COMPASS Note 2013-8
Using Bayesian Methods for Particle Identification in the CEDARs

Tobias Weisrock* and Eva-Maria Kabuß
Institut für Kernpyhsik, Johannes Gutenberg-Universität Mainz
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So far a multiplicity cut was used to identify beam particles with the CEDARs[1]. This method does not work for divergent beams as the kaon ring leaves the acceptance of the diaphragm. Only 50% of the kaons in the beam are identified that way with a purity of around 85%. An improved method has to take the beam divergence into account. Jan Friedrich proposed a method[2] for Primakoff data which uses a certain grouping of the photomultipliers and likelihood fits. This method does not work properly for 2008/2009 hadron runs. Therefore a new method using a likelihood approach based on Bayes' theorem is presented here.

All methods described here implemented in the current version of the CEDAR helper. The usage of the functions is described in appendix A.

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I. THE BAYESIAN APPROACH

A multiplicity cut does not work properly for particle identification in the CEDARs due to beam divergence. We will present an improved method based on 2008 hadron data which takes the beam divergence into account. The method is based on likelihood calculations using Bayes' theorem and uses the responses of every single photomultiplier to get rid of geometric effects and correlations induced by a multiplicity cut.
To use the method five steps have to be taken:

1. Measure the beam divergence.
2. Create a pure kaon sample and a pure pion sample for calibration.
3. Determine probabilities to get signals in the photomultipliers for pions and for kaons separately.
4. Calculate likelihoods for pions and kaons using these probabilities.
5. Use likelihoods to distinguish between pions and kaons.

A. Measurement of the beam divergence

To calculate the beam divergence in the CEDAR region the reconstructed beam particles are propagated backwards from the target region. Two points upstream and downstream of the CEDARs are used. The beam divergence in x and y is then given by

$$\theta_x = \arctan\left(\frac{\Delta x}{\Delta z}\right) \approx \frac{\Delta x}{\Delta z}$$
$$\theta_y = \arctan\left(\frac{\Delta y}{\Delta z}\right) \approx \frac{\Delta y}{\Delta z}$$

with the distance between the two points

$$\Delta z = 1283.4\,\mathrm{cm}\ .$$

The propagation of the beam through the beamline magnets is not fully correctly implemented, thus the divergence is not a priori centered around $\theta_x = \theta_y = 0$. To correct for this displacement events with a multiplicity of 8 are used. These should have a small divergence and thus should be centered around $\theta_x = \theta_y = 0$.

The distributions of θ_x and θ_y for such events can be found in figure 1 where the offsets are determined using Gaussian fits. The two CEDARs are not totally aligned with each other and have a displacement of about $5\,\mu$rad with respect to each other. This offset is below the step width of the servomotor, so that the CEDARs cannot be aligned better. In the following all given values for divergences θ_x and θ_y are corrected for the initial offsets.

