Parton distribution functions

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Introductions

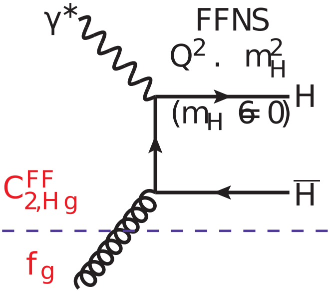
- Description of structure functions F_i by
  - $F_i(x, Q^2) = \sum_{a=q,g} C_{i,a} \otimes f_{a/A}(x, Q^2)$
  - PDFs $f_{a/A}$ of flavour a in a hadron A
  - Coefficient functions $C_{i,a}$
- Hadron cross sections σ_{AB} in hadron-hadron collisions
  - $\sigma_{AB} = \sum_{a,b=q,g} \hat{\sigma}_{ab} \otimes f_{a/A}(x_1, Q^2) \otimes f_{b/B}(x_2, Q^2)$
  - Process-dependent partonic cross sections $\hat{\sigma}_{ab}$
- Scale dependence given by DGLAP evolution equations
  - $\frac{\partial f_{a/A}}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes f_{a'/A}$
  - Splitting functions $P_{a,a'}$
- $C_{i,a}, \hat{\sigma}_{a,b}, P_{a,a'}$ perturbative series in α_s
- Number sum rules:

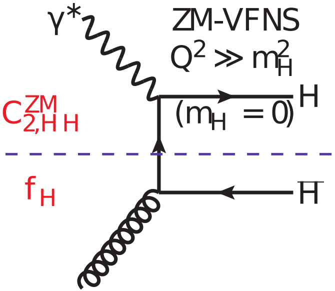
$$\int_0^1 \mathrm{d}x \; u_v(x, Q_0^2) = 2 \quad \int_0^1 \mathrm{d}x \; d_v(x, Q_0^2) = 1 \quad \int_0^1 \mathrm{d}x \; s_v(x, Q_0^2) = 0$$

- Momentum sum rule:

$$\int_0^1 \mathrm{d}x \; x[u_v(x, Q_0^2) + d_v(x, Q_0^2) + S(x, Q_0^2) + g(x, Q_0^2)] = 1$$

- Important for precise measurements at hadron colliders
- Delicate issue to obtain proper treatment
  - Choices/mistakes lead to changes in PDFs
  - Larger than quoted uncertainties
- Two distinct regimes:
  - Fixed flavour number scheme (FFNS)
  - Zero mass variable number scheme (ZM-VFNS)





Fixed flavour number scheme

- Hard scale $\lesssim m_H$
- Describe heavy quark as final-state particle (not as parton)
- Only light quarks are partons $\Rightarrow$ flavour number is fixed
- Usually $n_f = 3$ but also possibility for $n_F = 4, 5$
- Structure functions are $F_i = \sum_k C_{i,k}^{FF,n_f}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$
- Contains all m_H dependent contributions
- Problems
  - Does not sum $\alpha_s^m \ln^j(Q^2/m_H^2)(j \leq m)$, problem for $Q^2 > m_H^2$
  - Calculations including full mass dependence complicated
  - FFNS coefficient functions, only known up to NLO for neutral current structure functions

Zero mass variable number scheme

- Problems of FFNS solved
  - Heavy quark evolves like massless quarks
  - Resumation of large logarithms is achieved by heavy quark PDF
- Assumption at $Q^2 \gg m_H^2$: heavy quark behaves like massless parton
  $\Rightarrow$ Same coefficient functions as in the massless limit
  $F_i = \sum_k C_{i,j}^{ZM,n_f} \otimes f_j^{n_f}(Q^2)$
- $n_f - 3$ number of active heavy flavour, switched on above transition point
- Typical at scales similar to m_H^2
- Some mass dependence included in boundary condition (evolution)
- PDFs in different quark number regimes related
  $f_j^{n+1} = \sum_k A_{jk}(Q^2/m_H^2) \otimes f_k^n(Q^2)$
- Perturbative matrix element A_{jk} contains ln terms
- Guaranteeing correct evolution in both regimes

