t_2) \Psi_q^L W_\mu^\dagger \\ &\quad - g \bar{\Psi}_q^L \gamma^\mu t_3 \Psi_q^L W_{3\mu} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{INT}}^{SU(2)} &= -\frac{g}{\sqrt{2}} \bar{U}^L \gamma^\mu d_c^L W_\mu - \frac{g}{\sqrt{2}} \bar{d}_c^L \gamma^\mu U^L W_\mu^\dagger \\ &\quad - g \bar{\Psi}_q^L \gamma^\mu t_3 \Psi_q^L W_{3\mu} \end{aligned}$$

FIRST 2 TERMS ARE CHARGED CURRENT WEAK INTERACTION

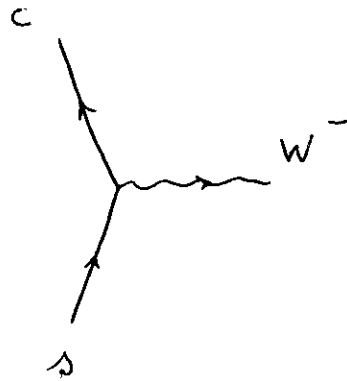
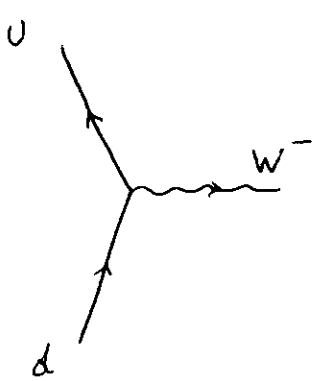
NOTE: $\bar{U}^L \gamma^\mu d_c^L = \frac{1}{2} \underbrace{\bar{U} \gamma^\mu (1 - \gamma_5)}_{V-A} d_c$

• FEYNMAN RULES :

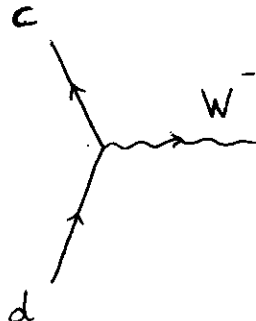
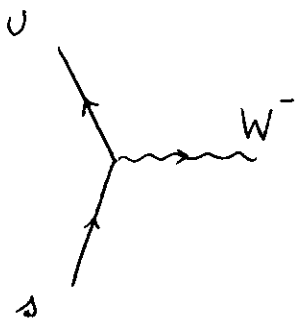
CHARGED CURRENT WEAK INTERACTION

$$d_c = \cos \theta_c d + \sin \theta_c s$$

$$s_c = -\sin \theta_c d + \cos \theta_c s$$



$$-\frac{i g}{2\sqrt{2}} \cdot \cos \theta_c \cdot \gamma^\mu (1 - \gamma_5)$$



$$-\frac{i g}{2\sqrt{2}} (\pm \sin \theta_c) \cdot \gamma^\mu (1 - \gamma_5)$$

(+sin θ_c)

(-sin θ_c)

o_c CC INTERACTION IS

1/ OF TYPE V-A

2/ FLAVOR CHANGING

• WEAK HYPERCHARGE U(1)

$$\mathcal{L}_0 = \bar{\Psi}_q^L i \gamma^\mu \partial_\mu \Psi_q^L + \bar{U}^R i \gamma^\mu \partial_\mu U^R + \bar{d}^R i \gamma^\mu \partial_\mu d^R$$

$$\Downarrow \quad \partial_\mu \Rightarrow \underline{\underline{D_\mu = \partial_\mu + ig t_i W_{i\mu} + ig' \frac{Y}{2} B_\mu}}$$

$\begin{matrix} \uparrow & \uparrow \\ SU(2) & U(1) \end{matrix}$

$$\mathcal{L}_{INT}^{U(1)} = \left\{ -g' \bar{\Psi}_q^L \gamma^\mu \underbrace{\frac{Y}{2}}_{\frac{1}{6}} \Psi_q^L - g' \bar{U}^R \gamma^\mu \underbrace{\frac{Y}{2}}_{+\frac{2}{3}} U^R - g' \bar{d}^R \gamma^\mu \underbrace{\frac{Y}{2}}_{-\frac{1}{3}} d^R \right\} B_\mu$$

