Baryonspektroskopie – $\mathbf{pp} \rightarrow \mathbf{p}\pi^0\mathbf{p}$
Partialwellenanalyse – Erste Ansätze

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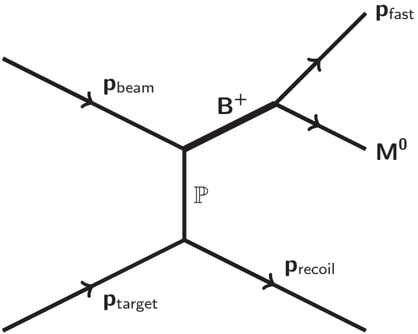
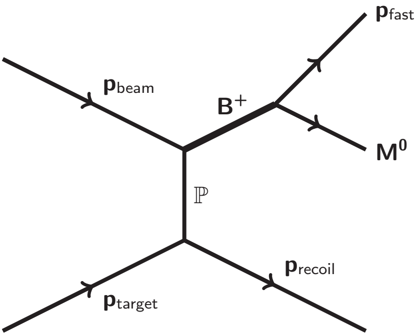
Starting Point

- 2-body final state $\mathbf{p}_{\text{fast}}\mathbf{M}^0$
- pomeron exchange dominates
- no isospin exchange $\Rightarrow$ only $\mathbf{I} = \frac{1}{2}$ baryons
- many different final state mesons accessible
  - $\pi^0 \rightarrow \gamma\gamma$
  - $\eta \rightarrow \gamma\gamma$
  - $\eta \rightarrow \pi^+\pi^-\pi^0$
  - $\eta' \rightarrow \pi^+\pi^-\eta$
  - $\omega \rightarrow \pi^+\pi^-\pi^0$
  - ...



Simplifying the Process

- target/recoil proton does not participate in the reaction



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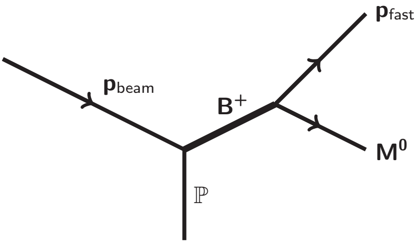

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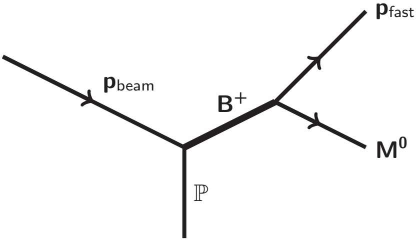

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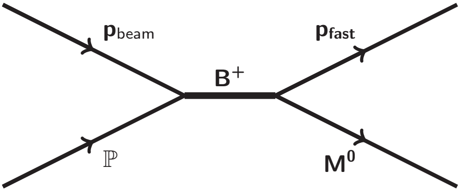

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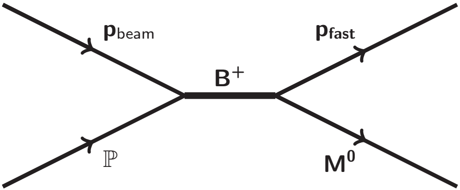


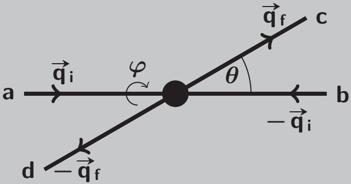
Some Constraints

- Neglect everything but spin-0 pomeron exchange
- Concentrate on spin-0 mesons (π^0, η)

2-Particle Scattering Amplitudes

Cross Section

$$\frac{d\sigma}{d\Omega}(s,\theta,\varphi) = \frac{1}{64\pi^2 s}\frac{|\vec{q}_i|}{|\vec{q}_f|}\,|\langle f|T|i\rangle|^2$$



with

- $s = M_{ab}^2 = M_{cd}^2$
- $|i\rangle = |q_i, \theta_i, \phi_i, \lambda_a, \lambda_b\rangle$

