# Using Bayesian Methods for Particle Identification in the CEDARs

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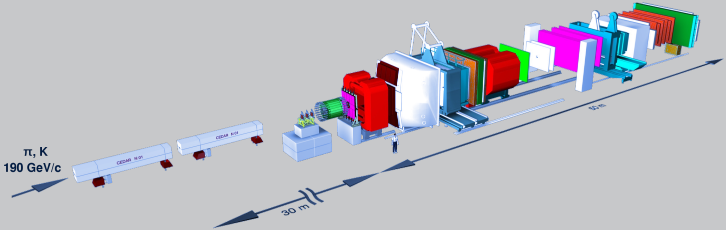
## Outline

## CEDARs at COMPASS

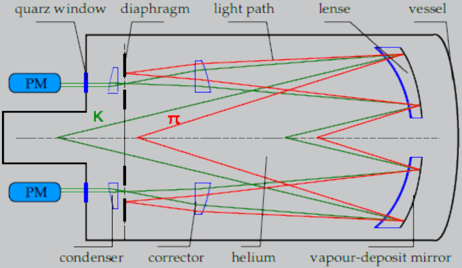
- CEDAR = ČErenkov Differential counters with Acromatic Ring focus
- Two CEDARs at COMPASS beamline about 30 m upstream of the target



# How does a CEDAR work?

- Fast charged particles emit Čerenkov light with angle $\boldsymbol{\theta}$

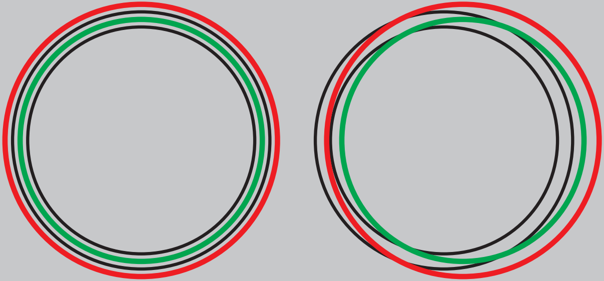
$$\cos(\theta) = \frac{1}{n\beta}$$




- Čerenkov light detected with 8 PMTs
- Particle identification using multiplicities, e.g. 6 of 8 PMTs

# Influence of Beam Divergence

- Kaon ring leaves acceptance, pion ring enters



⇒ Multiplicity method does not work for divergent beams

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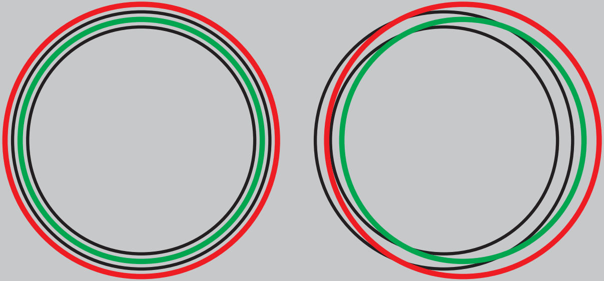

⇒ Multiplicity method does not work for divergent beams

## Goal

Find a better method to take divergence into account

## General Idea

- Look at PMT response for Kaon and Pion seperately
- Take beam divergence into account
- Identify beam particles using likelihoods

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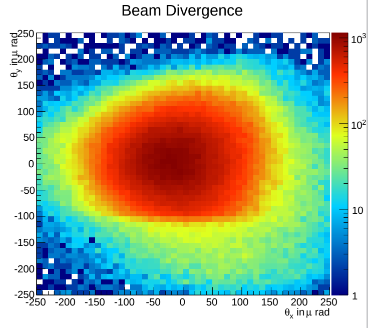
## 5 steps to take

1. Measure beam divergence
2. Create a pure Kaonsample and a pure Pionsample
3. Determine probabilities to have hits in PMTs for Pion and Kaon
4. Calculate likelihoods from probabilities
5. Use likelihoods to identify particles