NEUTRAL CURRENT

$$\mathcal{L}_{INT}^{NC} = -g \bar{\Psi}_q^L \gamma^\mu t_3 \Psi_q^L W_{3\mu} - g' \bar{\Psi}_q^L \gamma^\mu \frac{Y}{2} \Psi_q^L B_\mu - g' \bar{U}^R \gamma^\mu \frac{Y}{2} U^R B_\mu - g' \bar{d}^R \gamma^\mu \frac{Y}{2} d^R B_\mu$$

\Downarrow REPLACE $\frac{Y}{2} = Q - t_3$

$$\mathcal{L}_{INT}^{NC} = - \bar{\Psi}_q^L \gamma^\mu t_3 \Psi_q^L (g W_{3\mu} - g' B_\mu) - g' B_\mu (\bar{U}^L \gamma^\mu Q U^L + \bar{U}^R \gamma^\mu Q U^R + \bar{d}^L \gamma^\mu Q d^L + \bar{d}^R \gamma^\mu Q d^R)$$

$$J_{em}^\mu = \underbrace{\bar{U} \gamma^\mu Q U}_{+\frac{2}{3}} + \underbrace{\bar{d} \gamma^\mu Q d}_{-\frac{1}{3}}$$

$$\mathcal{L}_{\text{INT}}^{\text{NC}} = - \bar{\Psi}_q^L \gamma^\mu t_3 \Psi_q^L (g W_{3\mu} - g' B_\mu) - g' B_\mu J_{em}^\mu$$

INTRODUCE Z^0, γ FIELDS

$$\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

$$g \sin \theta_W = g' \cos \theta_W = e$$

$$\begin{aligned} \hookrightarrow g W_{3\mu} - g' B_\mu &= (g \cos \theta_W + g' \sin \theta_W) Z_\mu \\ &= \frac{g}{\cos \theta_W} Z_\mu \end{aligned}$$

$$\begin{aligned} \hookrightarrow g' B_\mu &= -g' \sin \theta_W Z_\mu + g' \cos \theta_W A_\mu \\ &= -\frac{g}{\cos \theta_W} \sin^2 \theta_W Z_\mu + e A_\mu \end{aligned}$$

∴

$$\mathcal{L}_{\text{INT}}^{\text{NC}} = - e A_\mu J_{em}^\mu - \frac{g}{\cos \theta_W} (J_3^\mu - \sin^2 \theta_W J_{em}^\mu) Z_\mu$$

WITH

$$J_{em}^\mu = +\frac{2}{3} \bar{u} \gamma^\mu u - \frac{1}{3} \bar{d} \gamma^\mu d$$

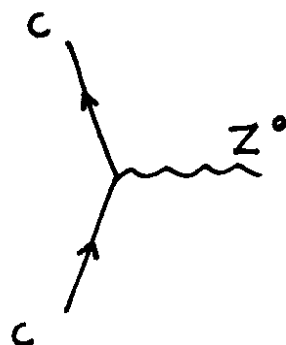
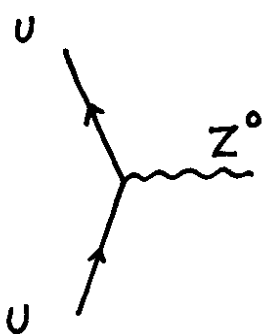
$$J_3^\mu = \frac{1}{4} \bar{u} \gamma^\mu (1 - \gamma_5) u - \frac{1}{4} \bar{d} \gamma^\mu (1 - \gamma_5) d$$

$$J_3^\mu - \sin^2 \theta_W J_{em}^\mu$$

$$= \frac{1}{4} \bar{u} \gamma^\mu \left(\left[1 - \frac{8}{3} \sin^2 \theta_W \right] - \gamma_5 \right) u$$