Partial Wave Decomposition

$$\langle f|T|i\rangle = 4\pi\sqrt{\frac{s}{q_i q_f}}\sum_{J=0}^{\infty}(2J+1)D^{*J}_{\lambda_i\lambda_f}(\varphi,\theta,0)T^J_{\lambda_a\lambda_b\lambda_c\lambda_d}(s)$$

with $\lambda_i = \lambda_a - \lambda_b$ and $\lambda_f = \lambda_c - \lambda_d$

Intermezzo: Wigner D-Functions

$$D^{*J}_{\lambda_i \lambda_f}(\varphi, \theta, 0) = e^{i\lambda_i \varphi} d^J_{\lambda_i \lambda_f}(\theta)$$

- d^J are (mixed) polynomials in $\sin(\frac{\theta}{2})$ and $\cos(\frac{\theta}{2})$
- $J = \frac{1}{2}$:
  $$d^{\frac{1}{2}}_{\frac{1}{2}\frac{1}{2}} = \cos(\tfrac{\theta}{2}) \qquad d^{\frac{1}{2}}_{\frac{1}{2}-\frac{1}{2}} = \sin(\tfrac{\theta}{2})$$
  $$d^{\frac{1}{2}}_{-\frac{1}{2}\frac{1}{2}} = -\sin(\tfrac{\theta}{2}) \qquad d^{\frac{1}{2}}_{-\frac{1}{2}-\frac{1}{2}} = \cos(\tfrac{\theta}{2})$$
- $J = \frac{3}{2}$:
  $$d^{\frac{3}{2}}_{\frac{1}{2}\frac{1}{2}} = \tfrac{1}{2}\cos(\tfrac{\theta}{2})(3\cos(\theta) - 1) \qquad d^{\frac{3}{2}}_{\frac{1}{2}-\frac{1}{2}} = \tfrac{1}{2}\sin(\tfrac{\theta}{2})(3\cos(\theta) + 1)$$
  $$d^{\frac{3}{2}}_{-\frac{1}{2}\frac{1}{2}} = -\tfrac{1}{2}\sin(\tfrac{\theta}{2})(3\cos(\theta) + 1) \qquad d^{\frac{3}{2}}_{-\frac{1}{2}-\frac{1}{2}} = \tfrac{1}{2}\cos(\tfrac{\theta}{2})(3\cos(\theta) - 1)$$
- with $\cos(\theta) = \cos^2(\frac{\theta}{2}) - \sin^2(\frac{\theta}{2})$

2-Particle Scattering Amplitudes

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with $\lambda_i = \lambda_a - \lambda_b$ and $\lambda_f = \lambda_c - \lambda_d$

For $p\mathbb{P} \rightarrow p\pi^0$:

- $\lambda_b = \lambda_d = 0 \Rightarrow \lambda_i = \lambda_a$ and $\lambda_f = \lambda_c$
- $\lambda_{i,f} = \pm\frac{1}{2}$
- $J = L \pm \frac{1}{2}$

$$\langle f|T|i\rangle = 4\pi\sqrt{\frac{s}{q_i q_f}}\sum_{J=0}^{\infty}(2J+1){D^*}^J_{\lambda_i\lambda_f}(\varphi,\theta,0)T^J_{\lambda_i\lambda_f}(s)$$

2-Particle Scattering Amplitudes

- unpolarized beam and target ⇒ average over initial state spins
- final state spins not measured ⇒ sum over final state spins

Full Cross Section

$$\frac{d\sigma}{d\Omega}(s,\theta,\varphi) = \frac{1}{4}\frac{1}{|\vec{q}_f|^2}\frac{1}{2}\sum_{\substack{\lambda_i=\pm\frac{1}{2}\\ \lambda_f=\pm\frac{1}{2}}}\left|\sum_{J=0}^{\infty}(2J+1)D^{*J}_{\lambda_i\lambda_f}(\varphi,\theta,0)T^J_{\lambda_i\lambda_f}(s)\right|^2$$

⇒ Four complex amplitudes for each **J**
Complicated angular dependence, $D^{*J}_{\lambda_i\lambda_f}$ not directly available in ROOT

Simplification

Ignore the spin of the proton!

