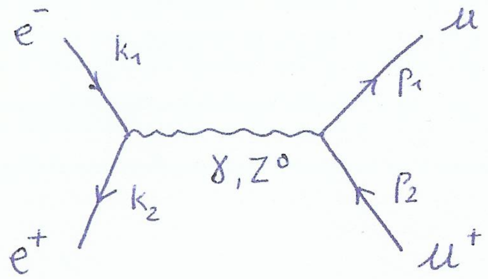
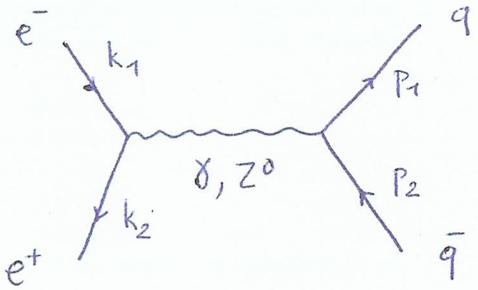
QUARK INTERACTIONS

$e^+e^- \to$ HADRONS

* $e^+e^- \to \mu^+\mu^-$ & $e^+e^- \to q\bar{q}$ ANNIHILATION



$e^-e^+ \to \mu^-\mu^+$



$e^-e^+ \to q\bar{q}$

CONSIDER e^-e^+ CM SYSTEM (cf. e^+e^- COLLIDERS)

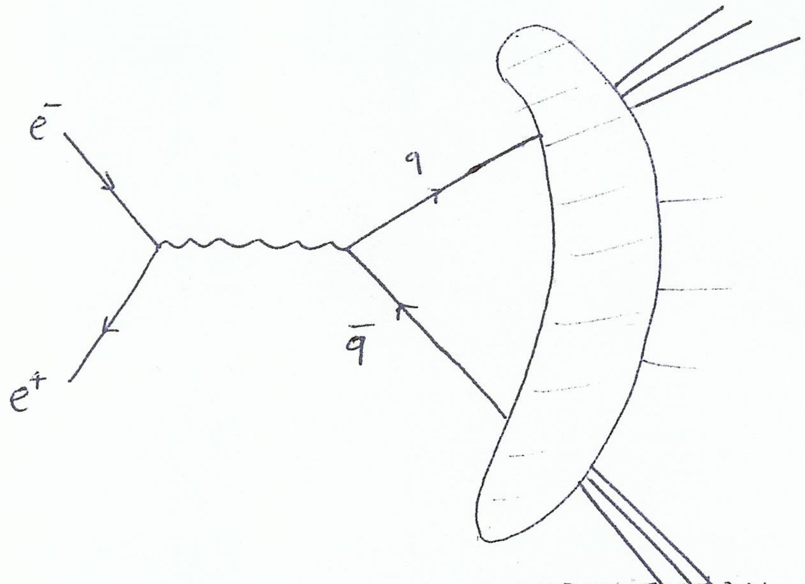
AT HIGH C.M ENERGY $\sqrt{s}$

$s = (k_1 + k_2)^2$ LEP-II : $\sqrt{s} = 200$ GeV

↳ PRODUCTION OF QUARKS : SHORT DISTANCE PROCESS :
INVOLVES TIME / LENGTH SCALES $\sim \frac{1}{\sqrt{s}}$

e.g. $\sqrt{s} = 200$ GeV $\leftrightarrow \frac{1}{\sqrt{s}} \simeq 10^{-3}$ fm

↳ QUARKS HADRONIZE ON THEIR WAY OUT
THIS HAPPENS AT MUCH LONGER LENGTH SCALES ~ 1 fm



HADRONIZATION

IN THIS CASE:

