* DECAYS AND RESONANCES

- **MEAN LIFETIME** OF DECAYING STATE

$$\tau = \frac{1}{W} \qquad W: \text{ TRANSITION PROBABILITY / UNIT TIME}$$

- **DECAY RATE / WIDTH** OF DECAYING STATE

$$\Gamma = \frac{\hbar}{\tau} = \hbar W$$

$N(t)$: # PARTICLES PRESENT AT TIME t

$$\Gamma = -\hbar \frac{dN}{dt} \frac{1}{N}$$

$$\downarrow$$

$$N(t) = N(0)\, e^{-\Gamma t/\hbar} = N(0)\, e^{-t/\tau}$$

- WAVE FUNCTION

STATIONARY STATE, ENERGY E_R : $\psi(t) = \psi(0)\, e^{-iE_R t}$

NON-STATIONARY (i.e. DECAYING) STATE : $\psi(t) = \psi(0)\, e^{-iE_R t}\, e^{-t/2\tau}$

$$= \psi(0)\, e^{-i\left(E_R - \frac{i}{2}\Gamma_R\right)t}$$

FOURIER TF.

$$\chi(E) = \int_0^\infty dt\, e^{iEt}\, \psi(t)$$

$$= \psi(0) \int_0^\infty dt\, e^{i\left[E - \left(E_R - \frac{i}{2}\Gamma_R\right)\right]t}$$

