Exercise sheet 7
Theoretical Physics 2: SS2016
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Exercise 1 (20 points): Lorentz invariants

(a) (7 points)
Compute

$$F_{\mu\nu}F^{\mu\nu}, \quad \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}, \quad F_{\mu\nu}\tilde{F}^{\mu\nu},$$

in terms of the fields **E** and **B**.
(b) (5 points)
What can be concluded from these results with regard to the Lorentz invariance of the 3 quantities?
(c) (5 points)
Show that $\varepsilon_{\mu\nu\rho\sigma}$ is invariant with respect to proper Lorentz transformations.
(d) (3 points)
Show that the contraction of a symmetric tensor $S_{\mu\nu}$ with an antisymmetric tensor $A_{\mu\nu}$ is zero.

Exercise 2 (35 points): Moving magnetic moment

A magnetic dipole moment **m** is located at the origin of an inertial system S' that moves with speed **v** with respect to S. In S' the potential is given by

$$\mathbf{A}' = \frac{\mathbf{m} \times \mathbf{r}'}{4\pi r'^3} \quad \text{and} \quad \phi' = 0. \tag{1}$$

(a) (15 points)
Find the scalar potential ϕ in S.
(b) (10 points)
Find ϕ in the nonrelativistic limit.
(c) (10 points) Calculate $\mathbf{E}$ from ϕ.

Exercise 3 (20 points): Electric and magnetic fields in different reference frames

In an inertial system S the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ are neither parallel nor perpendicular.
(a) (5 points)
Show that $\mathbf{E} \cdot \mathbf{B}$ is in general relativistically invariant.
(b) (5 points)
Is it possible to have an electromagnetic field that appears as a purely electric field in one inertial frame and as a purely magnetic field in some other inertial frame ?
(c) (10 points)
Show (independent of the choice of S), that in an inertial system S', moving relative to S with velocity $\mathbf{v}$ given by

$$\frac{\mathbf{v}}{1 + v^2/c^2} = \frac{c\mathbf{E} \times \mathbf{B}}{E^2 + B^2}$$

the fields $\mathbf{E}$ and $\mathbf{B}$ are parallel.

Exercise 4 (25 points): Motion of a charge in a constant uniform electric field

A particle with q moves in an electromagnetic field with the field tensor $F_{\mu\nu}$. The covariant equation of motion is given by

$$m\frac{du^\mu}{d\tau} = \frac{q}{c} F^\mu{}_\nu u^\nu,$$

with $u^\mu = dx^\mu/d\tau = \gamma dx^\mu/dt$ and the relativistic momentum $p^\mu = mu^\mu$.

(a) (10 points)

Derive equations of motion for the energy W and the momentum $\mathbf{p}$ using the explicit representation of $F_{\mu\nu}$ and the covariant equation of motion.

(b) (15 points)

Solve these equations and find the path $\mathbf{r}(t)$ of the particle in a uniform electric field $\mathbf{E} = E_0\mathbf{e}_x$, where the particle starts at the beginning in the origin with velocity $\mathbf{v} = v_0\mathbf{v}_x$.