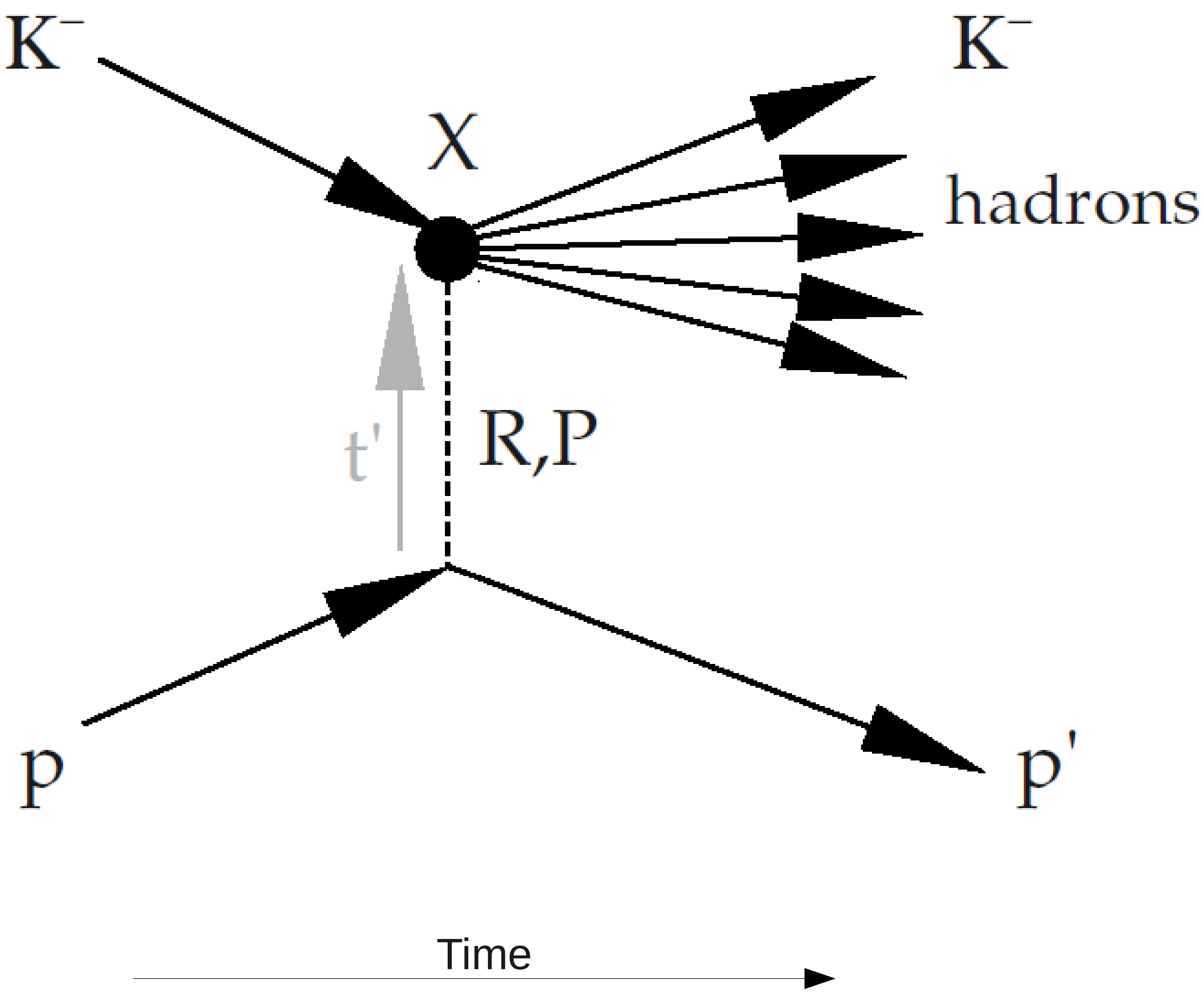
PWA of

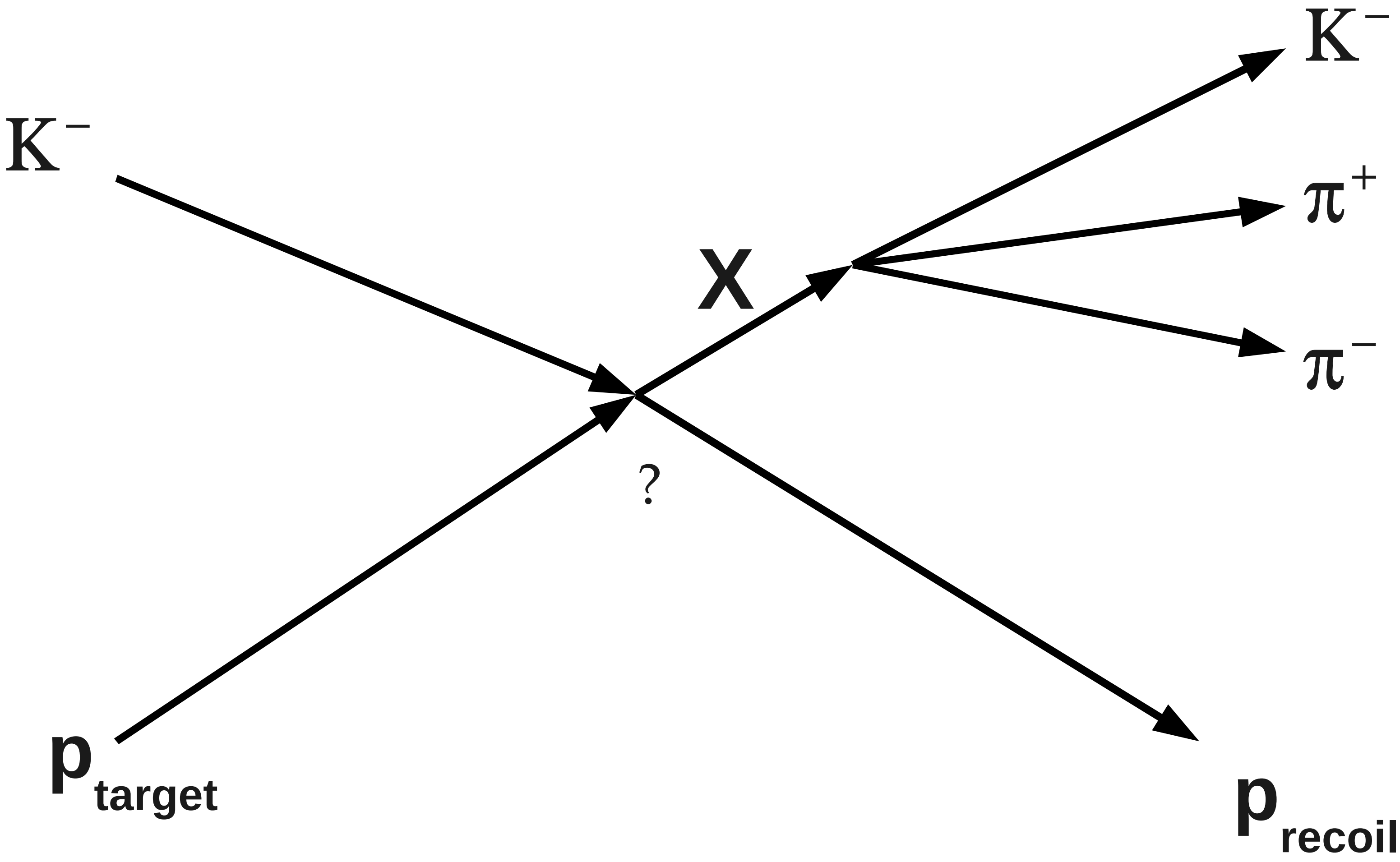
$$K^- p \rightarrow K^- \pi^+ \pi^- p_{recoil}$$