- $\lambda_i = \lambda_f = 0$
- $D^{*J}_{00}(\varphi, \theta, 0) = P_J(\cos(\theta))$ (Legendre polynomials)
- no φ dependence $\rightarrow$ integrate over φ
- no spin $\rightarrow$ $J = L$

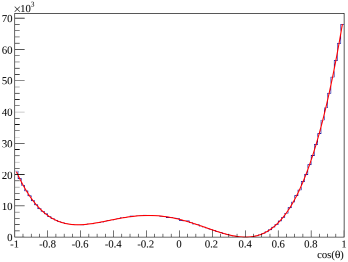
Partial Wave Decomposition

$$\langle f|T|i\rangle = 8\pi^2 \sqrt{\frac{s}{q_i q_f}} \sum_{L=0}^{\infty} (2L+1) f_L(s) P_L(\cos(\theta))$$

- Only one complex amplitude for each L
- Legendre polynomials directly available in ROOT

First Test

- Take squared sum of the first three Legendre polynomials with fixed amplitudes as test function
- Sample histogram from this function
- Fit histogram



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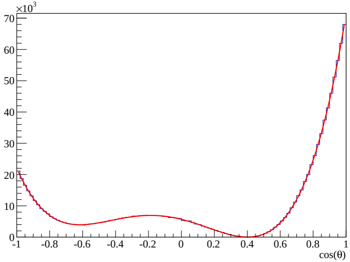

	Sampling	Fit
f_0	$0.1 - 0.8i$	$0.003 - 1.1i$
f_1	$0.9 + 0.2i$	$-1.1 + 0.1i$
f_2	$0.9 - 0.5i$	$-1.1 - 0.7i$

$\chi^2/\text{NDF} = 109/94$

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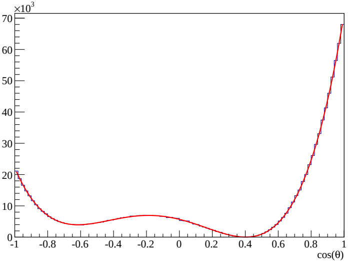

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What went wrong?

Barrelet Zeros

- Fit function is square of absolute value: $f(z) = |a(z)|^2$
- $a(z)$ is a sum of Legendre polynomials ⇒ polynomial of order L in $z = \cos(\theta)$
- $a(z)$ can be written as product of zeros:

$$a(z) = a_0 \cdot (z - z_1) \cdot (z - z_2) \cdot \ldots$$

  where the number of zeros is given by L
- z_i are complex numbers (just as the f_L)
- $f(z) = |a(z)|^2 = a(z) \cdot a^*(z)$
  with $a^*(z) = a_0 \cdot (z - z_1^*) \cdot (z - z_2^*) \cdot \ldots$
- taking complex conjugate of a zero does not change the fit

⇒ 2^L possible solutions

- complex conjugates in z_i but more complicated in f_L

Next Steps

1. Get fit running on real data (in bins of $\sqrt{s} = M(p\pi^0)$)
2. Include acceptance correction
3. Get Barrelet ambiguities under control
4. Switch to description with spin

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Interesting analysis based on this method

Coupled channel analysis of

$$pp \longrightarrow p\eta\, p_s \qquad \text{with } \eta \rightarrow \pi^+\pi^-\pi^0$$

$$pp \longrightarrow \Sigma^+ K^0\, p_s \qquad \text{with } \Sigma^+ \rightarrow p\pi^0,\ K^0 \rightarrow \pi^+\pi^-$$

- same final state $p\pi^+\pi^-\pi^0\, p_s$
- poorly known decay channels in PDG

BACKUP