General mass variable number scheme

- ZM-VFNS ignores $\mathcal{O}(m_H^2/Q^2)$ corrections ($\sim 6\%$ error in light quark PDFs at small-x)
- Connection between FFNS and ZM-VFNS
- No unique definition
- Possibility:
  - Equivalence of $n_f = n(FFNS)$ and $n_f = n+1(GM - VFNS)$ above transition point

$$
\begin{aligned}
F_i(x, Q^2) &= \sum_k C_{i,k}^{FF,n}(Q^2/m_H^2) \otimes f_k^n(Q^2) \\
&= \sum_j C_{i,j}^{VF,n+1}(Q^2/m_H^2) \otimes f_j^{n+1}(Q^2) \\
&= \sum_{j,k} C_{i,j}^{VF,n+1}(Q^2/m_H^2) \otimes A_{j,k}(Q^2/m_H^2) \otimes f_k^n(Q^2)
\end{aligned}
$$

- Problems of ZM-VFNS are thrown into sharp relief at NNLO
- $A_{j,k} \neq 0$ at $Q^2 = m_H^2$, discontinuities in PDFs

Heavy flavour treatment

	Heavy flavour	**Intrinsic charm**
MSTW	GM-VFNS (TR′)	–
MMHT	GM-VFNS ('optimal')	–
NNPDF	GM-VFNS (FONLL-B)	possible, small
HERAPDF	GM-VFNS (RTOPT)	–
CTEQ	GM-VFNS (S-ACOT-χ)	–
CJ	GM-VFNS (S-ACOT)	–

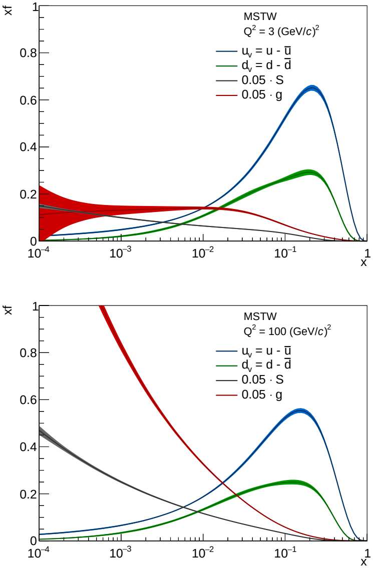
Input data

Experiment	NNPDF	MSTW (MMHT)	CTEQ	CJ	HERApdf
NMC	F_2^d/F_2^p, $\sigma^{NC,p}$	F_2^d/F_2^p, F_2^p, F_2^d, F_L	F_2^d/F_2^p, $\sigma^{NC,p}$	F_2^d/F_2^p, F_2^p	—
SLAC	F_2^p, F_2^d	F_2^p, F_2^d, F_L	—	F_2^p, F_2^d	—
BCDMS	F_2^p, F_2^d	F_2^p, F_2^d, F_L	F_2^p, F_2^d	F_2^p, F_2^d	—
E665	—	F_2^p, F_2^d	—	—	—
HERMES	—	—	—	F_2^p, F_2^d	—
JLab	—	—	—	F_2^p, F_2^d, F_2^n/F_2^d	—
CHORUS	σ_ν^{CC}, $\sigma_{\bar{\nu}}^{CC}$	F_2, xF_3	—	—	—
NuTev	σ_ν^{CC}, $\sigma_{\bar{\nu}}^{CC}$	F_2, xF_3 dimuon	dimuon	—	—
CDHSW	—	—	F_2, F_3	—	—
CCFR	—	dimuon	F_2, xF_3, dimuon	—	—
HERA	$\sigma_p^{NC,CC}$, σ_{NC}^c, F_2^b	$\sigma_p^{NC,CC}$, F_2^c(, F_L), jets	$\sigma_p^{NC,CC}$, σ_{NC}^c, σ^b, F_L	$\sigma_p^{NC,CC}$	$\sigma_p^{NC,CC}$
EMC	F_2^c	—	—	—	—
E866	σ_{DY}^p, $\sigma_{DY}^d/\sigma_{DY}^p$	σ_{DY}^p, $\sigma_{DY}^d/\sigma_{DY}^p$	σ_{DY}^p, $\sigma_{DY}^d/\sigma_{DY}^p$	σ_{DY}^p, $\sigma_{DY}^d/\sigma_{DY}^p$	—
E605	σ_{DY}^p	—	σ_{DY}^p	—	—
CDF	Z^0, jets	Z^0, $W(asy)$, jets	Z^0, $W(asy)$, jets	Z^0, $W(asy)$, jets	—
D0	Z^0, $W(asy)$	Z^0, $W(asy)$, jets	Z^0, $W(asy)$, jets	Z^0, $W(asy)$, jets	—
LHC	W, Z, DY, $t\bar{t}$, jets	(W, Z, DY)	W, Z, jets	—	—