B. Creation of calibration samples

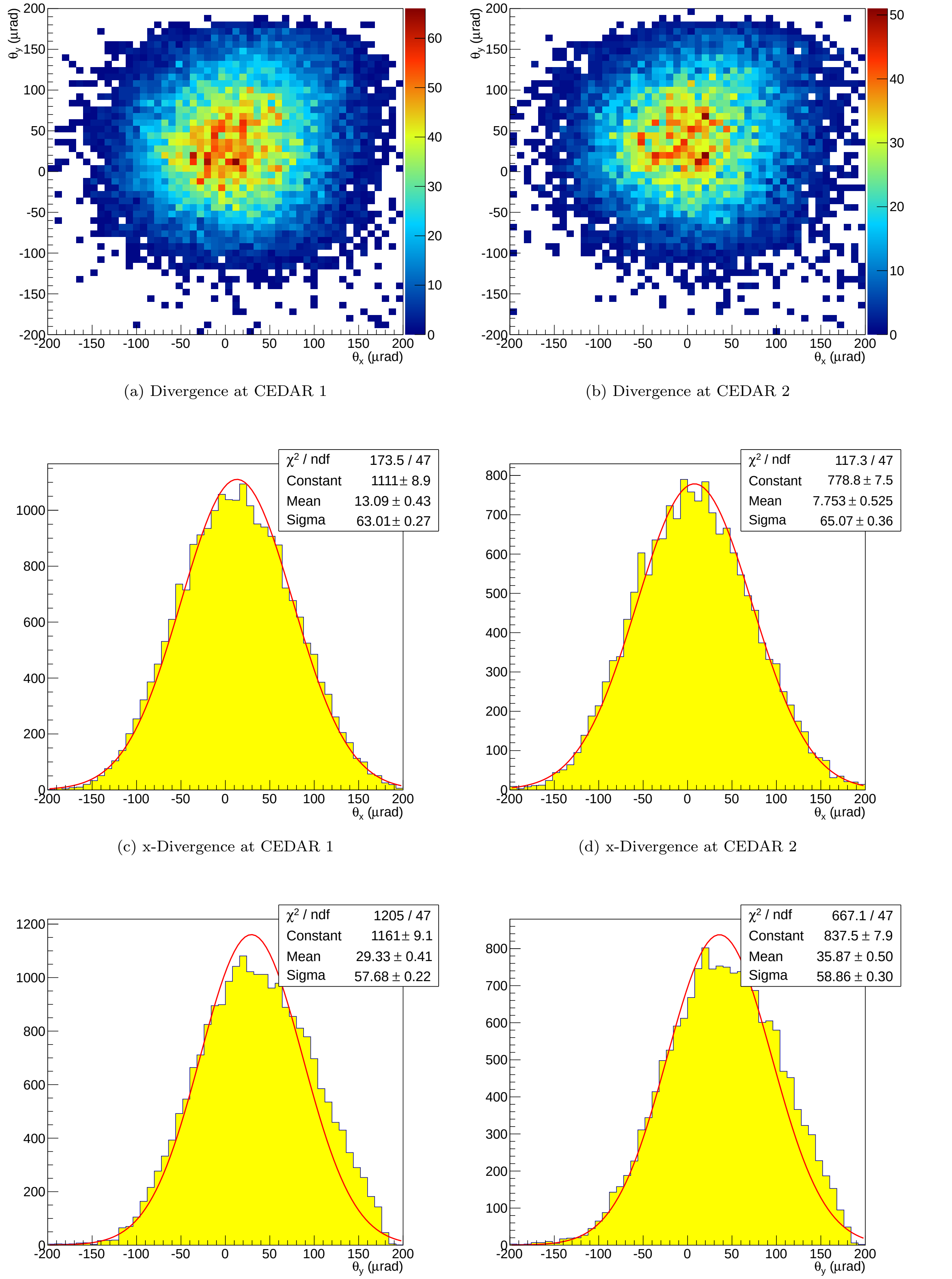
The samples used for the calibration of this method were taken from W33, W35, W37 of 2008.

1. Kaon sample

The kaon sample was prepared using free kaon decays into three charged pions. The following cuts were applied:

- primary vertex outside of the target (to get rid of diffractively produced pions)
- 3 outgoing charged particles with $\sum_i q_i = -1$
- Exclusivity cut $|190\,\mathrm{GeV} - p_{3\pi}| \leq 4\,\mathrm{GeV}$
- Combined cut on 3-pion mass and transverse momentum relative to the beam (cf. figure 2)

$$\frac{|M_K - m_{3\pi}|^2}{(10\,\mathrm{MeV})^2} + \frac{p_T^2}{(20\,\mathrm{MeV})^2} \leq 1$$



(a) Divergence at CEDAR 1

(b) Divergence at CEDAR 2

(c) x-Divergence at CEDAR 1

(d) x-Divergence at CEDAR 2

(e) y-Divergence at CEDAR 1

(f) y-Divergence at CEDAR 2

FIG. 1: Determination of the offset of the beam divergence at the CEDARs using events with signals in all eight photomultipliers. The given fit values are obtained from a Gaussian fit. The two CEDARs are aligned up to 5.3 μrad in x and 6.6 μrad in y, respectively. The beam is slightly wider in x.