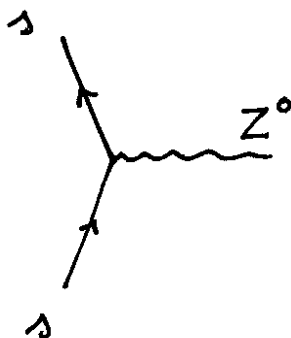
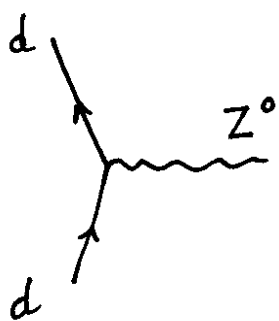
$$- \frac{1}{4} \bar{d} \gamma^\mu \left(\left[1 - \frac{4}{3} \sin^2 \theta_W \right] - \gamma_5 \right) d$$

• FEYNMAN RULES: NEUTRAL CURRENT WEAK INTERACTION



$$-i \frac{g}{4 \cos \theta_W} \gamma^\mu \left(\left[1 - \frac{8}{3} \sin^2 \theta_W \right] - \gamma_5 \right)$$

$$V/A \approx 38\%$$



$$+i \frac{g}{4 \cos \theta_W} \gamma^\mu \left(\left[1 - \frac{4}{3} \sin^2 \theta_W \right] - \gamma_5 \right)$$

$$V/A \approx 69\%$$

∴ NC WEAK INTERACTION IS
FLAVOR CONSERVING \checkmark
0

⇒ HIGGS MECHANISM

• PROBLEM

$SU(2) \times U(1)_Y$ GAUGE SYMMETRY OF STANDARD ELECTROWEAK THEORY

↳ 4 GAUGE BOSONS: γ , W^+ , W^- , Z^0

MANIFEST GAUGE SYMM REQUIRES GAUGE BOSONS TO BE MASSLESS

BUT EXPERIMENTALLY $M_W \approx 80.4 \text{ GeV}$

$M_Z \approx 91.2 \text{ GeV}$

⇒ ADDING MASS TERMS IN LAGRANGIAN YIELDS INCONSISTENT THEORY
UNRENORMALIZABLE DIVERGENCES APPEAR → EXPLICIT SYMM. BREAKING

⇒ HAVE TO INTRODUCE MASS WITHOUT BREAKING GAUGE INV.
SPONTANEOUS SYMMETRY BREAKING

• SPONTANEOUS SYMMETRY BREAKING (DISCRETE SYMMETRY)

CONSIDER SIMPLE EXAMPLE: SCALAR FIELD THEORY

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} \mu^2 \phi^2 - \frac{\lambda}{4} \phi^4$$

↑
INTERACTION TERM ($\lambda > 0$)

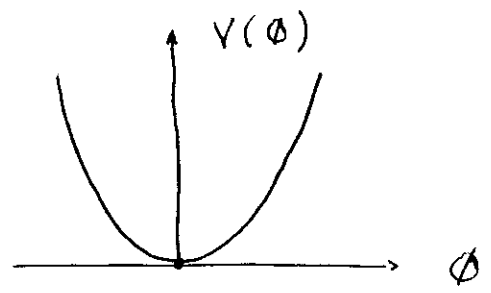
$$= T - V$$

$$V = \frac{1}{2} \mu^2 \phi^2 + \frac{\lambda}{4} \phi^4$$

\mathcal{L} HAS SYMMETRY w.r.t $\phi \rightarrow -\phi$

↳ IF $\underline{\mu^2 > 0}$

⇓
MINIMUM OF V



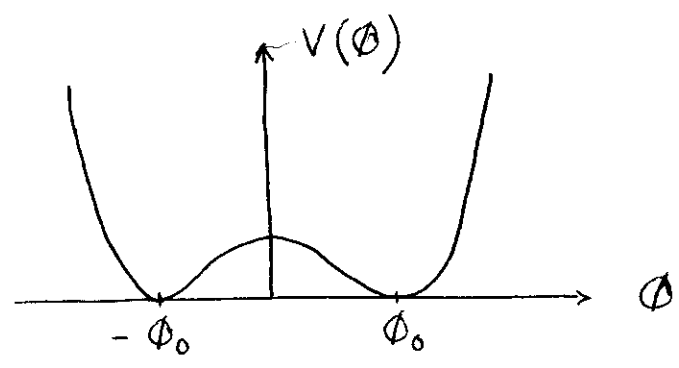
$\phi = 0$

⇒ GROUND STATE ($\phi = 0$) REFLECTS THE SYMMETRY ($\phi \rightarrow -\phi$) OF \mathcal{L}