- A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt
- DIS cuts: $Q^2 \geq 2\,\mathrm{GeV}^2$, $W^2 \geq 15\,\mathrm{GeV}^2$
- Parametrisation $u_v, d_v, S, s+\bar{s}$:

$$xq = Ax^{\eta_1}(1-x)^{\eta_2}\left(1+\epsilon\sqrt{x}+\gamma x\right)$$
$$x(\bar{d}-\bar{u}) = Ax^{\eta_1}(1-x)^{\eta_2}\left(1+\gamma x+\delta x^2\right)$$
$$xg = Ax^{\eta_1}(1-x)^{\eta_2}\left(1+\epsilon\sqrt{x}\gamma x\right) + A'x^{\delta'}(1-x)^{\eta'}$$
$$x(s-\bar{s}) = Ax^{\eta_1}(1-x)^{\eta_2}(1-x/x_0)$$

- 30 Parameters (including α_s)
- $\mu_0^2 = 1\,\mathrm{GeV}^2$



MMHT

- A.D. Martin, P. Motylinski, L.A. Harland-Lang, R.S. Thorne
- DIS cuts: $Q^2 \geq 2\,\mathrm{GeV}^2$, $W^2 \geq 15\,\mathrm{GeV}^2$
- Parametrisation $u_v, d_v, S, s+\bar{s}$:

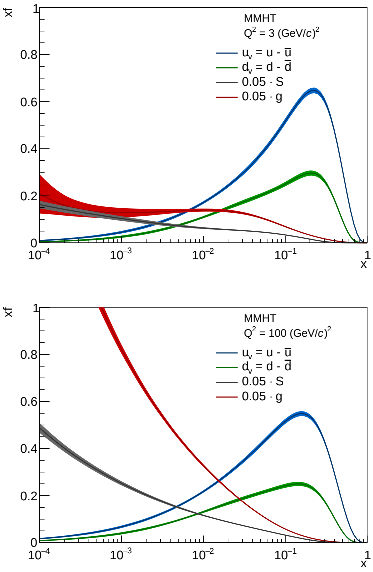
$$xq = A\,(1-x)^\eta\, x^\delta [1 + \sum_{i=1}^{4} a_i\, T_i^{Ch}(1-2\sqrt{x})]$$

$$x(\bar{d}-\bar{u}) = A\,(1-x)^\eta\, x^\delta\, \left(1+\gamma x + \delta x^2\right)$$

$$xg = A\,(1-x)^\eta\, x^\delta [1 + \sum_{i=1}^{2} a_i\, T_i^{Ch}(1-2\sqrt{x})] + A'\,(1-x)^{\eta'}\, x^{\delta'}$$

$$x(s-\bar{s}) = A\,(1-x)^\eta\, x^\delta\, (1 - x/x_0)$$

- 37 Parameters (including α_s)
- $\mu_0^2 = 1\,\mathrm{GeV}^2$

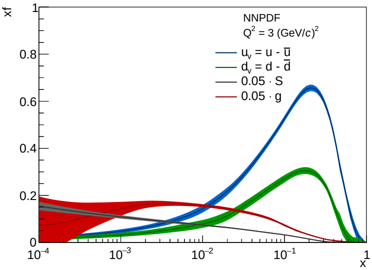


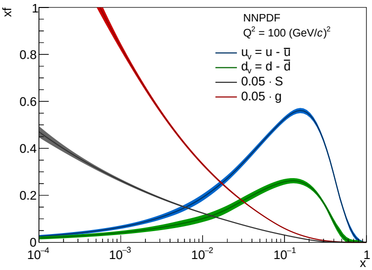
NNPDF

- Neural Network PDF
- DIS cuts: $Q^2 \geq 3.5\,\mathrm{GeV}^2$, $W^2 \geq 12.5\,\mathrm{GeV}^2$
- Neural network: $f_i = A_i x^\alpha (1-x)^\beta NN_i(x)$
- Preprocessing term for speed up