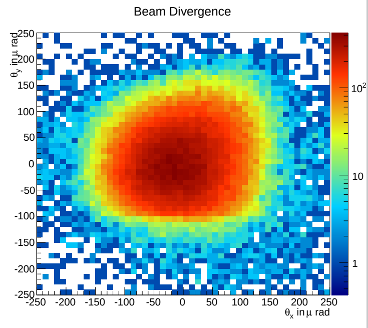
# Step 1: Measure beam divergence

- Measure beam position in front of $(\mathbf{x_1}, \mathbf{y_1})$ and behind $(\mathbf{x_2}, \mathbf{y_2})$ CEDARs
- Calculate relative displacement $\mathbf{\Delta_x} = \frac{\mathbf{x_2 - x_1}}{\mathbf{1283{,}4\,cm}}$
- Divergence $\theta_x = \mathbf{arctan(\Delta_x)} \approx \mathbf{\Delta_x}$

Events with at least 6 of 8 PMTs




Events with 8 of 8 PMTs



## Step 2: Create a pure Kaonsample and a pure Pionsample

Create a Kaonsample and a Pionsample

- Kaonsample

→ Use free Kaon decay $\mathbf{K^- \to \pi^-\pi^-\pi^+}$

  - 3 outgoing particles with correct charged
  - Primary vertex outside of the target
  - Cut on beam momentum and Kaon mass

- Pionsample

→ Use diffractive production $\mathbf{\pi^- p \to \pi^-\pi^-\pi^+ p}$

  - 3 outgoing particles with correct charge
  - Primary vertex inside the target
  - Small angle to beam direction

*In addition:* Produce a Beamsample without any filtering for testing the method

## Step 3: Determine probabilities to have hits in PMTs for $\pi$ and **K**

*Example:* Probability that a particle with divergence $\theta_x, \theta_y$ that produces a signal in PMT **i** is a Kaon

$\rightarrow$ Use Bayes' Theorem:

$$\mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Kaon}|\text{Signal}) = \frac{\mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Signal}|\text{Kaon}) \cdot \mathbf{P}_{\theta_x,\theta_y}(\text{Kaon})}{\mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Signal})}$$

Here:

| | | |
|---|---|---|
| $\mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Signal}\vert\text{Kaon})$: | Probability that Kaon at $\theta_x$, $\theta_y$ produces signal in PMT **i** | ($\rightarrow$ Kaonsample) |
| $\mathbf{P}_{\theta_x,\theta_y}(\text{Kaon})$: | Probability that Kaon has divergence $\theta_x$ and $\theta_y$ | ($\rightarrow$ Kaonsample) |
| $\mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Signal})$: | Probability that signal in PMT **i** ist produced at $\theta_x$, $\theta_y$ | ($\rightarrow$ Beamsample) |

## Step 3 continued

We know that Pions and Kaons have the same divergence distribution:

$$\mathbf{P}_{\theta_x,\theta_y}(\text{Kaon}) = \mathbf{P}_{\theta_x,\theta_y}(\text{Pion}) = \mathbf{P}_{\theta_x,\theta_y}(\text{Beam})$$

$\Rightarrow \mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Kaon}|\text{Signal})$ and $\mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Pion}|\text{Signal})$ have same normalization

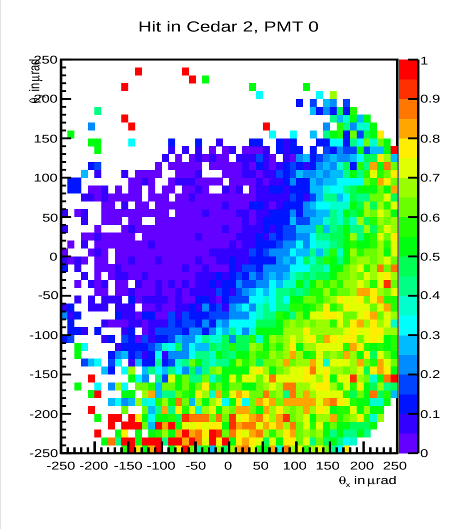
factor $\frac{\mathbf{P}_{\theta_x,\theta_y}(\text{Beam})}{\mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Signal})}$, thus

$$\mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Kaon}|\text{Signal}) \propto \mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Signal}|\text{Kaon})$$
$$\mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Pion}|\text{Signal}) \propto \mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Signal}|\text{Pion})$$