∴ MANIFEST SYMMETRY (SYMM OF DYNAMICS = SYMM OF GROUND STATE)

↳ IF $\underline{\mu^2 < 0}$

⇓
MINIMUM OF V



$$\frac{\Delta V}{\Delta \phi} = \mu^2 \phi + \lambda \phi^3 = \phi (\mu^2 + \lambda \phi^2) = 0$$

$\phi = 0$
MAXIMUM

$\phi = \pm \phi_0 = \pm \sqrt{-\frac{\mu^2}{\lambda}}$ MINIMUM

2 DEGENERATE MINIMA / GROUND STATES $\phi_0, -\phi_0$

∴ HIDDEN SYMMETRY (SYMM. OF DYNAMICS IS NOT SYMM OF GROUND STATE)

SPONTANEOUS BREAKING OF SYMMETRY

↳ PERTURBATIVE EXPANSION AROUND $\Phi_0 = +\sqrt{-\frac{\mu^2}{\lambda}}$ SM 30

$$\Phi(x) = \Phi_0 + \eta(x)$$



$$\mathcal{L}' = \frac{1}{2} (\partial_\mu \eta) (\partial^\mu \eta) - \frac{1}{2} \mu^2 \eta^2 - \cancel{\mu^2 \Phi_0 \eta} - \frac{\lambda}{4} [\cancel{4 \Phi_0^3 \eta} + 6 \Phi_0^2 \eta^2 + 4 \Phi_0 \eta^3 + \eta^4] + \text{Const.}$$

$$\mathcal{L}' = \frac{1}{2} (\partial_\mu \eta) (\partial^\mu \eta) - \frac{1}{2} \underbrace{(-2\mu^2)}_{m_\eta^2} \eta^2 - \lambda \Phi_0 \eta^3 - \frac{\lambda}{4} \eta^4$$

↳ WE HAVE GENERATED A MASS m_η
 IN A MASSLESS THEORY
 BY REQUIRING SPONTANEOUS SYMM BREAKING

↳ PHYSICAL EXAMPLES :

FERROMAGNETISM : SPINS CAN ALIGN ALONG ONE DIRECTION

DEGENERATE GROUND STATES

↑↑↑↑↑

↓↓↓↓↓

• SPONTANEOUS BREAKING OF GLOBAL GAUGE SYMMETRY

\mathcal{L} COMPLEX SCALAR FIELD Φ , Φ^*

$$\Phi = \frac{1}{\sqrt{2}} (\Phi_1 + i\Phi_2)$$

$$\mathcal{L} = (\partial_\mu \Phi)^* (\partial^\mu \Phi) - \mu^2 \Phi^* \Phi - \lambda (\Phi^* \Phi)^2$$

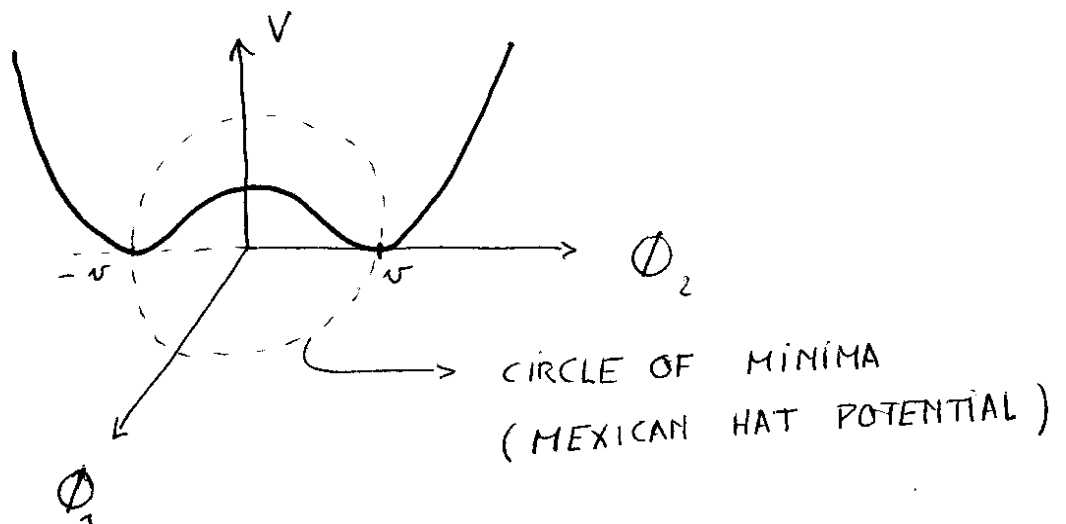
\mathcal{L} HAS GLOBAL GAUGE SYMM $U(1)$