(behind the curtains)

The diffractive process

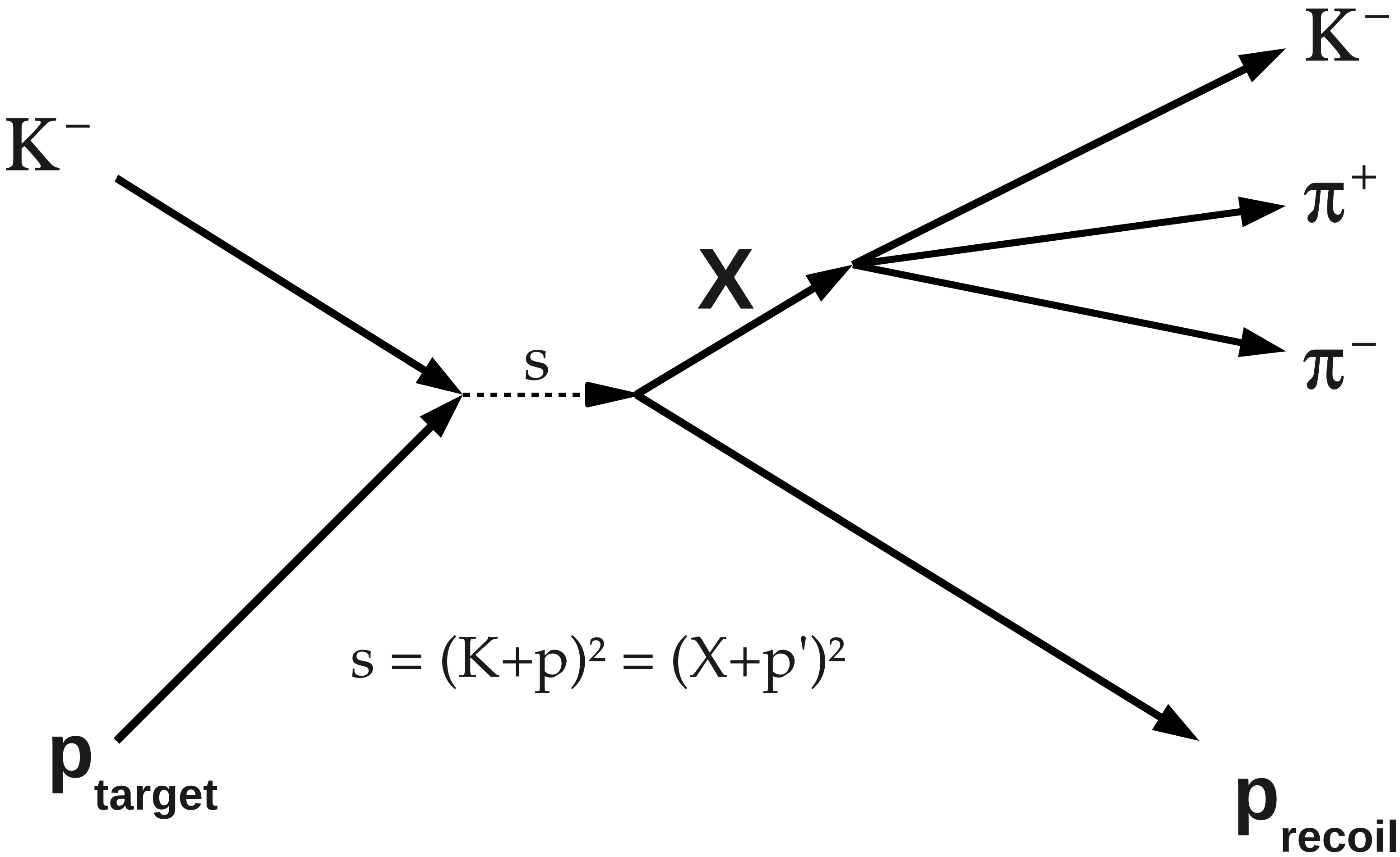


Dynamics and kinematics



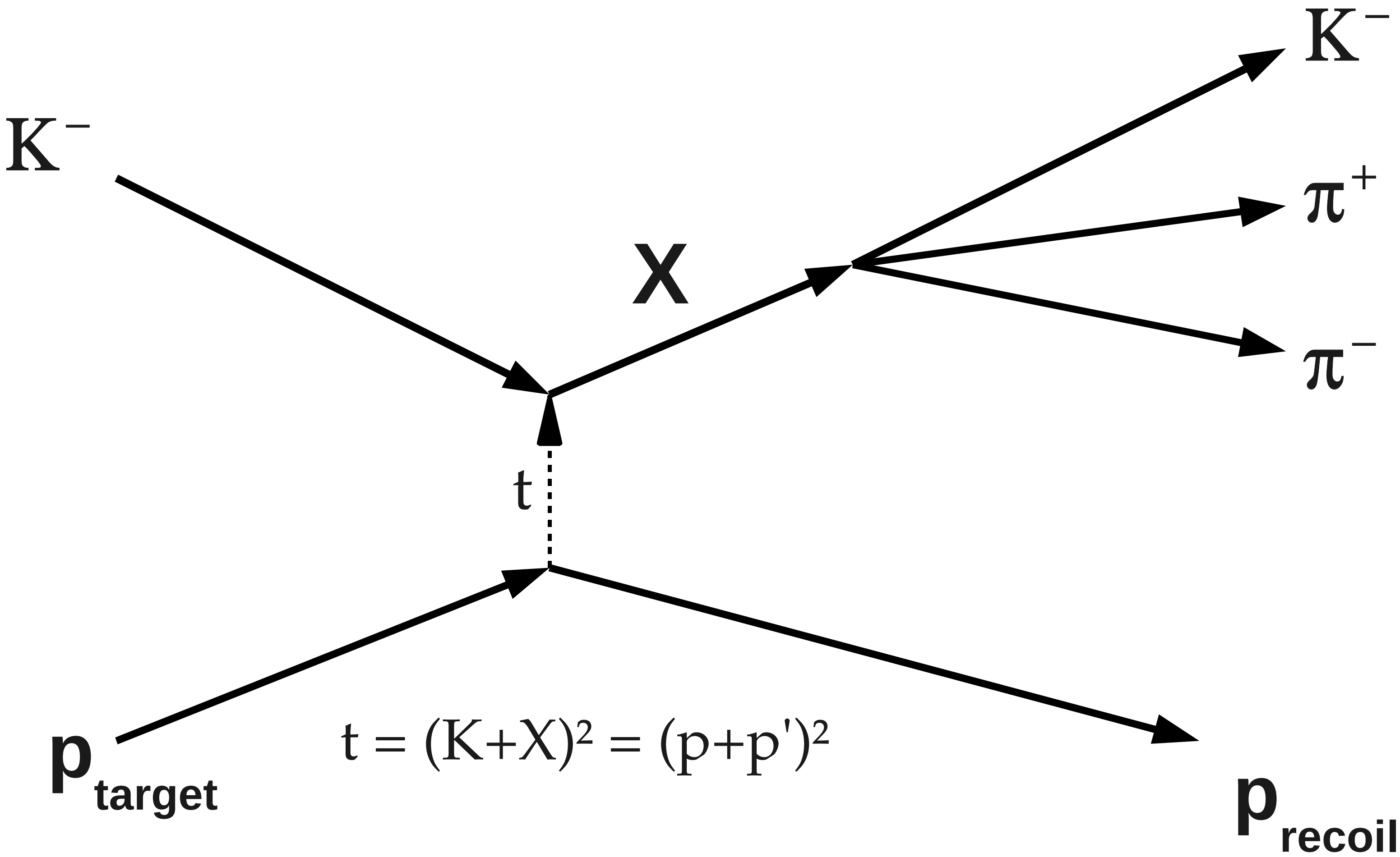