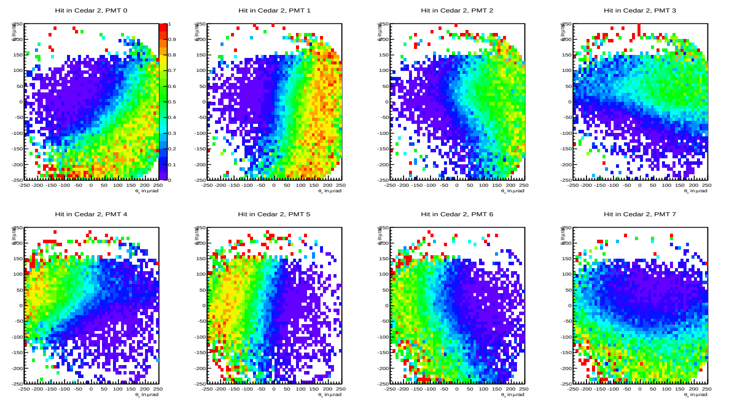
Also calculate

$$\mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Kaon}|\overline{\text{Signal}}) \text{ and } \mathbf{P}^{\mathbf{i}}_{\theta_x,\theta_y}(\text{Pion}|\overline{\text{Signal}})$$

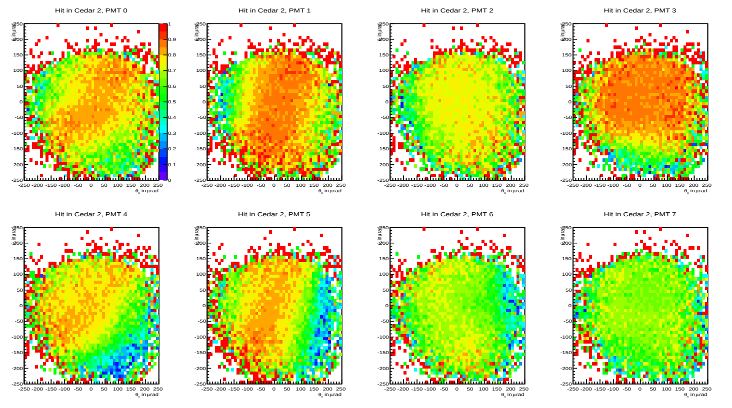
## $P^i_{xy}(\text{Signal}|\text{Pion}) \propto P^i_{xy}(\text{Pion}|\text{Signal})$



# $P^i_{xy}(\text{Signal}|\text{Pion}) \propto P^i_{xy}(\text{Pion}|\text{Signal})$



# $P^i_{xy}(\text{Signal}|\text{Kaon}) \propto P^i_{xy}(\text{Kaon}|\text{Signal})$



## Step 4: Calculate likelihoods from probabilities

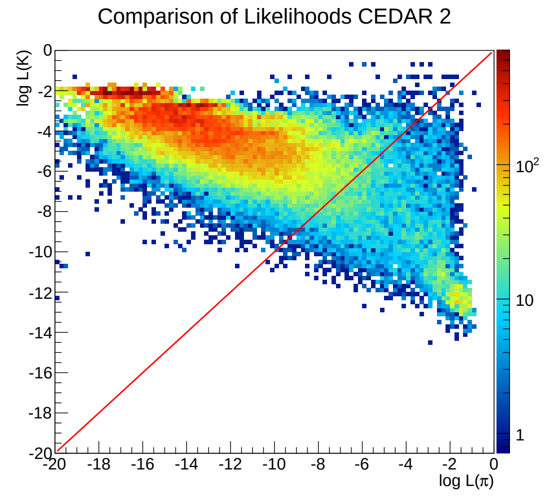
To obtain the log likelihood just add logarithms of probabilities

$$\log \mathbf{L}(\text{Kaon}) = \sum_{i=1}^{8} \log \mathbf{P}^{i}_{\theta_x,\theta_y}(\text{Kaon}|\text{Signal}) \cdot \eta^{i} + \sum_{i=1}^{8} \log \mathbf{P}^{i}_{\theta_x,\theta_y}(\text{Kaon}|\overline{\text{Signal}}) \cdot (1 - \eta^{i})$$