$$\Phi \xrightarrow{U(1)} e^{i\alpha} \Phi \quad (\text{GLOBAL PHASE TF.})$$

IN TERMS OF Φ_1, Φ_2

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} (\partial_\mu \Phi_1)^2 + \frac{1}{2} (\partial_\mu \Phi_2)^2 \\ & - \left[\frac{1}{2} \mu^2 (\Phi_1^2 + \Phi_2^2) + \frac{\lambda}{4} (\Phi_1^2 + \Phi_2^2)^2 \right] \\ & \underbrace{\hspace{15em}}_{V(\Phi_1, \Phi_2)} \end{aligned}$$

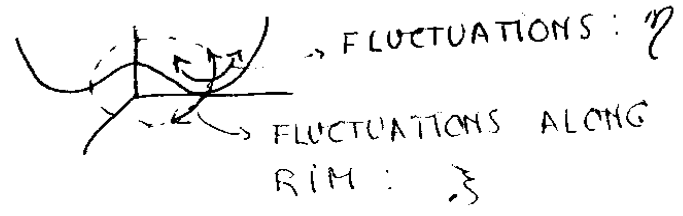
FOR $\frac{\mu^2 < 0}{\lambda > 0}$: MINIMA OF V : $\Phi_1^2 + \Phi_2^2 = v^2 = -\frac{\mu^2}{\lambda}$



↳ PERTURBATIVE EXPANSION

$$\Phi(x) = \frac{1}{\sqrt{2}} \left[v + \eta(x) + i \xi(x) \right]$$

$$v = \sqrt{-\frac{\mu^2}{\lambda}}$$



$$\begin{aligned} \mathcal{L}' &= \frac{1}{2} (\partial_\mu \xi)^2 + \frac{1}{2} (\partial_\mu \eta)^2 \\ &+ 0 - \frac{1}{2} \underbrace{(-2\mu^2)}_{m_\eta^2 = 2v^2\lambda} \eta^2 \\ &+ \text{CUBIC \& QUARTIC TERMS IN } \eta, \xi \end{aligned}$$

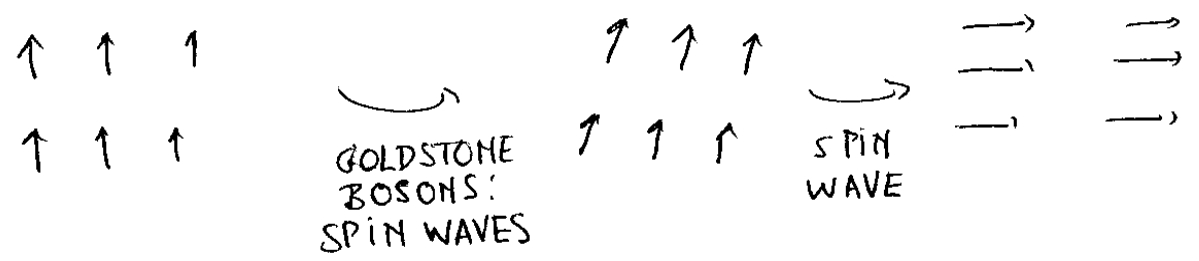
η FIELD HAS ACQUIRED MASS $m_\eta = \sqrt{-2\mu^2}$