HADRONS WHICH COME OUT FOR A 'SPRAY' OF COLLINEAR HADRONS : JET

↳ TOTAL MOMENTUM

↳ FLAVOR QUANTUM NUMBERS

CORRESPOND WITH THAT OF FRAGMENTING QUARK

⇓

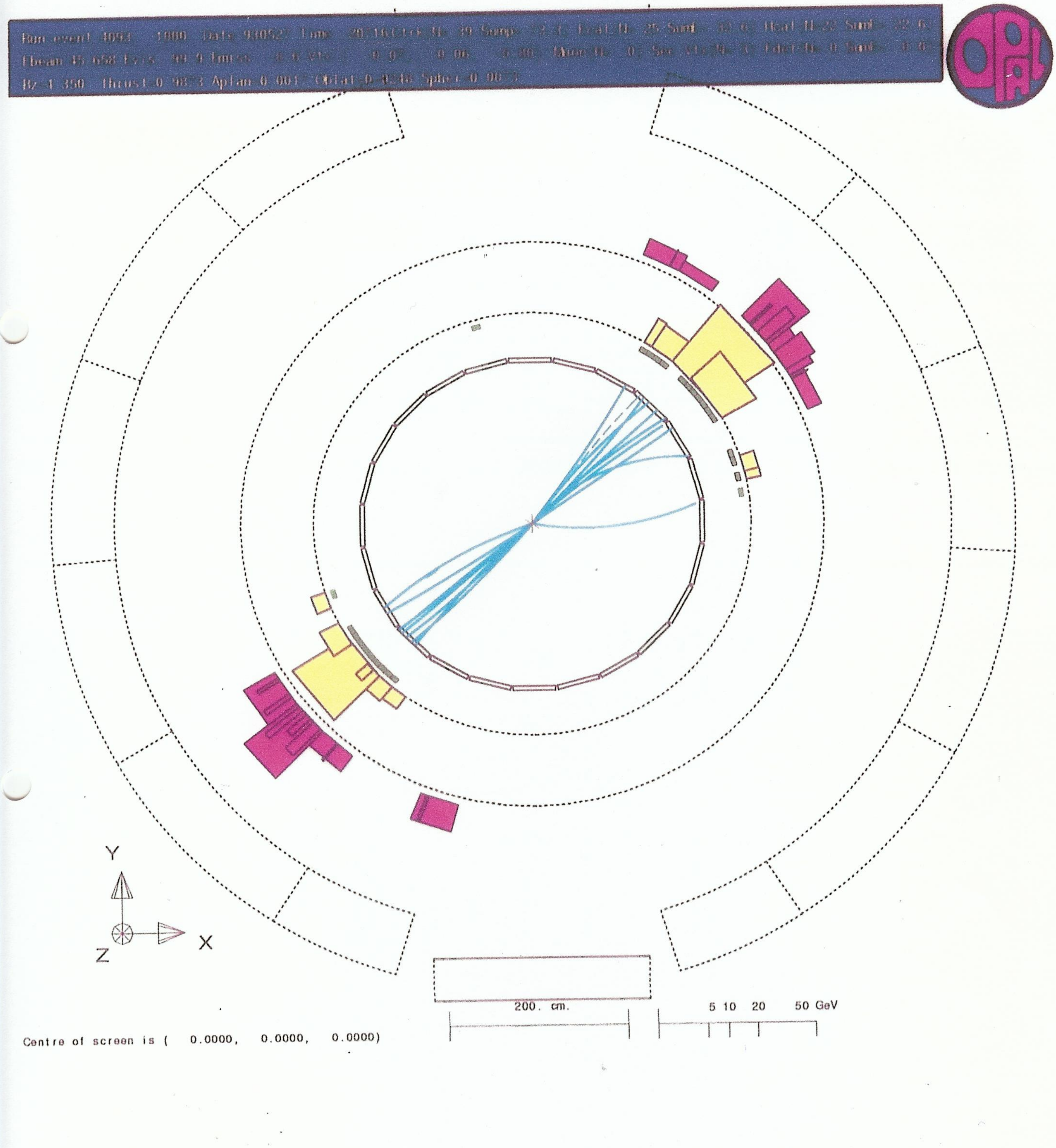
JET'S CAN BE SEEN AS "FOOTPRINT" OF QUARK

⇒ TOTAL INCLUSIVE CROSS SECTION

$\equiv \sum$ OVER ALL PARTONIC SUBPROCESSES.

$$\sigma(e^+e^- \to \text{HADRONS}) = \sum_q \sigma(e^+e^- \to \underset{\substack{\uparrow \\ \text{2 JETS}}}{q\bar{q}}, \underset{\substack{\uparrow \\ \text{3 JETS}}}{q\bar{q}g}, \ldots)$$