$$\begin{aligned}
\Sigma &= u + \bar{u} + d + \bar{d} + s + \bar{s} \\
T_3 &= u + \bar{u} - d - \bar{d} \\
T_8 &= u + \bar{u} + d + \bar{d} - 2s - 2\bar{s} \\
V &= u - \bar{u} + d - \bar{d} + s - \bar{s} \\
V_3 &= u - \bar{u} - d + \bar{d} \\
V_8 &= u - \bar{u} + d - \bar{d} - 2s + 2\bar{s} \\
&\quad + charm
\end{aligned}$$

- $\mu_0 = 1.65\,\mathrm{GeV}$

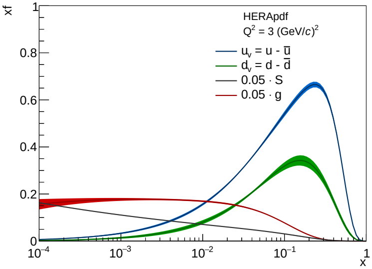


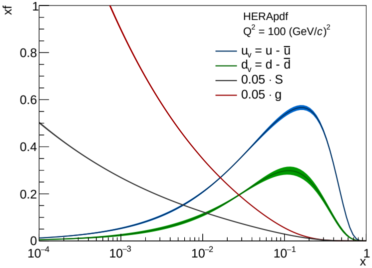


- Only HERA data
- DIS cuts: $Q^2 \geq 2.5\,\mathrm{GeV}^2$
- Parametrisation

$$\begin{aligned} xg &= Ax^B(1-x)^C - A'x^{B'}(1-x)^{C'} \\ xu_V &= Ax^B(1-x)^C(1+Ex^2) \\ xd_V &= Ax^B(1-x)^C \\ x\bar{U} &= Ax^B(1-x)^C(1+Dx) \\ x\bar{D} &= Ax^B(1-x)^C \end{aligned}$$

- $\mu_0^2 = 1.9\,\mathrm{GeV}^2$
- Up-type $(xU = xu + xc)$ distributions
- Down-type $(xD = xd + xs)$ distributions





CTEQ

- The Coordinated Theoretical-Experimental project on QCD
- DIS cuts: $Q \geq 2\,\mathrm{GeV}$, $W \geq 3.5\,\mathrm{GeV}$
- Parametrisation: $x^{a_1}(1-x)^{a_2}P(x)$

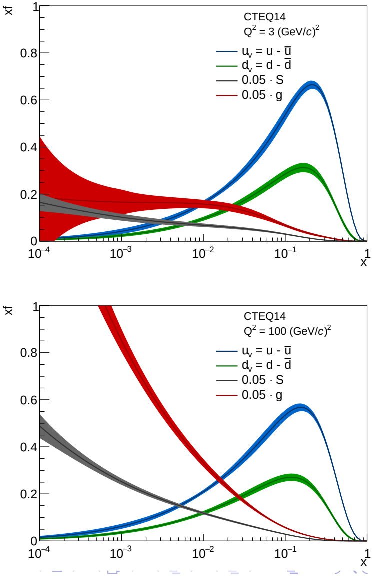
$$P_{u_v,d_v} = d_0 p_0(\sqrt{x}) + d_1 p_1(\sqrt{x}) + d_2 p_2(\sqrt{x}) + d_3 p_3(\sqrt{x}) + d_4 p_4(\sqrt{x})$$

