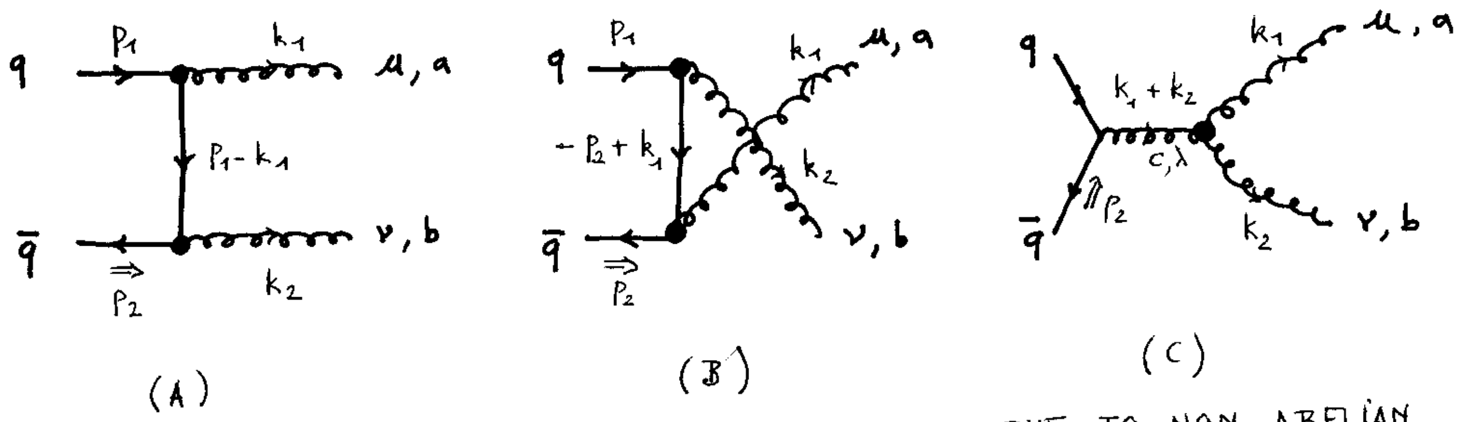
QUANTIZATION OF A NON-ABELIAN GAUGE THEORY (QCD)

$\Rightarrow$ GAUGE INVARIANCE

e.g. 2 GLUON PROCESS $q\bar{q} \to gg$



AS IN QED

DUE TO NON-ABELIAN NATURE OF GAUGE GROUP SU(3)

- $$\left(\mathcal{M}_A^{\mu\nu}\right)_{ab} = \bar{v}(P_2, s_2)\left[-ig\frac{\lambda_b}{2}\gamma^\nu\right]\frac{i(\not{P}_1 - \not{k}_1 + m)}{(P_1 - k_1)^2 - m^2}\left[-ig\frac{\lambda_a}{2}\gamma^\mu\right]u(P_1, s_1)$$

$$= -ig^2\frac{\lambda_b}{2}\frac{\lambda_a}{2}\,\bar{v}(P_2, s_2)\,\gamma^\nu\frac{(\not{P}_1 - \not{k}_1 + m)}{(P_1 - k_1)^2 - m^2}\,\gamma^\mu\,u(P_1, s_1)$$

- $$\left(\mathcal{M}_B^{\mu\nu}\right)_{ab} = -ig^2\frac{\lambda_a}{2}\frac{\lambda_b}{2}\,\bar{v}(P_2, s_2)\,\gamma^\mu\frac{(-\not{P}_2 + \not{k}_1 + m)}{(-P_2 + k_1)^2 - m^2}\,\gamma^\nu\,u(P_1, s_1)$$

- PHYSICAL AMPLITUDE

$$\mathcal{M}_{ab} = \mathcal{M}_{ab}^{\mu\nu} \cdot \varepsilon_\mu^*(k_1,\lambda_1)\,\varepsilon_\nu^*(k_2,\lambda_2)$$

GLUON POLARIZATION VECTORS

PHYSICAL GLUONS ARE TRANSVERSE ($\lambda_1 = \pm 1$)

$$\Rightarrow k_1^2 = 0, \quad k_1^\mu \cdot \varepsilon_\mu(k_1, \lambda_1 = \pm 1) = 0$$

$$\& \quad k_2^2 = 0, \quad k_2^\nu \cdot \varepsilon_\nu(k_2, \lambda_2 = \pm 1) = 0$$

