Exercise sheet 10
Theoretical Physics 2: SS2016
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Exercise 1 (25 points): Skin effect

(a) (10 points)
Silver is a very good and expensive conductor. How thick should be the silver coating of a microwave experiment with the frequency of waves 10^{10} Hz? (The resistivity of silver is $\rho_{Ag} = 1/\sigma_{Ag} = 1.59 \cdot 10^{-8}\ \Omega\text{m}$)
(b) (15 points)
Calculate the wavelength and the velocity of the radiowaves with the frequency 1 MHz in copper and compare these with the values in air (or in vacuum). Calculate the Skin depth for the wave with frequency 60 Hz, 1 MHz, 100 MHz. (The resistivity of copper is $\rho_{Cu} = 1/\sigma_{Cu} = 1.68 \cdot 10^{-8}\ \Omega\text{m}$)

Exercise 2 (35 points): TEM mode

The coaxial transmission line consists of two concentric conducting hollow cylinders of radius a and b ($a < b$). The wave propagates in the space between two cylinders. With this geometry the transmission of TEM-waves is possible.
(a) (10 points)
Consider the following wave

$$\mathbf{E} = \mathbf{E}_0(x,y)e^{i(kz-\omega t)}, \qquad \mathbf{B} = \mathbf{B}_0(x,y)e^{i(kz-\omega t)}.$$

From the Maxwell equations with $E_z = B_z = 0$ find

$$\begin{aligned} \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = 0, \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0 \\ \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0, \quad \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = 0 \\ \omega = kc, \quad E_y = -B_x, \quad \text{and} \quad E_x = B_y. \end{aligned} \tag{1}$$

(b) (5 points)
Using the fact that E_x and E_y are only functions of x, y, introduce an scalar function $\phi(x, y)$ to show that Eq.(**??**) reduces to the Laplace equation in two dimensions. Write down the equation in cylindrical coordinates.
(c) (20 points)
Find the electric and magnetic field in cylindrical coordinates. Prove that the fields **E** and **B** satisfy the boundary conditions.
Hint: A solution of the laplace equation with the given symmetry is $\phi = A + B\ln(r)$

Exercise 3 (40 points): TM Mode

The magnetic field for the transverse magnetic (TM) mode has no longitudinal component ($B_z = 0$) and the electric field in the rectangular wave guide is given by

$$E_z = E_0 \sin\frac{\pi m x}{a} \sin\frac{\pi n y}{b} e^{i(kz-\omega t)}.$$

(a)(5 points)
What TM modes will propagate in the wave guide with dimensions $2.28\text{cm} \times 1.01\text{cm}$, if the driving frequency is $1.7 \times 10^{10}\text{Hz}$?
(b)(10 points)
Suppose you wanted to excite only one TM mode; what range of frequencies could you use? What are the corresponding wavelengths (in open space)?
(c)(25 points)
Confirm that the energy in the TM_{mn} mode travels at the group velocity.
Hint: Find the time averaged Poynting vector (**S**) and the energy density $< u >$. Integrate over the cross section of the wave guide to get the energy per unit time and per unit length carried by the wave, and take the ratio.