Decay constants

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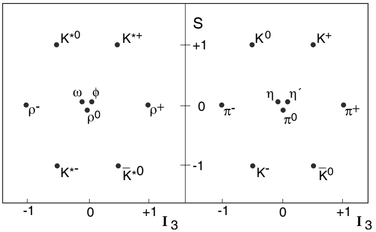
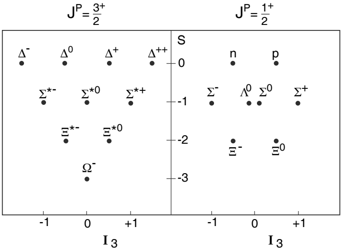
7^{th} June 2017

SU(2) flavour symmetries

- Isospin symmetry
- Observation: Proton and neutron same mass
- Two representations of the same particle (nucleon) with different isospin
- Based on flavour independence of strong force, u and d mass nearly the same
- $m_u, m_d \ll m_{\mathrm{baryon}}$

- Sorting the particle zoo
- Grouping by same spin j and parity P
- Sorted by isospin I_3 and hypercharge $Y = B + S$
- Charge given by $Q = I_3 + Y/2$
- Baryon decuplet wit $J^P = \frac{3}{2}^+$
- Baryon octet wit $J^P = \frac{1}{2}^+$
- Meson octets with $J^P = 1^-$ and 0^-





Helicity distributions

- The Δq can be expressed by $2MS_\mu \Delta q = \langle P, S|\bar{q}\gamma_\mu\gamma_5 q|P, S\rangle$ for a proton with momentum P and spin S
- This allows to introduce the singlet and axial currents

$$J^0_{5\mu} = \bar{\psi}\gamma_\mu\gamma_5\psi \qquad \text{with} \quad \psi = \begin{pmatrix} \psi_u \\ \psi_d \\ \psi_s \end{pmatrix}$$

$$J^i_{5\mu} = \bar{\psi}\gamma_\mu\gamma_5\frac{\lambda_i}{2}\psi$$

- Connected to the axial charges

$$\langle P, S|J^0_{5\mu}|P, S\rangle = 2MS_\mu a_0$$
$$\langle P, S|J^i_{5\mu}|P, S\rangle = MS_\mu a_i$$

- These are connected to F, D: $a_3 = F + D$, $a_8 = 3F - D$

Cabibbo theory

- Lagrangian full weak interaction: $\mathcal{L}_W = \frac{G}{\sqrt{2}} J^\mu \times J_\mu^+$
- SU(3) symmetry: Three Noether currents

$$V_\lambda^{\mathrm{lept}}(\Delta Q = 0)$$
$$V_\lambda^{(1)} + iV_\lambda^{(2)}(\Delta S = 0, \Delta Q = 1)$$
$$V_\lambda^{(4)} + iV_\lambda^{(5)}(\Delta S = \Delta Q = 1)$$

- Cabibbo:
  - Ignore $\Delta S = -\Delta Q$ component
  - Ignore problems of normalisation of non-leptonic processes
  - Universality between leptonic current and **only one** hadronic current

$$V_\lambda^{(\mathrm{had})} = a\left[V_\lambda^{(1)} + iV_\lambda^{(2)}\right] + a\left[V_\lambda^{(4)} + iV_\lambda^{(5)}\right]$$
$$1 = a^2 + b^2$$

Cabbibo theory

- Adding these hypothesis to the V-A formalism of weak interactions:

$$
\begin{aligned}
J_\lambda^{\text{lept}} &= \bar{\nu}_\mu \gamma_\lambda (1-\gamma_5)\mu + \bar{\nu}_e \gamma_\lambda (1-\gamma_5) e \\
J_\lambda^{\text{hadr}} &= \cos\theta \left[J_\lambda^{(1)} + i J_\lambda^{(2)} \right] + \sin\theta \left[J_\lambda^{(4)} + i J_\lambda^{(5)} \right] \\
J_\lambda^{(i)} &= V_\lambda^{(i)} - A_\lambda^{(i)}
\end{aligned}
$$

- θ: Cabibbo angle
- Octet of axial current A not conserved
- Octet normalisation are free parameters
- Vector currents: Normalisation fixed by strangeness non changing β decays

Cabbibo theory

- J_μ^{hadr} complicated in terms of individual baryon fields

$$
\begin{aligned}
J_\mu^{\text{hadr}} &= \cos\theta\,\bar{p}\gamma_\mu\left[1-(F+D)\gamma_5\right]n \\
&+\sin\theta\left\{-\sqrt{\frac{3}{2}}\bar{p}\gamma_\mu\left[1-\left(F+\frac{1}{3}D\right)\gamma_5\right]\Lambda\right\} \\
&+\sin\theta\left\{-\bar{n}\gamma_\mu\left[1-(F-D)\gamma_5\right]\Sigma^- - \bar{\Sigma}^+\gamma_\mu\left[1-(F+D)\gamma_5\right]\Xi^0\right\} \\
&+\sin\theta\left\{\sqrt{\frac{3}{2}}\bar{\Lambda}\gamma_\mu\left[1-\left(F-\frac{1}{3}D\right)\gamma_5\right]\Xi^-\right\} \\
&+\ldots
\end{aligned}
$$