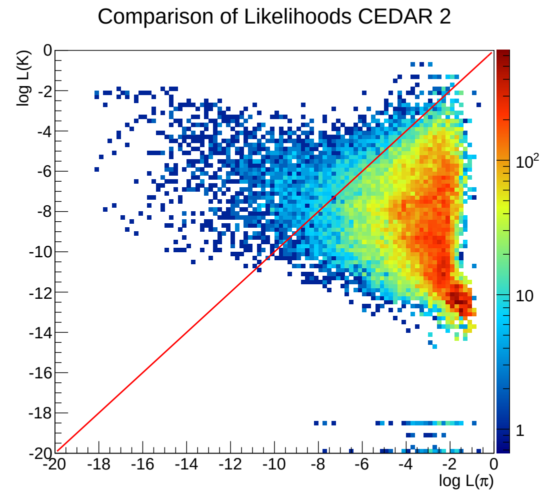
Where:

$$\eta^{i} = \begin{cases} 1 & \text{Signal in PMT i} \\ 0 & \text{no Signal in PMT i} \end{cases}$$

# Kaonsample


Comparison of Likelihoods CEDAR 2
log L(K)
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-20
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
log L(π)
$10^2$
10
1

# Pionsample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-16
-18
-20
-20 -18 -16 -14 -12 -10 -8 -6 -4 -2 0
10²
10
1

## Step 5: Use likelihoods to identify particles

- Compare log likelihoods to get an ID for each CEDAR:
  - $\log L^K > \log L^\pi + A \Rightarrow$ PID $K$
  - $\log L^\pi > \log L^K + B \Rightarrow$ PID $\pi$
  - else no PID given
- Tune **A** and **B** due to efficiency/purity.
- Preliminary choice: $A = B = 0.1$ in the following slides, equivalent to $P^i(\text{Kaon}) = 1.01 \cdot P^i(\text{Pion})$ (and vice versa)

## Step 5: Use likelihoods to identify particles

- Compare log likelihoods to get an ID for each CEDAR:
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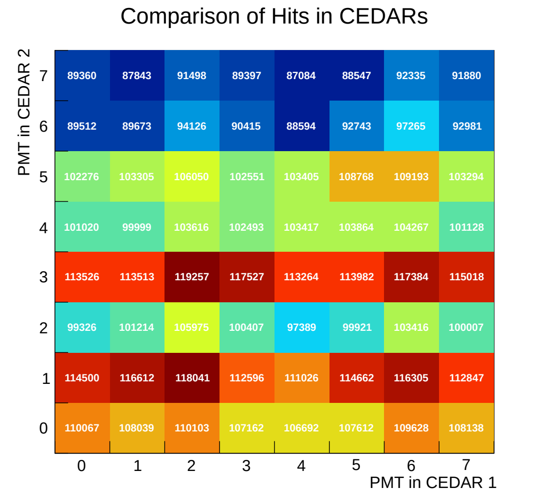
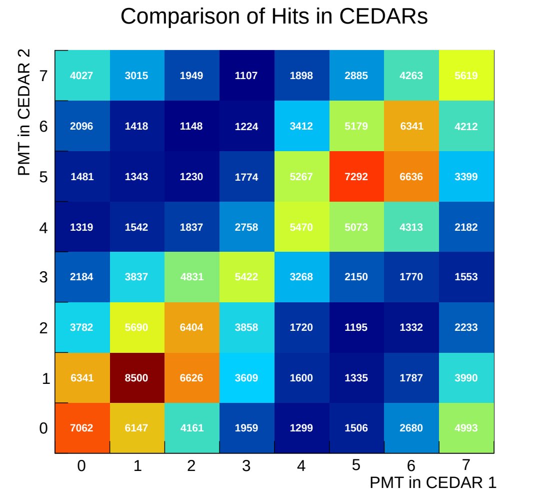
- Tune **A** and **B** due to efficiency/purity.
- Preliminary choice: $\mathbf{A = B = 0.1}$ in the following slides, equivalent to $\mathbf{P^i(}\text{Kaon}\mathbf{) = 1.01 \cdot P^i(}\text{Pion}\mathbf{)}$ (and vice versa)
- Combine CEDARs afterwards