GAUGE INVARIANCE : $\varepsilon_\mu(k_1,\lambda_1) \to \varepsilon_\mu(k_1,\lambda_1) - a(k_1)_\mu$

$$\Downarrow$$

$$(k_1)_\mu \cdot \mathcal{M}_{ab}^{\mu\nu} = 0$$

ANALOGOUSLY $(k_2)_\nu\, \mathcal{M}_{ab}^{\mu\nu} = 0$

- CHECK OF GAUGE INVARIANCE

$$\rightarrow (k_1)_\mu \left(\mathcal{M}_A^{\mu\nu}\right)_{ab} = -ig^2 \frac{\lambda_b}{2}\frac{\lambda_a}{2}\,\bar{v}(p_2,s_2)\,\gamma^\nu \frac{(\not{p}_1 - \not{k}_1 + m)}{(p_1-k_1)^2 - m^2}\,\not{k}_1\, u(p_1,s_1)$$

$$\not{k}_1\, u(p_1,s_1) = -(\not{p}_1 - \not{k}_1 - m)\, u(p_1,s_1)$$

$$\downarrow$$

$$= +ig^2 \frac{\lambda_b}{2}\frac{\lambda_a}{2}\,\bar{v}(p_2,s_2)\,\gamma^\nu\, u(p_1,s_1)$$

$$\rightarrow (k_1)_\mu \left(\mathcal{M}_B^{\mu\nu}\right)_{ab} = -ig^2 \frac{\lambda_a}{2}\frac{\lambda_b}{2}\,\bar{v}(p_2,s_2)\,\not{k}_1 \frac{(-\not{p}_2 + \not{k}_1 + m)}{(-p_2+k_1)^2 - m^2}\,\gamma^\nu\, u(p_1,s_1)$$

$$\bar{v}(p_2,s_2)\,\not{k}_1 = \bar{v}(p_2,s_2)\,(-\not{p}_2 + \not{k}_1 - m)$$

$$\downarrow$$

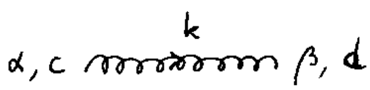
$$= -ig^2 \frac{\lambda_a}{2}\frac{\lambda_b}{2}\,\bar{v}(p_2,s_2)\,\gamma^\nu\, u(p_1,s_1)$$

$$\boxed{\begin{aligned} \rightarrow\ & (k_1)_\mu \left(\mathcal{M}_A^{\mu\nu} + \mathcal{M}_B^{\mu\nu} \right)_{ab} \\ &= -ig^2 \underbrace{\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2} \right]_-}_{i f_{abc} \frac{\lambda_c}{2}} \bar{v}(p_2, s_2)\, \gamma^\nu\, u(p_1, s_1) \\ &= g^2 f_{abc} \frac{\lambda_c}{2}\, \bar{v}(p_2, s_2)\, \gamma^\nu\, u(p_1, s_1) \end{aligned}}$$

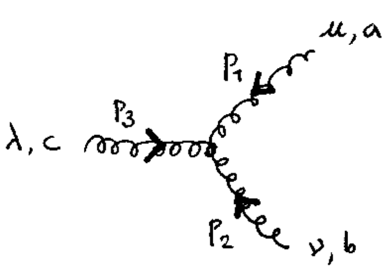
$\nearrow$ QED ("$f_{abc} = 0$") : $(k_1)_\mu \cdot \left(\mathcal{M}_A^{\mu\nu} + \mathcal{M}_B^{\mu\nu} \right) = 0$

$\searrow$ QCD ($f_{abc} \neq 0$) : $(k_1)_\mu \cdot \left(\mathcal{M}_A^{\mu\nu} + \mathcal{M}_B^{\mu\nu} \right) \neq 0$

- IN QCD CASE WE HAVE TO ADD DIAGRAM C



FEYNMAN GAUGE . $\dfrac{-i g^{\alpha\beta}}{k^2} \cdot \delta_{cd}$



$$- g f_{abc} \left\{ g^{\mu\nu} (p_1 - p_2)^\lambda + g^{\nu\lambda} (p_2 - p_3)^\mu + g^{\lambda\mu} (p_3 - p_1)^\nu \right\}$$

$$\left(\mathcal{M}_c^{\mu\nu}\right)_{ab} = \bar{v}(p_2, s_2)\left[-ig\frac{\lambda_c}{2}\gamma_\lambda\right]u(p_1, s_1)$$

$$\cdot \frac{(-i)}{(k_1+k_2)^2}$$

$$\cdot(-g f_{abc})\left\{g^{\mu\nu}(-k_1+k_2)^\lambda + g^{\nu\lambda}(-k_1-2k_2)^\mu + g^{\lambda\mu}(2k_1+k_2)^\nu\right\}$$

$$\longrightarrow (k_1)_\mu\left(\mathcal{M}_c^{\mu\nu}\right)_{ab} = g^2 f_{abc}\frac{\lambda_c}{2}\bar{v}(p_2, s_2)\gamma_\lambda u(p_1, s_1)$$

$$\cdot\frac{1}{2k_1\cdot k_2}\left\{\underset{\sim}{k_1^\nu}(-k_1+k_2)^\lambda + g^{\nu\lambda}(-2k_1\cdot k_2) + k_1^\lambda(\underset{\sim}{2k_1}+k_2)^\nu\right\} \qquad (k_1^2=0)$$