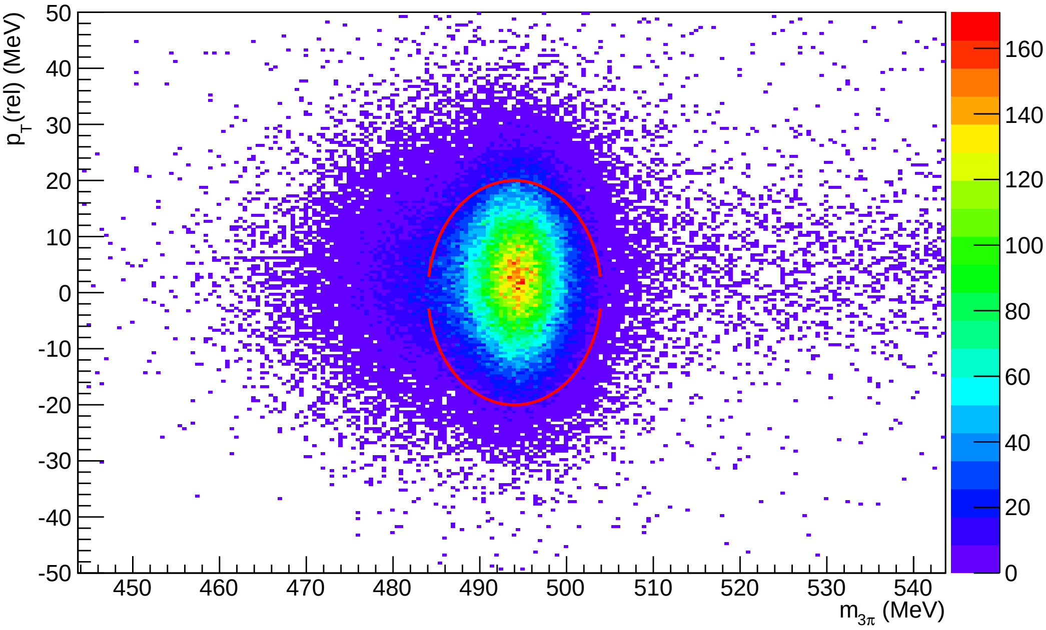


FIG. 2: Illustration of the combined cut on 3-pion mass and transverse momentum. The region where mostly kaons are expected ($m_{3\pi} \approx M_K$, low p_T) is selected by the cut.

2. *Pion sample*

The pion sample is prepared by selecting diffractively produced pions. These are identified by selecting three charged particles with similar momenta (and thus the same mass) and small angles to the beam direction (to filter out higher excitations). The following cuts were applied:

- DT0-Trigger
- primary vertex inside of the target
- 3 outgoing charged particles with $\sum_i q_i = -1$
- Exclusivity cut $|190\,\mathrm{GeV} - p_{3\pi}| \leq 4\,\mathrm{GeV}$
- Angle between outgoing particles $\alpha \leq 0.2\,\mathrm{rad}$
- Momentum difference between outgoing particles $|p_i - p_j| < 10\,\mathrm{GeV}$

3. *Beam sample*

Here some beamtrigger events (about 3.7 million) of W35 are used.

C. Determination of probabilities

In the next step the probabilities for pions and kaons to produce signals in the photomultipliers have to be determined. There are two possibilities for each beamtrack and each photomultiplier:

- signal in photomultiplier
- no signal in photomultiplier

Given this information for each PMT we can calculate the probability that the distribution of hits is produced by an incoming Kaon or an incoming Pion using Bayes' theorem.

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} \tag{1}$$

The theorem states that the probabilitiy $P(A|B)$ of A provided that B occured before is given by the product of the probabilitiy $P(B|A)$ of B provided that A occured before and the a priori probability $P(A)$ of A divided by the a priori probability of B.

