11. Übungsblatt
Theoretische Physik 6: WS2014/15
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Exercise 1 (60 points): Electron-positron annihilation into a scalar quark-antiquark pair $e^+e^- \to q\bar{q}$

Consider electron-positron annihilation into a quark-antiquark pair $e^+(k_2)e^-(k_1) \to q(p_2)\bar{q}(p_1)$. Treat the electron as a massless Dirac particle and the quark as a massless Klein-Gordon particle.

(a) (20 points) The squared matrix element obtained as an average over electron and positron spin configurations can be expressed as

$$|M|^2 = \frac{e^4 e_q^2}{s^2} L^{\mu\nu} Q_{\mu\nu}$$

with the quark charge - $e_q e$.

Find the expressions for the quark tensor $Q_{\mu\nu}$ and the lepton tensor $L_{\mu\nu}$ in terms of momenta k_1, k_2, p_1, p_2.

Solution:

$$L^{\mu\nu} = k_2^\mu k_1^\nu + k_1^\mu k_2^\nu - \frac{s}{2} g^{\mu\nu}$$
$$Q^{\mu\nu} = (p_2 - p_1)_\mu (p_2 - p_1)_\nu$$

(b) (10 points) Calculate $L^{\mu\nu} Q_{\mu\nu}$ in terms of the Mandelstam variable s and the angle between the initial electron momenta and the final anti-quark momenta in the center-of-mass frame.

Solution:

$$L^{\mu\nu} Q_{\mu\nu} = \frac{s^2}{2} \sin^2\theta$$

(c) (10 points) Express the result for $L^{\mu\nu}Q_{\mu\nu}$ in terms of the Mandelstam variable $s = (k_1 + k_2)^2$ and variable $t = (k_1 - p_1)^2$.

Solution:

$$L^{\mu\nu}Q_{\mu\nu} = -2t(s + t)$$

(d) (5 points) Write down the expression for the differential cross-section in the center-of-mass frame in terms of the matrix element $|M|^2$.

(e) (10 points) Integrate over the quark phase space and the antiquark momentum and obtain the expression for the differential cross section in terms of the energy in the center-of-mass frame squared $s = (k_1 + k_2)^2$ and the angle between k_1 and p_1.

Solution:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{1}{32\pi s}|M|^2$$

(f) (5 points) Find the result for the differential cross section in the center of mass frame.

Exercise 2 (40 points): Electron-positron annihilation: $e^+e^- \to \mu^+\mu^-$

Consider electron-positron annihilation into a muon-antimuon pair $e^+(k_2)e^-(k_1) \to \mu^+(p_2)\mu^-(p_1)$. Treat the electron and muon as massless Dirac particles.
Starting from

$$\mathrm{d}\sigma = \frac{1}{2s}\frac{1}{(2\pi)^2}\int d\Omega_{\vec{p}_1}\frac{\mathrm{d}|\vec{p}_1|\,|\vec{p}_1|^2}{4(p_1^0)^2}\delta(\sqrt{s} - 2|\vec{p}_1|)\frac{e^4}{4s^2}\,\mathrm{Tr}(\not{k}_2\gamma_\mu\not{k}_1\gamma_\nu)\,\mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu),$$

derive the differential cross section in the center of mass frame:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}(e^+e^- \to \mu^+\mu^-) = \frac{\pi\alpha^2}{2s}(1 + \cos^2\theta).$$