$$= g^2 f_{abc}\frac{\lambda_c}{2}\cdot\frac{1}{2k_1\cdot k_2}\cdot\bar{v}(p_2, s_2)\left\{\underset{\sim}{k_1^\nu}(\not{k}_1+\not{k}_2) + \not{k}_1 k_2^\nu - 2k_1\cdot k_2\,\gamma^\nu\right\}u(p_1, s_1)$$

$$\downarrow$$

$$\bar{v}(p_2, s_2)(\not{k}_1+\not{k}_2)u(p_1, s_1)$$

$$= \bar{v}(p_2, s_2)(\not{p}_2+\not{p}_1)u(p_1, s_1)$$

$$= \bar{v}(p_2, s_2)(-m+m)u(p_1, s_1) = 0$$

$$\therefore \boxed{(k_1)_\mu\left(\mathcal{M}_c^{\mu\nu}\right)_{ab} = -g^2 f_{abc}\frac{\lambda_c}{2}\bar{v}(p_2, s_2)\left\{\gamma^\nu - \frac{k_2^\nu\not{k}_1}{2k_1\cdot k_2}\right\}u(p_1, s_1)}$$

- SUM OF A + B + C

$$\boxed{\begin{aligned}&(k_1)_\mu \left(\mathcal{M}_A^{\mu\nu}+\mathcal{M}_B^{\mu\nu}+\mathcal{M}_C^{\mu\nu}\right)_{ab}\\ &= g^2 f_{abc}\,\frac{\lambda_c}{2}\,\bar{v}(p_2,s_2)\,\frac{k_2^\nu\,\not{k}_1}{2k_1\cdot k_2}\,u(p_1,s_1)\end{aligned}}$$

$\downarrow$

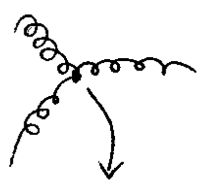
ZERO WHEN CONTRACTING WITH REAL GLUON POLARIZATION VECTOR $\varepsilon^*_\nu(k_2,\lambda_2)$

BECAUSE $(k_2)^\nu_{\bullet}\ \varepsilon_\nu(k_2,\lambda_2)=0$ $\quad \lambda_2=\pm 1$

$\therefore$ DIAGRAM C IS NEEDED IN QCD CASE TO PRESERVE GAUGE INVARIANCE

$\downarrow$

CANCELLATION $A+B \Leftrightarrow C$
REQUIRES THAT STRONG COUPLING CONSTANT

IN  AND 

g $\qquad$ g

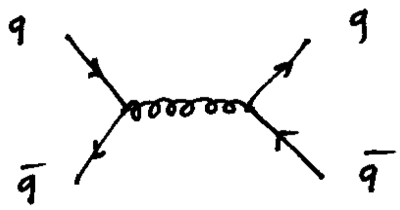
ARE SAME !

$\hookrightarrow$ ANALOGOUSLY $(k_2)_\nu\left(\mathcal{M}_A^{\mu\nu}+\mathcal{M}_B^{\mu\nu}+\mathcal{M}_C^{\mu\nu}\right)\cdot(\varepsilon_1)_\mu=0$

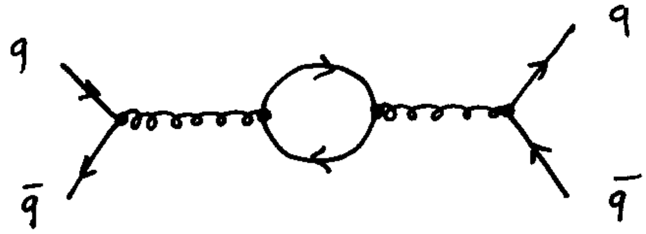
⇒ LOOPS

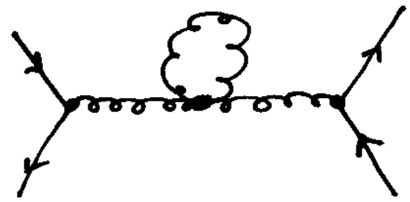
PROCESS $q\bar{q} \to q\bar{q}$ WITH $2g$ (VIRTUAL) INTERMEDIATE STATE