| C2 \ C1 | ? | $\pi$ | **K** |
|---|---|---|---|
| ? | ? | $\pi$ | **K** |
| $\pi$ | $\pi$ | $\pi$ | ? |
| **K** | **K** | ? | **K** |

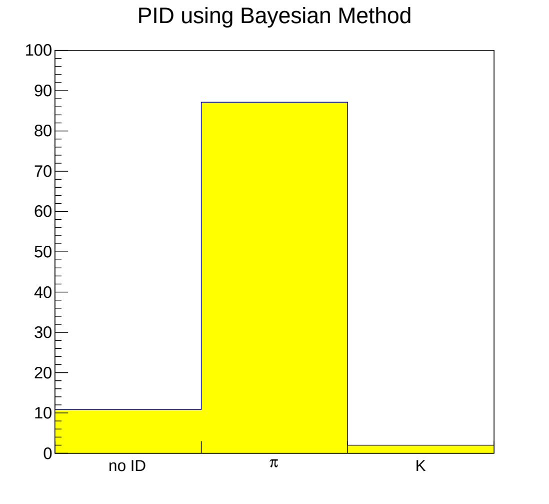
# PMT efficiencies



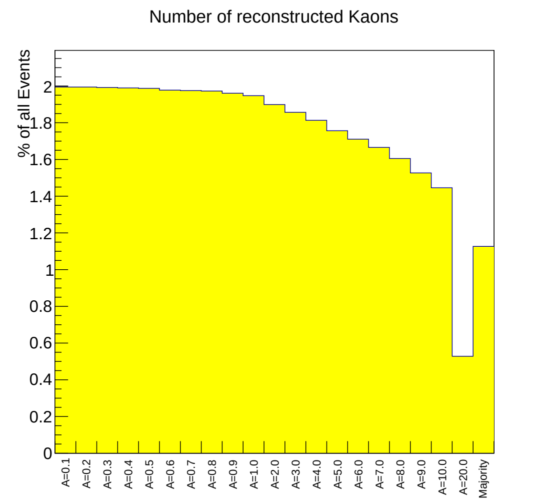
# PMT correlation between CEDAR 1 and CEDAR 2



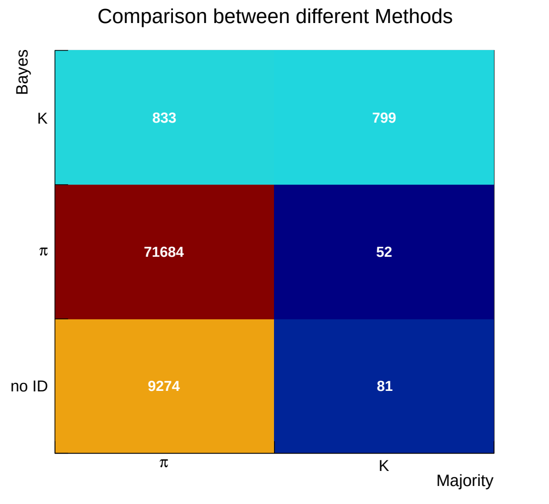
# Particle Identification in the Beamsample