$$P_g = g_0(e_0 q_0(2\sqrt{x}-x) + e_1 q_1(2\sqrt{x}-x) + q_2(2\sqrt{x}-x))$$

$$P_{\bar{u},\bar{d}} = 4^{th}\ order\ in\ 2\sqrt{x}-x$$

$$P_{s+\bar{s}} = const$$

- Bernstein polynomials p_i
- $\mu_0 = 1.4\,\mathrm{GeV}$



CTEQ-Jefferson Lab

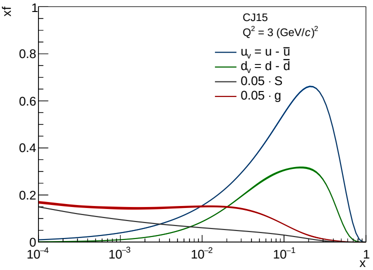
- DIS cuts:
  $Q^2 > 1.69\,\mathrm{GeV}^2$, $W^2 > 3\,\mathrm{GeV}^2$
- Parametrisation $u_v, d_v, \bar{u} + \bar{d}, g$

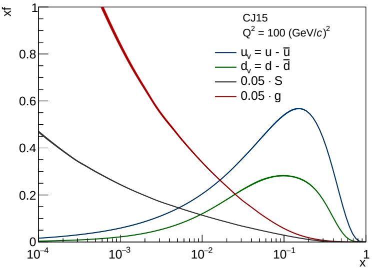
$$xq = a_0 x^{a_1}(1-x)^{a_2}(1 - a_3\sqrt{x} a_4 x)$$

$$d_v \to a_0 \left(\frac{d_v}{a_0} + b x^c u_v \right)$$

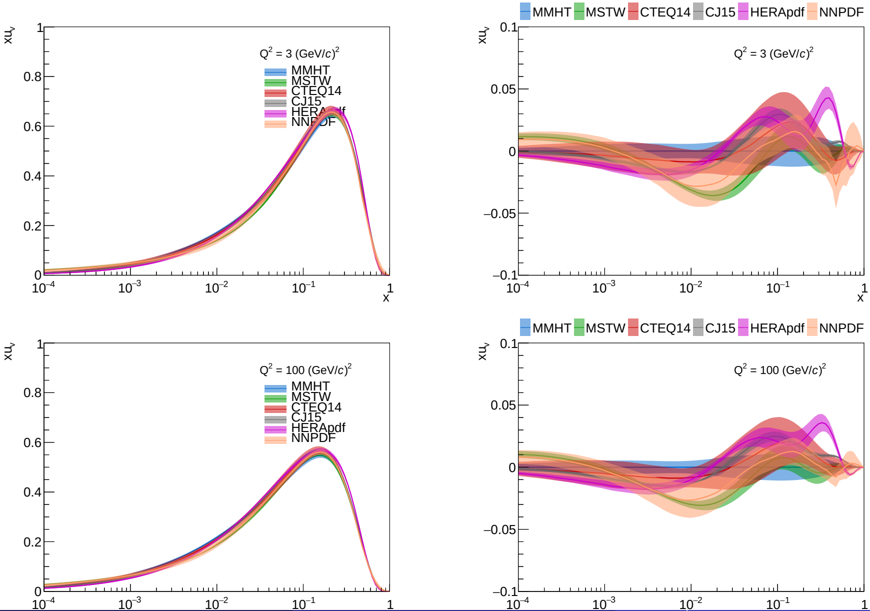
$$\frac{\bar{d}}{\bar{u}} = a_0 x^{a_1}(1-x)^{a_2} + 1 - a_3 x (1-x)^{a_4}$$