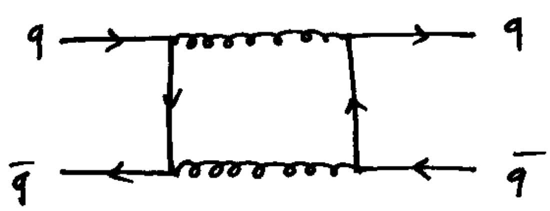
- $q\bar{q} \to q\bar{q}$ TO ORDER g^2



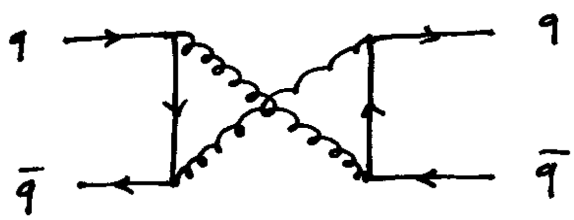
- $q\bar{q} \to q\bar{q}$ TO ORDER g^4



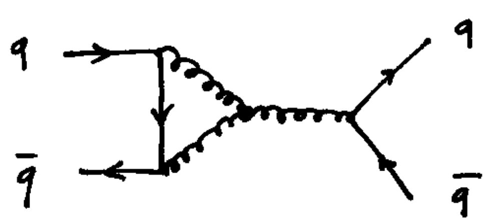




(A)



(B)



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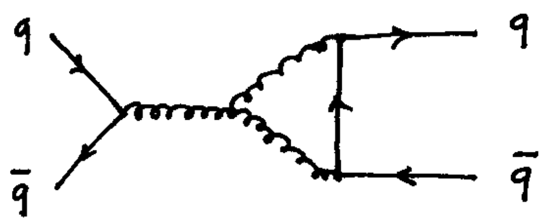


(D)

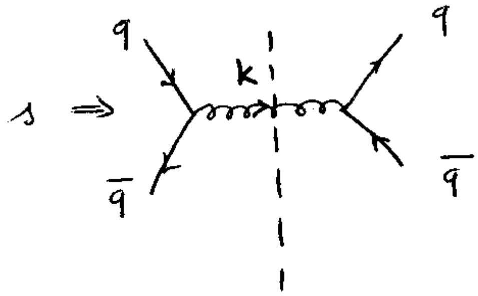
(E)

- **UNITARITY** ('CONSERVATION' OF PROBABILITY)

CALCULATE 'ABSORPTIVE' PART (IMAGINARY PART)

OF $q\bar{q} \to q\bar{q}$

↳ **TO ORDER g^2**

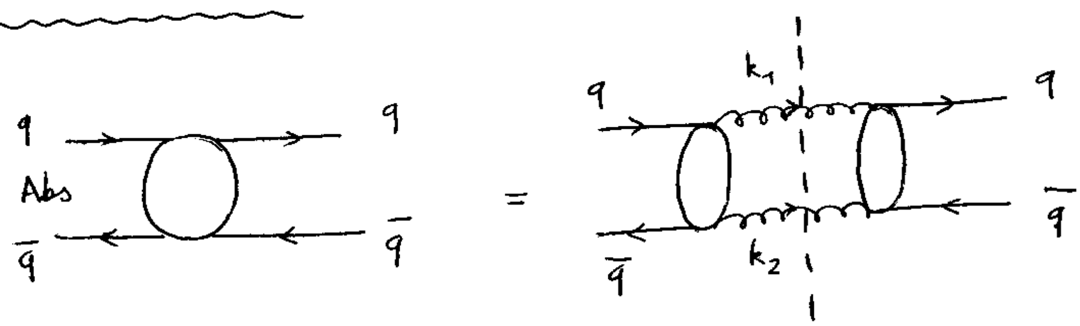


TREE LEVEL
↓
REAL
↓
NO ABSORPTIVE PART

ABS PART
⇕
PUT INTERMEDIATE STATE ON-SHELL ($k^2 = 0$)

GLUON CANNOT BE PUT ON SHELL
IN PHYSICAL REGION AS $s = (P_1 + P_2)^2 > 4m^2 > 0$

↳ **TO ORDER g^4**



CAN BE PUT ON-SHELL
($k_1^2 = k_2^2 = 0$)
⇓
ABSORPTIVE PART

↳ UNITARITY IN GENERAL

$|i\rangle$  $|f\rangle$

SCATTERING PROCESS : DESCRIBED BY S-MATRIX ELEMENT S_{fi}

S-MATRIX IS UNITARY (TOTAL PROBABILITY OF SCATTERING IS 1)