$$= \frac{C}{E - E_R + i\frac{\Gamma_R}{2}}$$

$$\sigma \sim \chi^*(E)\,\chi(E) = \frac{|c|^2}{(E-E_R)^2 + \Gamma_R^2/4}$$

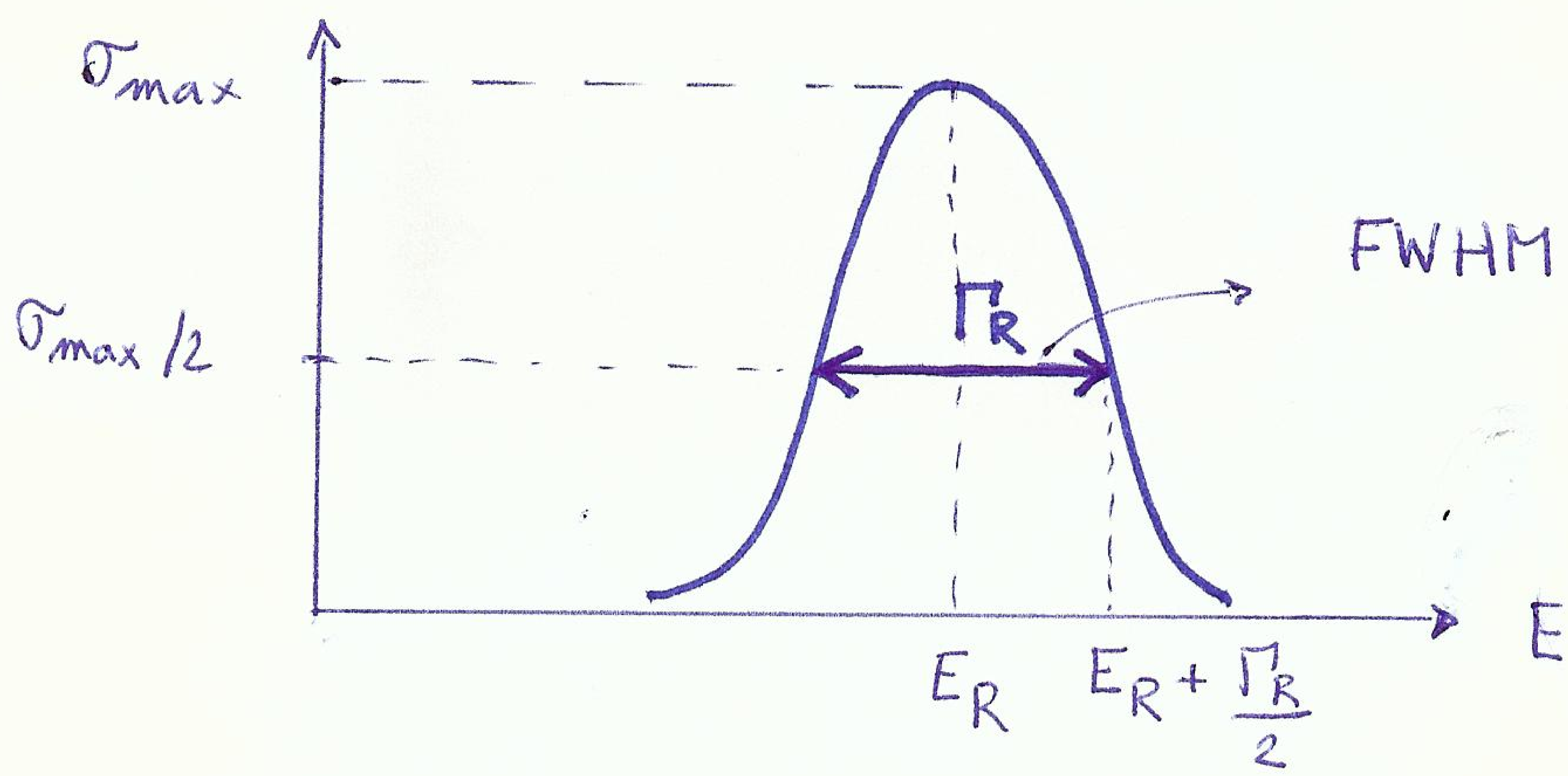
$\Downarrow$

CROSS SECTION (PROBABILITY) TO FORM RESONANCE $a + b \rightarrow R$ (e.g. $\gamma + p \rightarrow \Delta^+$, $\pi^+ + p \rightarrow \Delta^{++}$)

$$\boxed{\sigma(E) = \sigma_{max} \cdot \frac{\Gamma_R^2/4}{(E-E_R)^2 + \Gamma_R^2/4}}$$

$\Downarrow$

BREIT – WIGNER RESONANCE FORMULA



FWHM (FULL WIDTH AT HALF MAXIMUM)

- SEVERAL DECAY CHANNELS

$$\begin{aligned} R &\rightarrow a_1 + b_1 \qquad && \Gamma_1 \\ &\rightarrow a_2 + b_2 && \Gamma_2 \\ &\quad \vdots \end{aligned}$$

$$\Gamma_{tot} = \sum_i \Gamma_i$$

$\uparrow$ TOTAL WIDTH

PARTIAL WIDTHS

EXAMPLES

- **BARYON RESONANCE** (qqq)