$e^+e^- \to q\bar{q}$ (2 JET EVENT)



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SOME DIRACOLOGY

HELP 1

- SPINOR

$$u(p,s) = \sqrt{E+M} \begin{pmatrix} \chi_s \\ \frac{\bar{\sigma}.\bar{p}}{E+M} \chi_s \end{pmatrix}$$

$$\bar{u}u = 2M$$

- ENERGY PROJECTOR

$$\boxed{\sum_s u(p,s)\,\bar{u}(p\,s) = \not{p} + M}$$

PROOF

$$u(p\,s)\;\bar{u}(p\,s) = (E+M) \begin{pmatrix} \chi_s \\ \frac{\bar{\sigma}.\bar{p}}{E+M} \chi_s \end{pmatrix} \left(\chi_s^+ \quad -\chi_s^+ \frac{\bar{\sigma}.\bar{p}}{E+M} \right)$$

$$= (E+M) \begin{pmatrix} \chi_s \chi_s^+ & -\chi_s \chi_s^+ \frac{\bar{\sigma}.\bar{p}}{E+M} \\ \frac{\bar{\sigma}.\bar{p}}{E+M} \chi_s \chi_s^+ & -\frac{\bar{\sigma}.\bar{p}}{E+M} \chi_s \chi_s^+ \frac{\bar{\sigma}.\bar{p}}{E+M} \end{pmatrix}$$

$$\downarrow \quad \sum_s \chi_s \chi_s^+ = \mathbb{1}_{2\times 2}$$

$$\sum_s u(p\,s)\;\bar{u}(p\,s) = (E+M) \begin{pmatrix} \mathbb{1}_{2\times 2} & -\frac{\bar{\sigma}.\bar{p}}{E+M} \\ \frac{\bar{\sigma}.\bar{p}}{E+M} & -\frac{\bar{p}^2}{(E+M)^2} \end{pmatrix}$$

$$\sum_s u(p\,s)\,\bar{u}(p\,s) = \begin{pmatrix} E+M & -\bar{\sigma}.\bar{p} \\ \bar{\sigma}.\bar{p} & -(E-M) \end{pmatrix}$$

$$= \gamma^0 . E - \bar{\gamma}.\bar{p} + M$$

$$\stackrel{!}{=} \not{p} + M$$

- FOR MASSLESS SPINOR $(M=0)$ $(M \ll E)$

$$\sum_s u(p,s)\,\bar{u}(p,s) = \not{p}$$

ANALOGOUSLY FOR ANTI-PARTICLE SPINOR $(M=0)$

$$\sum_s v(p\,s)\,\bar{v}(p,s) = \not{p}$$

- TRACE THEOREM

$$\mathrm{Tr}\left\{\not{a}\,\not{b}\,\not{c}\,\not{d}\right\} = 4\left\{a.b\ c.d - a.c\ b.d + a.d\ b.c\right.$$

PROOF

$$\hookrightarrow\quad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$$

$$\Big\downarrow \quad \times\, a_\mu\, b_\nu$$

$$\not{a}\,\not{b} + \not{b}\,\not{a} = 2\,a.b$$

$$\hookrightarrow\quad \mathrm{Tr}(\not{a}\,\not{b}) = a.b\ \mathrm{Tr}\left(\mathbb{I}_{4\times 4}\right) = 4\,a.b$$

$$\hookrightarrow\quad \mathrm{Tr}\left\{\not{a}\,\not{b}\,\not{c}\,\not{d}\right\}$$

$$= 2\,a.b\ \underbrace{\mathrm{Tr}\left\{\not{c}\,\not{d}\right\}}_{4\,c.d} - \mathrm{Tr}\left\{\not{b}\,\not{a}\,\not{c}\,\not{d}\right\}$$

$$= 8(a.b)(c.d) - 2\,a.c\ \mathrm{Tr}\left\{\not{b}\,\not{d}\right\} + \mathrm{Tr}\left\{\not{b}\,\not{c}\,\not{a}\,\not{d}\right\}$$

$$= 8(a.b)(c.d) - 8(a.c)(b.d) + 2\,a.d\ \mathrm{Tr}\left\{\not{b}\,\not{c}\right\}$$