cms

Mandelsteam variables

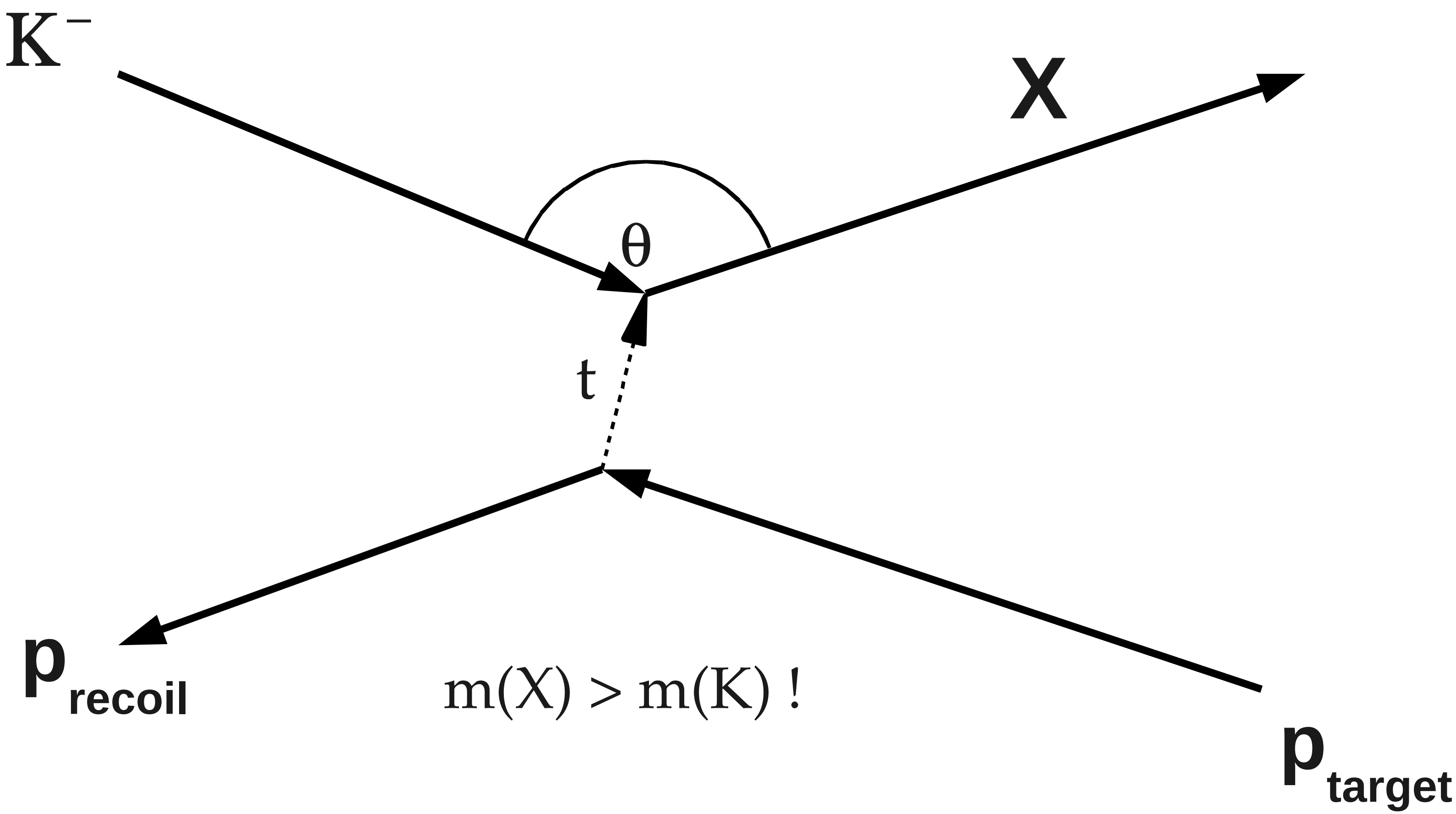


$$s = (K+p)^2 = (X+p')^2$$

Mandelsteam variables



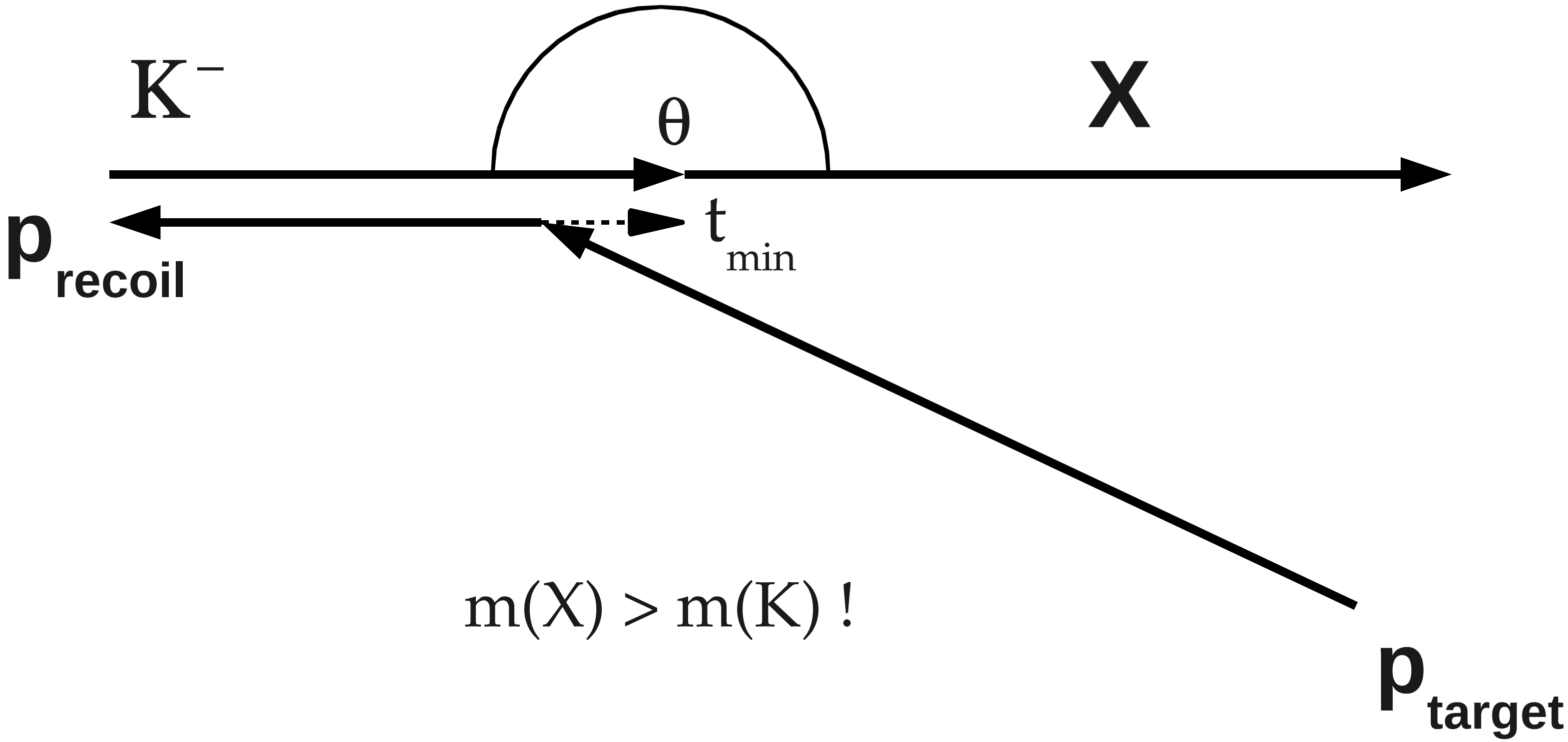