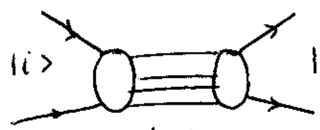
$$S^{+} S = \mathbb{1}$$

$$S = \mathbb{1} - i\,T$$

↳ T-MATRIX ↔ SCATTERING AMPLITUDE

$$(S^{+} S)_{fi} = \delta_{fi}$$

$$(S^{+})_{fn} (S)_{ni} = \delta_{fi}$$

$|i\rangle$  $|f\rangle$

$|n\rangle$

$$S^{*}_{nf} \; S_{ni} = \delta_{fi}$$

$\sum_n |n\rangle\langle n|$: COMPLETE SET OF INTERMEDIATE STATES

$$(1 + i\,T^{*})_{nf} \, (1 - i\,T)_{ni} = \delta_{fi}$$

$$\delta_{fi} - i\,T_{fi} + i\,T^{*}_{if} + T^{*}_{nf} \, T_{ni} = \delta_{fi}$$

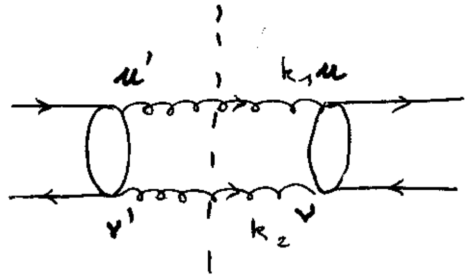
$$\boxed{i\,(T_{fi} - T^{*}_{if}) = (T^{+})_{fn} \, (T)_{ni}}$$

FOR $|i\rangle = |f\rangle$ ELASTIC PROCESS

$$i\,\underbrace{(T_{ii} - T^{*}_{ii})}_{2i\,\mathrm{Im}\,T_{ii}} = (T^{+})_{in} \, (T)_{ni}$$

$$\boxed{\mathrm{Abs}\,T_{ii} \equiv +\,2\,\mathrm{Im}\,T_{ii} = -\,(T^{+})_{in} \, (T)_{ni}}$$

↳ $q\bar{q} \to q\bar{q}$ TO ORDER g^4



$|n\rangle$: 2 GLUON INTERMEDIATE STATE

$$\sum_n \iff \frac{1}{2}\int d\rho^{(2)} \sum_{\lambda_1 = \pm 1}\sum_{\lambda_2 = \pm 1} = \frac{1}{2}\sum_{a,b}\sum_{\lambda_1 = \pm 1}\int \frac{d^3\vec{k}_1}{(2\pi)^3\, 2|\vec{k}_1|} \sum_{\lambda_2 = \pm 1}\int \frac{d^3\vec{k}_2}{(2\pi)^3\, 2|\vec{k}_2|}$$

2 IDENTICAL GLUONS IN INTERMEDIATE STATE

PHASE SPACE OF 2-GLUON STATE

BOTH GLUONS ARE TRANSVERSE (PHYSICAL d.o.f.)

(1) → RHS OF UNITARITY EQUATION

$$\mathrm{Abs}\, T^{\text{PHYS}}_{q\bar{q}\to q\bar{q}} = -\frac{1}{2}\int d\rho^{(2)} \sum_{\lambda_1 = \pm 1}\sum_{\lambda_2 = \pm 1} \left(T^{+\mu\nu}_{ab}\, \varepsilon_\mu(1)\, \varepsilon_\nu(2)\right) \left(T^{\mu'\nu'}_{ab}\, \varepsilon^*_{\mu'}(1)\, \varepsilon^*_{\nu'}(2)\right)$$

$$\sum_{\lambda_1 = \pm 1} \varepsilon_\mu(1)\, \varepsilon^*_{\mu'}(1) \equiv \left(-g_T\right)_{\mu\mu'}$$

⇑

PROJECTOR FOR TRANSVERSE GLUONS

⇓

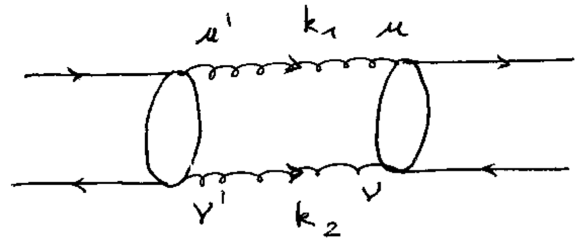
$$\sum_{\lambda_2 = \pm 1} \varepsilon_\nu(2)\, \varepsilon^*_{\nu'}(2) \equiv \left(-g_T\right)_{\nu\nu'}$$

$$\therefore \quad \boxed{\mathrm{Abs}\, T^{\text{PHYS}}_{q\bar{q}\to q\bar{q}} = -\frac{1}{2}\int d\rho^{(2)}\; T^{+\mu\nu}_{ab} \left(-g_T\right)_{\mu\mu'} \left(-g_T\right)_{\nu\nu'} T^{\mu'\nu'}_{ab}}$$

$q\bar{q} \to 2g$ AMPL.

