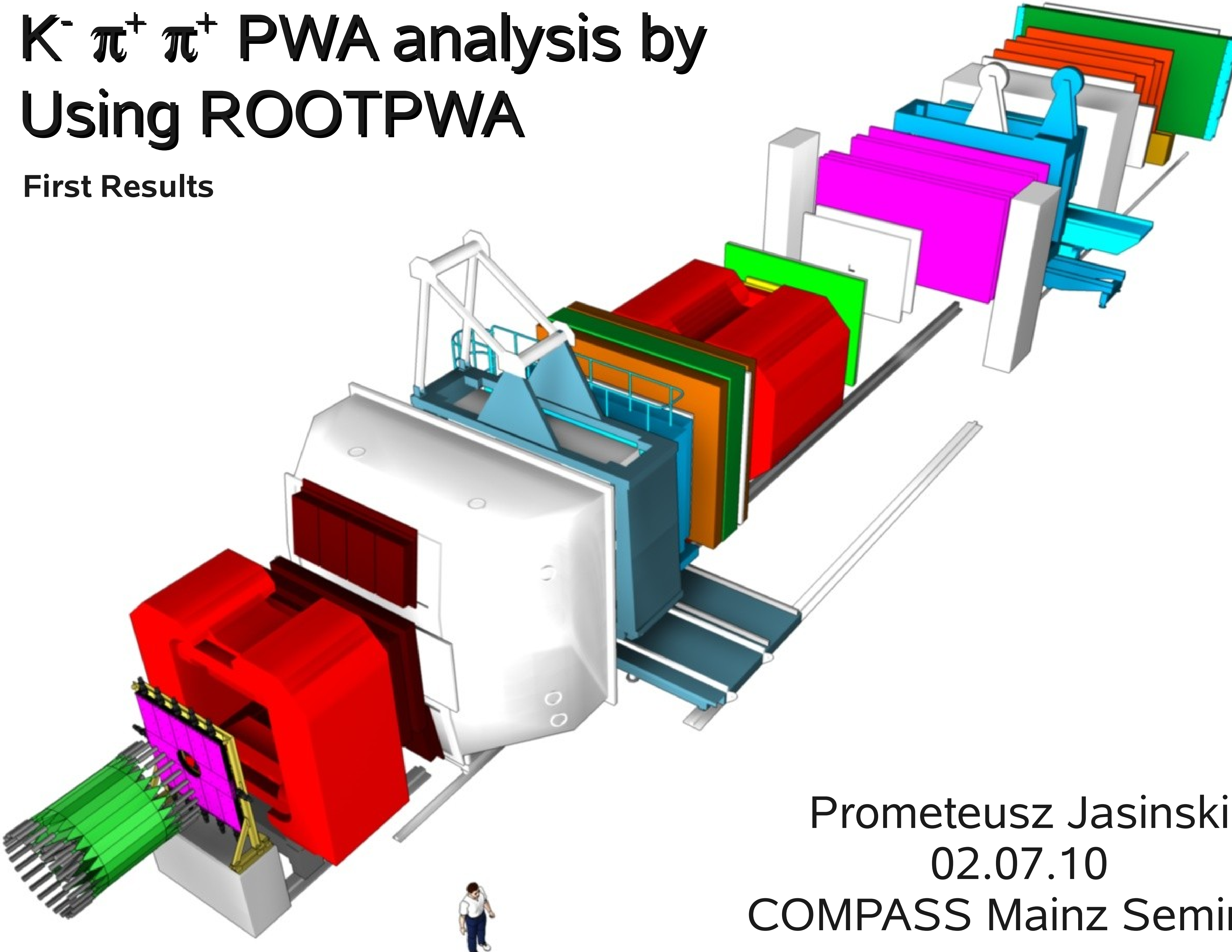
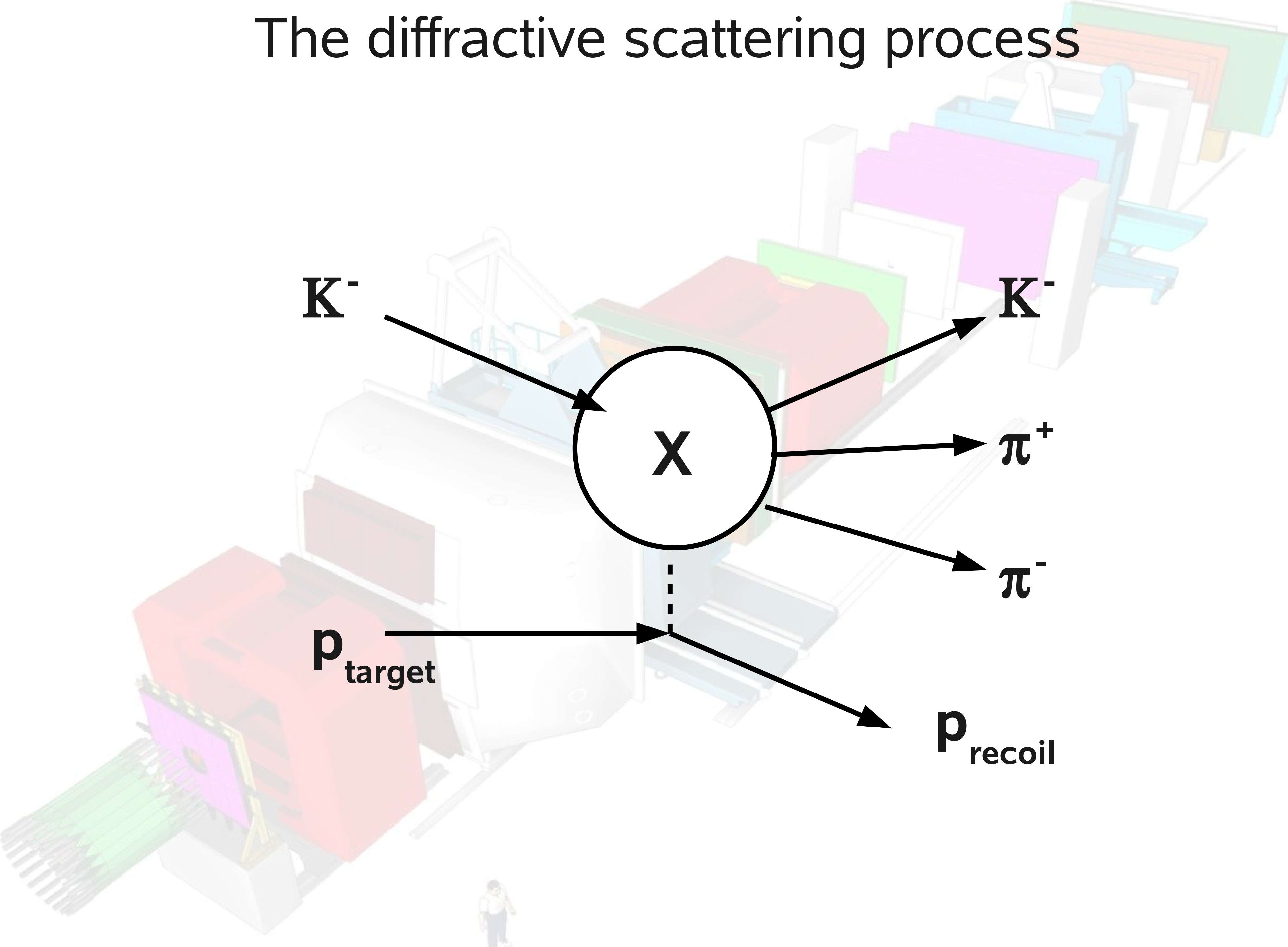
$K^- \pi^+ \pi^+$ PWA analysis by Using ROOTPWA

First Results

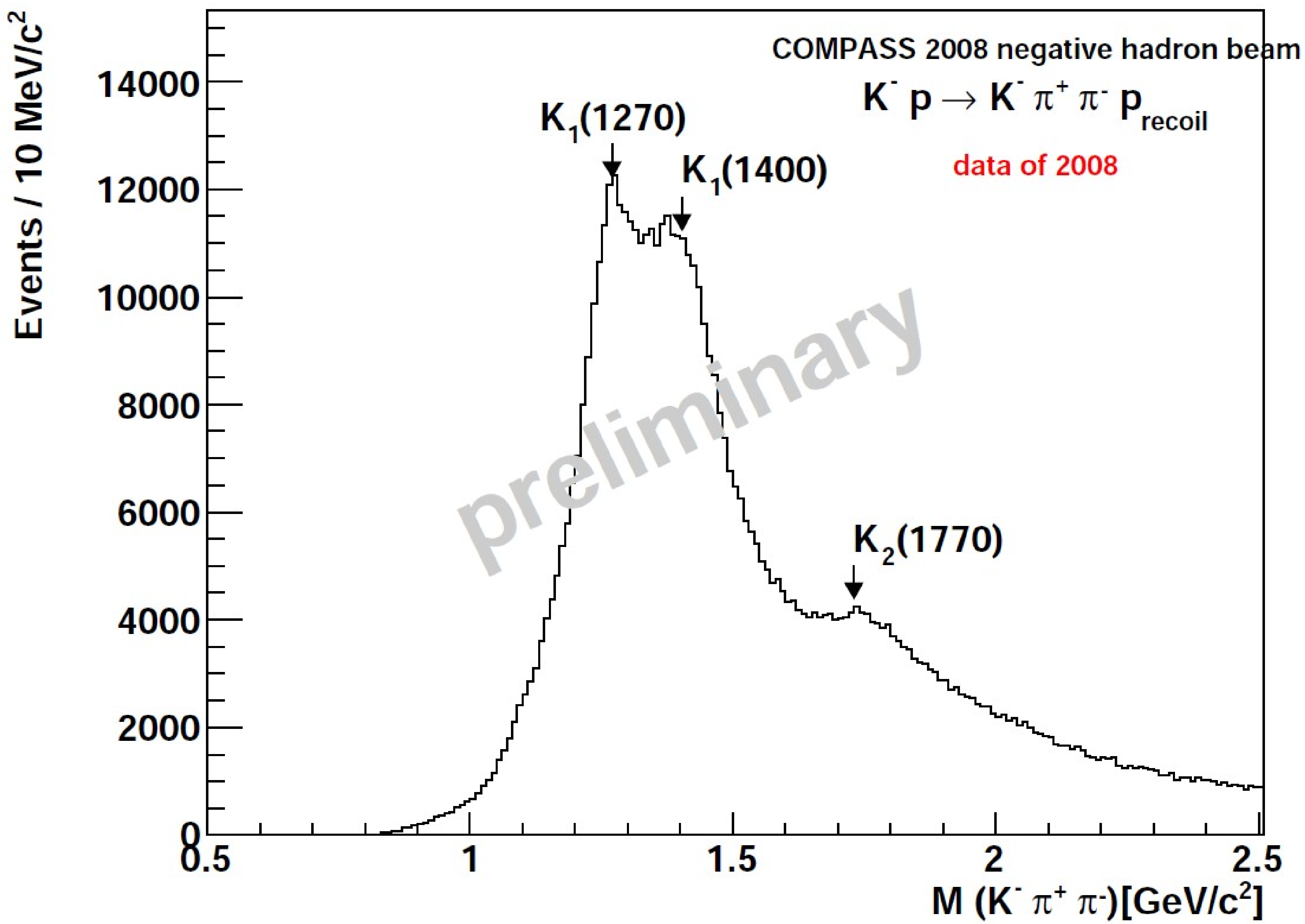


Prometeusz Jasinski
02.07.10
COMPASS Mainz Seminar

The diffractive scattering process

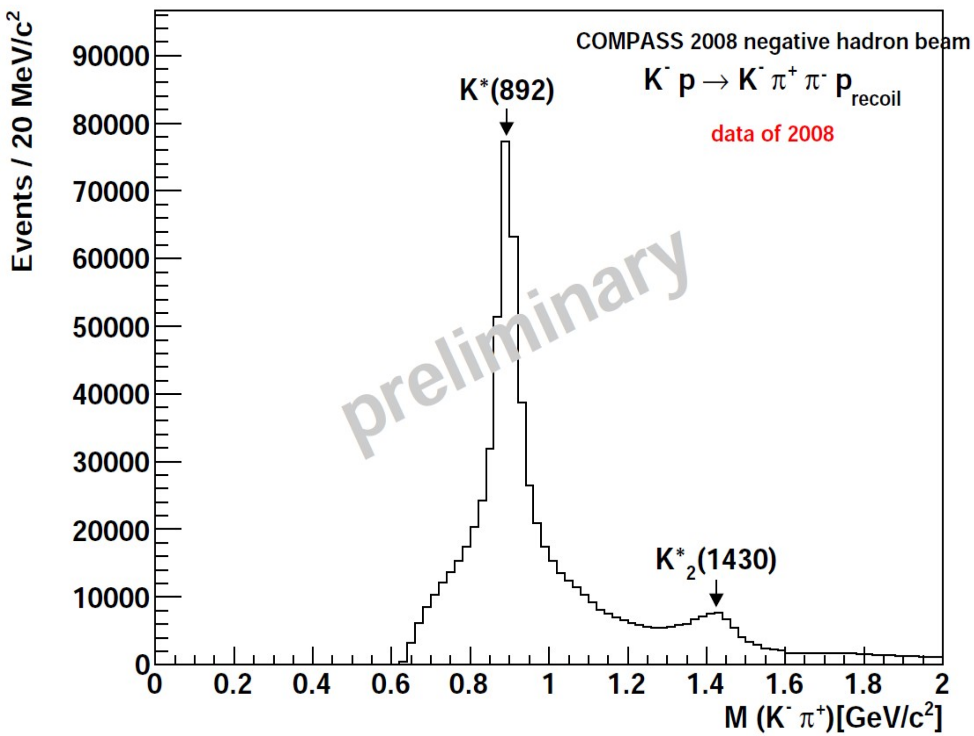


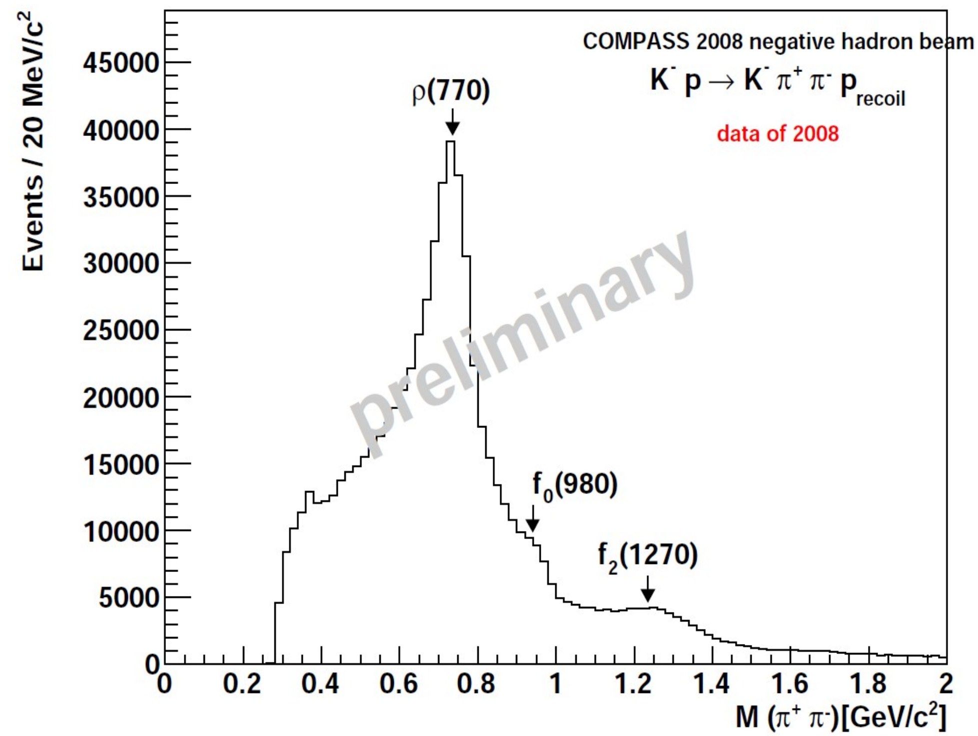
Invariant mass distributions ($K^- \pi^+ \pi^-$)



What about subsystems? Let's have a look...