Timelike description of t → t'



m(X) > m(K) !

cms

Timelike description of $t \rightarrow t'$

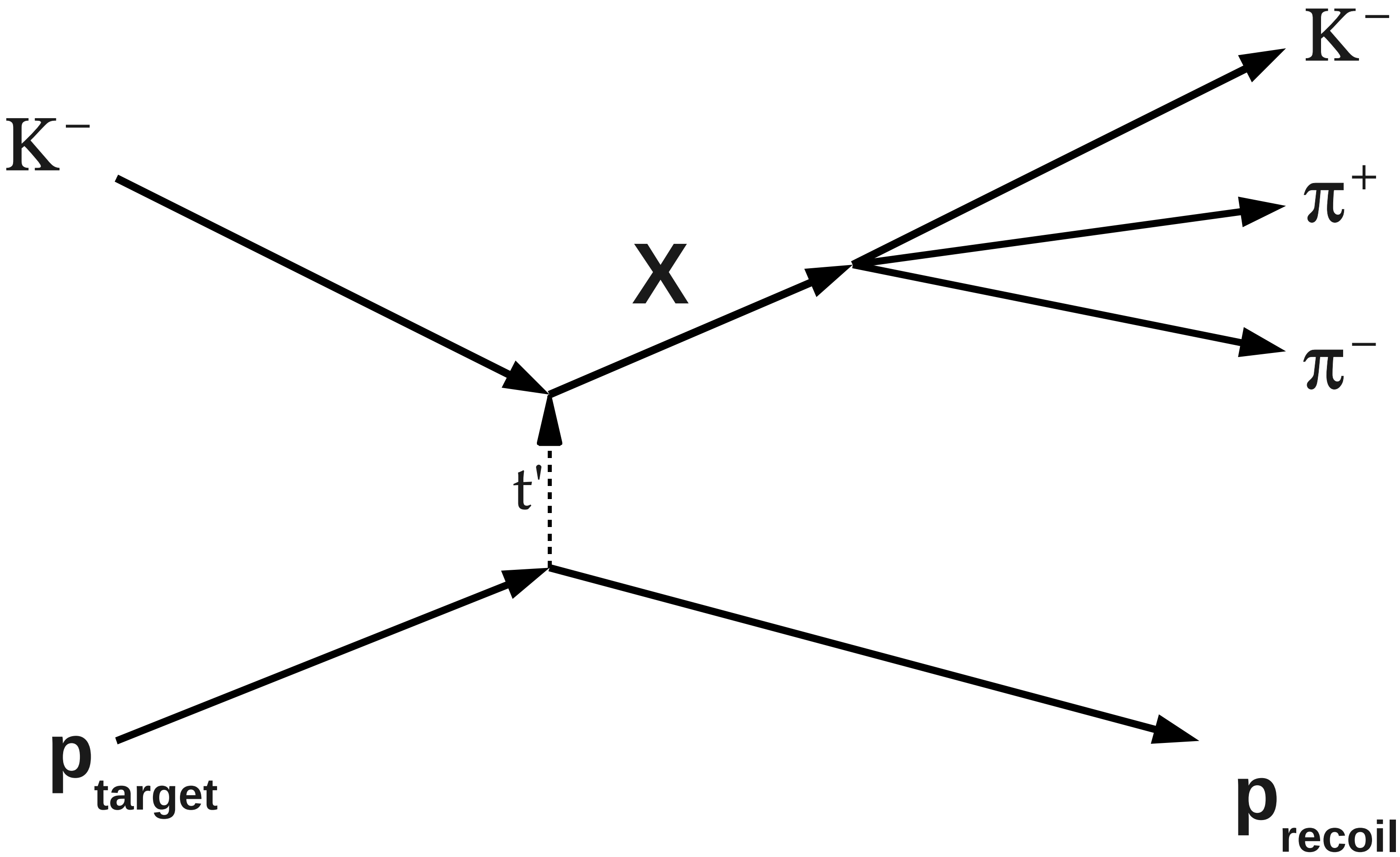


$m(X) > m(K)$!

