Exercise sheet 9
Theoretical Physics 2: SS2016

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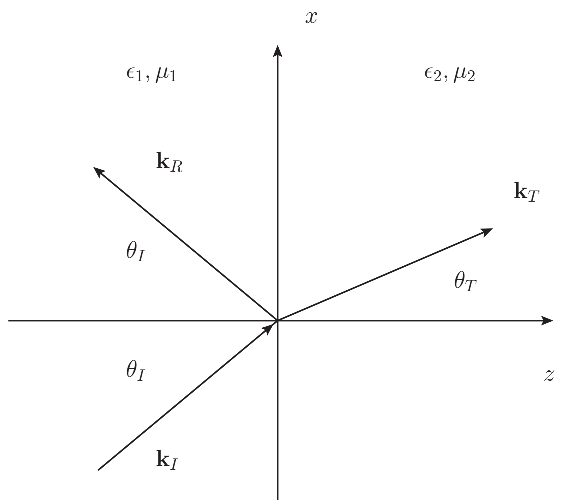
Exercise 1 (35 points): Fresnel equations

Consider a wave travelling from the left which strikes the interface between two media with the angle θ_I to the normal of xy-plane (see Fig. below).

(a) (10 points)
Write down the equation for the Brewster's angle. What will you have for $\mu_1 \approx \mu_2$? What is the angle between the reflected wave and the transmitted one?

(b) (10 points)
Find the angle when the amplitude for the reflected wave will be the same as the amplitude for the transmitted wave. What will you have for $\mu_1 \approx \mu_2$?

(c) (15 points)
The diamond refractive index is 2.42. Find the numerical value of the Brewster's angle, the critical angle of the total reflection, and the angle when the amplitude for the reflected wave will be the same as the amplitude for the transmitted wave.



Exercise 2 (40 points): Rainbow

The rainbow can be observed after the rain in the opposite direction of the sunlight. The sunlight can enter a water drop, then it can be reflected a few times inside a drop and refracted when leaving the drop. The primary rainbow is created by the light reflected once inside a water drop (see Fig. below) and the secondary rainbow is created by the light reflected twice.

(a)(10 points)
Find the expression for the light scattering angle θ between the incident ray and outgoing ray after N reflections inside the drop as a function of the light ray impact parameter ρ.

(b)(10 points)
Neglecting the wave nature of the light, find the expression for the radiant light intensity $dI/d\Omega$ angular dependence as a function of the scattering angle θ after N reflections inside the water drop.

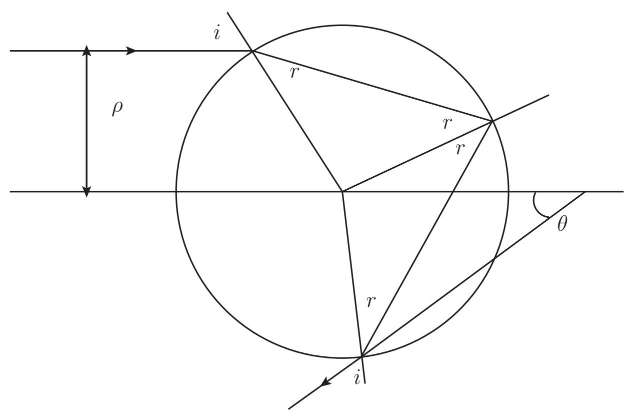
(c)(10 points)
Find the scattering angle which maximizes the intensity of the light after N reflections inside the water droplet. Evaluate numerically the angle for the primary (θ_1) and the secondary (θ_2) rainbows, assuming the water refractive index $n = 4/3$.

(d)(5 points)

Can the primary or secondary rainbows be observed if the sun is 47^0 above the horizon? How does the answer change for the sun at 32^0, 60^0 above the horizon?

(e)(5 points)

The refractive index for the red light is $n = 1.331$ and for the blue light it is $n = 1.340$. Find the scattering angles for the red and blue bows of the primary and secondary rainbows. What is the order of colours in these rainbows? Explain the presence of the dark region between two rainbows.



Exercise 3 (25 points): Nonlocal connection of the displacement and electric field

A consequence of the dependence of $\epsilon(\omega)$ is a temporally nonlocal connection between the displacement $\mathbf{D}(\mathbf{x}, t)$ and the electric field $\mathbf{E}(\mathbf{x}, t)$. If the monochromatic components of the frequency ω are related as

$$\mathbf{D}(\mathbf{x}, \omega) = \epsilon(\omega)\mathbf{E}(\mathbf{x}, \omega). \tag{1}$$

Using Fourier analysis (*faltung* theorem) the time dependence can be expressed as

$$\mathbf{D}(\mathbf{x},t) = \mathbf{E}(\mathbf{x},t) + \int\limits_{-\infty}^{\infty} G(\tau)\mathbf{E}(\mathbf{x},t-\tau)d\tau, \tag{2}$$

where $G(\tau)$ is the Fourier transform of $\chi = \epsilon(\omega) - 1$. Show that the in the case of one damped resonant frequency

$$G(\tau) = \frac{Nq^2 f_0}{m} e^{\gamma_0 \tau/2} \frac{\sin \nu_0 \tau}{\nu_0} \theta(\tau), \tag{3}$$

where $\nu_0 = \omega_0^2 - \frac{\gamma_0^2}{4}$ and $\theta(\tau)$ is the step function. Hint.

$$G(\tau) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} d\omega \chi(\omega) e^{i\omega\tau}.$$