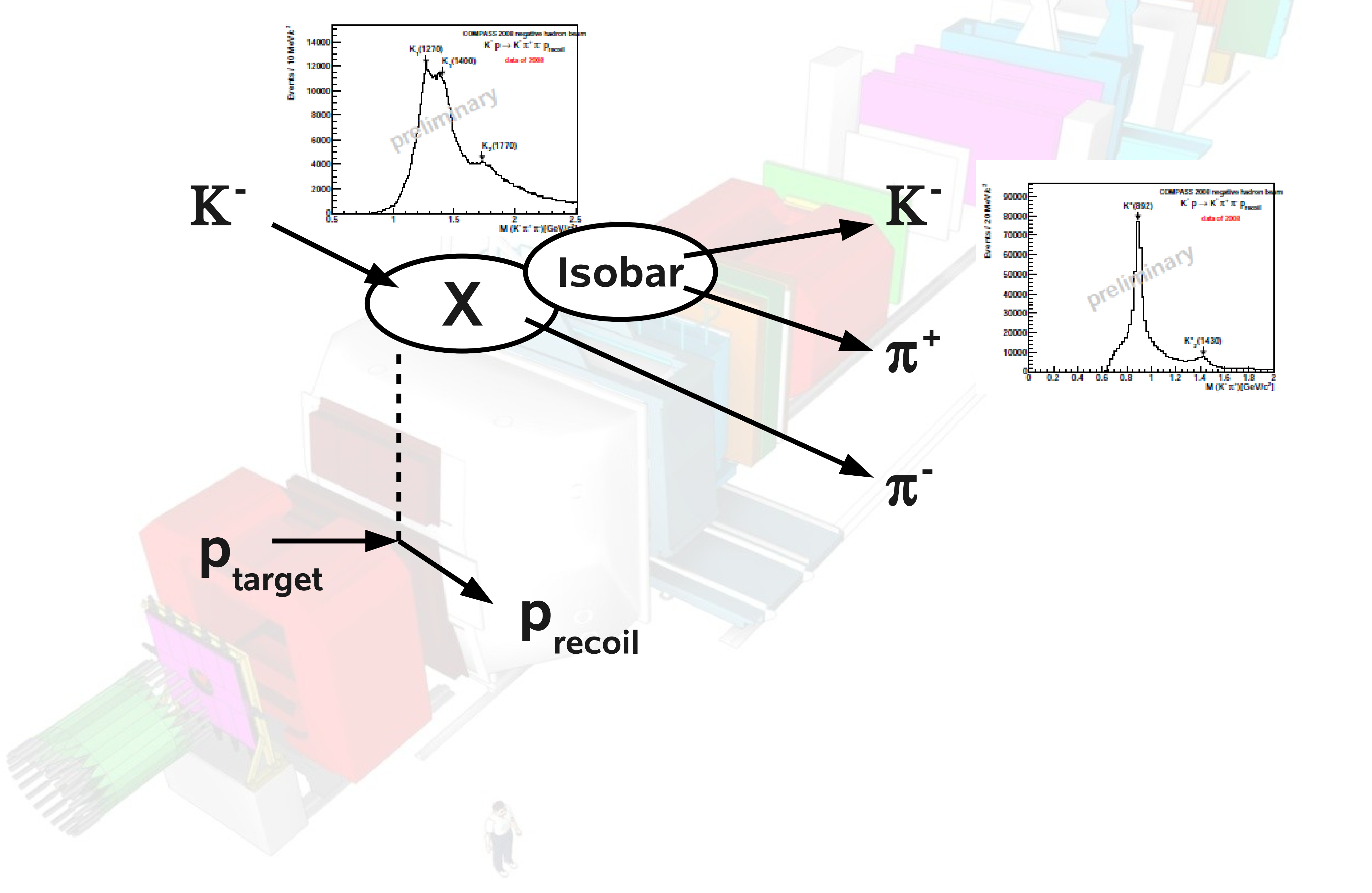
Invariant mass distributions ($K^- \pi^+$) and ($\pi^+ \pi^-$)



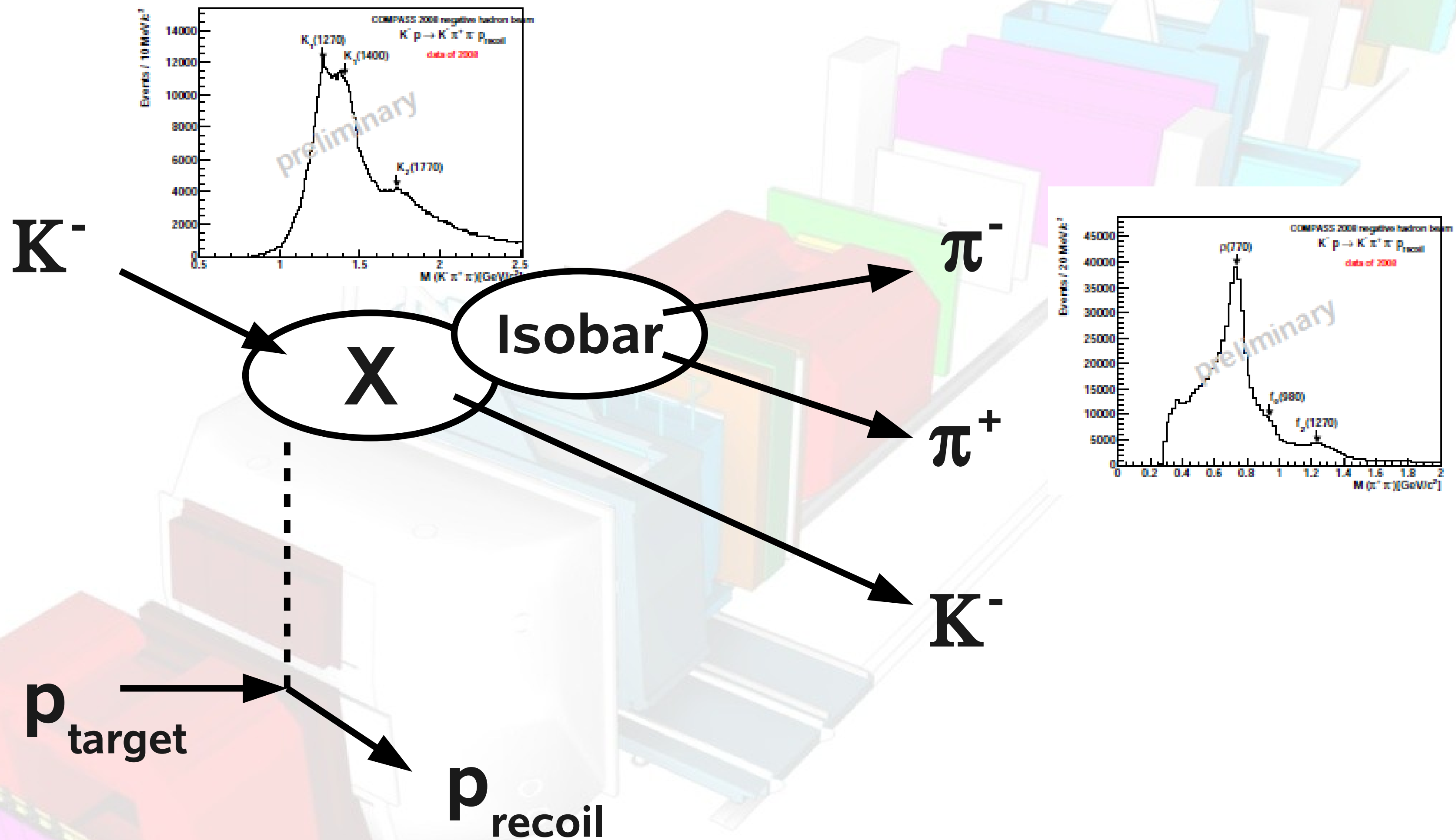


Aha, there is structure, too → Assuming an isobar decay chain...

Isobar model assumption: A decay chain

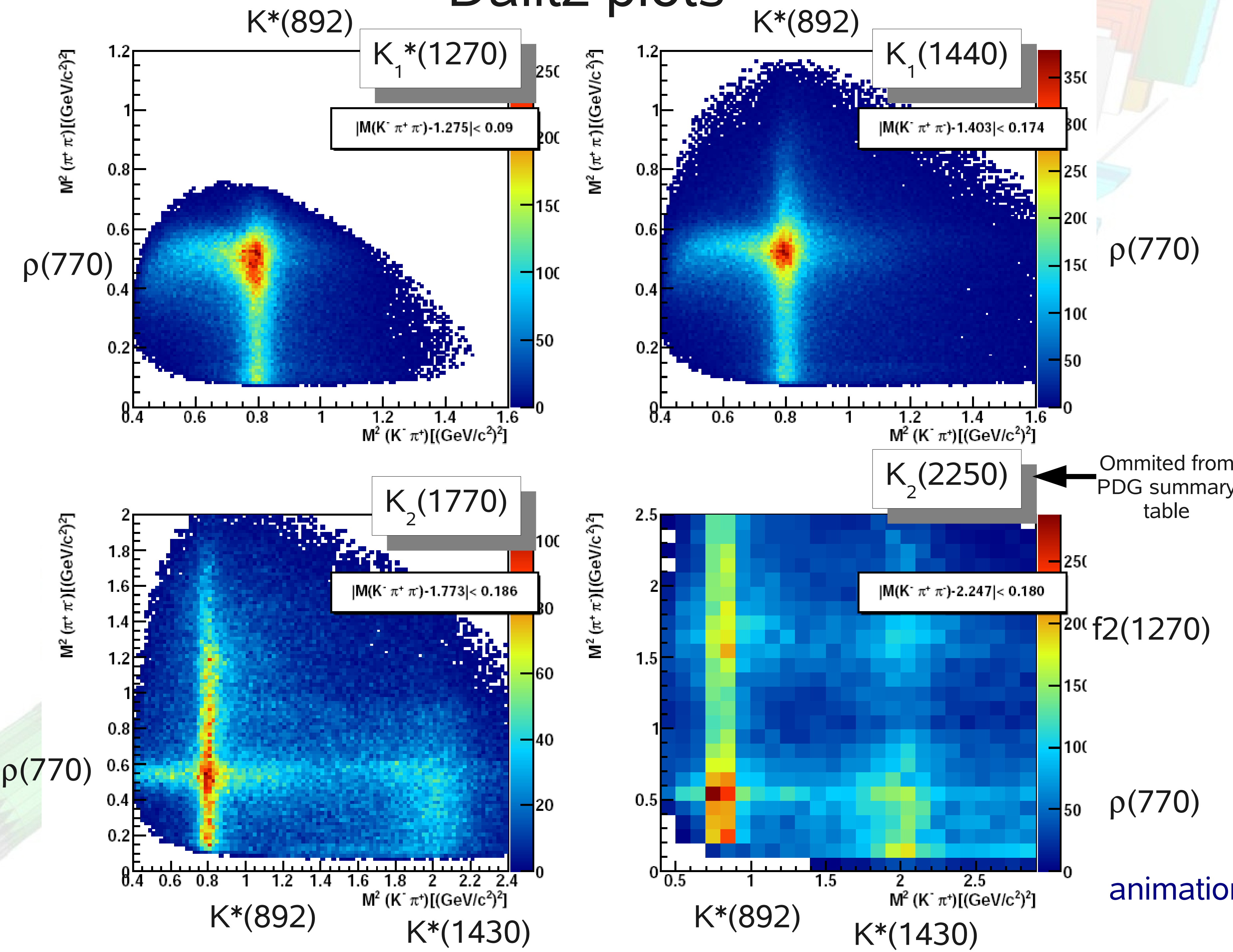


Isobar model assumption: A decay chain



Note: Assuming no direct 3 particle decay

Dalitz plots



animation

Quantum numbers to deal with

14. QUARK MODEL

Revised December 2007 by C. Amsler (University of Zürich), T. DeGrand (University of Colorado, Boulder), and B. Krusche (University of Basel).

14.1. Quantum numbers of the quarks

Quarks are strongly interacting fermions with spin 1/2 and, by convention, positive parity. Antiquarks have negative parity. Quarks have the additive baryon number 1/3, antiquarks -1/3. Table 14.1 gives the other additive quantum numbers (flavors) for the three generations of quarks. They are related to the charge Q (in units of the elementary charge e) through the generalized Gell-Mann-Nishijima formula

$$\mathsf{Q} = \mathsf{I}_z + \frac{\mathcal{B} + \mathsf{S} + \mathsf{C} + \mathsf{B} + \mathsf{T}}{2}\ , \qquad (14.1)$$

where $\mathcal{B}$ is the baryon number. The convention is that the *flavor* of a quark (I_z, S, C, B, or T) has the same sign as its *charge* Q. With this convention, any flavor carried by a charged meson has the same sign as its charge, *e.g.*, the strangeness of the K^+ is +1, the bottomness of the B^+ is +1, and the charm and strangeness of the D_s^- are each −1. Antiquarks have the opposite flavor signs.

Table 14.1: Additive quantum numbers of the quarks.

Property \ Quark	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	−1	0	0	0
C – charm	0	0	0	+1	0	0
B – bottomness	0	0	0	0	−1	0
T – topness	0	0	0	0	0	+1

14.2. Mesons

Mesons have baryon number $\mathcal{B} = 0$. In the quark model, they are $q\overline{q}'$ bound states of quarks q and antiquarks $\overline{q}'$ (the flavors of q and q' may be different). If the orbital angular momentum of the $q\overline{q}'$ state is ℓ, then the parity P is $(-1)^{\ell+1}$. The meson spin J is given by the usual relation $|\ell - s| < J < |\ell + s|$, where s is 0 (antiparallel quark spins) or 1 (parallel quark spins). The charge conjugation, or C-parity

$$\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}\ . \qquad (14.2)$$

A fourth quark such as charm c can be included by extending SU(3) to SU(4). However, SU(4) is badly broken owing to the much heavier c quark. Nevertheless, in an SU(4) classification, the sixteen mesons are grouped into a 15-plet and a singlet:

$$\mathbf{4} \otimes \overline{\mathbf{4}} = \mathbf{15} \oplus \mathbf{1}\ . \qquad (14.3)$$

The *weight diagrams* for the ground-state pseudoscalar (0^{-+}) and vector (1^{--}) mesons are depicted in Fig. 14.1. The light quark mesons are members of nonets building the middle plane in Fig. 14.1(a) and (b).

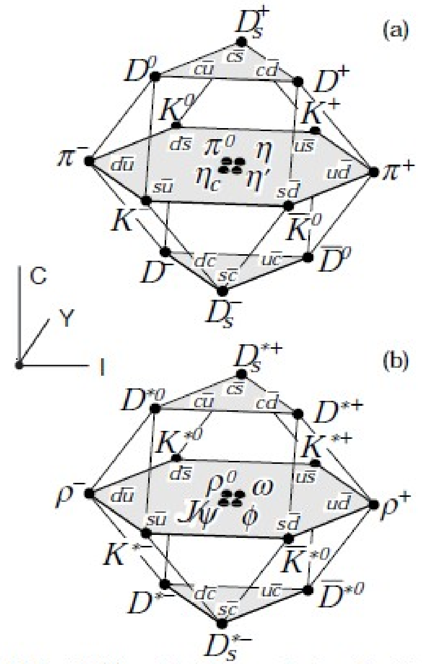
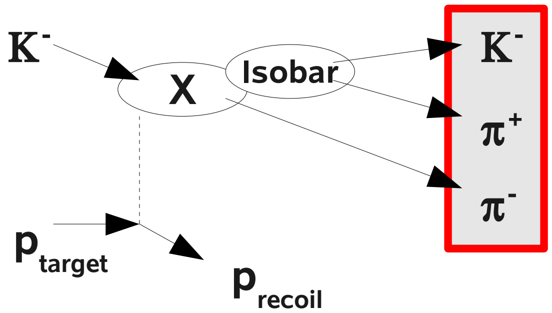


Figure 14.1: SU(4) weight diagram showing the 16-plets for the pseudoscalar (a) and vector mesons (b) made of the u, d, s, and c quarks as a function of isospin I, charm C, and hypercharge $\mathsf{Y} = \mathsf{S} + \mathcal{B} - \frac{\mathsf{C}}{3}$. The nonets of light mesons occupy the central planes to which the $c\overline{c}$ states have been added.

http://pdg.lbl.gov/2009/download/rpp-2008-plB667.pdf Page 172

Decomposition of the decay chain (reversely)

final states (more or less stable particles)



For us relevant quantum numbers:

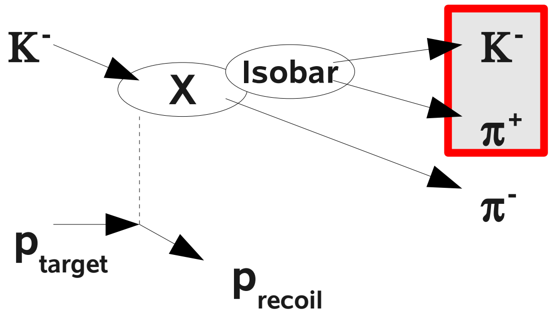
Quark spin: S = +- ½ Parity: P = +1

Table 14.1: Additive quantum numbers of the quarks.