(2) LHS OF UNITARITY EQUATION

$$\mathrm{Abs}\, T^{\mathrm{LOOP}}_{q\bar{q}\to q\bar{q}} = 2\, \mathrm{Im}\, T_{q\bar{q}\to q\bar{q}}$$



IN FEYNMAN DIAGRAM

FOR EACH INTERMEDIATE GLUON (FEYNMAN GAUGE)

$$\int \frac{d^4 k_1}{(2\pi)^4} \frac{i(-g_{\mu\mu'})}{k_1^2 + i\varepsilon}$$

$\downarrow$ PUT GLUON ON-SHELL (ABS. PART = 2 IM PART)

$$\int \frac{d^4 k_1}{(2\pi)^4}\, i\,(-g_{\mu\mu'})\,(-2\pi i)\,\delta(k_1^2)\,\theta(k_1^0)$$

↳ POS. ENERGY PART

$$= (-g_{\mu\mu'}) \int \frac{d^4 k_1}{(2\pi)^4} \frac{2\pi}{2|\vec{k}_1|} \delta(k_1^0 - |\vec{k}_1|)$$

$$= (-g_{\mu\mu'}) \underbrace{\int \frac{d^3 \vec{k}_1}{(2\pi)^3\, 2|\vec{k}_1|}}_{\text{PHASE SPACE INTEGRAL FOR 1 GLUON}}$$

$$\therefore \boxed{\begin{aligned}&\text{Abs } T^{\text{LOOP}}_{q\bar q \to q\bar q} \\ &= -\frac{1}{2}\int d\rho^{(2)}\; T^{+\,\mu\nu}_{ab}\,(-g_{\mu\mu'})\,(-g_{\nu\nu'})\; T^{\mu'\nu'}_{ab}\end{aligned}}$$

FOR A THEORY WHICH RESPECTS UNITARITY

BOTH WAYS ~~TO~~ TO CALCULATE Abs T SHOULD BE EQUAL !

↳ IN GLUON PROPAGATOR NUMERATOR $(-g_{\mu\mu'})$

THERE IS A SUM OVER BOTH T & L GLUON POLARIZATIONS

CONSIDER c.m. SYSTEM OF $2g$ SYSTEM

$$(k_1+k_2)^2 = s = 2\,k_1\cdot k_2$$

$$|\vec{k}_1| = |\vec{k}_2| = \frac{\sqrt{s}}{2}$$

DEFINE UNIT VECTORS (SUDAKOV VECTORS)

$$(1,0,0,1) \qquad \hat{k}_1^{\mu} \equiv \frac{2}{\sqrt{s}}\, k_1^{\mu}$$

$$\to \quad \hat{k}_1\cdot\hat{k}_2 = 2$$

$$(1,0,0,-1) \qquad \hat{k}_2^{\nu} \equiv \frac{2}{\sqrt{s}}\, k_2^{\nu}$$

WE CAN USE THESE UNIT VECTORS TO DEFINE LONG. POL.

$$\boxed{-g^{\mu\mu'} = \sum_{\lambda_1=\pm 1} \varepsilon^{\mu}(k_1,\lambda_1)\,\varepsilon^{\mu'*}(k_1,\lambda_1) - \frac{1}{2}\left(\hat{k}_1^{\mu}\hat{k}_2^{\mu'} + \hat{k}_1^{\mu'}\hat{k}_2^{\mu}\right)}$$

CHECK :

$$-g^{\mu\mu'}\,(k_1)_{\mu} = 0 - \frac{\sqrt{s}}{2}\,\frac{1}{2}\Big(\underbrace{\cancel{\hat{k}_1^{2}}}_{0}\,\hat{k}_2^{\mu'} + \hat{k}_1^{\mu'}\,\underbrace{\hat{k}_1\cdot\hat{k}_2}_{\underbrace{\frac{4}{s}\cdot\frac{s}{2}}_{2}}\Big)$$

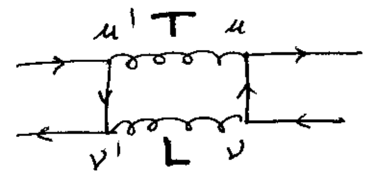
$$-(k_1)^{\mu'} \stackrel{!}{=} (-k_1)^{\mu'}$$

$$\therefore \quad \boxed{-g^{\mu\mu'} = (-g_T)^{\mu\mu'} - \frac{1}{2}\left(\hat{k}_1^{\mu}\hat{k}_2^{\mu'} + \hat{k}_1^{\mu'}\hat{k}_2^{\mu}\right)}$$

T POLARIZATION L. POLARIZATION

ΔT SHOULD BE 0 FOR UNITARY THEORY

- $\Delta T \equiv \text{Abs}\, T^{\text{PHYS}}_{q\bar{q}\to q\bar{q}} - \text{Abs}\, T^{\text{LOOP}}_{q\bar{q}\to q\bar{q}}$