$$t' = |t| - |t_{min}|$$

cms

Further variables



Definition of an observed particle state

$$|\vec{p}\, m J M\rangle$$

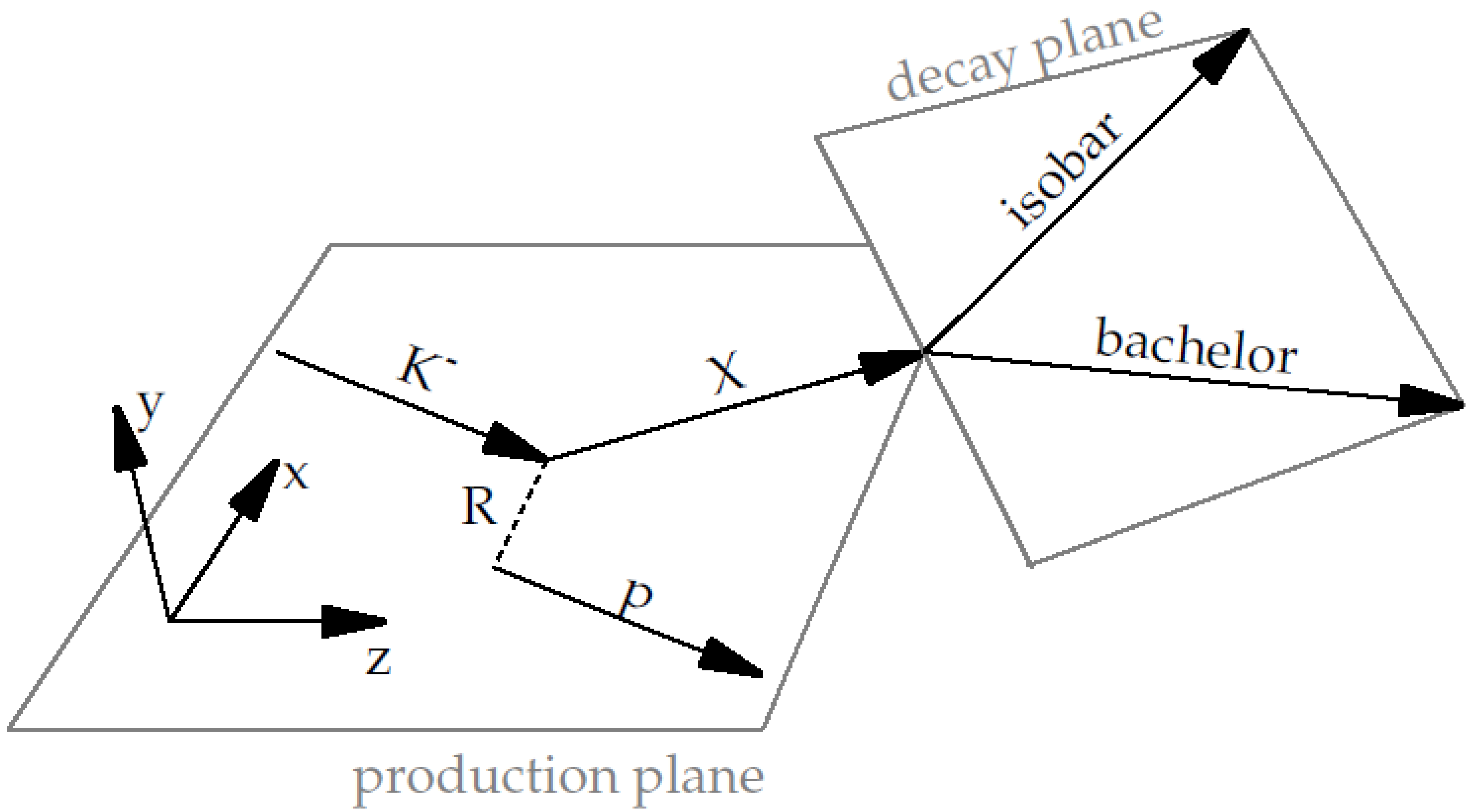
Canonical (orbital) basis

Definition of an observed particle state

$$|\vec{p}mJ\lambda\rangle$$

Helicity basis

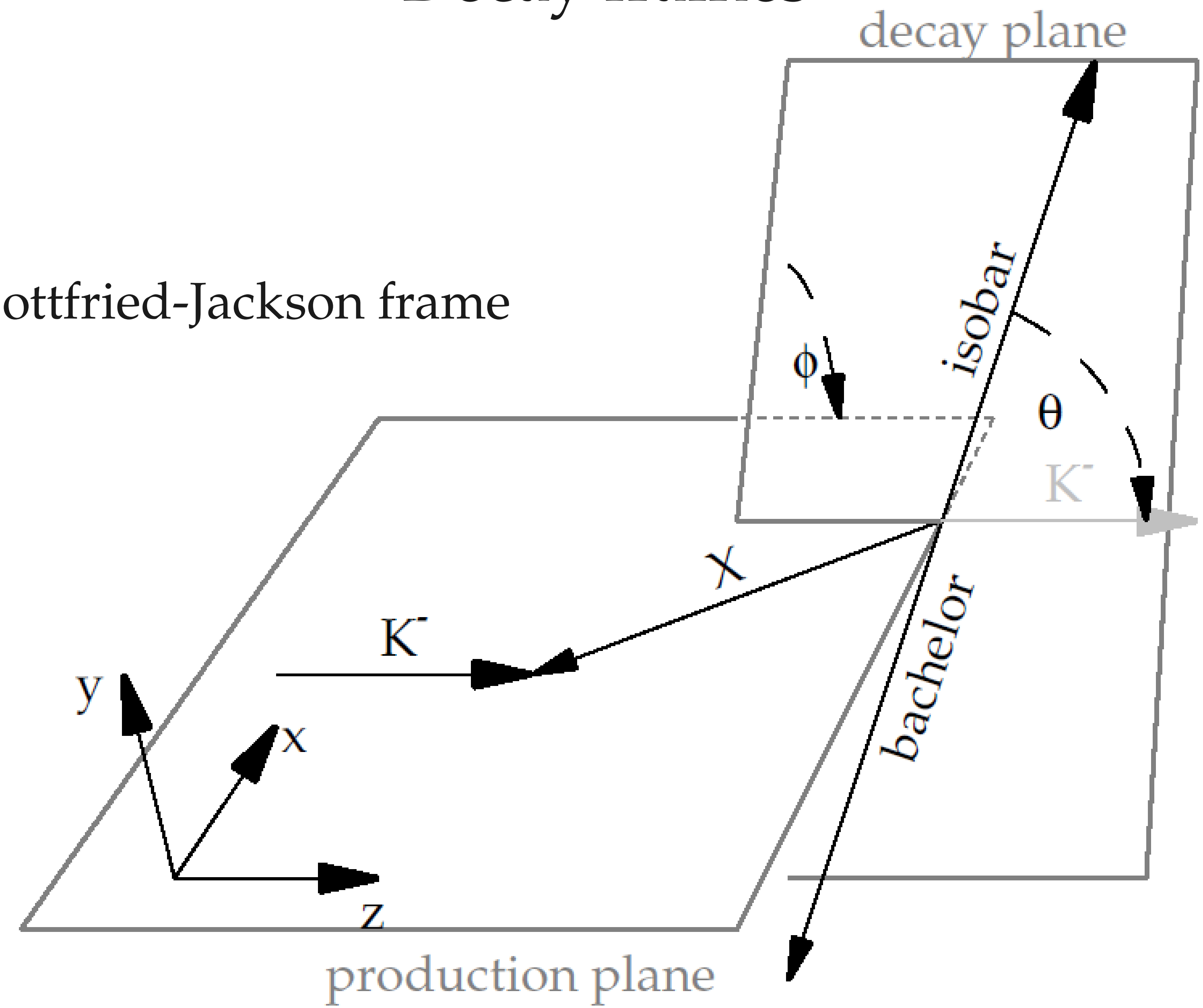
The production plane



Boosting into the decay rest frame...