Property \ Quark	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	−1	0	0	0
C – charm	0	0	0	+1	0	0
B – bottomness	0	0	0	0	−1	0
T – topness	0	0	0	0	0	+1

Decomposition of the decay chain (reversely)

final states (more or less stable particles)



For us relevant quantum numbers:

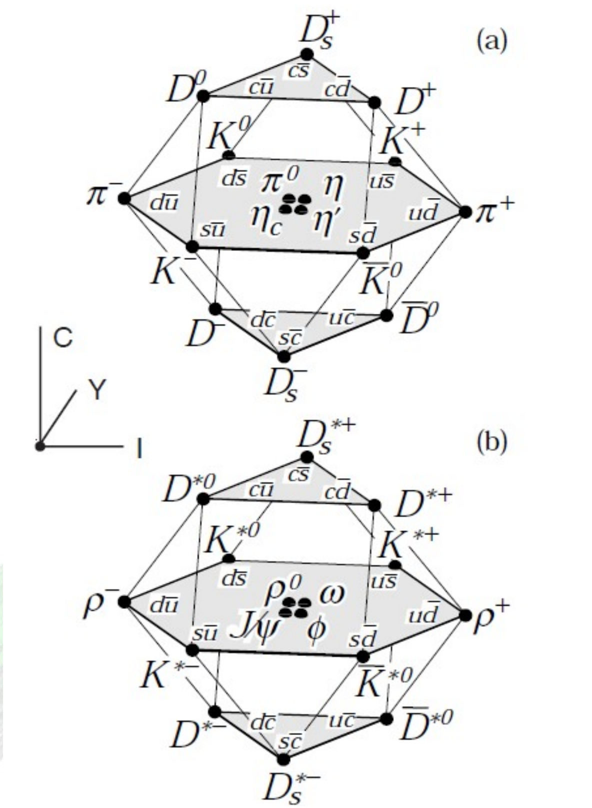
Quark spin: $s = \pm \frac{1}{2}$ Parity: $P = +1$

The spin of a quark pair couples to

$$|s_1 - s_2| \leq J \leq |s_1 + s_2| \rightarrow J = 0 \text{ or } 1$$

J = 0 : pseudoscalar mesons

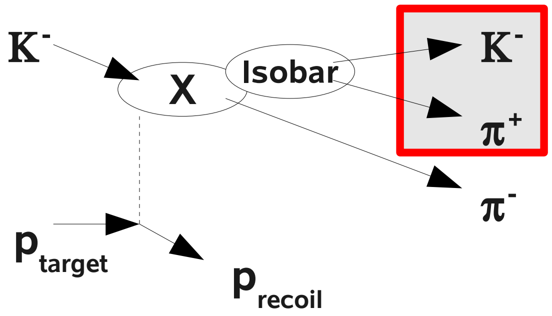
J = 1 : vector mesons



$\boxed{\pi^{\pm}}$	$I^G(J^P) = 1^-(0^-)$
$\boxed{K^{\pm}}$	$I(J^P) = \frac{1}{2}(0^-)$
$\boxed{K^*(892)}$	$I(J^P) = \frac{1}{2}(1^-)$

Decomposition of the decay chain (reversely)

final states (more or less stable particles)



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The spin of a quark pair couples to
$|s_1 - s_2| \leq J \leq |s_1 + s_2| \rightarrow J = 0$ or 1

J = 0 : pseudoscalar mesons
J = 1 : vector mesons

Introducing additional inner
Quantum number:
Orbital angular momentum: $\ell = 0, 1, \ldots$

Parity of the quark pair becomes
$P = P_1 \times P_2 \times (-1)^{\ell+1} = -1$ (for $\ell=0$)

$\boxed{\pi^\pm}$ $I^G(J^P) = 1^-(0^-)$

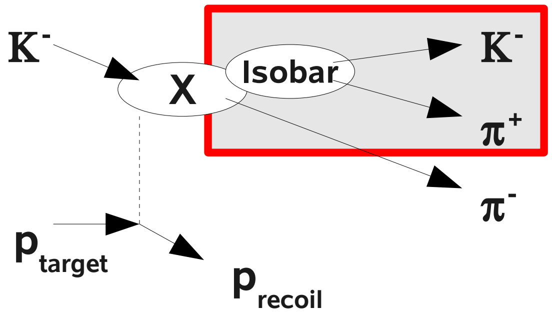
$\boxed{K^\pm}$ $I(J^P) = \frac{1}{2}(0^-)$

$\boxed{K_2^*(1430)}$ $I(J^P) = \frac{1}{2}(2^+)$

$\boxed{K^*(892)}$ $I(J^P) = \frac{1}{2}(1^-)$

Decomposition of the decay chain (reversely)

Isobar states (resonances in the subsystem)



For us relevant quantum numbers:
Quark spin: $s = \pm \frac{1}{2}$ Parity: $P = +1$

The spin of a quark pair couples to
$|s_1 - s_2| \leq J \leq |s_1 + s_2| \rightarrow J = 0$ or 1
J = 0 : pseudoscalar mesons
J = 1 : vector mesons

Introducing additional inner Quantum number:
Orbital angular momentum: $\ell = 0, 1, \ldots$

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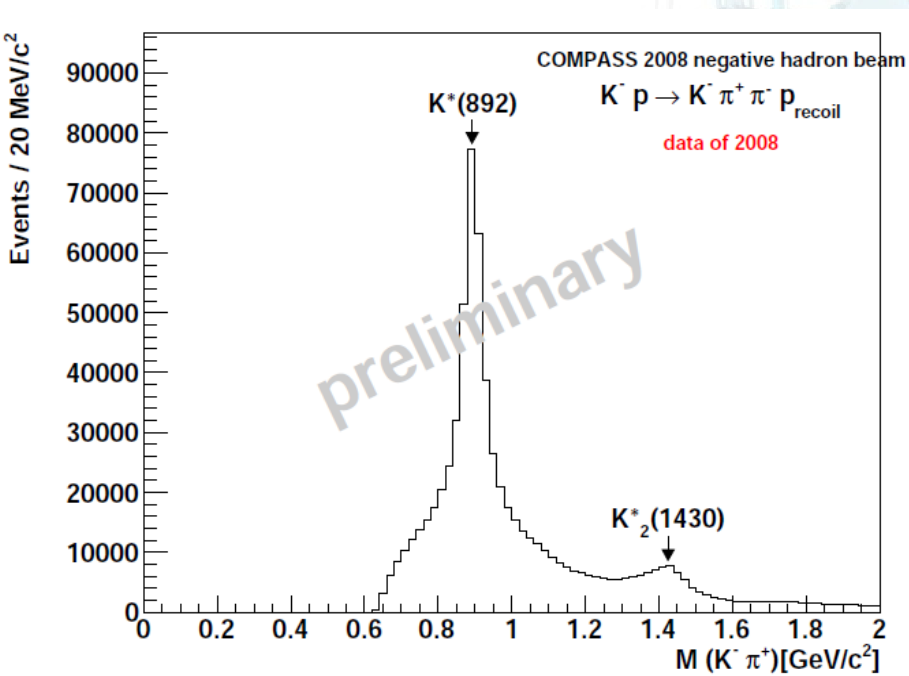

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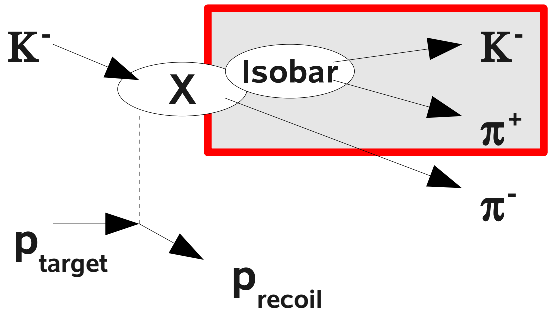
Decomposition of the decay chain (reversely)

Isobar states (resonances in the subsystem)

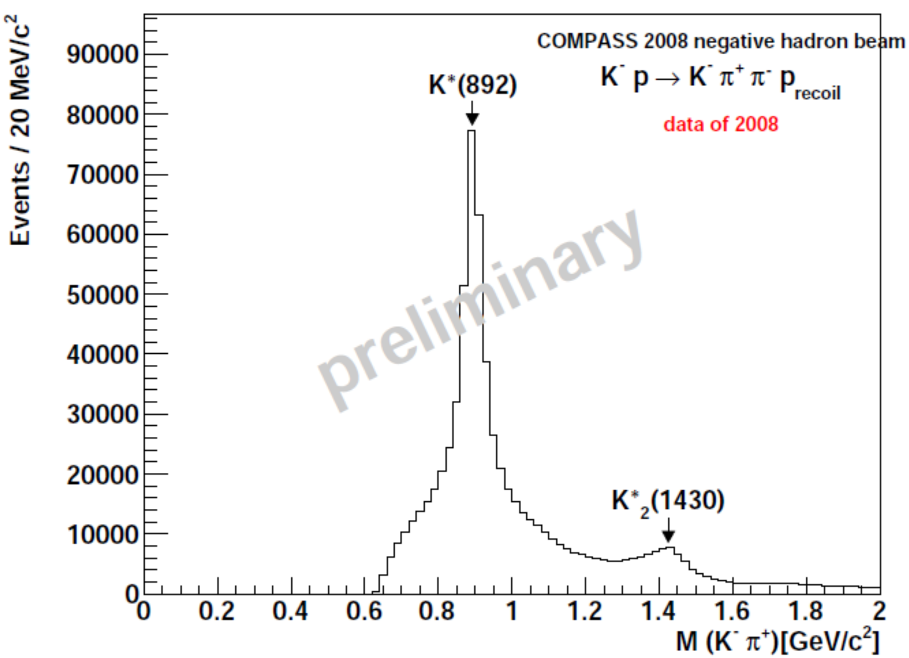


From now on forget about the quark pair interpretation!

Look only at the final states and combine quantum numbers:

$\boxed{\pi^{\pm}}$ $I^G(J^P) = 1^-(0^-)$

$\boxed{K^{\pm}}$ $I(J^P) = \frac{1}{2}(0^-)$



$\boxed{K^*_2(1430)}$ $I(J^P) = \frac{1}{2}(2^+)$

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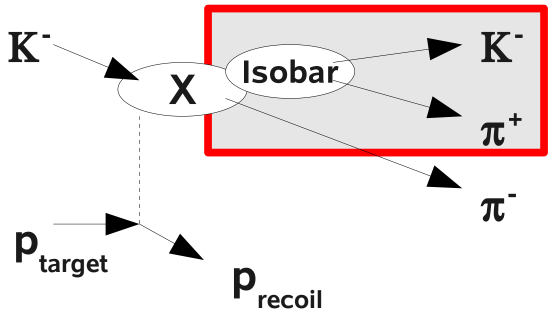
Decomposition of the decay chain (reversely)

Isobar states (resonances in the subsystem)



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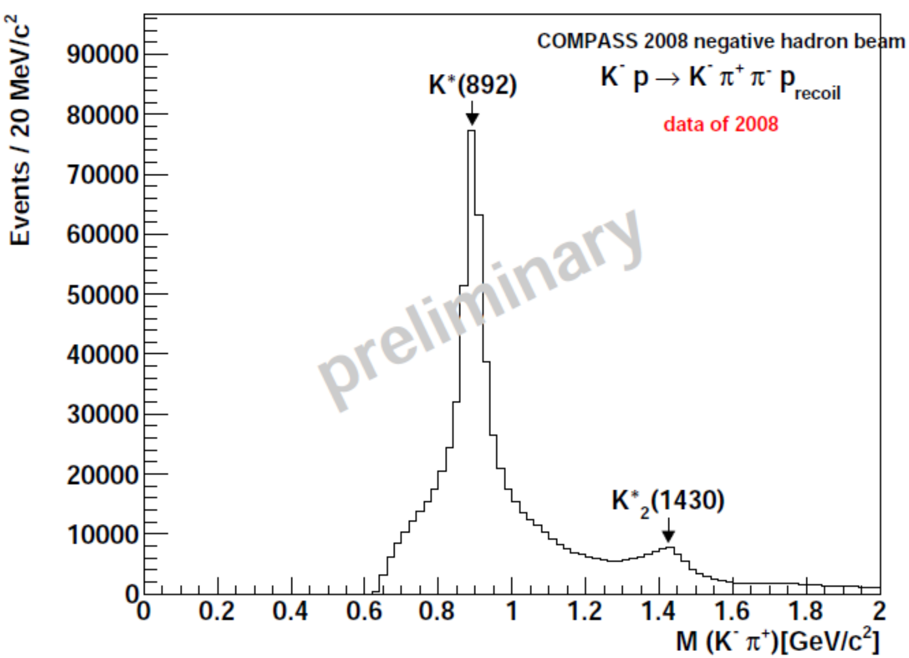

Renaming the total spin J → s

The spin of a final state particle pair couples to
$|s_1 - s_2| \leq s \leq |s_1 + s_2|$, here s = 0

$\boxed{K_2^*(1430)}$ $I(J^P) = \frac{1}{2}(2^+)$

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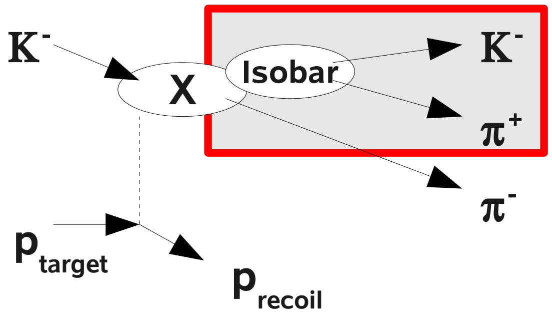
Decomposition of the decay chain (reversely)

Isobar states (resonances in the subsystem)



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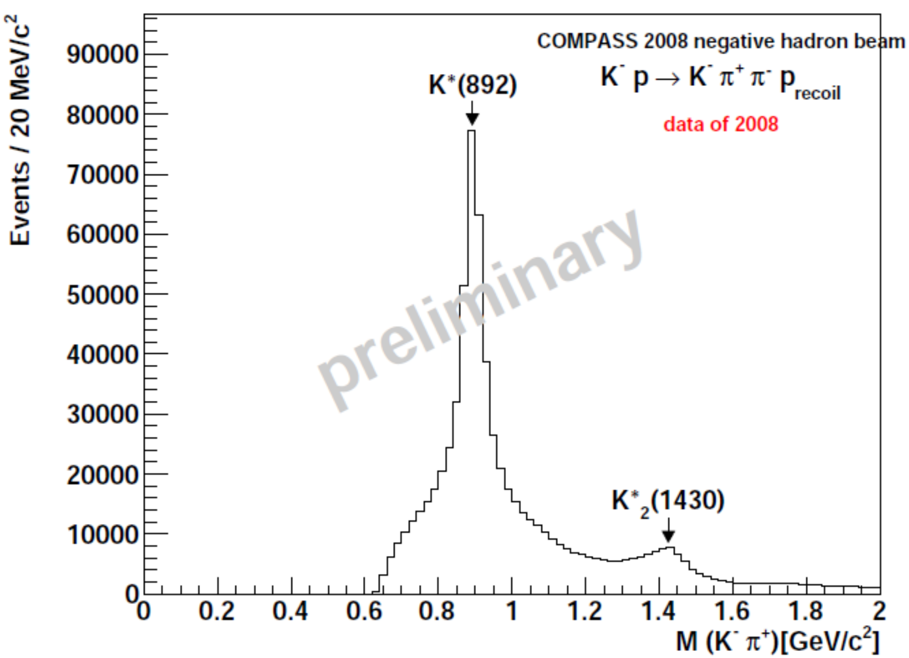


Renaming the total spin J → s

The spin of a final state particle pair couples to

$|s_1 - s_2| \leq s \leq |s_1 + s_2|$, here s = 0

Add orbital angular momentum ℓ = 0,1,2,...