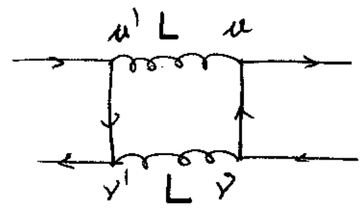


$$= \frac{1}{2}\int d\rho^{(2)}\; T^{+\mu\nu}_{ab}\Big\{(-g_T)_{\mu\mu'}\left(-\tfrac{1}{2}\right)\left(\hat{k}_{1\nu}\underline{\hat{k}_{2\nu'}} + \hat{k}_{1\nu'}\underline{\hat{k}_{2\nu}}\right)$$

$$+\left(-\tfrac{1}{2}\right)\left(\underline{\hat{k}_{1\mu}}\hat{k}_{2\mu'} + \underline{\hat{k}_{1\mu'}}\hat{k}_{2\mu}\right)(-g_T)_{\nu\nu'}$$

$$+\left(-\tfrac{1}{2}\right)\left(\hat{k}_{1\mu}\hat{k}_{2\mu'} + \hat{k}_{1\mu'}\hat{k}_{2\mu}\right)$$

$$\cdot\left(-\tfrac{1}{2}\right)\left(\hat{k}_{1\nu}\hat{k}_{2\nu'} + \hat{k}_{1\nu'}\hat{k}_{2\nu}\right)\Big\}\; T^{\mu'\nu'}_{ab}$$



- WE KNOW (FROM PART 1 OF LECTURE) THAT

$$(\hat{k}_1)_{\mu'}\; T^{\mu'\nu'}_{ab}\cdot \varepsilon_{\nu'}(2) = 0$$

$$(\hat{k}_2)_{\nu'}\; T^{\mu'\nu'}_{ab}\; \varepsilon_{\mu'}(1) = 0$$

$\Rightarrow$ THIS ELIMINATES ALL <u>TL TERMS</u> !

- FOR LL TERMS $\quad (\mathcal{M} = -i T)$

$$(\hat{k}_1)_{\mu'} \; T_{ab}^{\mu'\nu'} = i g^2 f_{abc} \frac{\lambda_c}{2} \; \bar{v}(p_2, s_2) \frac{\hat{k}_2^{\nu'} \not{k}_1}{2 k_1 \cdot k_2} \; u(p_1, s_1)$$

$$\downarrow \qquad \bar{v} \left(\not{k}_1 + \not{k}_2 \right) u = 0$$

$$\boxed{(\hat{k}_1)_{\mu'} \; T_{ab}^{\mu'\nu'} = \hat{k}_2^{\nu'} \cdot \underbrace{i g^2 f_{abc} \frac{\lambda_c}{2} \; \bar{v}(p_2, s_2) \frac{\not{k}_1 - \not{k}_2}{4 k_1 \cdot k_2} \; u(p_1, s_1)}_{\equiv \; T_{ab}}}$$

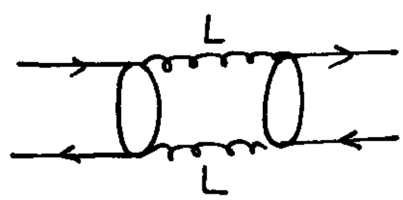
(SYMMETRIC UNDER EXCHANGE OF 2 GLUONS)

ANALOGOUSLY

$$(\hat{k}_2)_{\nu'} \; T_{ab}^{\mu'\nu'} = \hat{k}_1^{\mu'} \; T_{ab}$$

$$\therefore \quad \Delta T = \frac{1}{2} \int d\rho^{(2)} \cdot \frac{1}{4} \; T_{ab}^{+} \left(\Big(\underbrace{(\hat{k}_1 \cdot \hat{k}_2)}_{4}\Big)^2 + \Big(\underbrace{(\hat{k}_1 \cdot \hat{k}_2)}_{4}\Big)^2 + 0 + 0 \right) T_{ab}$$

$$\boxed{\Delta T = \int d\rho^{(2)} \; T_{ab}^{+} \; T_{ab}}$$



$\Downarrow$

UNITARITY VIOLATED BY SUM OF DIAGRAMS (A) – (E)

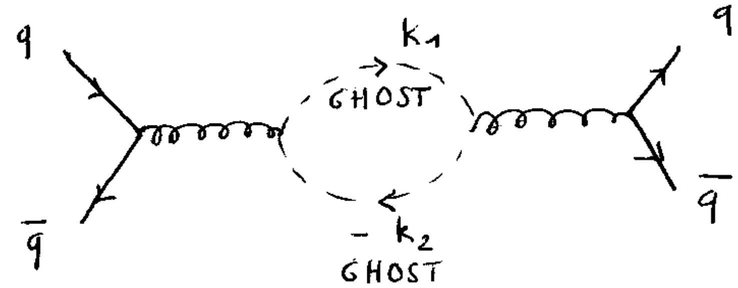
FOR <u>NON</u> – ABELIAN GAUGE THEORY !