ξ FIELD IS MASSLESS
 ↓
 FLUCTUATIONS ALONG RIM
 (FROM ONE MINIMUM TO ANOTHER)
 COST NO ENERGY
 ⇓
 GOLDSTONE BOSONS

↳ PHYSICAL EXAMPLE: FERROMAGNETISM

$$H = -\vec{S}_i \cdot \vec{S}_j$$

ROTATIONAL SYMM IS SPONTANEOUSLY BROKEN



• SPONTANEOUS SYMMETRY OF LOCAL GAUGE SYMMETRY

$\mathcal{L}_0 = (\partial_\mu \psi)^\dagger (\partial^\mu \psi) - \mu^2 \psi^\dagger \psi$ SYMMETRY . .

REQUIRE

\mathcal{L} HAS U(1) LOCAL GAUGE SYMM

$\phi(x) \xrightarrow{U(1)} e^{i\alpha(x)} \phi(x)$

$D^\mu = \partial^\mu + ie A^\mu$

$A^\mu \xrightarrow{U(1)} A^\mu - \frac{1}{e} \partial^\mu \alpha$

$\mathcal{L} = [(\partial_\mu - ie A_\mu) \phi^\dagger] [(\partial^\mu + ie A^\mu) \phi] - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

↳ FOR $\mu^2 > 0$: THIS IS QED LAGRANGIAN FOR PARTICLE WITH MASS μ + INTERACTION TERM $\lambda \phi^4$

↳ $\mu^2 < 0$: SPONTANEOUS SYMM BREAKING

$$\Phi(x) = \frac{1}{\sqrt{2}} \left[v + \eta(x) + i \xi(x) \right]$$

$$\mathcal{L}' = \frac{1}{2} (\partial_\mu \xi)^2 + \frac{1}{2} (\partial_\mu \eta)^2 - \frac{1}{2} \underbrace{(2v^2 \lambda)}_{m_\eta^2} \eta^2$$

$$+ \frac{1}{2} e^2 v^2 A_\mu A^\mu + e v A_\mu (\partial^\mu \xi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

+ INTERACTION TERMS

↳ GAUGE BOSON (A^μ FIELD)

HAS ACQUIRED A MASS $m_A = e v$!

BUT • MASSIVE SPIN 1 PARTICLE HAS 3 POLARIZATIONS (2 TRANSVERSE + 1 LONGITUDINAL)

• MASSLESS SPIN 1 PARTICLE HAS ONLY 2 TRANSVERSE POLARIZATIONS

↳ GAUGE TF. TO ELIMINATE UNPHYSICAL FIELDS ?

$$\Phi(x) \approx \frac{1}{\sqrt{2}} (v + \eta(x)) e^{i\xi/v}$$

TO LOWEST ORDER IN ξ

GAUGE
TF

$$\Phi \rightarrow \frac{1}{\sqrt{2}} (v + h(x)) e^{i\theta/v}$$

$$A^\mu \rightarrow A^\mu - \frac{1}{e} \frac{1}{v} \partial^\mu \theta(x)$$

WORK WITH h & θ , A^μ FIELDS

$$(\partial^\mu + ie A^\mu) \Phi$$


$$\begin{aligned} \mathcal{L}'' = & \frac{1}{2} (\partial_\mu h)^2 - \lambda v^2 h^2 - \lambda v h^3 - \frac{1}{4} \lambda h^4 \\ & + \frac{1}{2} e^2 v^2 A^\mu A_\mu + e^2 v A^\mu A_\mu h + \frac{1}{2} e^2 A^\mu A_\mu h^2 \\ & - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \end{aligned}$$

GOLDSTONE BOSON (§ FIELD)) HIGGS MECHANISM
 DOES NOT APPEAR IN THEORY
 HAS BEEN TRANSFORMED AWAY

PHYSICALLY : DEGREE OF FREEDOM CORRESPONDS WITH
 LONGITUDINAL POL. OF A^μ FIELD

⇒ GAUGE BOSONS HAVE ACQUIRED MASS

$$m_A = e v$$

⇒ EXCITATIONS : h 

⇓
 HIGGS FIELD : MASS $m_h^2 = 2 \lambda v^2$

(MASSIVE SCALAR PARTICLE)