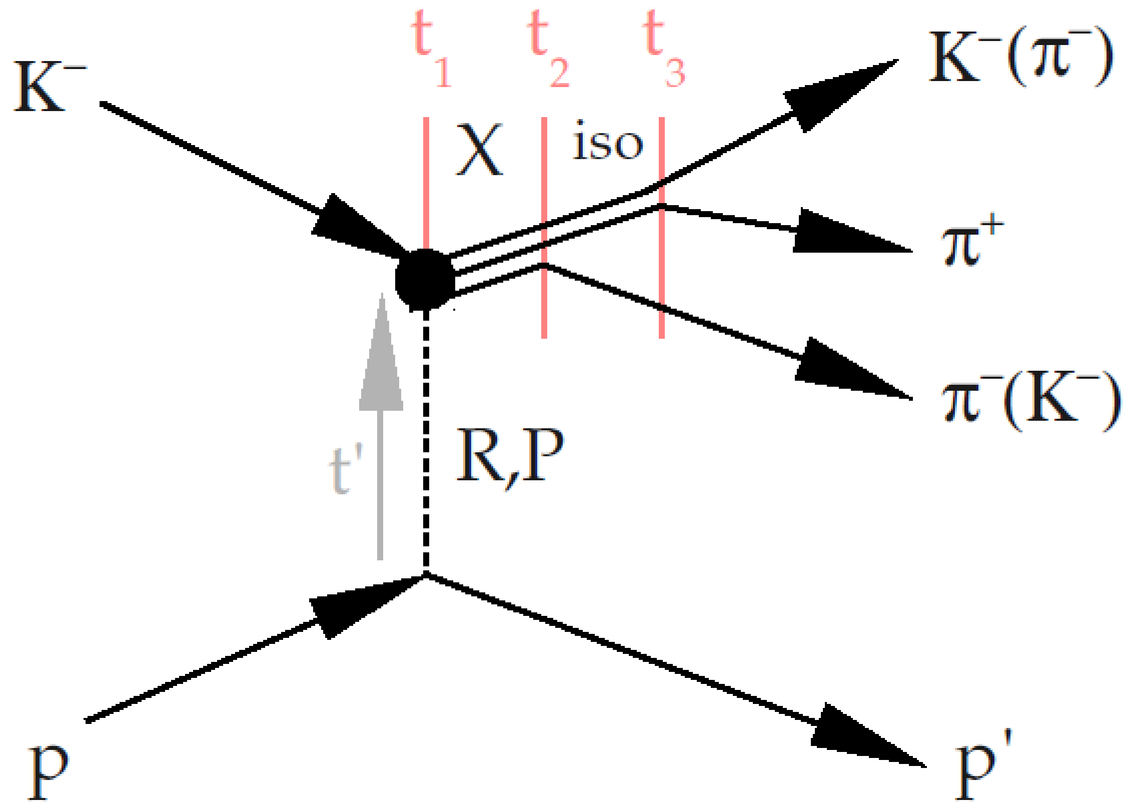
Decay frames

The Gottfried-Jackson frame



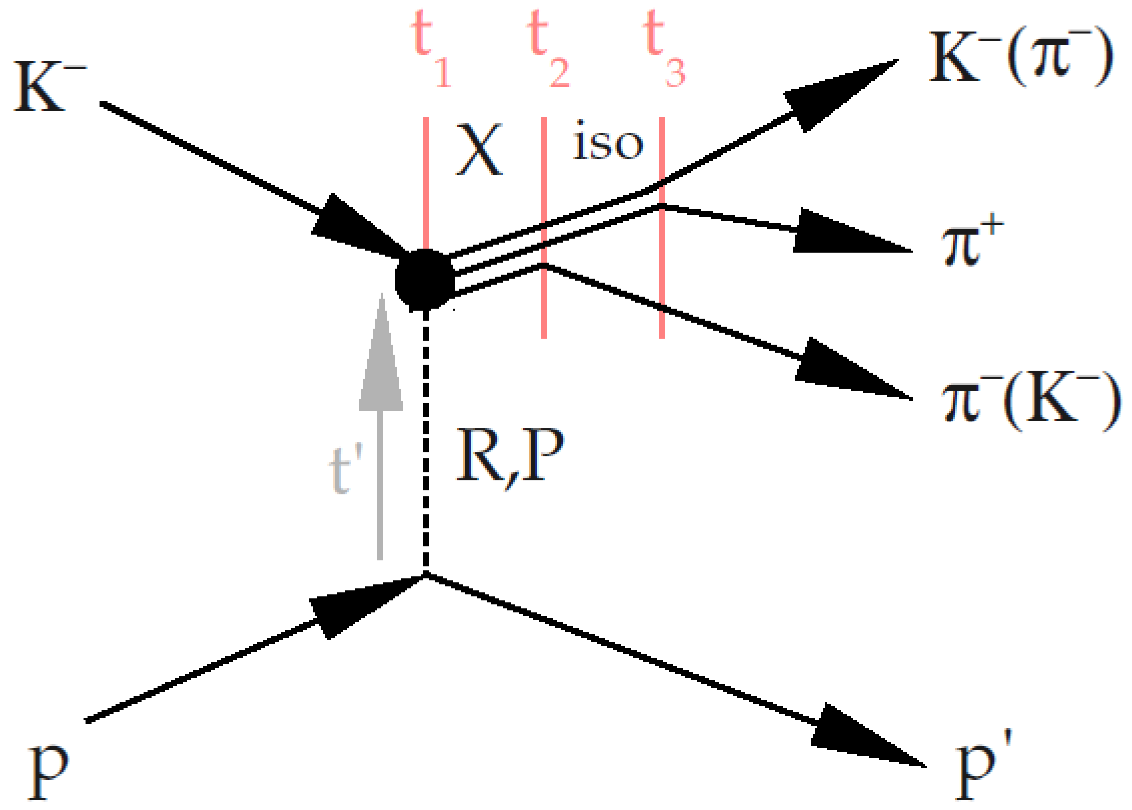
How about the helicity frame?