Parity of the resonance becomes

$P = P_1 \times P_2 \times (-1)^{\ell+1}$

Total Spin couples then to

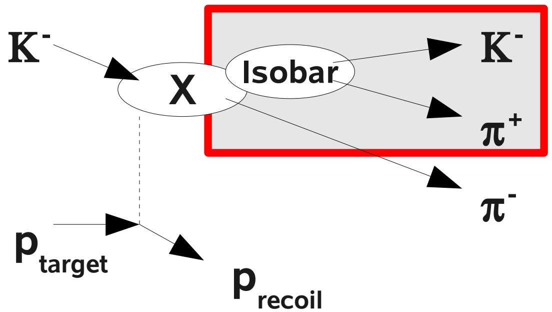
$|s - \ell| \leq J \leq |s + \ell|$

$\boxed{K_2^*(1430)}$ $\quad I(J^P) = \frac{1}{2}(2^+)$

$\boxed{K^*(892)}$ $\quad I(J^P) = \frac{1}{2}(1^-)$

Decomposition of the decay chain (reversely)

Isobar states (resonances in the subsystem)



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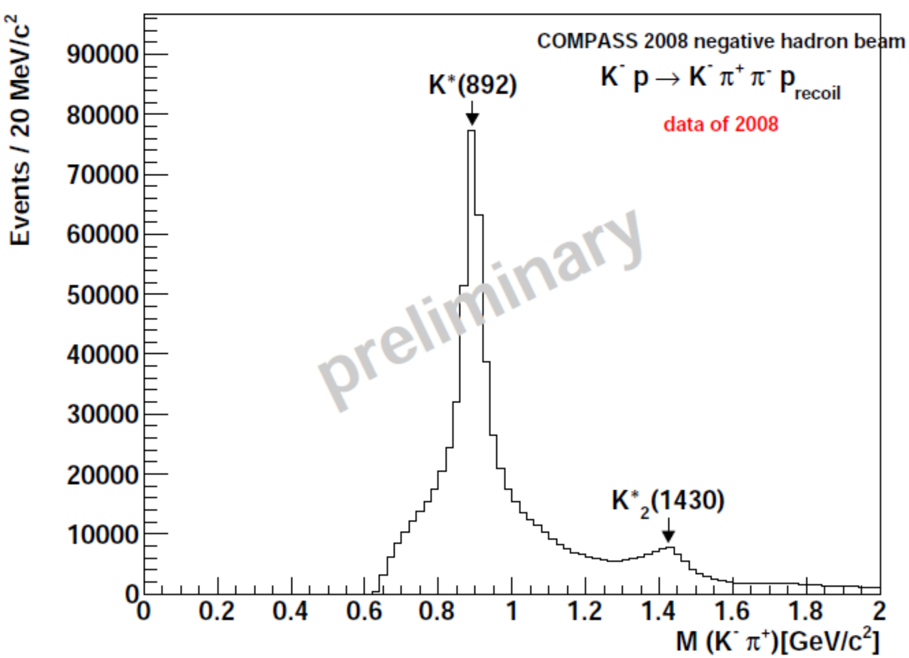


Important notation:

$$particle_1 \left[\begin{matrix} l \\ s \end{matrix} \right] particle_2$$

here:

$$K^- \left[\begin{matrix} l = 0,1,2,\ldots \\ |s_1 - s_2| \le s \le |s_1 + s_2| = 0\ldots 0 \end{matrix} \right] \pi^+$$

Total Spin still couples

$|s-\ell| \le J \le |s+\ell|$

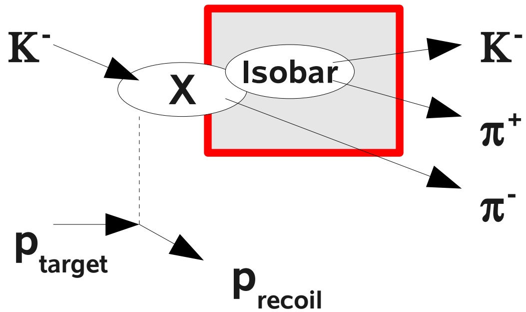
and thus every J bigger than 0 is possible
due to the orbital angular momentum ℓ= 0,1,2,...
But not all J are observed!

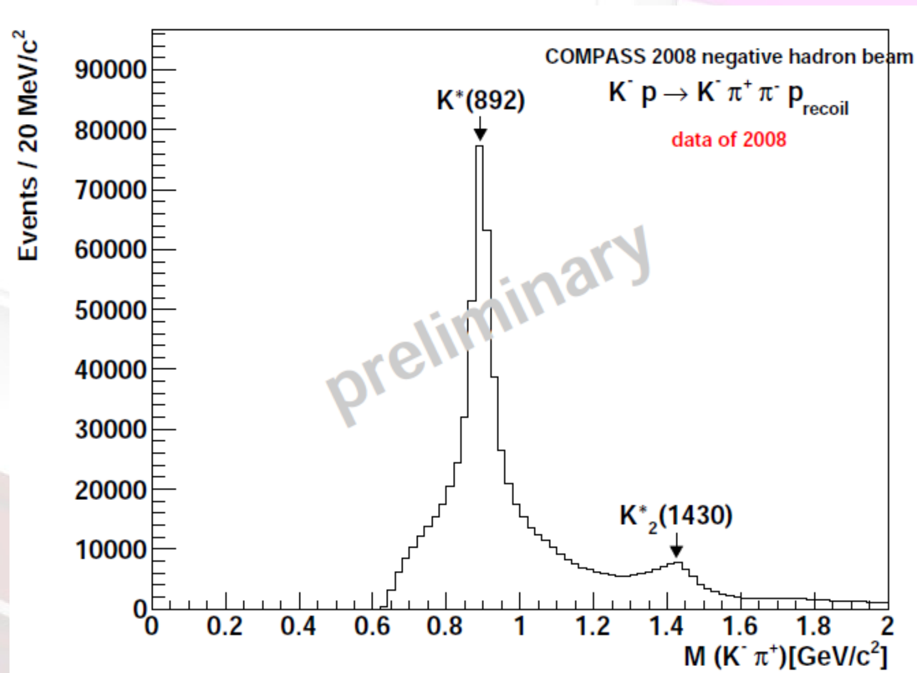
$\boxed{K_2^*(1430)}$ $I(J^P) = \frac{1}{2}(2^+)$

$\boxed{K^*(892)}$ $I(J^P) = \frac{1}{2}(1^-)$

Decomposition of the decay chain (reversely)

Isobar states (resonances in the subsystem)



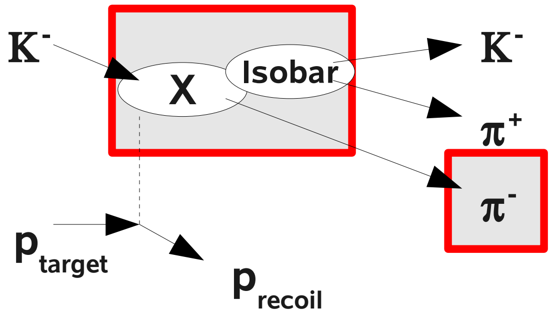


It is now assumed to have a good knowledge about the Isobars appearing in the subsystems.
(Of course the subsystems are not fully understood at all!)
2 particle subsystems are usually already analyzed in further experiments and we use this Knowledge to setup now a partial wave set based on the observed Isobars.
As an example we use the clearly visible resonances:

$\boxed{K_2^*(1430)}$ $I(J^P) = \frac{1}{2}(2^+)$ $\boxed{K^*(892)}$ $I(J^P) = \frac{1}{2}(1^-)$

Decomposition of the decay chain (reversely)

The diffractively produced resonance



$\boxed{\pi^{\pm}}$ $I^G(J^P) = 1^-(0^-)$

$\boxed{K^*(892)}$ $I(J^P) = \frac{1}{2}(1^-)$

$\boxed{K_2^*(1430)}$ $I(J^P) = \frac{1}{2}(2^+)$

Renaming the total spin J $\rightarrow$ s

The spin of a final state particle pair couples to

$$|s_1 - s_2| \leq s \leq |s_1 + s_2| \longrightarrow s = 1$$

Add orbital angular momentum $\ell = 0,1,2,\ldots$

Parity of the resonance becomes

$$P = P_1 \times P_2 \times (-1)^{\ell+1}$$

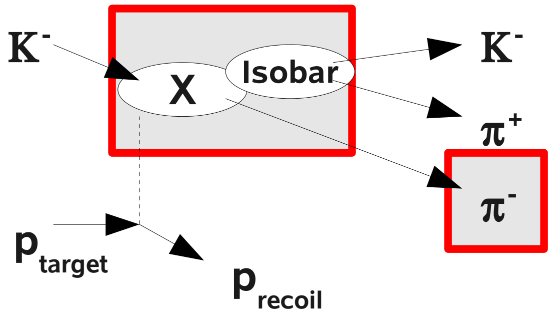
Total Spin couples then to

$$|s-\ell| \leq J \leq |s+\ell|$$

Projection of J is the quantum number M = -J,-J+1..,+J

Decomposition of the decay chain (reversely)

The diffractively produced resonance



$\boxed{\pi^{\pm}}$ $I^G(J^P) = 1^-(0^-)$

$\boxed{K^*(892)}$ $I(J^P) = \frac{1}{2}(1^-)$ $\boxed{K_2^*(1430)}$ $I(J^P) = \frac{1}{2}(2^+)$

Renaming the total spin J → s

$$K*(892)^0 \left[\begin{matrix} l=0,1,2,\dots \\ |s_1-s_2| \le s \le |s_1+s_2| = 1\dots1 \end{matrix} \right] \pi^-$$

The spin of a final state particle pair couples to

$|s_1-s_2| \le s \le |s_1+s_2| \longrightarrow s = 1$

Add orbital angular momentum ℓ= 0,1,2,...

Parity of the resonance becomes

$P = P_1 \times P_2 \times (-1)^{\ell+1}$

Total Spin couples then to

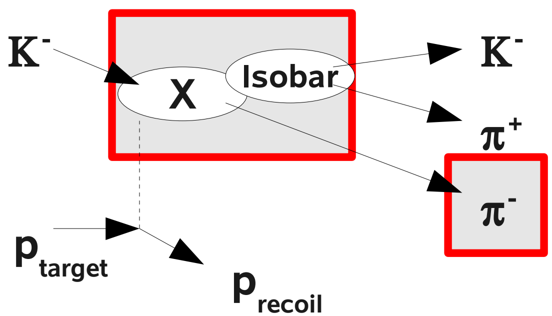
$|s-\ell| \le J \le |s+\ell|$

J	P	L	M
1	+	0	-1,0,1
0	-	1	0
1	-	1	-1,0,1
2	-	2	-2,-1,0,1,2
1	+	2	-1,0,1
2	+	2	-2,-1,0,1,2
3	+	2	-3,..,3

Projection of J is the quantum number M = -J,-J+1..,+J

Decomposition of the decay chain (reversely)

The diffractively produced resonance



$\boxed{\pi^{\pm}}$

$I^G(J^P) = 1^-(0^-)$

$\boxed{K^*(892)}$

$I(J^P) = \frac{1}{2}(1^-)$

$\boxed{K_2^*(1430)}$

$I(J^P) = \frac{1}{2}(2^+)$

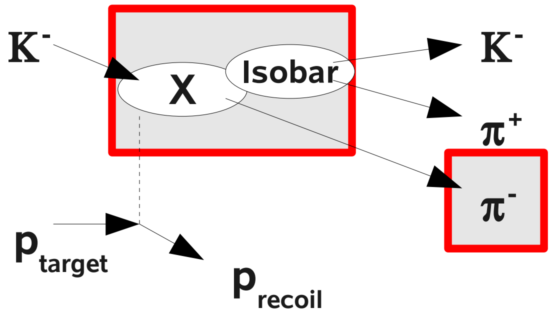
$$K*(892)^0 \left[\begin{array}{c} l=0,1,2,\dots \\ |s_1-s_2| \le s \le |s_1+s_2| = 1\dots1 \end{array} \right] \pi^-$$

J	P	L	M
0	-	1	0
1	+	0	-1,0,1
1	+	2	-1,0,1
1	-	1	-1,0,1
2	+	2	-2,-1,0,1,2
2	-	1	-2,-1,0,1,2
3	+	2	-3,..,3

Regroup to values of same JP(C) since
It is an observed fact that same JP(C) states mix and are therefore dealt as one and the same resonance in the PDG.

Decomposition of the decay chain (reversely)

The diffractively produced resonace



$\boxed{\pi^{\pm}}$ $I^G(J^P) = 1^-(0^-)$

$\boxed{K^*(892)}$ $I(J^P) = \frac{1}{2}(1^-)$

$\boxed{K_2^*(1430)}$ $I(J^P) = \frac{1}{2}(2^+)$

$$K_2(1430)^0\left[\begin{matrix} l=0,1,2,\ldots \\ |s_1-s_2|\le s\le|s_1+s_2|=2\ldots2\end{matrix}\right]\pi^-$$

Renaming the total spin J $\longrightarrow$ s

The spin of a final state particle pair couples to

$|s_1-s_2| \leq s \leq |s_1+s_2| \longrightarrow s = 1$

Add orbital angular momentum ℓ= 0,1,2,...

Parity of the resonance becomes

$P = P_1 \times P_2 \times (-1)^{\ell+1}$

Total Spin couples then to

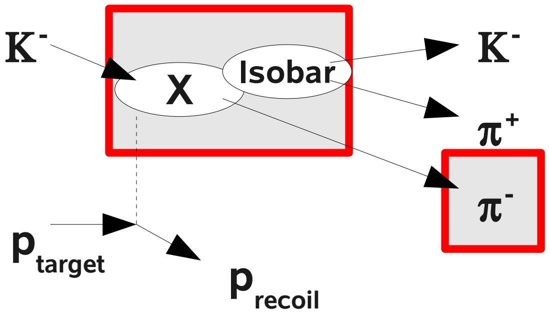
$|s-\ell| \leq J \leq |s+\ell|$

Projection of J is the quantum number M = -J,-J+1..,+J

J	P	L	M
2	-	0	-2,..,2
1	+	1	-1,..,1
2	+	1	-2,..,2
3	+	1	-3,..,3
0	-	2	0
1	-	2	-1,..,1
2	-	2	-2,..,2
3	-	2	-3,...,3
4	-	2	-4,...,4

Decomposition of the decay chain (reversely)

The diffractively produced resonace



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$\boxed{K^*(892)}$ $I(J^P) = \frac{1}{2}(1^-)$

$\boxed{K_2^*(1430)}$ $I(J^P) = \frac{1}{2}(2^+)$

$$K_2(1430)^0 \begin{bmatrix} l=0,1,2,\dots \\ |s_1-s_2| \leq s \leq |s_1+s_2| = 2\dots2 \end{bmatrix} \pi^-$$

Grouped again to same JP(C)

J	P	L	M
0	-	2	0
1	+	1	-1,..,1
1	-	2	-1,..,1
2	+	1	-2,..,2
2	-	0	-2,..,2
2	-	2	-2,..,2
3	+	1	-3,..,3
3	-	2	-3,...,3
4	-	2	-4,...,4

Listed resonances in the PDG

$$K^*(892)^0 \begin{bmatrix} l=0,1,2,\ldots \\ 1 \end{bmatrix} \pi^- \qquad K_2(1430)^0 \begin{bmatrix} l=0,1,2,\ldots \\ 2 \end{bmatrix} \pi^-$$

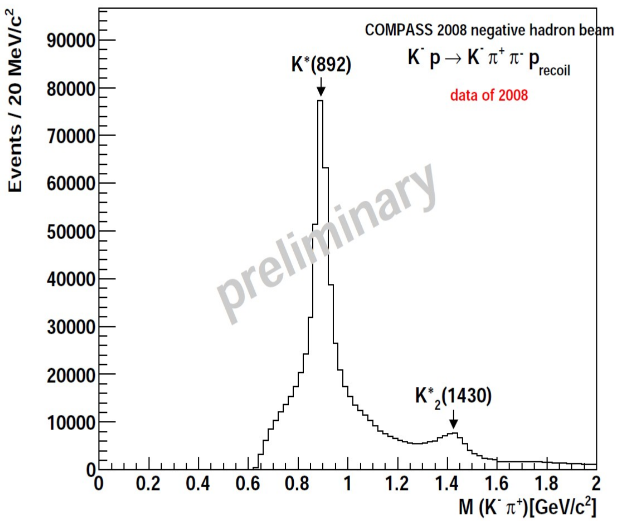
J	P
0	-
1	+
1	-
2	+
2	-
3	+

J	P	Observed resonances listed in the PDG
0	-	K(1460)
1	+	K1(1270),K1(1400)
1	-	*K*(1410),K*(1680)*
2	+	*K*2(1430)*,K*(1980)
2	-	K2(1580),K2(1770),K2(1820),*K*2(1980)*,...
3	+	-
3	-	K*3(1780)
4	-	-

Only resonances listed with observed decays containing these two example isobars
Italic if K(892) decays were observed* but no K*2(1430) contribution.