$$\Delta^{++} \to \pi^+ p$$

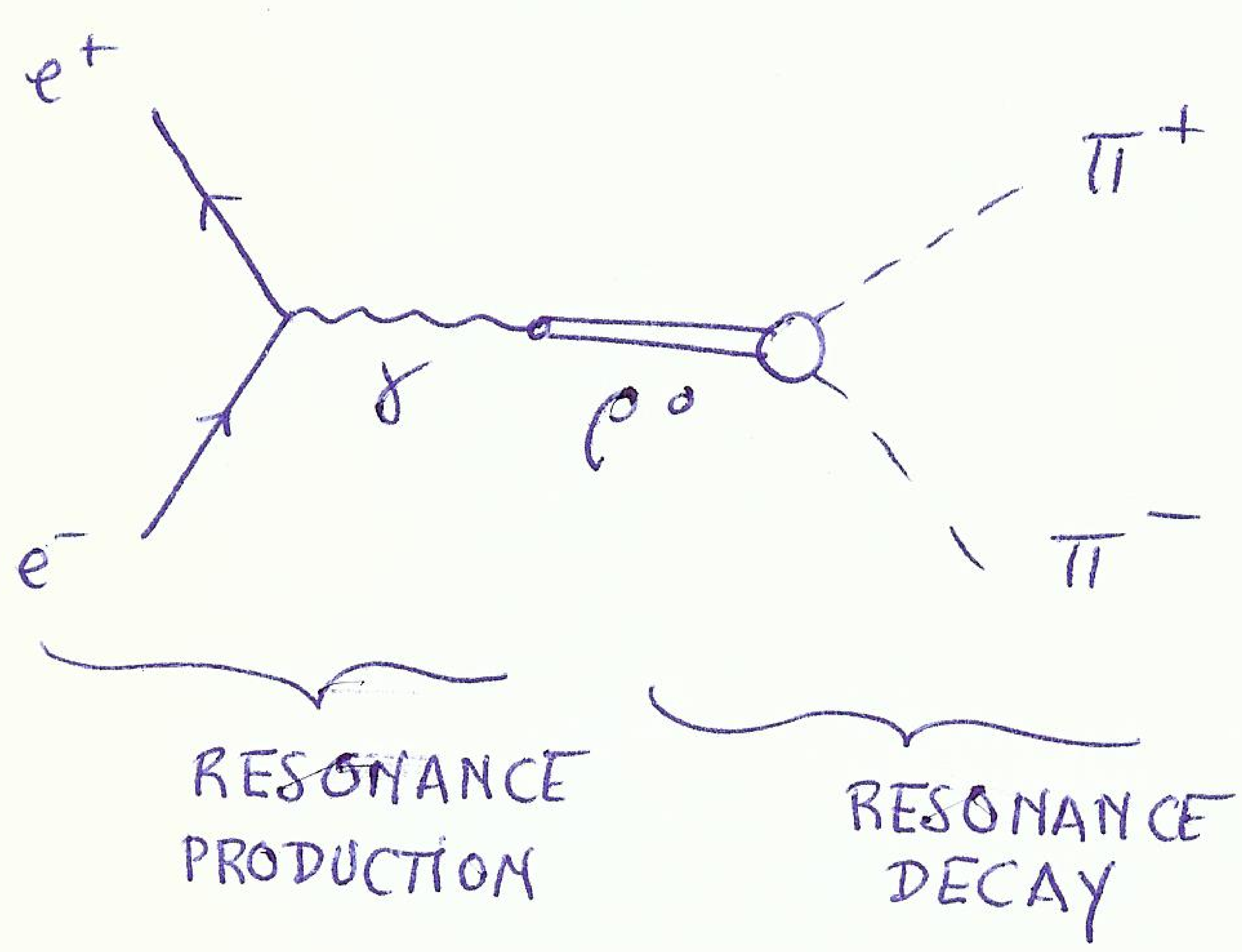
$$M_\Delta = 1.232 \text{ GeV}$$

- **MESON RESONANCES** $(q\bar{q})$

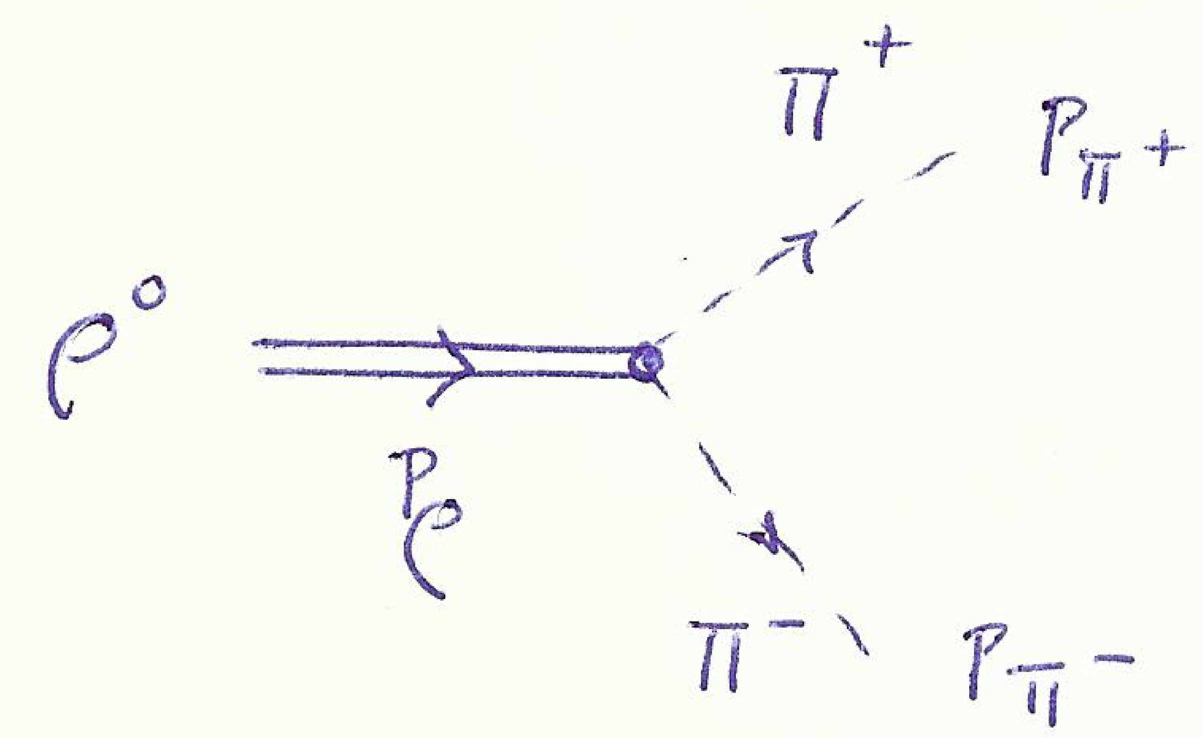
$$\rho^0 \to \pi^+ \pi^-$$

$$\to e^+ e^- \quad \left(\text{SMALL BECAUSE } \sim \alpha = \tfrac{1}{137}\right)$$

$$M_{\rho^0} \simeq 0.770 \text{ GeV}$$



WORK OUT $\rho^0 \to \pi^+ \pi^-$ DECAY WIDTH



FEYNMAN RULE (cf $\gamma\pi\pi$)

$$\boxed{-i\, g_{\rho\pi\pi} \left(P_{\pi^+} - P_{\pi^-} \right)^\mu}$$

ρ^0 SPIN 1 : 3 POLARIZATION STATES

$$\rightsquigarrow \underbrace{\frac{1}{3} \sum_{\lambda_\rho}}_{\text{AVERAGE OVER } \rho \text{ POL STATES}} |\mathcal{M}|^2 = \frac{1}{3} g^2_{\rho\pi\pi} \left(P_{\pi^+} - P_{\pi^-} \right)^\mu \left(P_{\pi^+} - P_{\pi^-} \right)^\nu \sum_{\lambda_\rho = -1,0,1} \varepsilon_\mu(\lambda_\rho)\, \varepsilon^*_\nu(\lambda_\rho)$$

FOR PARTICLE (ρ^0) AT REST

$$\varepsilon^\mu(\lambda_\rho = +1) = \left(0, -\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right)$$

$$\varepsilon^\mu(\lambda_\rho = -1) = \left(0, +\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0\right)$$

$$\varepsilon^\mu(\lambda_\rho = 0) = (0, 0, 0, 1)$$

$$\sum_{\lambda_\rho = -1,0,1} \varepsilon_\mu(\lambda_\rho)\, \varepsilon^*_\nu(\lambda_\rho) = -g_{\mu\nu} + \frac{(P_\rho)_\mu (P_\rho)_\nu}{m_\rho^2}$$

$$\therefore \frac{1}{3} \sum_{\lambda_\rho} |\mathcal{M}|^2 = -\frac{1}{3} g_{\rho\pi\pi}^2 \left(P_{\pi^+} - P_{\pi^-} \right)^2$$