- $\mu_0 = m_c$





Comparison PDFs



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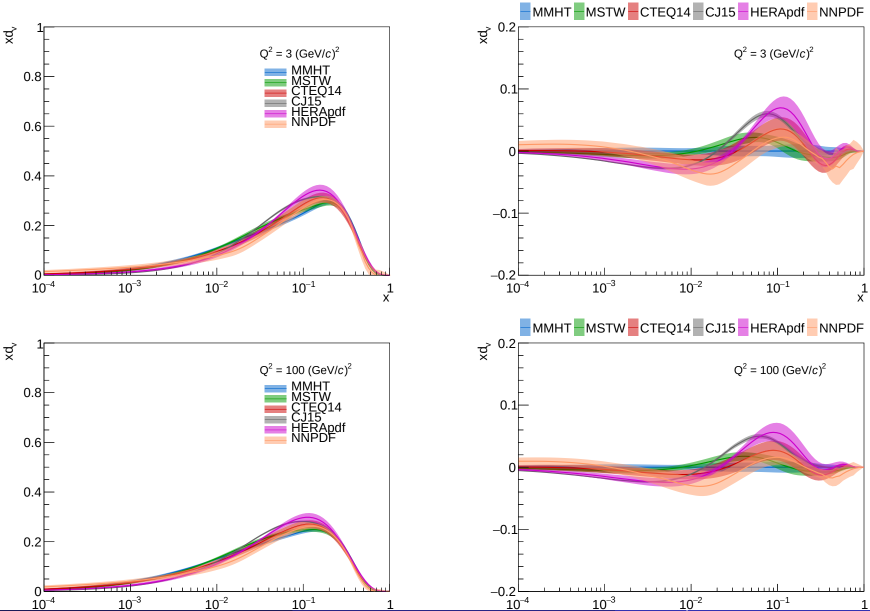

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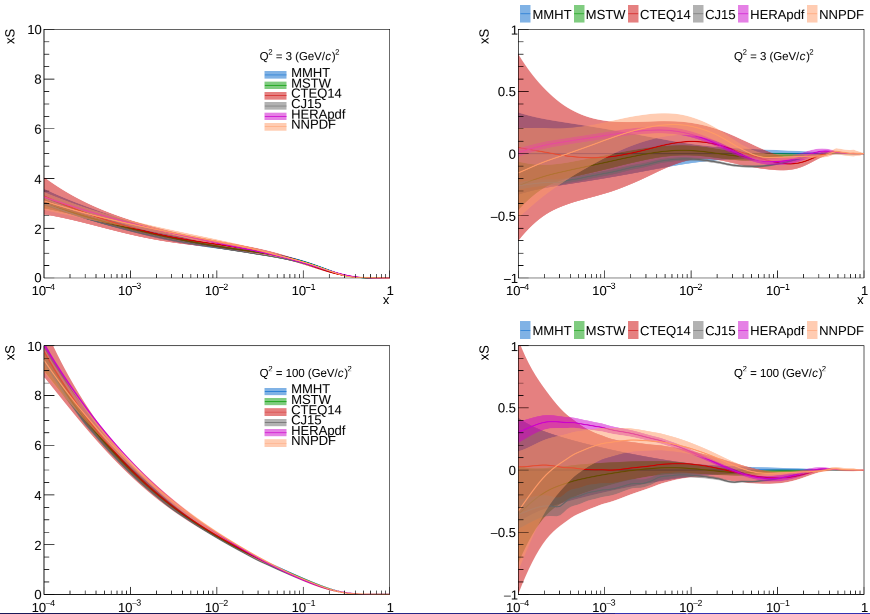

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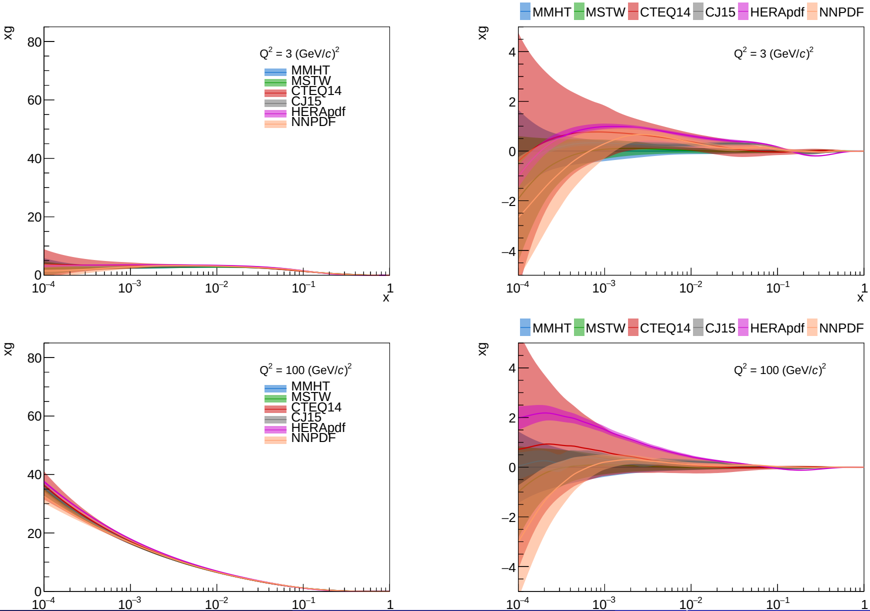