$$- \mathrm{Tr}\left\{\not{b}\,\not{c}\,\not{d}\,\not{a}\right\}.$$

$$\Big\downarrow$$

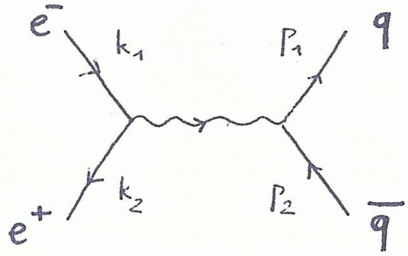
$$\mathrm{Tr}\left\{\not{a}\,\not{b}\,\not{c}\,\not{d}\right\} + \mathrm{Tr}\left\{\not{b}\,\not{c}\,\not{d}\,\not{a}\right\}$$

$$= 2\left\{4(a.b)(c.d) - 4(a.c)(b.d) + 4(a.d)(b.c)\right\}$$

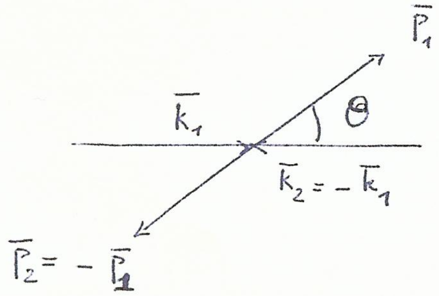
$$= 2\,\mathrm{Tr}\left\{\not{a}\,\not{b}\,\not{c}\,\not{d}\right\}.$$

e^+e^-

$\Rightarrow$ CROSS SECTION FOR $e^+e^- \to q\bar{q}$



IN C.M.



$$|\bar{p}_1| = |\bar{k}_1| = \frac{\sqrt{s}}{2}$$

$$k_1\left(\frac{\sqrt{s}}{2}, \bar{k}_1\right) \qquad p_1\left(\frac{\sqrt{s}}{2}, \bar{p}_1\right)$$

$$k_2\left(\frac{\sqrt{s}}{2}, -\bar{k}_1\right) \qquad p_2\left(\frac{\sqrt{s}}{2}, -\bar{p}_1\right)$$

$$d\sigma(e^+e^- \to q\bar{q}) = \frac{1}{(2k_1^0)(2k_2^0)\underbrace{v_{rel}}_{2}} \int \frac{d^3\bar{p}_1}{(2\pi)^3 2p_1^0} \int \frac{d^3\bar{p}_2}{(2\pi)^3 2p_2^0} (2\pi)^4 \delta^4(k_1 + k_2 - p_1 - p_2)$$

$$\cdot e^4 e_q^2 \frac{1}{4} \sum_{s_{e^-} s_{e^+}} \sum_{s_q s_{\bar{q}}} \frac{1}{s^2} \Big| \bar{v}(k_2, s_{e^+}) \gamma_\mu u(k_1, s_{e^-})$$

$$\cdot \bar{u}(p_1, s_q) \gamma^\mu v(p_2, s_{\bar{q}}) \Big|^2$$

$$d\sigma = \frac{1}{2s} \cdot \frac{1}{(2\pi)^2} \int d\Omega_{\bar{p}_1} \frac{d|\bar{p}_1| \, |\bar{p}_1|^2}{4(p_1^0)^2} \underbrace{\delta\left(\sqrt{s} - 2|\bar{p}_1|\right)}_{\frac{1}{2}\delta(|\bar{p}_1| - \sqrt{s}/2)}$$

$$\cdot e^4 \cdot e_q^2 \frac{1}{4} \frac{1}{s^2} \cdot \underbrace{Tr\{\not{k}_2 \gamma_\mu \not{k}_1 \gamma_\nu\}}_{4\{k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu} - (k_1 \cdot k_2) g_{\mu\nu}\}} \cdot Tr\{\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu\}$$

$$4\{p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - (p_1 \cdot p_2) g^{\mu\nu}\}$$

DIFFERENTIAL CROSS SECTION

$$\boxed{\frac{d\sigma}{d\cos\theta}(e^+e^- \to q\bar{q}) = \underbrace{\frac{\pi\alpha^2}{2s}(1+\cos^2\theta)}_{\downarrow \; \frac{d\sigma}{d\cos\theta}(e^+e^- \to \mu^+\mu^-)} \quad N_c \sum_q e_q^2}$$

$(1+\cos^2\theta)$ FACTOR IS CONSEQUENCE OF SPIN 1/2 NATURE OF QUARKS !

