Exercise sheet 6
Theoretical Physics 2: SS2016
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Exercise 1 (30 points): Lorentz matrix and Muon lifetime

a) (10 points)
Using the invariance of the 4-dimensional distance between space-time points under a Lorentz transformation

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu,$$

show that the Lorentz matrix satisfies

$$\eta_{\mu\nu}\Lambda^\mu{}_{\mu'}\Lambda^\nu{}_{\nu'} = \eta_{\mu'\nu'}, \tag{1}$$

and that this expression can be written shortly as

$$\Lambda^T\Lambda = 1.$$

b) (20 points)
In a laboratory experiment a muon is observed to travel a distance of 800 m before decay. Using the lifetime of a muon, 2×10^{-6} s, the velocity can be found as

$$v = \frac{800 \text{ m}}{2 \times 10^{-6} \text{ s}} = 4 \times 10^8 \ \frac{\text{m}}{\text{s}} > c.$$

Identify the error and the actual speed of the muon.

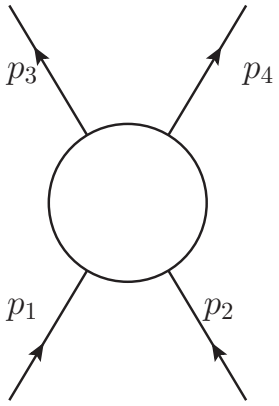
Exercise 2 (20 points): Example of the Lorentz matrix

The inertial system S' moves at constant velocity $\mathbf{v} = \beta c\,(\cos\phi\,\mathbf{e}_x + \sin\phi\,\mathbf{e}_y)$ with respect to S. Find the Lorentz transformation matrix Λ.

Exercise 3 (30 points): Kinematic invariants

Consider a scattering process of two incident particles of masses m_1, m_2 and two emerging particles of masses m_3 and m_4. The four momenta is labeled as p_i, $i = 1, 2, 3, 4$ (use units with $c = 1$). It is common to use the following variables to describe such a reaction:

$$s = (p_1 + p_2)^2, \qquad t = (p_1 - p_3)^2, \qquad u = (p_1 - p_4)^2. \tag{2}$$



a) (5 points)
Show that

$$s + t + u = \sum_{i=1}^{4} m_i^2 \tag{3}$$

b) (10 points)
Assume that the reaction is elastic and let

$$m_1 = m_3 = M, \qquad m_2 = m_4 = m. \tag{4}$$

In the Center of Momentum (C.M.) frame let the initial and final three-momenta of the particle M be $\mathbf{k}$ and $\mathbf{k}'$ respectively. Express s, t, and u in terms of $\mathbf{k}$ and $\mathbf{k}'$, simplifying as much as possible. Interpret s, t, and u.

c) (15 points)
Assume that in the laboratory frame the particle m is initially at rest. Express the initial and final laboratory energies of particle M, as well as the scattering angle, in terms of s, t, and u.

Exercise 4 (20 points): Electron-positron annihilation

In a pair annihilation experiment, an electron with mass m and momentum $\mathbf{p}_e$ hits a positron at rest. They annihilate, producing two photons. If one of the photons emerges at 60^0 to the incident electron direction, what is its energy (depending on the electron momentum $|\mathbf{p}_e|$)? Could electron and positron annihilate into one photon? Explain the answer.