- In terms of quarks:

$$
\begin{aligned}
J_\lambda &= \cos\theta\left[\bar{u}\gamma_\lambda(1-\gamma_5)(d+\tan\theta s)\right] \\
&= \bar{u}\gamma_\lambda(1-\gamma_5)d_C
\end{aligned}
$$

- Cabibbo angle: mixing angle

Decay constants

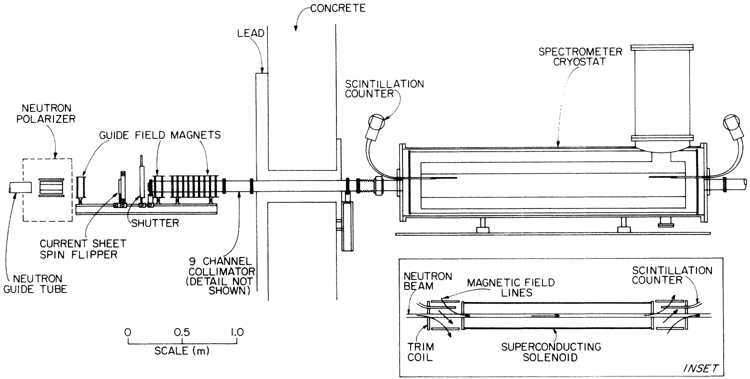
- Combinations of F, D in the β decays
- Two examples for their determination

Decay	Combination	Measurement
$n \rightarrow pe^-\bar{\nu}$	$F + D$	1.2723 ± 0.0023
$\Lambda \rightarrow pe^-\bar{\nu}$	$F + \frac{D}{3}$	0.718 ± 0.015
$\Sigma^- \rightarrow ne^-\bar{\nu}$	$F - D$	-0.340 ± 0.017
$\Xi^- \rightarrow \Lambda e^-\bar{\nu}$	$F - \frac{D}{3}$	0.25 ± 0.05
$\Xi^0 \rightarrow \Sigma^+ e^-\bar{\nu}$	$F + D$	1.22 ± 0.05

Neutron β decay

- Measure neutron β asymmetry
- Decay probability per time unit for emission of electron from polarised neutrons:
$$d\Gamma = (1 + \frac{v}{c} PA \cos\theta) d\Gamma(E)$$
- Neutron polarisation P
- θ: emission angle relative to the polarisation
- $d\Gamma(E)$: decay rate for unpolarised neutrons
- A: energy dependent, correction needed (recoil, induced currents)
- Largest deviation from A_0 smaller 1%
- $A_0 = -2\lambda(\lambda - 1)/(1 + 3\lambda^2)$
- $\lambda = |g_A/g_V|$

Neutron β decay



- 57 MW reactor in Grenoble, cold neutrons
- Measured polarisation $P > 97\%$
- Regular spin rotation (every 15 s)
- Decay electrons contained by magnetic field

Neutron - β decay

- Measure electron counting rate for two polarisations
- Result from 150 h of data
- $A_0 = -0.1146 \pm 0.0019 \rightarrow g_A/g_V = -1.262 \pm 0.005$

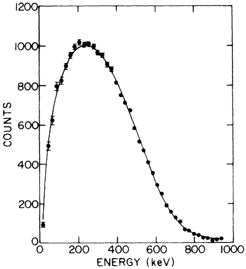


FIG. 2. β-decay energy spectrum from one run. Spectra for both polarization states are added and the solid curve is a fit to the resolution-corrected Fermi shape. The measured point at lowest energy is just at the threshold of the detectors.

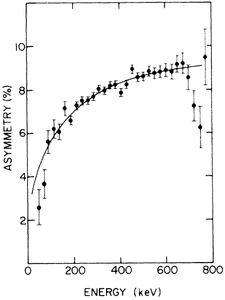


FIG. 3. Experimental β asymmetry as a function of β energy. The data for both detectors for all runs taken over a 150-h period are combined in the graph. The solid curve is the theoretical prediction after accounting for detector response.