- 'RESTORATION' OF UNITARITY

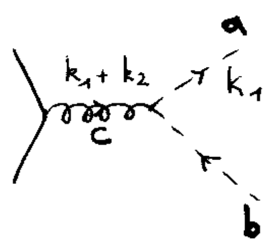
$$\Downarrow$$

ADD EXTRA UNPHYSICAL FIELDS ('GHOSTS')
SO THAT THEY RESTORE UNITARITY

ΔT CAN BE INTERPRETED AS GHOST CONTRIBUTION



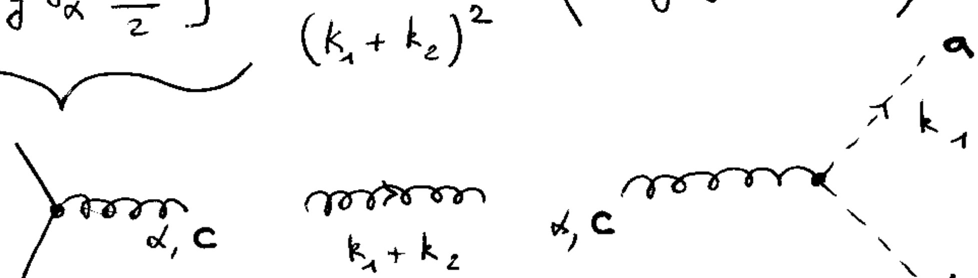
$$\Delta T = - \int d\rho^{(2)} \, (-1) \, T^{+}_{ab} \, \underbrace{T_{ab}}$$



$$T_{ab} = \bar{v} \left[-ig \gamma_\alpha \frac{\lambda_c}{2} \right] u \cdot \frac{1}{2 k_1 \cdot k_2} \cdot \left(-g f_{abc} k_1^\alpha \right)$$

$$= i \, \underbrace{\bar{v} \left[-ig \gamma_\alpha \frac{\lambda_c}{2} \right] u} \; \frac{-i}{(k_1 + k_2)^2} \left(-g f_{abc} k_1^\alpha \right)$$

TO CONVERT
T- MATRIX
TO S- MATRIX



GLUON- GHOST
VERTEX.

GHOSTS APPEAR AS SPINLESS PARTICLES

BUT SATISFY GRASSMANN ALGEBRA, i.e. ANTI-COMMUTE (cf. FERMION)

FACTOR (−1) FOR LOOP

↓

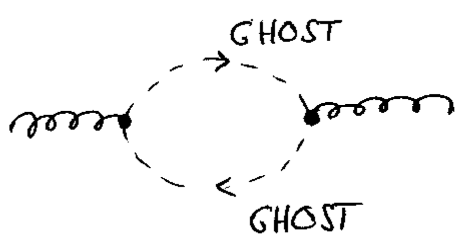
GHOSTS ARE UNPHYSICAL

DO NOT SATISFY SPIN-STATISTICS THEOREM !

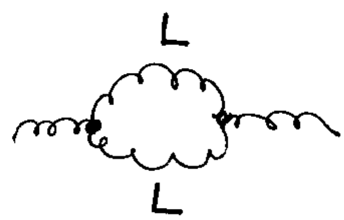
ROLE: SERVE TO CANCEL UNPHYSICAL L POL. DEGREES OF FREEDOM IN GLUON LOOPS

EACH TIME WE HAVE

WE SHOULD ADD



WHICH CANCELS

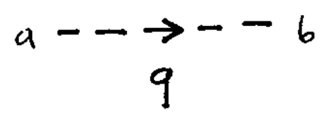


⟹ $\boxed{\text{UNITARITY IS RESTORED}}$!

(OR ALTERNATIVELY, USE UNITARY GAUGE ('T HOOFT) SO THAT ONLY T GLUON POL. STATES PROPAGATE)

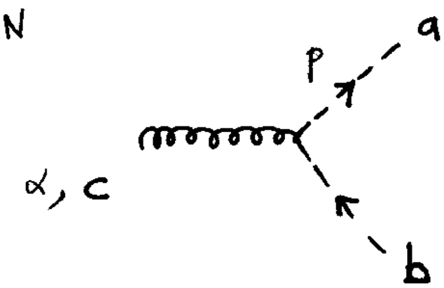
⇒ FEYNMAN RULES FOR GHOSTS

- PROPAGATOR



$$\boxed{\frac{i}{q^2}\,\delta_{ab}}$$

- GHOST-GLUON VERTEX



$$\boxed{-\,g\, f_{abc}\; p^{\alpha}}$$

OUTGOING GHOST LINE

- FOR EACH GHOST LOOP ⟶ FACTOR (-1)

AS FOR FERMIONS

∴ QUANTIZATION OF QCD AMOUNTS TO ADD TO $\mathcal{L}_{QCD}$

$$\boxed{\begin{aligned} &\mathcal{L}_{QCD} \longrightarrow \mathcal{L}_{QCD} + \mathcal{L}_{\text{GAUGE-FIXING}} + \mathcal{L}_{\text{GHOST}} \\ &\mathcal{L}_{\text{GHOST}} = i\left(\partial_\mu \chi_a\right)\left[\partial^\mu \delta_{ab} + g\, f_{abc}\, A^\mu_c\right]\chi_b \end{aligned}}$$