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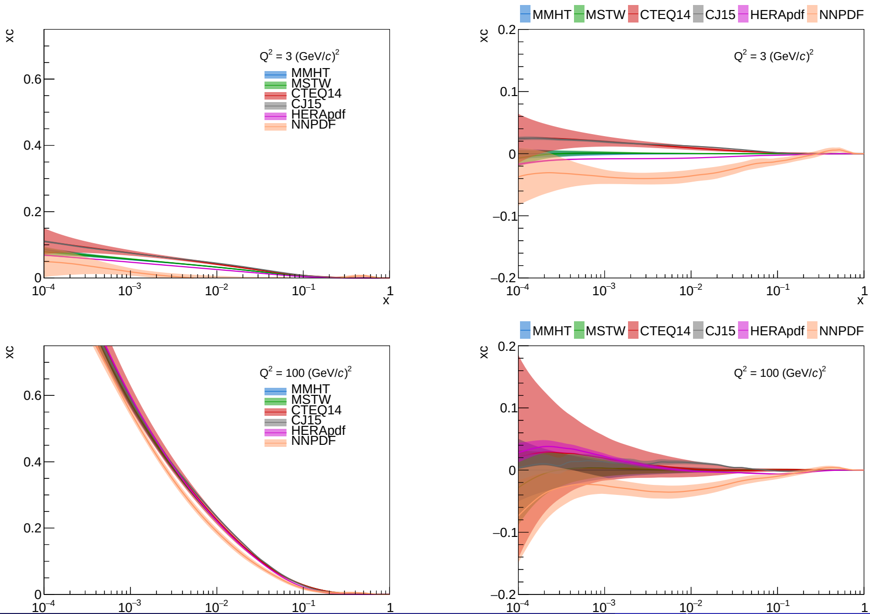

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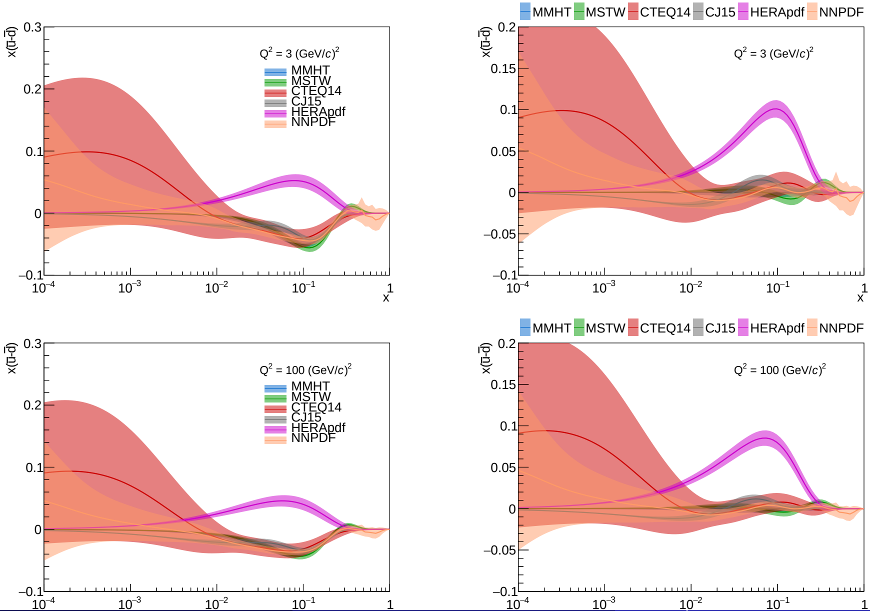

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