10. Übungsblatt
Theoretische Physik 6: WS 2014/15
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Exercise 1 (40 points): Energy-momentum operator

Express the following quantities in terms of creation and annihilation operators:

(a) (20 points) Energy: $H = c\int d^3x\,\bar{\psi}(-i\hbar\gamma^i\partial_i + m_0c)\psi$;

(b) (20 points) Momentum: $\mathbf{P} = -i\hbar\int d^3x\,\psi^\dagger\nabla\psi$.

Exercise 2 (20 points)

Calculate $[H, b_r^\dagger(\mathbf{p})b_r(\mathbf{p})]$.

Exercise 3 (40 points): Axial current

For a Dirac field, the transformations

$$\psi(x) \to \psi'(x) = e^{i\alpha\gamma_5}\psi(x), \qquad \psi^\dagger(x) \to \psi^{\dagger\prime}(x) = \psi^\dagger(x)e^{-i\alpha\gamma_5},$$

where α is here an arbitrary real parameter, are called chiral phase transformations.

(a) (20 points) Show that the Dirac Lagrangian density $\mathcal{L} = c\bar{\psi}(i\hbar\partial\!\!\!/ - m_0c)\psi$ is invariant under chiral phase transformations in the zero-mass limit $m_0 = 0$ only, and that the corresponding conserved current in this limit is the axial vector current $J_A^\mu \equiv \bar{\psi}(x)\gamma^\mu\gamma_5\psi(x)$.

(b) (20 points) Deduce the equations of motion for the fields

$$\psi_L(x) \equiv \frac{1}{2}(\mathbb{1} - \gamma_5)\psi(x), \qquad \psi_R(x) \equiv \frac{1}{2}(\mathbb{1} + \gamma_5)\psi(x),$$

for non-vanishing mass, and show that they decouple in the limit $m_0 = 0$.

Hence, the Lagrangian density $\mathcal{L} = i\hbar c\bar{\psi}_L \not{\partial} \psi_L$ describes massless fermions with negative helicity and massless anti-fermions with positive helicity only. This field is called the Weyl field and can be used to describe the neutrinos as far as the latter can be considered as massless.