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to 2\text{ JETS}) \sim (1+\cos^2\theta)$$

SAME ANGULAR DISTR AS UNDERLYING $e^+e^- \to q\bar{q}$ CROSS SECTION

TOTAL CROSS SECTION

$$\sigma_{total} = \int d\cos\theta \, \frac{d\sigma}{d\cos\theta}$$

$$\downarrow \qquad \int_{-1}^{1} d\cos\theta \, (1+\cos^2\theta) = \frac{8}{3}$$

$$\boxed{\sigma_{tot}(e^+e^- \to q\bar{q}) = \underbrace{\frac{4\pi\alpha^2}{3s}}_{\sigma_{tot}(e^+e^- \to \mu^+\mu^-)} \cdot N_c \sum_q e_q^2}$$

$$R \equiv \frac{\sigma_{tot}(e^+e^- \to q\bar{q})}{\sigma_{tot}(e^+e^- \to \mu^+\mu^-)} = N_c \cdot \sum_q e_q^2$$

$e^+e^- \longrightarrow$ 2 JETS

$e^+e^- \to q\bar{q} \to$ 2 JETS

TASSO $\Rightarrow$ PETRA e^+e^- R
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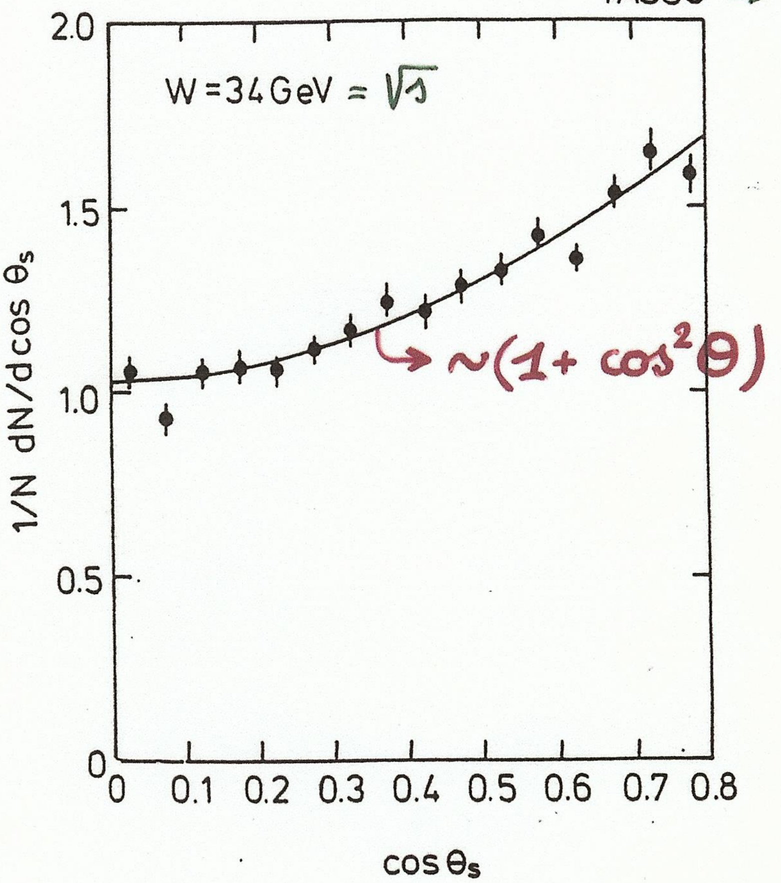


SPIN $\frac{1}{2}$ QUARKS : $\frac{d\sigma}{d\cos\theta} \sim (1+\cos^2\theta)$

SPIN 0 QUARKS : $\frac{d\sigma}{d\cos\theta} \sim \sin^2\theta$

⇒ RATIONALE FOR COLOR QUANTUM NUMBER

$$R = \frac{\sigma_{tot}(e^+e^- \to q\bar{q})}{\sigma_{tot}(e^+e^- \to \mu^+\mu^-)} = \sum_q e_q^2$$

↑ IF EACH QUARK (u, d, s, c, b, t) COMES IN ONE "CHARGE" STATE

* e.g. FOR $\sqrt{s} < M_{J/\psi}$

ONLY $e^+e^- \to u\bar{u}$
$\to d\bar{d}$
$\to s\bar{s}$ POSSIBLE

$$R = e_u^2 + e_d^2 + e_s^2 = \frac{4}{9} + \frac{1}{9} + \frac{1}{9} = \frac{2}{3}$$

EXPERIMENTALLY ONE OBSERVES $R = 2$!