$$= +\frac{1}{3} g^2_{\rho\pi\pi} \left(2 P_{\pi^+} \cdot P_{\pi^-} - 2 m_\pi^2 \right)$$

$$p_\rho^2 = m_\rho^2 = (p_{\pi^+} + p_{\pi^-})^2$$

$$= 2m_\pi^2 + 2p_{\pi^+}\cdot p_{\pi^-}$$

$$p_{\pi^+}\cdot p_{\pi^-} = \frac{m_\rho^2}{2} - m_\pi^2$$

$$\frac{1}{3}\sum_{\lambda_\rho} |\mathcal{M}|^2 = \frac{4}{3} g_{\rho\pi\pi}^2 \underbrace{\left(\frac{1}{4}m_\rho^2 - m_\pi^2\right)}_{|\bar{p}_\pi|^2}$$

$$\rightsquigarrow \quad \Gamma_{\rho^0\to\pi^+\pi^-} = \frac{1}{2m_\rho}\int \frac{d^3\bar{p}_{\pi^+}}{(2\pi)^3 2E_{\pi^+}} \int \frac{d^3\bar{p}_{\pi^-}}{(2\pi)^3 2E_{\pi^-}} (2\pi)^4 \delta^4(p_\rho - p_{\pi^+} - p_{\pi^-})$$

IN REST FRAME OF DECAYING PARTICLE

$$\downarrow \quad E_{\pi^+} = E_{\pi^-} = \frac{m_\rho}{2} \qquad \cdot \frac{1}{3}\sum_{\lambda_\rho} |\mathcal{M}|^2$$

$$= \frac{4}{3} g_{\rho\pi\pi}^2 \cdot \frac{1}{2m_\rho}\cdot\frac{1}{4\pi^2}\int \frac{d^3\bar{p}_{\pi^+}}{4E_{\pi^+}^2}\,\delta(m_\rho - 2E_{\pi^+}) \cdot |\bar{p}_{\pi^+}|^2$$

$$\downarrow \quad \delta(m_\rho - 2E_{\pi^+})$$

$$= \frac{1}{2}\delta\left(\frac{m_\rho}{2} - \sqrt{\bar{p}_\pi^2 + m_\pi^2}\right)$$

$$= \frac{1}{2}\frac{E_{\pi^+}}{|\bar{p}_{\pi^+}|}\,\delta\left(|\bar{p}_{\pi^+}| - \sqrt{\frac{m_\rho^2}{4} - m_\pi^2}\right)$$

$$\Gamma_{\rho^0 \to \pi^+\pi^-} = \frac{4}{3}\, g_{\rho\pi\pi}^2 \cdot \frac{1}{2m_\rho} \cdot \frac{1}{4\pi^2} \cdot \underbrace{4\pi}_{\text{SOLID ANGLE}}$$

$$\cdot \underbrace{\frac{1}{8E_{\pi^+}}}_{4m_\rho} \cdot \frac{|\bar{p}_{\pi^+}|^{\cancel{2}}}{\cancel{|\bar{p}_{\pi^+}|}} \cdot |\bar{p}_{\pi^+}|^2$$

$$\boxed{\Gamma_{\rho^0 \to \pi^+\pi^-} = \frac{2}{3} \cdot \frac{g_{\rho\pi\pi}^2}{4\pi} \cdot \frac{|\bar{p}_{\pi^+}|^3}{m_\rho^2}}$$

WITH $|\bar{p}_{\pi^+}|^2 = \frac{1}{4} m_\rho^2 - m_\pi^2$

FROM $\Gamma_{\rho^0 \to \pi^+\pi^-} = 149$ MeV

$\Downarrow$

DETERMINE STRENGTH OF COUPLING $\frac{g_{\rho\pi\pi}^2}{4\pi}$

$$\frac{g_{\rho\pi\pi}^2}{4\pi} \simeq 2.84 \quad , \quad g_{\rho\pi\pi} \simeq 6$$