Let's see whether we can observe them...

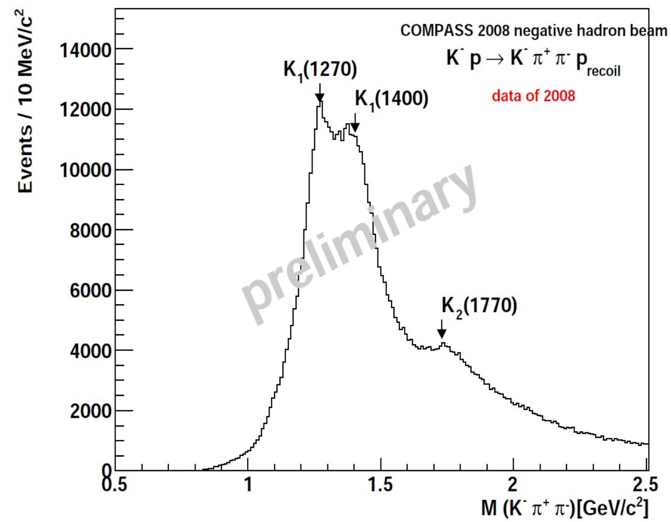
Basic partial wave set based on visible isobars

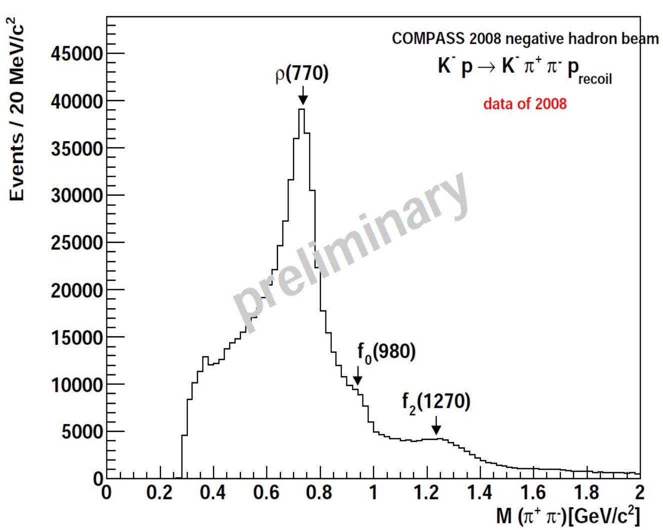


$$K*(892)^0\begin{bmatrix} l=0,1,2,\dots \\ 1 \end{bmatrix}\pi^-$$

$$K_2(1430)^0\begin{bmatrix} l=0,1,2,\dots \\ 2 \end{bmatrix}\pi^-$$

$$K_0(1430)^0\begin{bmatrix} l=0,1,2,\dots \\ 0 \end{bmatrix}\pi^-$$





$$K^-\begin{bmatrix} l=0,1,2,\dots \\ 1 \end{bmatrix}\rho(770)^0$$

$$K^-\begin{bmatrix} l=0,1,2,\dots \\ 0 \end{bmatrix}f_0(980)^0$$

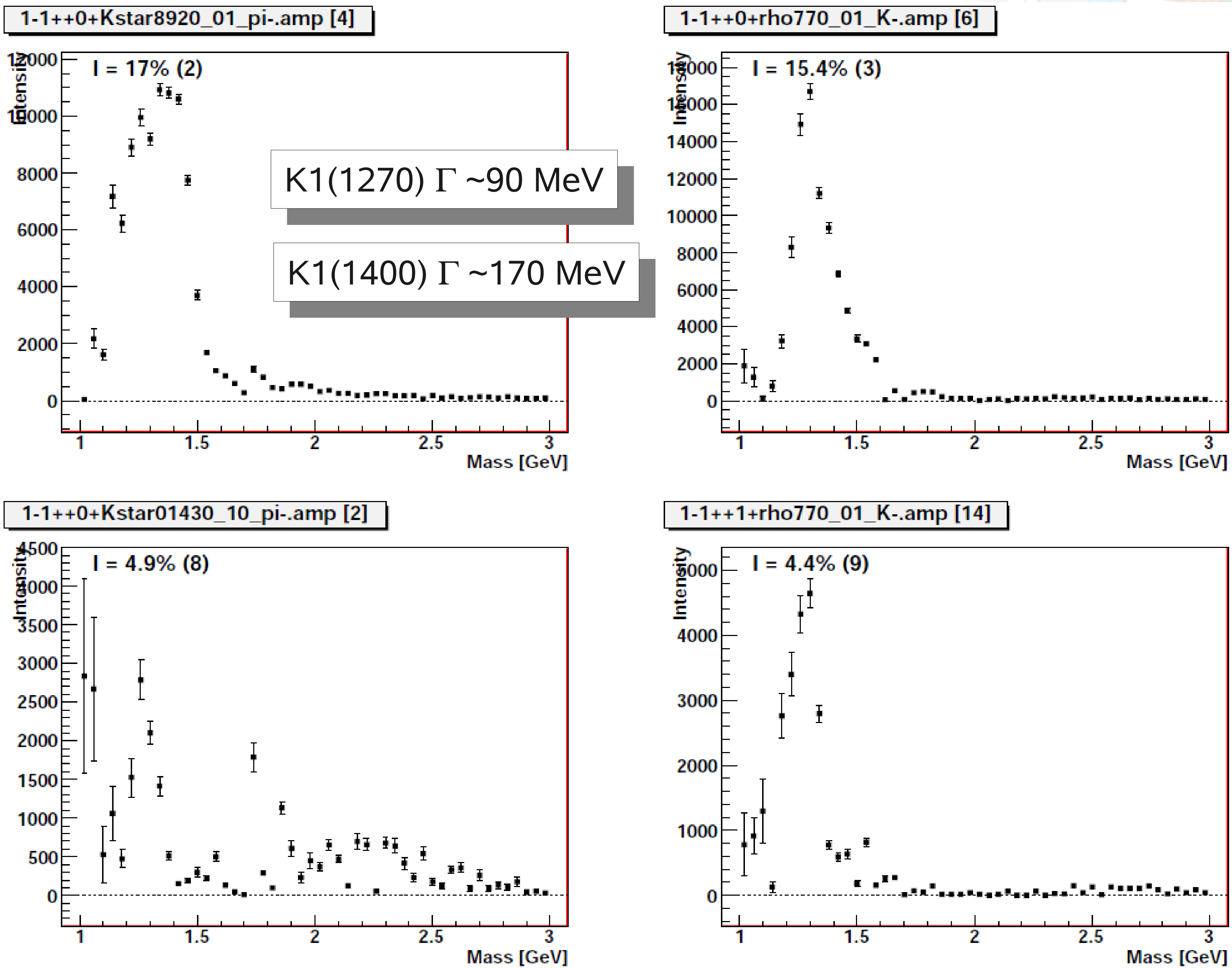
$$K^-\begin{bmatrix} l=0,1,2,\dots \\ 2 \end{bmatrix}f_2(1270)^0$$

- Take only M ≥ 0 (natural parity exchange, J = 0+,1-,2+,... since pomeron dominated)
- l ≤ 2
- Take only visible, known JP states, 1+ and 2 -

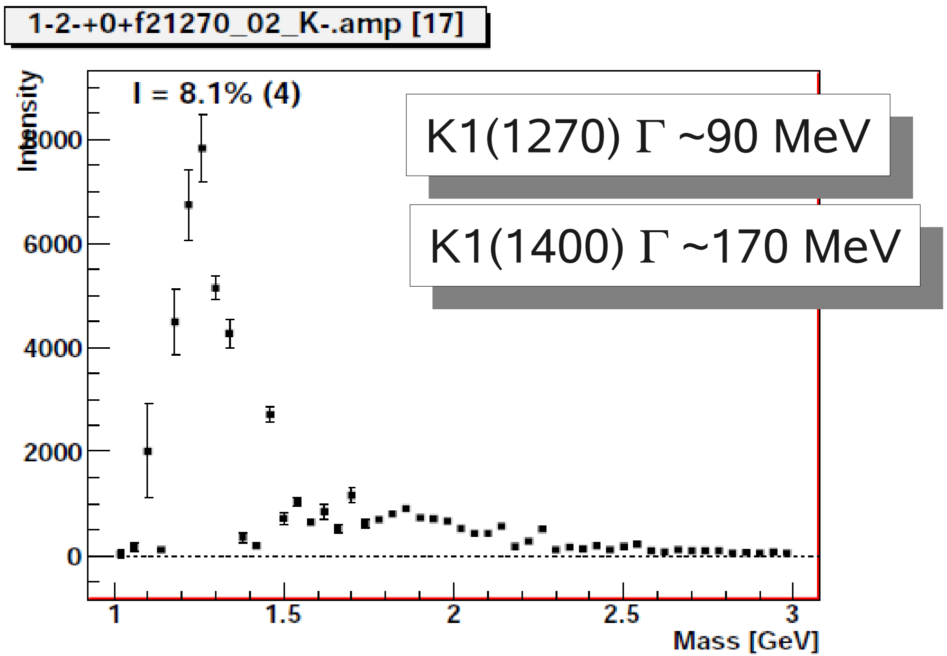
→ 32 waves + flat wave !

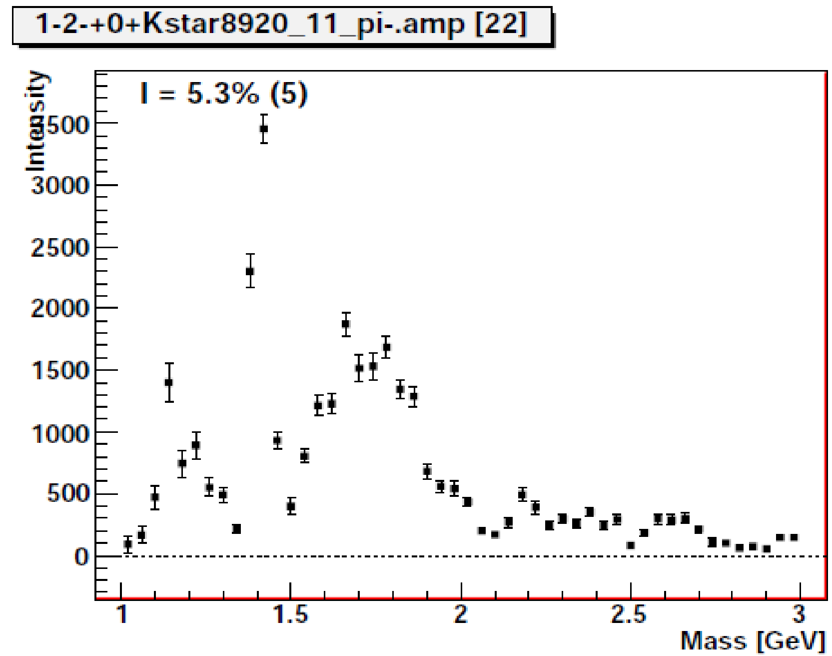
- Fit on 40 MeV bins

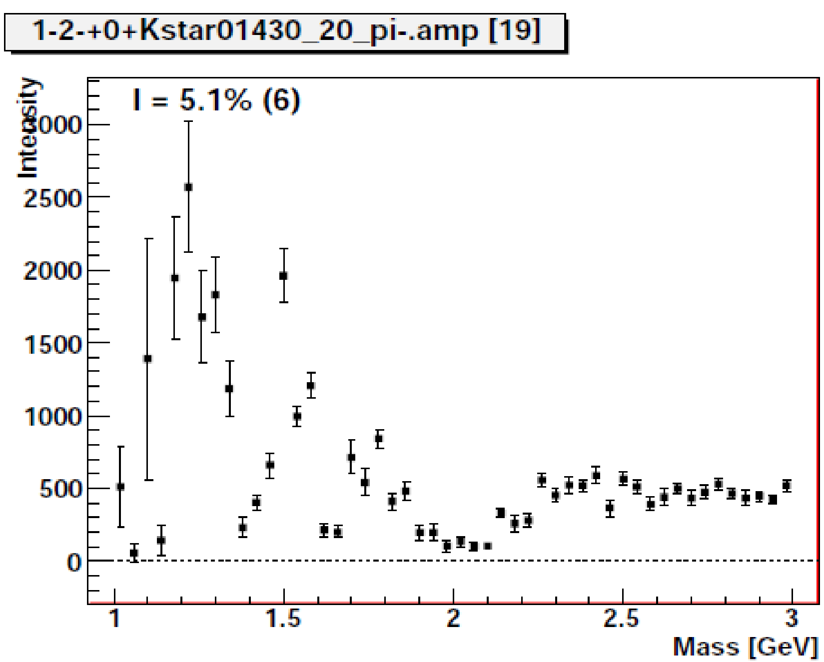
“ basic “ wave set results (JP=1+)

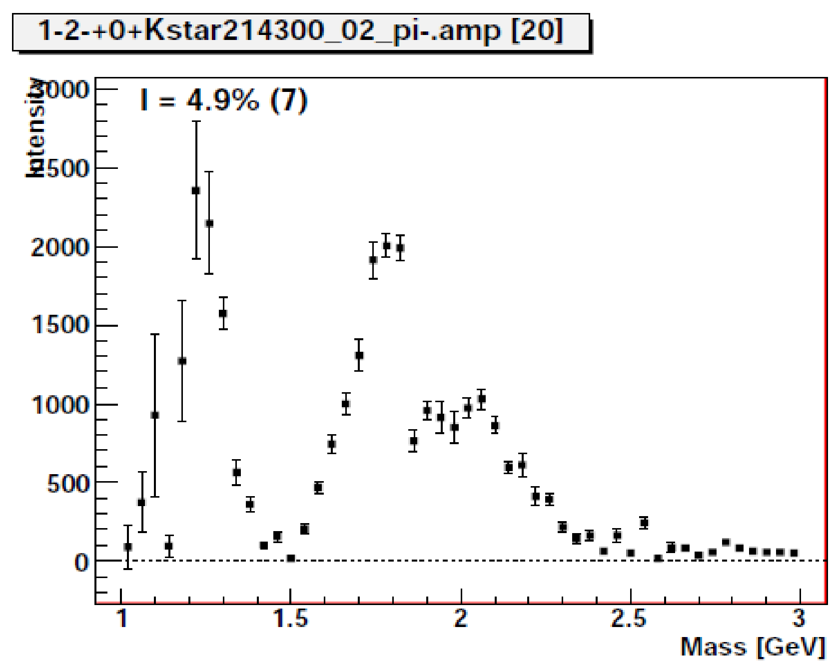


“ basic “ wave set results (JP=1+)