* FOR $\sqrt{s} > M_\Upsilon$ BUT $\sqrt{s} <$ $t\bar{t}$ THRESHOLD

$e^+e^- \to u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}$ POSSIBLE

$$R = e_u^2 + e_d^2 + e_s^2 + e_c^2 + e_b^2$$
$$= \frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} = \frac{11}{9}$$

EXP. ONE OBERVES $R = \frac{11}{3}$

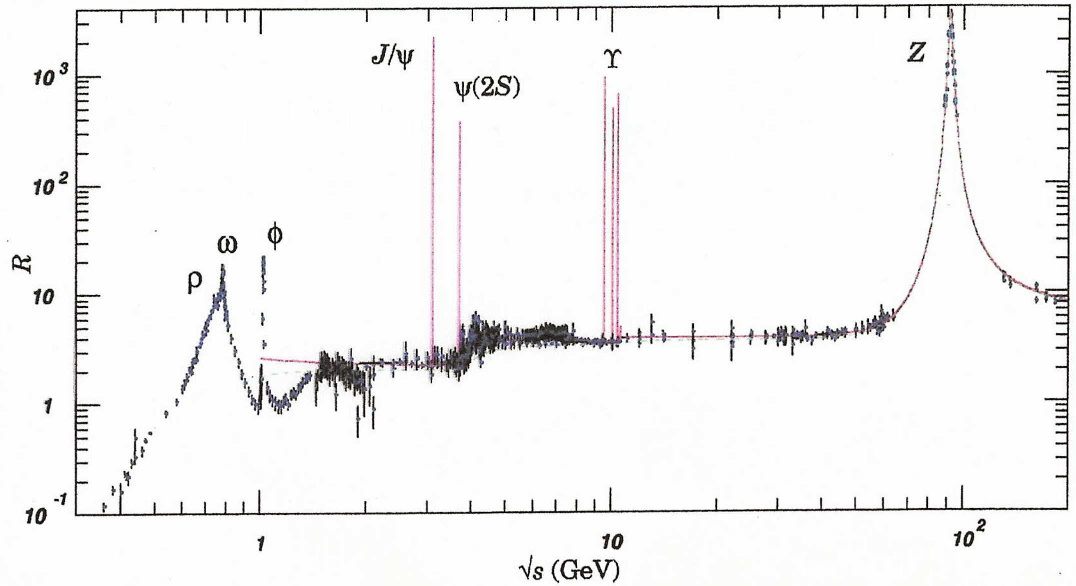
∴ EACH QUARK COMES IN 3 COLORS $(N_c = 3)$

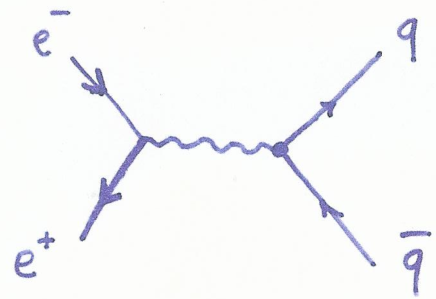
$$R = N_c \cdot \sum_q e_q^2$$

- Number of colours (N_c) can be tested in experiment:

e.g. $$R = \frac{\sigma[e^+e^- \to \text{hadrons}]}{\sigma[e^+e^- \to \mu^+\mu^-]} \propto N_c \quad \text{[see below]} \qquad (11)$$

$$= N_c \sum_q e_q^2$$





also $d\sigma[pp \to \mu^+\mu^- X]$, $\Gamma[\pi^0 \to \gamma\gamma]$

$\Rightarrow$ all measurements imply $N_c = 3.0 \pm \ldots$ $\quad\checkmark$