# A-Dependence of Kaon Identification



# Comparison with Majority Method



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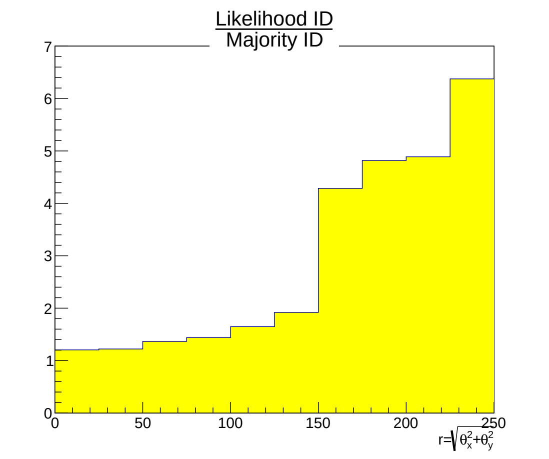
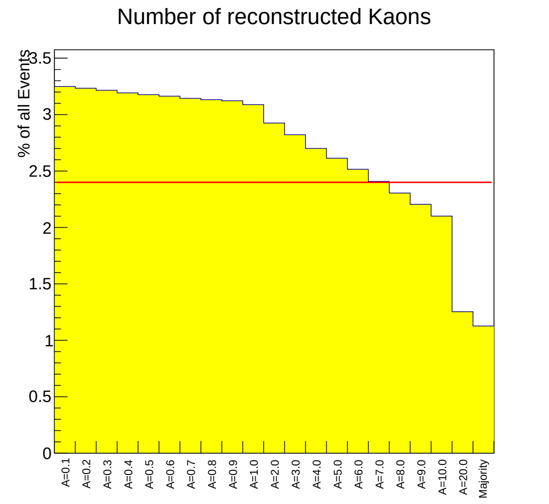


# Using 16 PMTs



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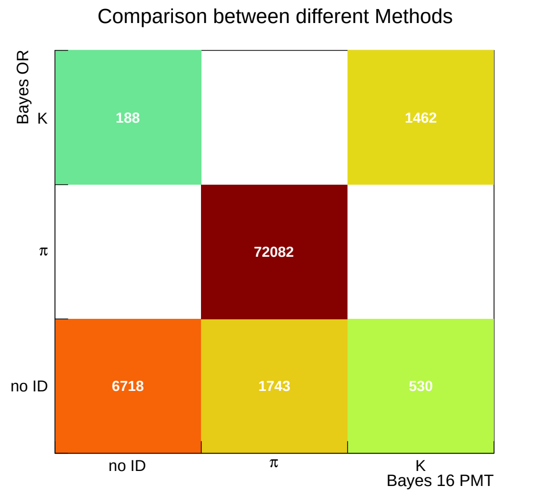
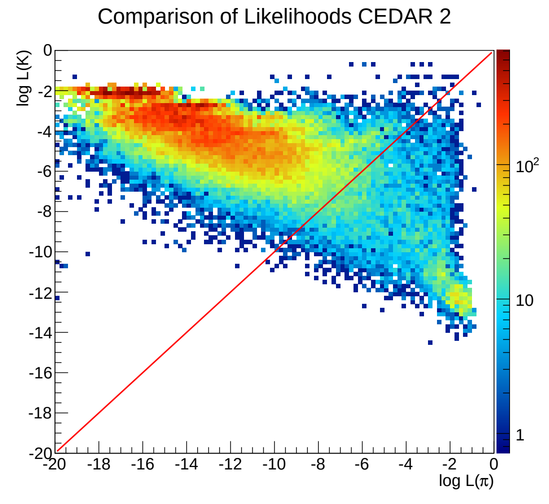


## Summary

- COMPASS hadron beam consists of 97% Pions and 2.4% Kaons
- Pions and Kaons have to be identified for analyses
- Majority method identifies 45% of the Kaons
  - Problems with divergent beams
- Likelihood method improves identification for divergent beams
  - Identifies 80% of the Kaons

# BACKUP

# Beamsample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-4
-2
0
10^2
10
1