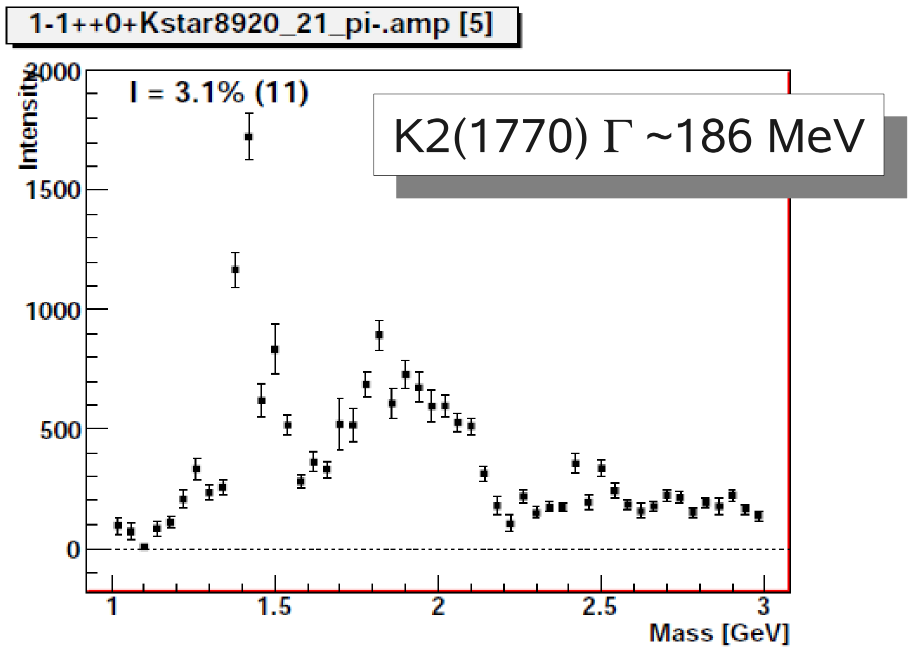


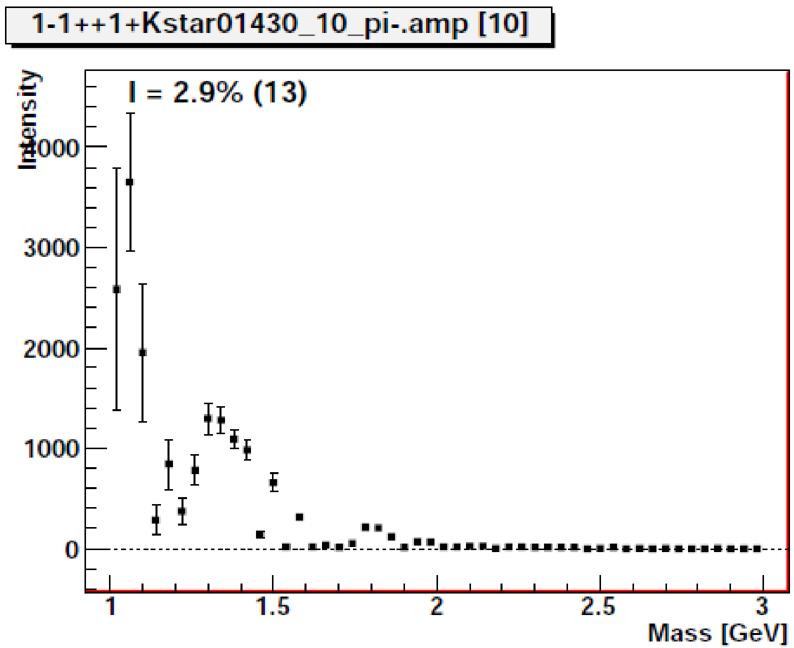


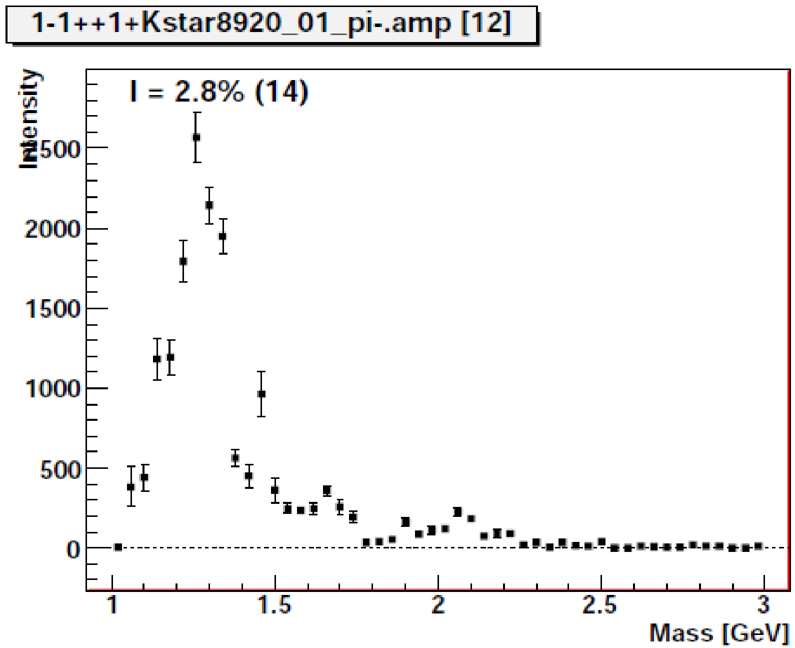


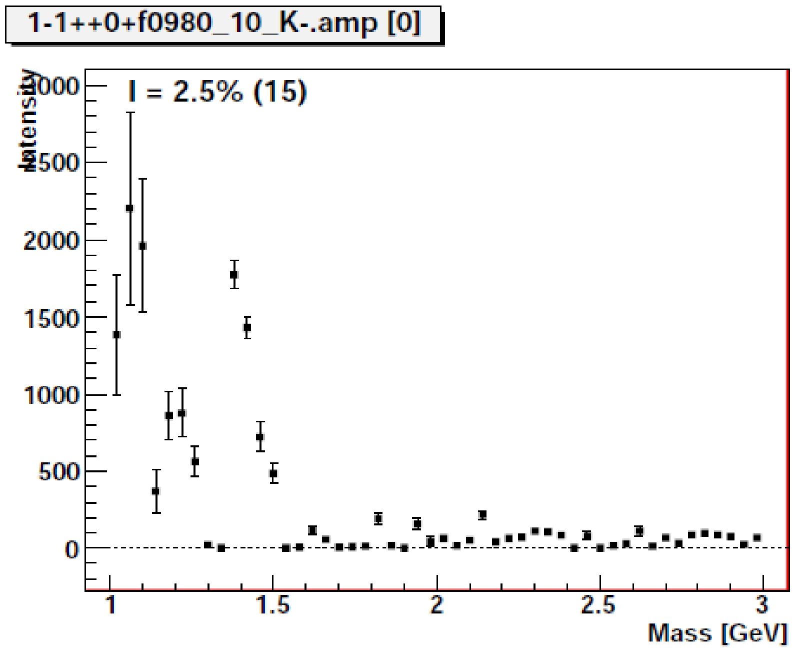


“ basic “ wave set results (JP=2-)

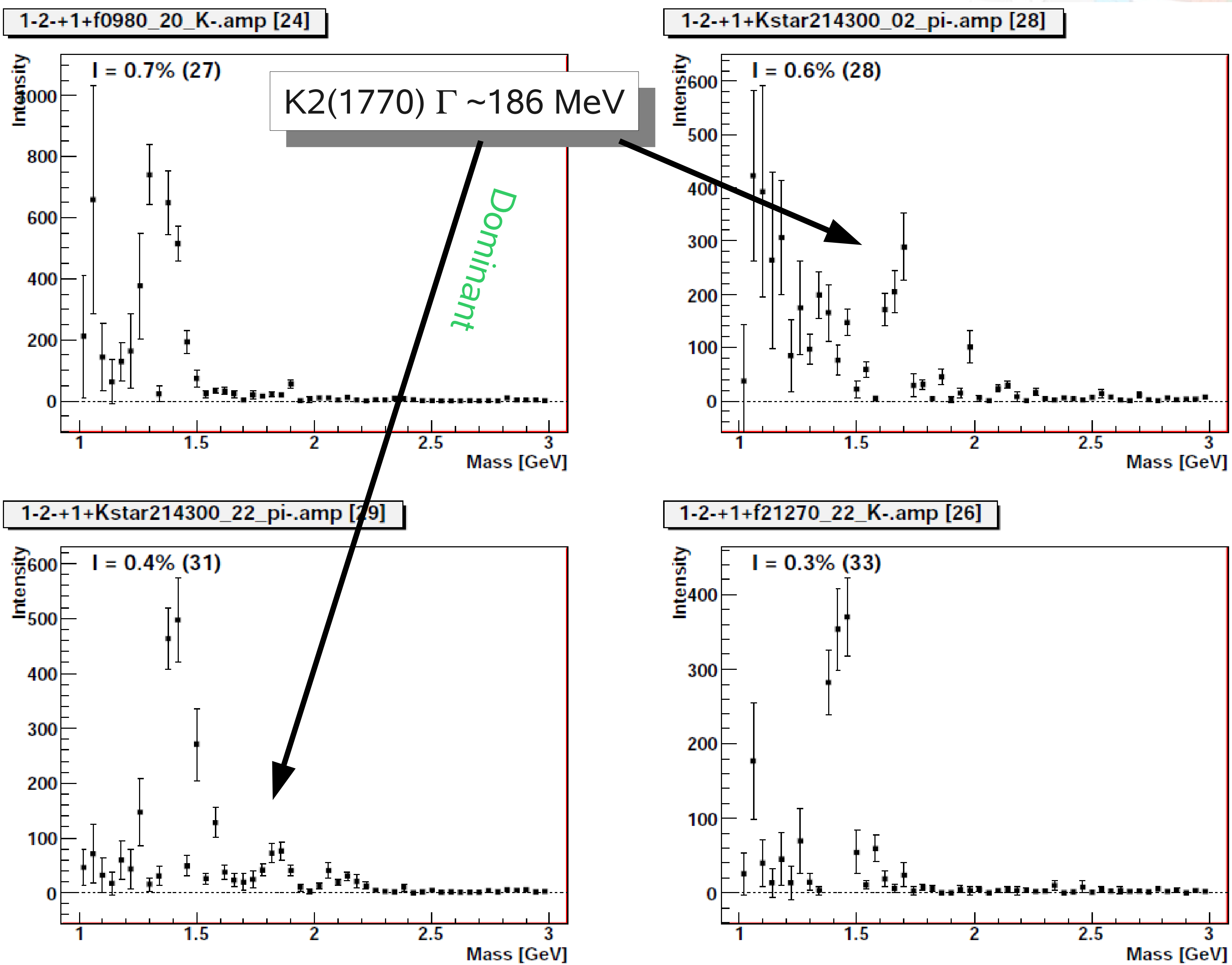




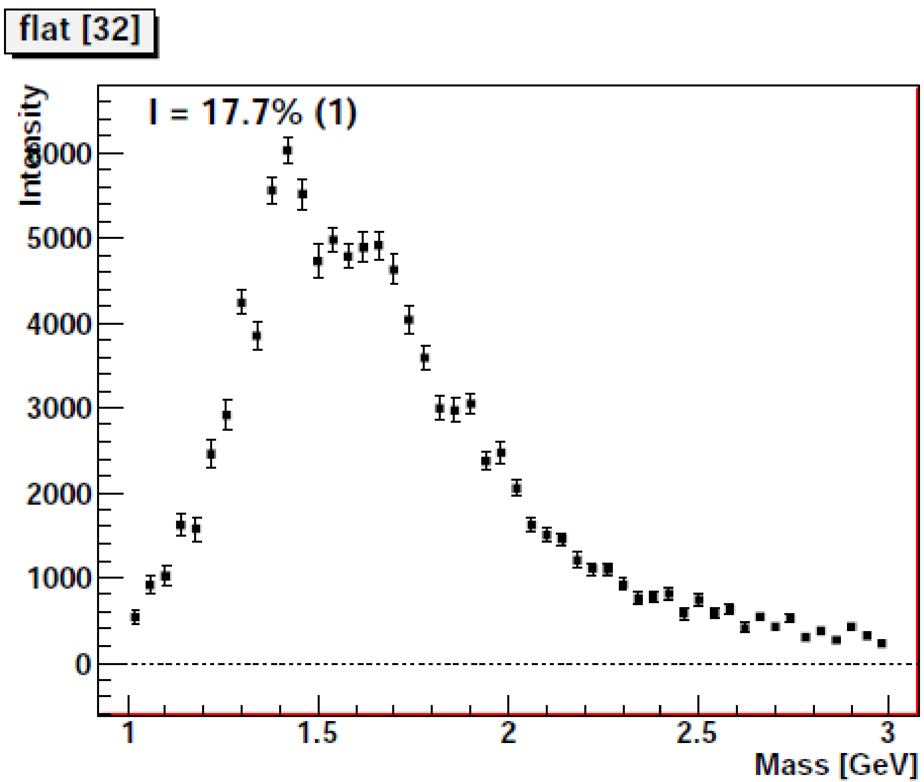




" basic " wave set results (JP=2-)



“ basic “ wave set results (flat wave)



Conclusion:

- initial partial wave set is still leaving too much structure in the flat wave → missing partial waves to describe

- Isobar system is not complete (missing for example flat $(\pi\ \pi)_s$ $(K\ \pi)_s$ waves)

- tuning of wave set is needed (thresholds, resonance description of $\rho(770)$ for example)

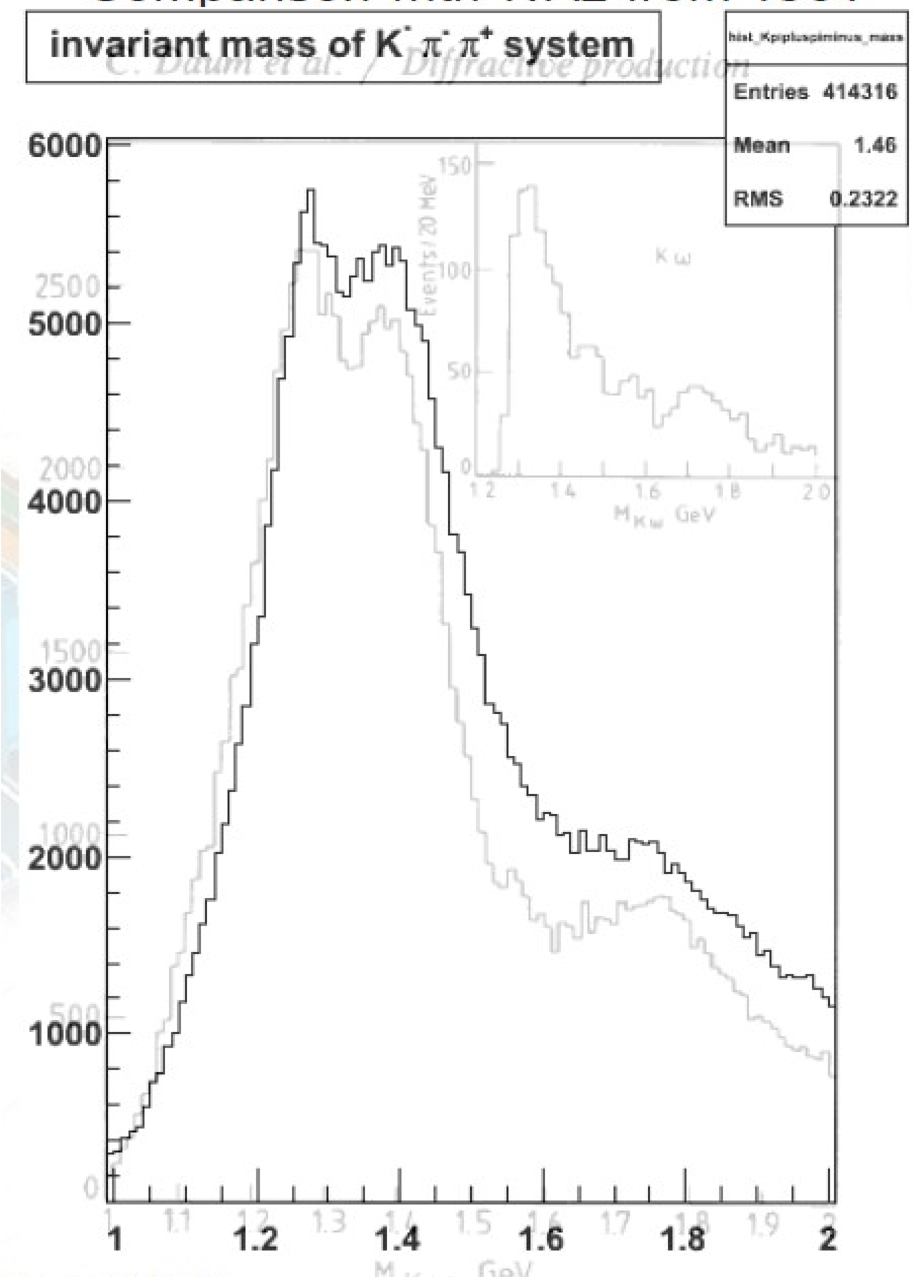
- A look at the phase motion is not shown here but needed to determine a resonance

How can we see that I'm going in the right direction? → Implement the wave set of WA03 (same channel) and compare to it...