Λ - β decay

- Matrix element for the decay

$$M = G/\sqrt{2}\langle p|J^{\mu}|\Lambda\rangle \bar{u}_e(p_e)\gamma_{\mu}(1+\gamma_5 u_{\nu}(p_{\nu}))$$

- Hadronic part:

$$\begin{aligned}\langle p|J^{\mu}|\Lambda\rangle =& \bar{u}_p[g_v(q^2)\gamma^{\mu} g_w(q^2)\sigma^{\mu\nu} q_{\nu} M_{\Lambda} \\ &+ g_a(q^2)\gamma^{\mu}\gamma_5 \\ &+ g_2(q^2)\sigma^{\mu\nu} q_{\nu}\gamma_5/M_{\Lambda}]u_{\Lambda}\sin\theta_C\end{aligned}$$

- Vector/Axial coupling g_v, g_a
- Assume usual q^2 dependence: $g_{a/v}(q^2) = g_{v,a}(0)[1+2(q/M_{v,a})^2]$
- Weak-magnetism term $g_w(q^2) = g_w(0)$
- Second class term $g_2 = 0$, neglect terms of order M_e^2/M_{Λ}^2

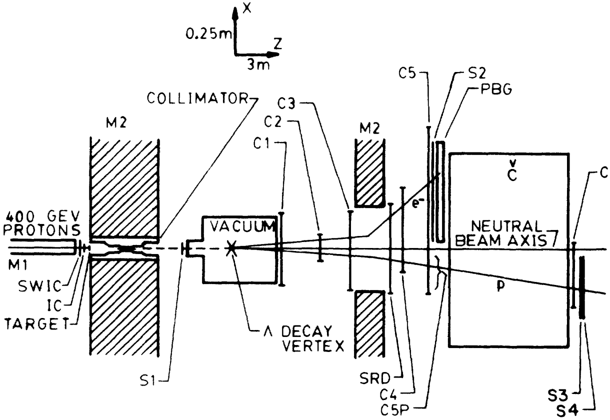
Λ - β decay

- Differential decay rate

$$\frac{d^2\Gamma}{dE_e^* \, d\cos\theta_{e\nu}^*} = W(E_e^*, \cos\theta_{e\nu}^*) \cdot T(E_e^*, \cos\theta_{e\nu}^*, \frac{g_a(0)}{g_v(0)}, \frac{g_w(0)}{g_v(0)})$$

- W contains kinematics
- T contains dependence on weak magnetism and axial-vector coupling constants
- Variables in the Λ rest frame

Λ - β decay



- 400 GeV CERN SPS proton beam
- Be target, 2.3 T magnet, decay region
- MWPC, scintillation counters, Cherenkov counter, lead glass array

Λ - β decay

- Four particles involved in $\Lambda \rightarrow p e^- \bar{\nu}$
- p, e^- fully reconstructed in spectrometer
- Λ: direction known but not the magnitude
- $\bar{\nu}$ not observed
- In centre of mass system: Two solutions for magnitude of p_Λ
  $\rightarrow$ Two solutions for $\cos\theta^*_{e\nu}$ (missing longitudinal component of p_ν)
- Approximation (neglect q^2 variation)

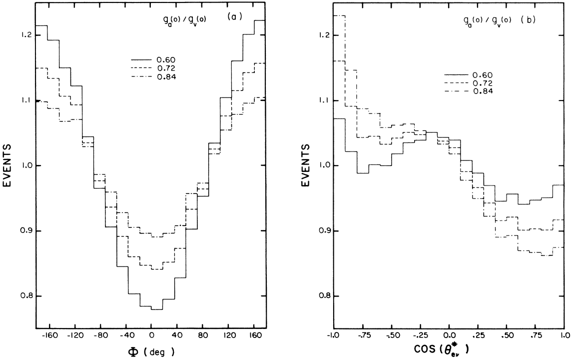
$$T = [g_v(0)]^2 \left[T_1(1-\cos\theta^*_{e\nu}) + T_2(1+\cos\theta^*_{e\nu}) \right]$$

$$T_1 = Y^2 \left[2(E^*_e - E^*_\nu) \right] X\, Y / M_\Lambda + \left[\frac{1}{2}(2+r)^2 \right]$$

$$T_2 = 1 + Y^2$$

- $r = M_p / M_\Lambda$, $Y = \frac{g_a(0)}{g_v(0)}$, $w = \frac{g_w}{g_v(0)}$, $X = 1 + (1+r)w$

- Angular distributions depend on $g_a(0)/g_v(0)$
- Compare with results from MC simulation
- $g_a(0)/g_v(0) = 0.720 + 0.018$
- Assume decay sequence: $\Lambda \to Q\bar{\nu}$, $Q \to pe$
- $g_a(0)/g_v(0) = 0.719 + 0.016$



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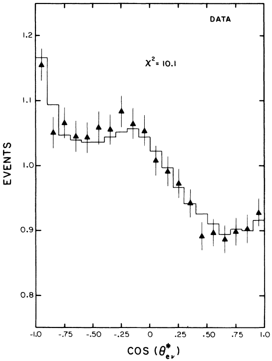

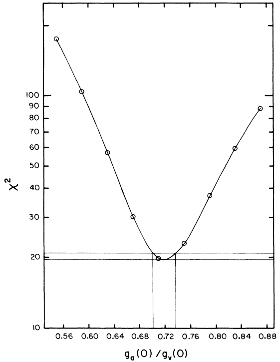

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