The amplitude of one channel



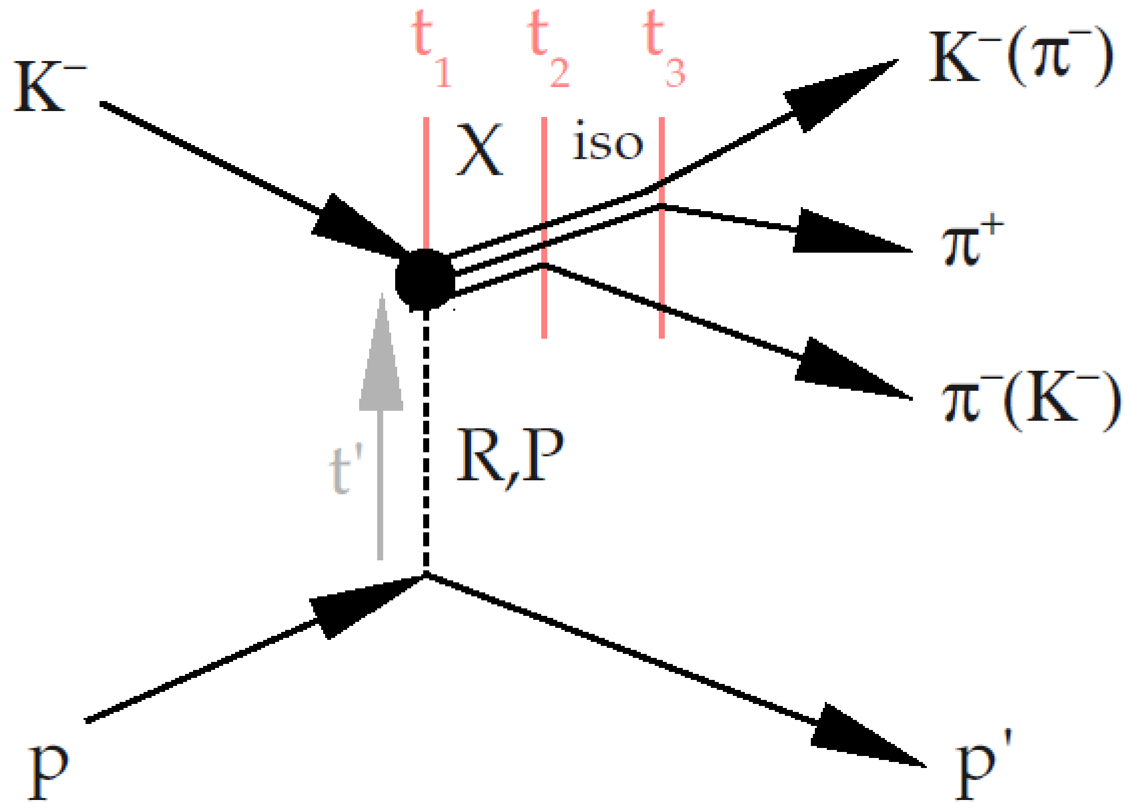
$$K^-_{beam} + p \rightarrow K^- + \pi^- + \pi^+ + p_{recoil}$$

The amplitude of one channel



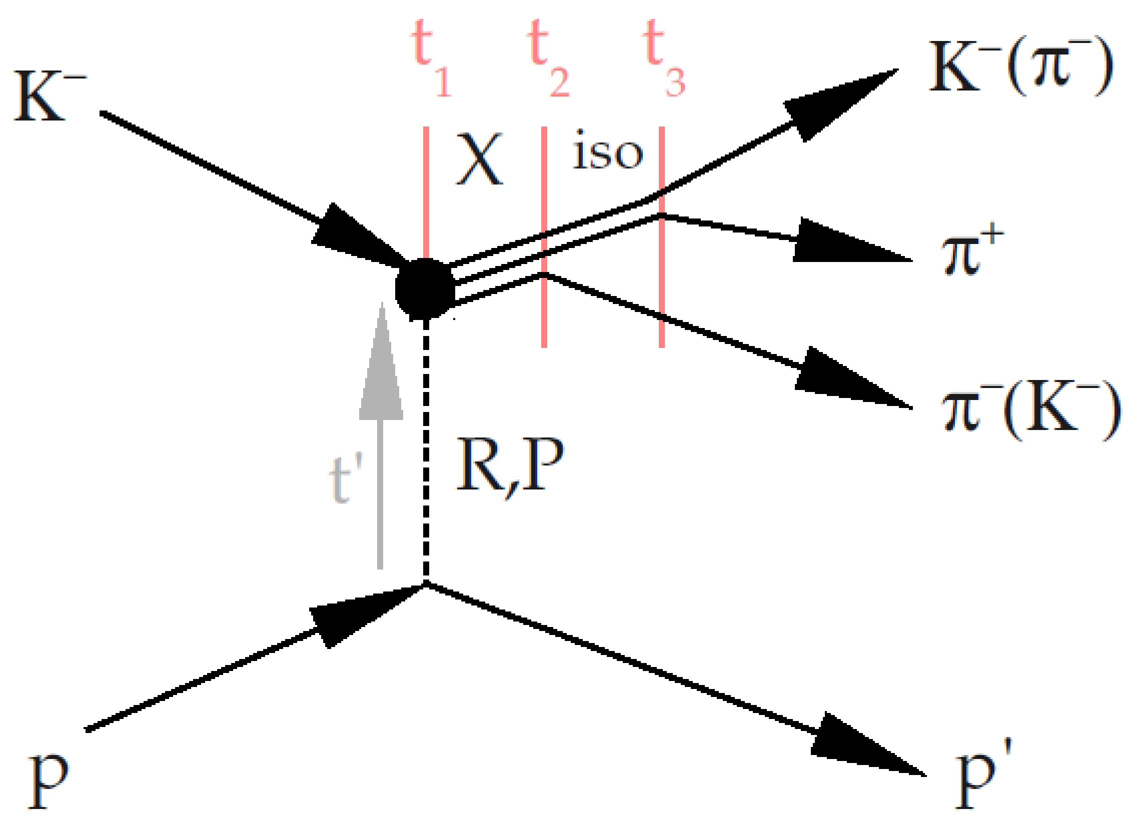
$$
\begin{aligned}
f_{fi} &\equiv \langle f|\mathcal{M}_{fi}|i\rangle \\
&= \langle \vec{p}_{K^-}\vec{p}_{\pi^+}\vec{p}_{\pi^-}, \vec{p}_{p_{recoil}}\lambda_{p_{recoil}}|\mathcal{M}_{fi}|\vec{p}_{K^-_{beam}}, \vec{p}_p\lambda_p\rangle
\end{aligned}
$$

The cross section for all channels



$$\sigma \propto \sum_i \sum_f |f_{fi}|^2$$

The cross section for all channels



$$\sigma(\tau, m) = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_{i}^{N_{waves}} T_{ir}^{\epsilon}(m_X) A_i^{\epsilon}(\tau) \right|^2$$

fit determine (partially)

Calculation of a decay amplitude

two particle decay state

$$|JM\lambda_1\lambda_2\rangle = \sqrt{\frac{2J+1}{4\pi}}\int\int d\phi\, d\cos\theta D^{J*}_{M\lambda}(\phi,\theta,0)|\phi\theta\lambda_1\lambda_2\rangle$$

recoupling the helicity and canonical frame

$$\langle JM\lambda_1\lambda_2|J'M'lS\rangle = \left(\frac{2l+1}{2J+1}\right)^{\frac{1}{2}}(l0s\lambda|J\lambda)\,(s_1\lambda_1 s_2\lambda_2|S\lambda)\,\delta_{JJ'}\delta_{MM'}$$

The amplitude for a two particle decay (canonical basis)

$$A = \langle\vec{p}\lambda_1 - \vec{p}\lambda_2|\mathcal{M}|JM\rangle$$

Calculation of a decay amplitude

The amplitude for a two particle decay

$$A = \langle \vec{p}\lambda_1 - \vec{p}\lambda_2 | \mathcal{M} | JM \rangle$$

Expansion to the helicity basis

$$\begin{aligned} A &= 4\pi \left(\frac{m}{p}\right)^{\frac{1}{2}} \langle \phi\theta\lambda_1\lambda_2 | JM\lambda_1\lambda_2 \rangle \langle JM\lambda_1\lambda_2 | \mathcal{M} | JM \rangle \\ &= \sqrt{\frac{2J+1}{4\pi}} F^J_{\lambda_1\lambda_2} D^{J*}_{M\lambda}(\phi, \theta, 0) \end{aligned}$$

$$F^J_{\lambda_1\lambda_2} = 4\pi \left(\frac{m}{p}\right)^{\frac{1}{2}} \langle JM\lambda_1\lambda_2 | \mathcal{M} | JM \rangle$$