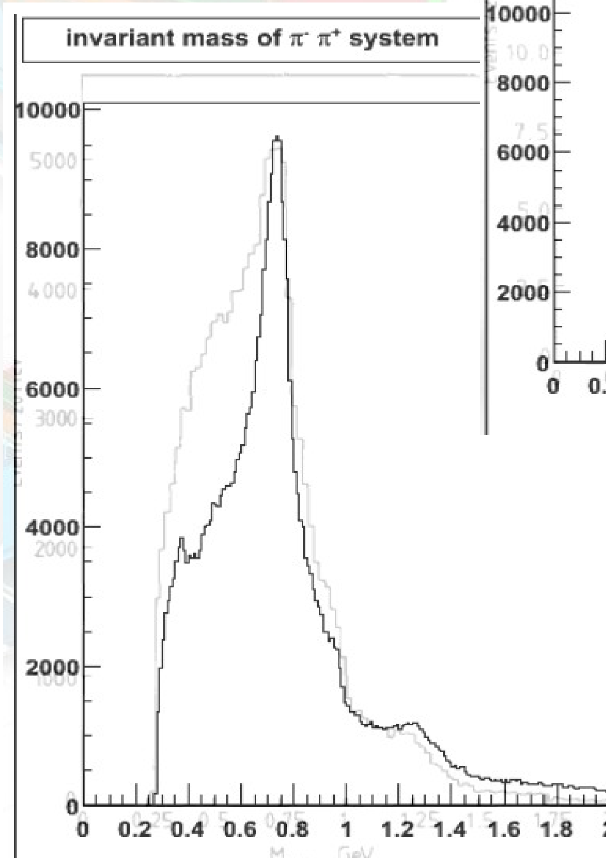
Note: I did not show all waves

COMPASS vs. WA03

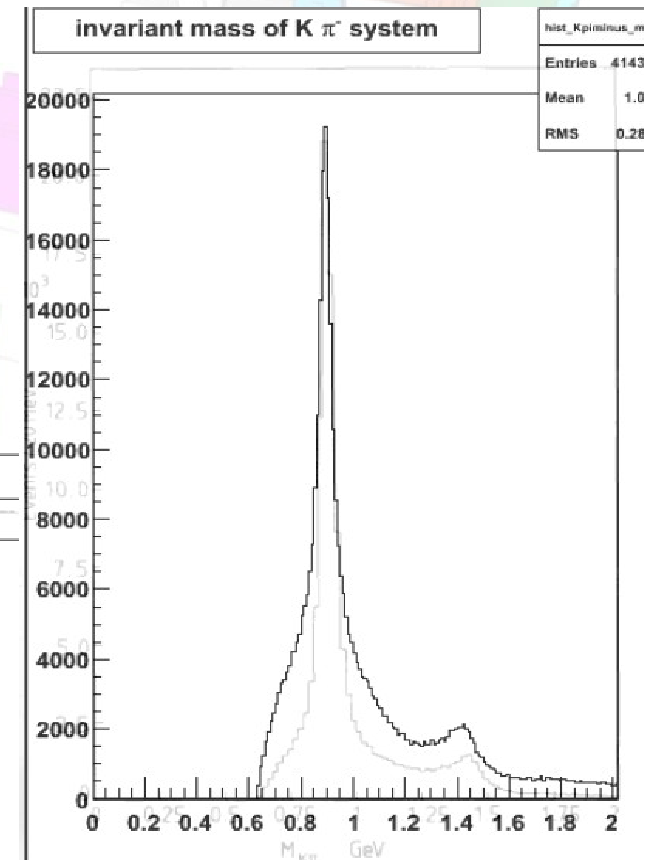
Comparison with WA2 from 1981



00), K2(1770)



Visible rho(770), f0(980), f2(1270)



The final wave set by ACCMOR-Collab. (WA03)

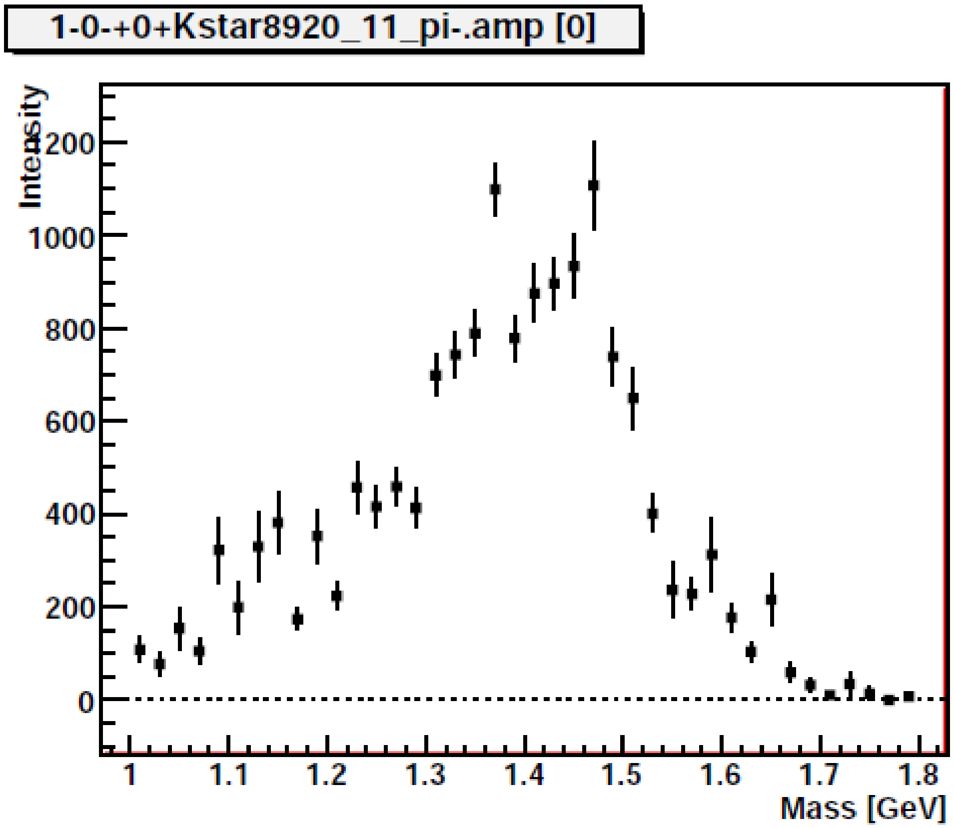
Using this wave set for a first try of the ROOTPWA software package

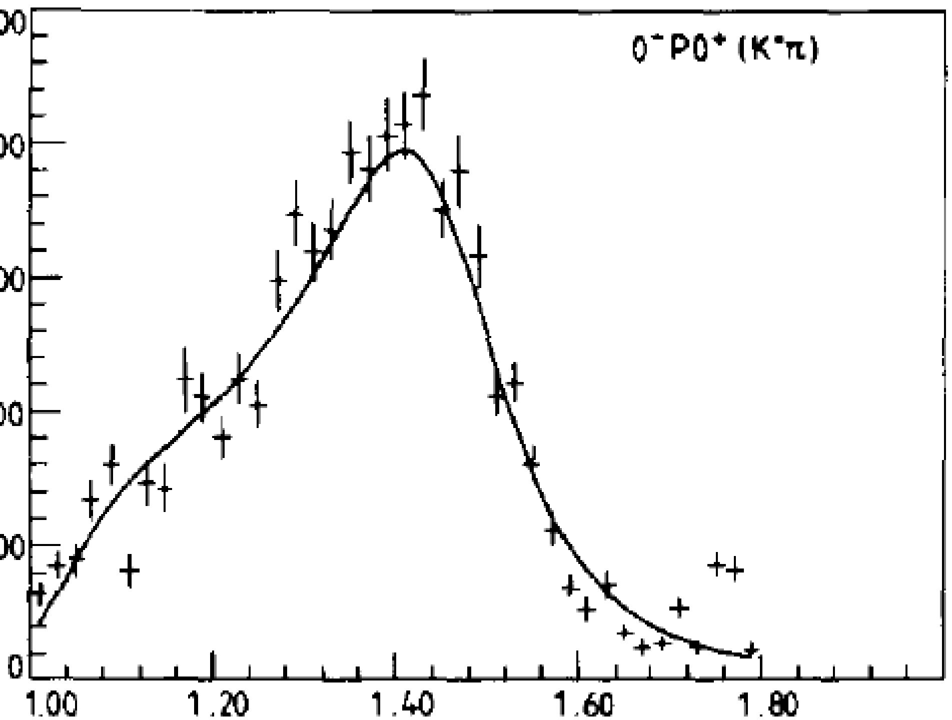
The notation is $J^P L M^\eta$

Mass range (GeV)	1.0–1.1	1.1–1.2	1.2–1.3	1.3–1.4	1.4–1.5	1.5–1.6	1.6–1.7	1.7–2.1
Non-flip waves	$0^-P0^+(K^*\pi)$							
	$1^+S0^+(K^*\pi)$							
		$0^-S0^+(\varepsilon K)$						
		$1^+P0^+(\varepsilon K)$						
		$1^+P0^+(\kappa\pi)$						
		$1^+S0^+(\rho K)$						
		$1^+S1^+(K^*\pi)$						
		$1^+S1^+(\rho K)$						
		$2^+D1^+(K^*\pi)$						
		$2^-P0^+(K^*\pi)$						
			$0^-P0^+(\rho K)$					
			$1^+D0^+(K^*\pi)$					
			$1^+P1^+(\kappa\pi)$					
				$2^+D1^+(\rho K)$				
							$1^+D0^+(\rho K)$	
							$2^-P0^+(\rho K)$	
						$2^-S0^+(K^{**}\pi)$		
							$2^-S0^+(fK)$	
Flip waves		$1^+S0^+(\rho K)$						
		$1^+S1^+(\rho K)$						
							$1^+P0^+(\varepsilon K)$	

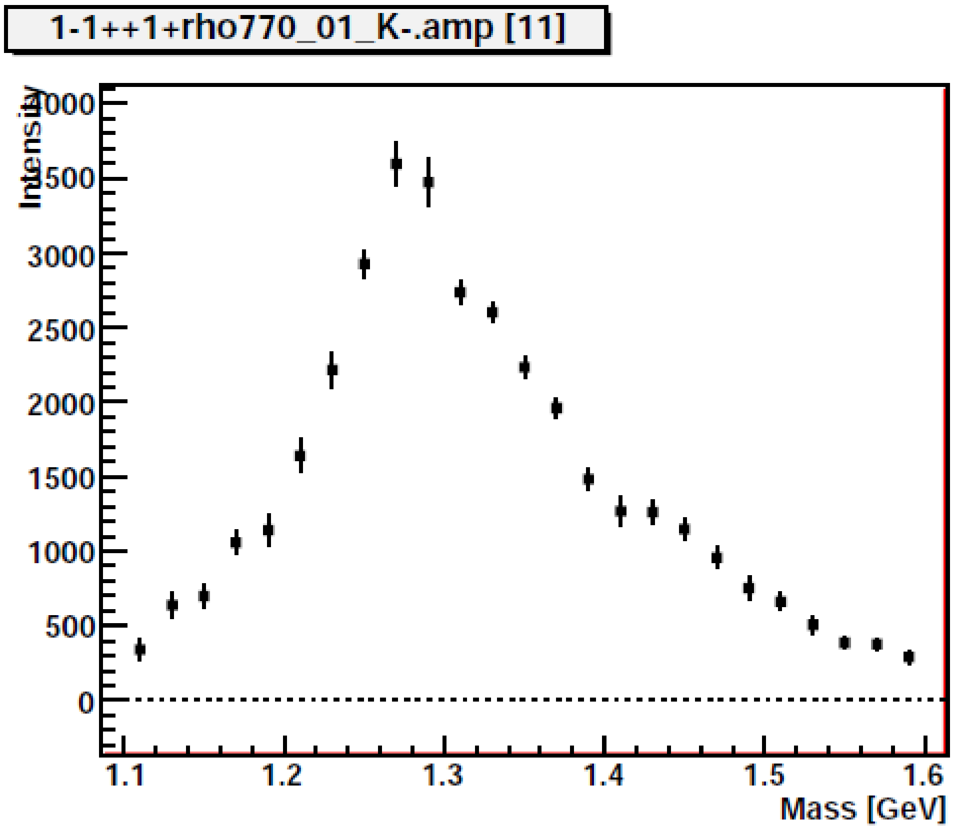
ε = (π π)s-wave I used σ ; κ = K(800) ; K** = K*(1430) ; f = f2(1270)

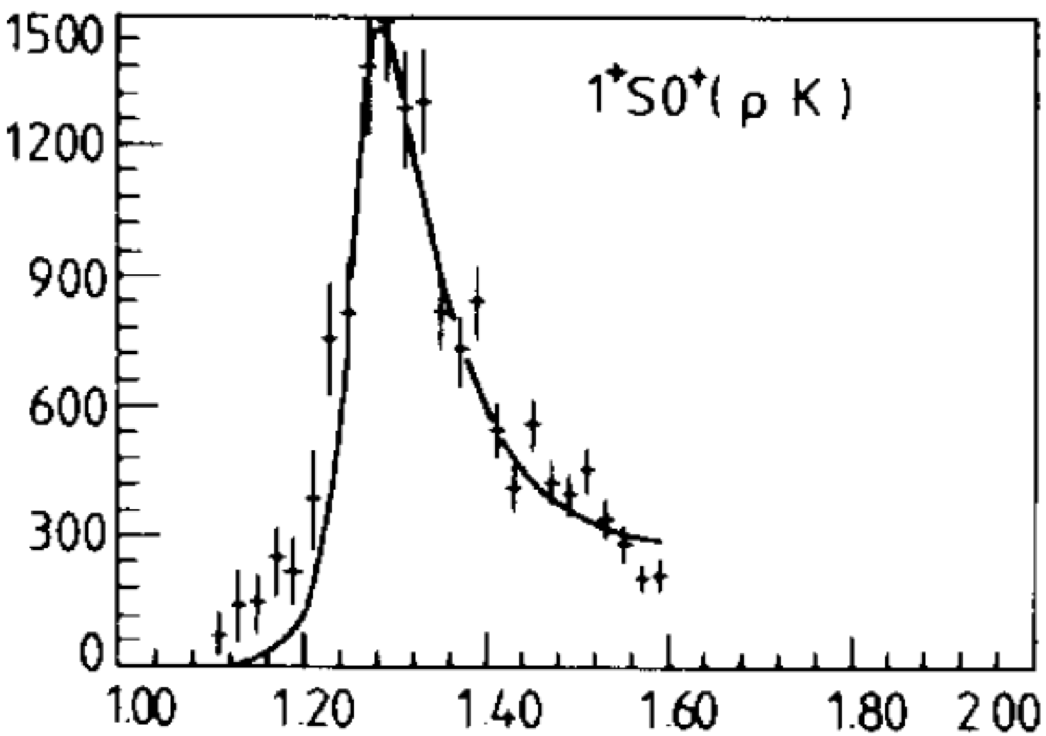
some results for COMPASS 2008 data compared to WA03



