Exercise sheet 1
Theoretical Physics 2: SS2016
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Exercise 1 (20 points): Divergence

Work out the following expression:

$$\nabla\cdot\left(\mathbf{r}f(r)\right), \tag{1}$$

in terms of f and df/dr.
Apply the previous result to the particular function $f(r) = r^{n-1}$.
For which value of n does the divergence vanish ?

Exercise 2 (40 points): Cylindrical coordinates

The Navier-Stokes equations of hydrodynamics contain a nonlinear term of the form

$$\nabla \times \left[\mathbf{v} \times (\nabla \times \mathbf{v})\right], \tag{2}$$

where $\mathbf{v}$ is the fluid velocity. Consider a fluid flowing through a cylindrical pipe in the z-direction with

$$\mathbf{v} = v(r)\mathbf{e_z}, \tag{3}$$

where $\mathbf{e_z}$ is the unit-vector in the z-direction. Make use of cylindrical coordinates (r, ϕ, z) to express

$$\begin{aligned} &\nabla \times \mathbf{v}, \\ &\mathbf{v} \times (\nabla \times \mathbf{v}), \\ &\nabla \times [\mathbf{v} \times (\nabla \times \mathbf{v})], \end{aligned} \tag{4}$$

in the cylindrical coordinates unit vectors $\mathbf{e_r}, \mathbf{e}_\phi, \mathbf{e_z}$.

Exercise 3 (40 points): Spherical coordinates

Consider a magnetic vector potential of the form

$$\mathbf{A} = C\frac{\mathbf{m} \times \mathbf{r}}{r^3}, \tag{5}$$

which corresponds to a point magnetic dipole. The constant vector $\mathbf{m}$ is called the magnetic dipole moment, and C is a constant factor.
For $\mathbf{m} = m\,\mathbf{e_z}$, express the components of the magnetic field $\mathbf{B}$ in spherical coordinates (r, θ, ϕ), with unit vectors $\mathbf{e_r}, \mathbf{e}_\theta, \mathbf{e}_\phi$.