Calculation of a decay amplitude

$$F^J_{\lambda_1\lambda_2} = 4\pi\left(\frac{m}{p}\right)^{\frac{1}{2}} \langle JM\lambda_1\lambda_2|\mathcal{M}|JM\rangle$$

Expansion to the canonical orbital basis

$$F^J_{\lambda_1\lambda_2} = \sum_{l,s}\left(\frac{2l+1}{2J+1}\right)^{\frac{1}{2}} a^J_{lS}(m,p)\,(l0S\lambda|J\lambda)\,(s_1\lambda_1 s_2-\lambda_2|S\lambda)$$

With partial wave amplitudes

$$a^J_{lS}(m,p) = 4\pi\left(\frac{m}{p}\right)^{\frac{1}{2}} \langle JMlS|\mathcal{M}|JM\rangle$$

Calculation of a decay amplitude

With partial wave amplitudes

$$a^J_{lS}(m,p) = 4\pi \left(\frac{m}{p}\right)^{\frac{1}{2}} \langle JMlS|\mathcal{M}|JM\rangle$$

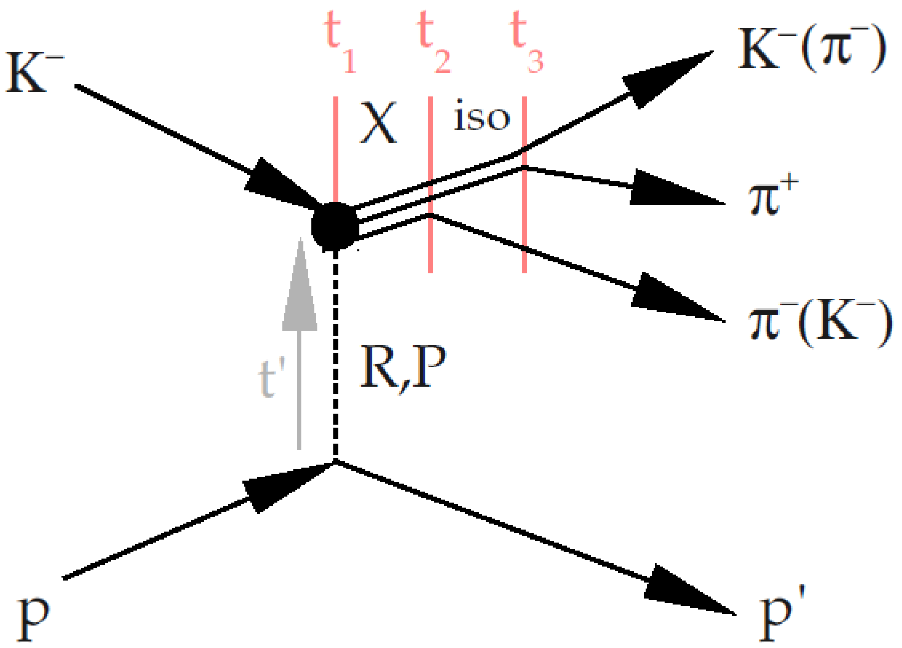
Parametrization in the simplest case:

$$a^J_{ls}(m,p) \equiv \frac{m_0\Gamma_0}{m_0^2 - m^2 - im_0\Gamma(m)}$$

with

$$\Gamma(m) = \Gamma_0 \left(\frac{F_l(p)}{F_l(p_0)}\right)^2 \left(\frac{m_0}{m}\frac{q}{q_0}\right).$$

The cross section for all channels



$$\sigma(\tau, m) = \sum_{\epsilon=\pm1} \sum_{r=1}^{N_r} \left| \sum_{i}^{N_{waves}} T_{ir}^{\epsilon}(m_X) A_i^{\epsilon}(\tau) \right|^2$$

$$A_i^{\epsilon} = A_{i,t_3}^{\epsilon} A_{i,t_2}^{\epsilon}$$

$$A_i^{\epsilon}(\tau) = \qquad {}_i a_{lS}^{J}(m_X, p) \Psi_i^{\epsilon}(\tau)$$

$$\sigma(\tau, m) = \sum_{\epsilon=\pm1} \sum_{r=1}^{N_r} \left| \sum_{i}^{N_{waves}} T_{ir}^{\epsilon}(m_X)\, {}_i a_{lS}^{J}(m_X, p) \Psi_i^{\epsilon}(\tau) \right|^2$$

The cross section for all channels

$$A_i^{\epsilon}(\tau) = \quad {}_i a^J_{lS}(m_X, p)\Psi_i^{\epsilon}(\tau)$$

$$\sigma(\tau, m) = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_{i}^{N_{waves}} T^{\epsilon}_{ir}(m_X)\, {}_i a^J_{lS}(m_X, p)\Psi_i^{\epsilon}(\tau) \right|^2$$

ascoli ansatz:

$$\rho^{\epsilon}_{ij}(m_X) \equiv \sum_{r=1}^{N_r} T^{\epsilon}_{ir}(m_X) T^{\epsilon *}_{ir}(m_X)$$

$$\sigma(\tau, m) = \sum_{\epsilon=\pm 1} \sum_{i,j}^{N_{waves}} \rho^{\epsilon}_{ij}(m_X)\, {}_i a^J_{lS}(m, p)\, {}_j a^{J*}_{lS}(m, p)\Psi_i^{\epsilon}\Psi_j^{\epsilon *}$$

Mass independent fit

$$\rho^{\epsilon}_{ij}(m_X) \equiv \sum_{r=1}^{N_r} T^{\epsilon}_{ir}(m_X) T^{\epsilon *}_{ir}(m_X)$$

$$\sigma(\tau, m) = \sum_{\epsilon=\pm 1} \sum_{i,j}^{N_{waves}} \rho^{\epsilon}_{ij}(m_X)_i a^{J}_{lS}(m,p)_j a^{J*}_{lS}(m,p) \Psi^{\epsilon}_i \Psi^{\epsilon *}_j$$

Constant for narrow mass bins

$$\sigma(\tau, m) = \sum_{\epsilon=\pm 1} \sum_{i,j}^{N_{waves}} \rho^{\epsilon}_{ij} \Psi^{\epsilon}_i \Psi^{\epsilon *}_j$$

Fit parameter

Summary

PWA tries to describe continuous observables as momenta in terms of spins and angular momenta between the decay particles.

Many assumptions are made and many free parameters are left but the ansatz works surprisingly well (see 5pi analysis).

The physical interpretation is sometimes not clear as influences like background, other channels opening and leakage effects have to be modeled.

You can believe in what we do or not. If not, so give me a better tool to work with!