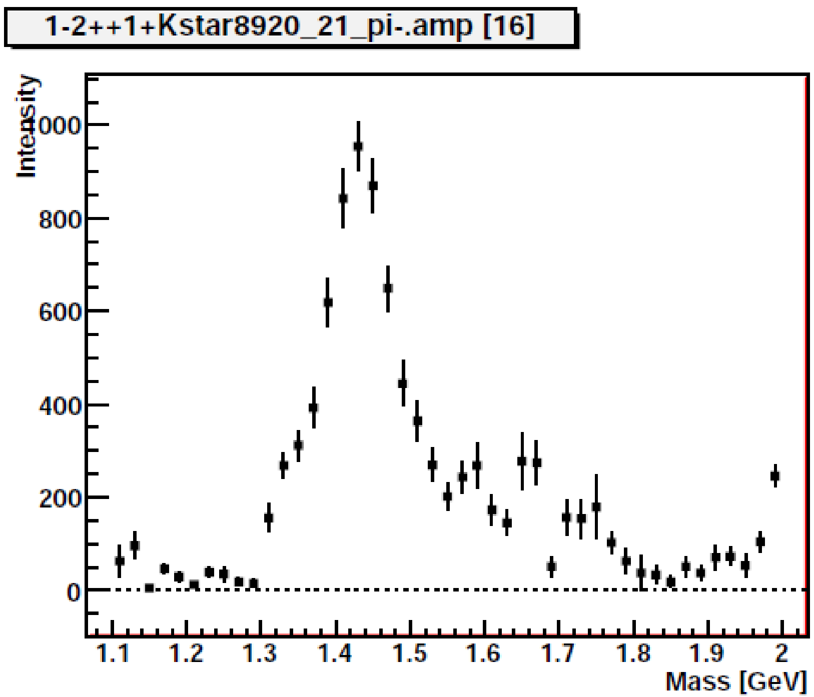


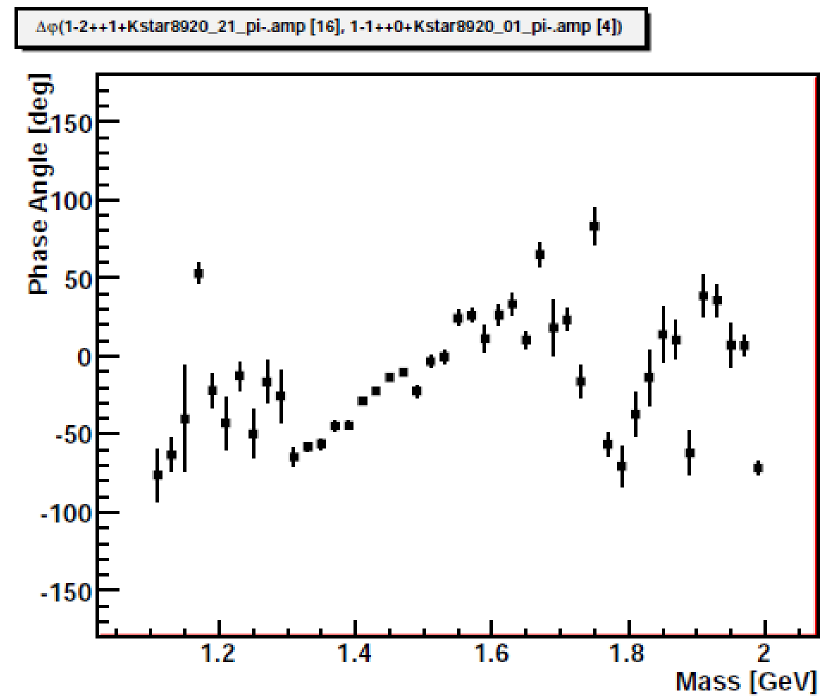
some results for COMPASS 2008 data compared to WA03

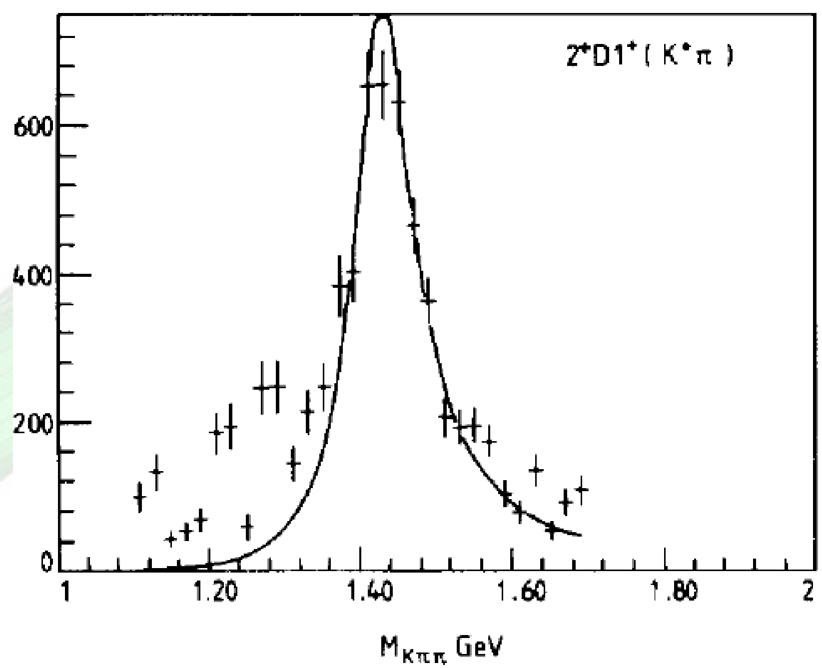


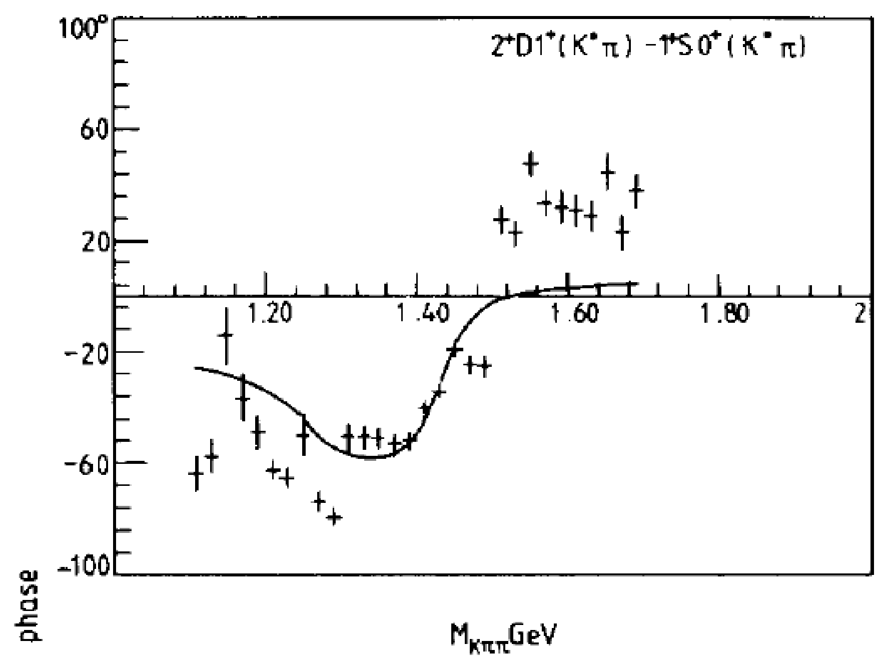


some results for COMPASS 2008 data compared to WA03

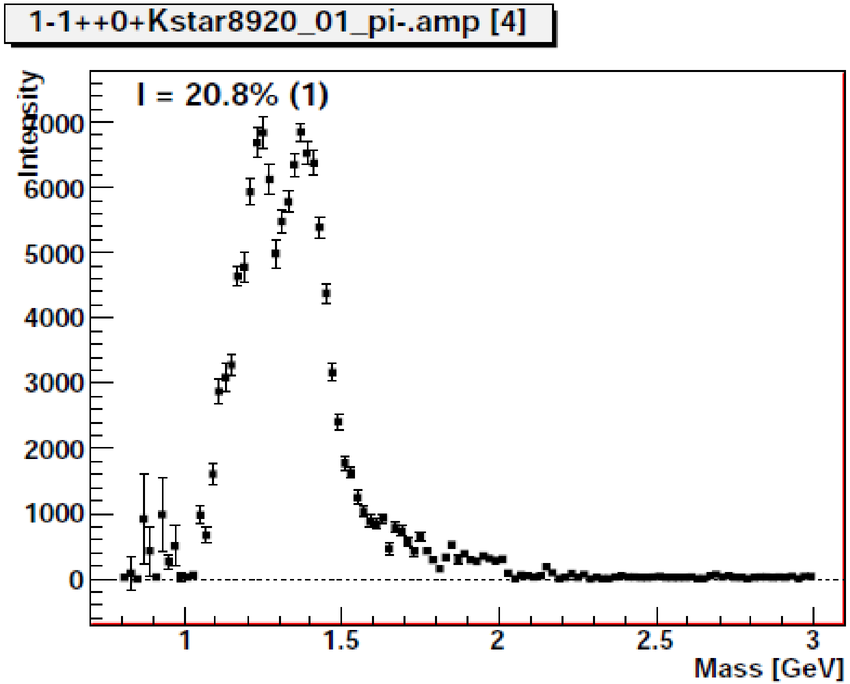


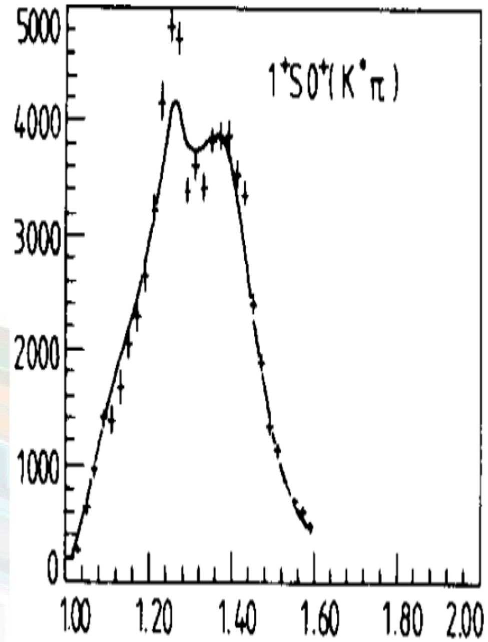


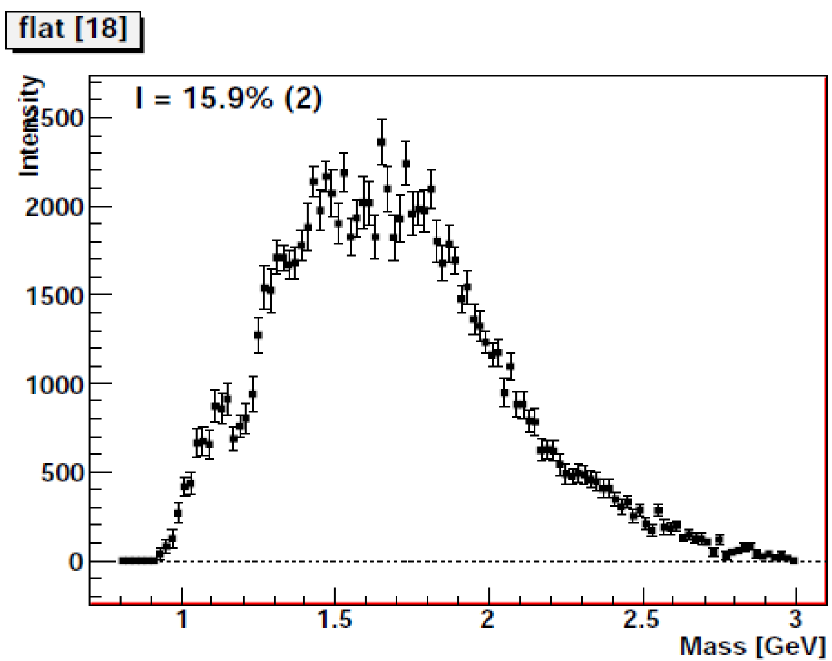




some results for COMPASS 2008 data compared to WA03





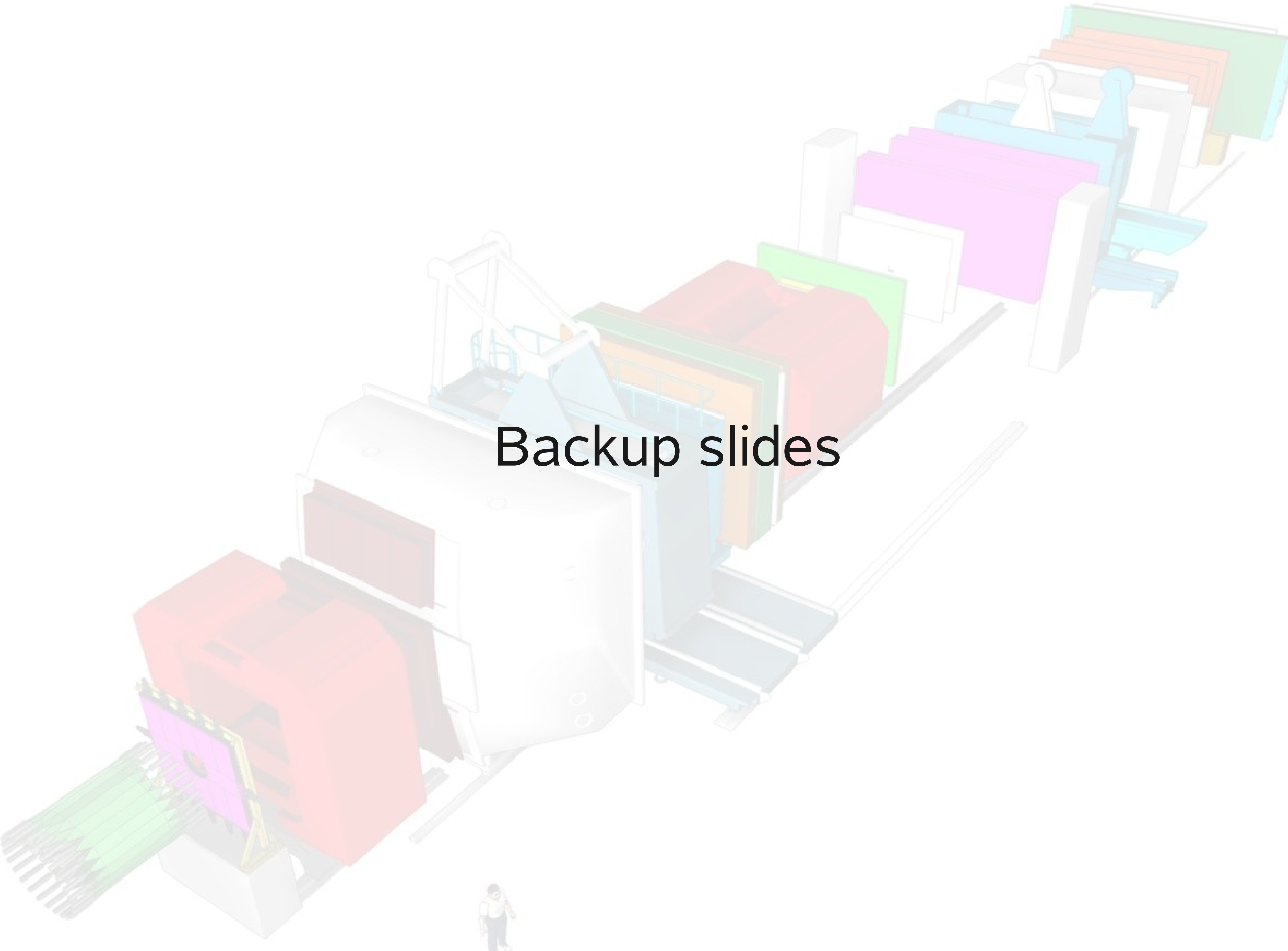


Major differences are due to:
- no spinflip waves included
- no mass cuts applied
- differences in the PDG value
- differences in the broad S-waves

Conclusion and outlook

- The basic work flow of the PWA program ROOTPWA, mainly maintained and developed by Sebastian Neubert and Boris Grube, is understood. I need to learn to use the correct “buttons” in a appropriate way.
- MC acceptance correction is still to be applied but expected to be flat.
- Studies on the background are being prepared. (3 π) background, (3 K) background, combinatorial background (~30%), leakage studies, deck effects, etc.
- And lot of other work to do... It's fun!

Thank you!



Backup slides

Mass independent PWA in a nutshell

Components of the LogLikelihood function:

Acceptance corrected
Phase space integral

Decay amplitudes

Kinematics

$$\ln L = \sum_{n=1}^{N_{\text{events}}} \ln \sum_{\epsilon} \sum_{i,j} \rho_{ij}^{\epsilon} \bar{\psi}_i^{\epsilon}(\tau_n) \bar{\psi}_j^{\epsilon}(\tau_n)^* - \sum_{\epsilon} \sum_{i,j} \rho_{ij}^{\epsilon} IA_{ij}^{\epsilon}$$

Spin density matrix (fit parameters)

Coherent sum over waves

Incoherent sum over reflectivities

Production amplitudes → Spin density matrix:

$$\rho_{ij}^{\epsilon} = \sum_{r} T_{ir}^{\epsilon} T_{jr}^{\epsilon *}$$

Normalized decay amplitudes:

$$\bar{\psi}_i^{\epsilon}(\tau) = \frac{\psi_i^{\epsilon}(\tau)}{\sqrt{\int |\psi_i^{\epsilon}(\tau')|^2 \mathrm{d}\tau'}}$$

Phase space integrals (with acceptance):

$$IA_{ij}^{\epsilon} = \int \bar{\psi}_i^{\epsilon}(\tau_n) \bar{\psi}_j^{\epsilon}(\tau_n)^* Acc(\tau) \mathrm{d}\tau$$

$$Acc(\tau) = \begin{cases} 0 \\ 1 \end{cases}$$

(from a talk by Sebastian Neubert)