1. *Example:*

A beam particle traverses the CEDAR region with a divergence (θ_x, θ_y) and produces a signal in photomultiplier i. The probability that this particle was a kaon is calculated as:

$$P^i_{\theta_x,\theta_y}(\text{kaon}|\text{signal}) = \frac{P^i_{\theta_x,\theta_y}(\text{signal}|\text{kaon}) \cdot P_{\theta_x,\theta_y}(\text{kaon})}{P^i_{\theta_x,\theta_y}(\text{signal})} \tag{2}$$

Thus the probability needed can be calculated as a combination of probabilities which can be easily determined:

- $\boldsymbol{P^i_{\theta_x,\theta_y}}$**(signal|kaon)** is the probabilitiy that a kaon with divergence (θ_x, θ_y) produces a signal in photomultiplier i. This probabilitiy is determined using the pure kaon sample. The events containing hits in PMT i are filled into a histogram with 250 bins each in θ_x and θ_y. Then the histogram is normalized bin by bin to the total number of particles with this divergence (which is the sum of particles with and without hits). Thus $P^i_{\theta_x,\theta_y}(\text{signal}|\text{kaon}) + P^i_{\theta_x,\theta_y}(\overline{\text{signal}}|\text{kaon}) = 1$ for every single bin.

- $\boldsymbol{P_{\theta_x,\theta_y}}$**(kaon)** is the probabilitiy that the beam contains a kaon with a divergence θ_x, θ_y. This is determined by normalizing the distribution of kaons in θ_x, θ_y to the total number of incoming particles. We observe that kaons and pions in the beam have the same distribution of divergence $P_{\theta_x,\theta_y}(\text{kaon}) = P_{\theta_x,\theta_y}(\text{pion}) = P_{\theta_x,\theta_y}(\text{beam})$. Thus this probabilitiy can be calculated using the beam sample. In addition - as this factor is the same for all cases - it can finally be dropped.

- $\boldsymbol{P^i_{\theta_x,\theta_y}}$**(signal)** is the probabilitiy that a signal in PMT i is produced by any particle with divergence (θ_x, θ_y). It is determined in the same way as $P^i_{\theta_x,\theta_y}(\text{signal}|\text{kaon})$ however using the beam sample instead of the kaon sample. As this factor is the same for all needed probabilities it can also be dropped for the calculation of the likelihoods.

Taking all this into account we only have to determine $P^i_{\theta_x,\theta_y}(\text{signal}|\text{kaon})$ and $P^i_{\theta_x,\theta_y}(\text{signal}|\text{pion})$ which is then proportional to the probabilities needed to calculate likelihoods:

$$\begin{aligned}
P^i_{\theta_x,\theta_y}(\text{kaon}|\text{signal}) &\propto P^i_{\theta_x,\theta_y}(\text{signal}|\text{kaon}) \\
P^i_{\theta_x,\theta_y}(\text{kaon}|\overline{\text{signal}}) &\propto \left(1 - P^i_{\theta_x,\theta_y}(\text{signal}|\text{kaon})\right) \\
P^i_{\theta_x,\theta_y}(\text{pion}|\text{signal}) &\propto P^i_{\theta_x,\theta_y}(\text{signal}|\text{pion}) \\
P^i_{\theta_x,\theta_y}(\text{pion}|\overline{\text{signal}}) &\propto \left(1 - P^i_{\theta_x,\theta_y}(\text{signal}|\text{pion})\right)
\end{aligned}$$

These probabilitiy distributions for both CEDARs can be found in the appendix (figures 10ff).

D. Calculation of likelihoods

Now likelihoods can be calculated from the obtained probabilities. Likelihoods are just the product of the single probabilities for signal or no signal, respectively. It is more convenient to calculate the logarithm of the likelihood as a sum of the logarithms of the single probabilities:

$$\log L(K) = \sum_{i=1}^{8} \log P^i_{\theta_x,\theta_y}(\text{kaon}|\text{signal}) \cdot \eta_i + \sum_{i=1}^{8} \log P^i_{\theta_x,\theta_y}(\text{kaon}|\overline{\text{signal}}) \cdot (1 - \eta_i) \tag{3}$$

where

$$\eta_i = \begin{cases} 1 & \text{signal in PMT } i \\ 0 & \text{no signal in PMT } i \end{cases} \quad . \tag{4}$$

A sum over all photomultipliers is performed using the corresponding probabilities depending on whether the photomultiplier has seen a signal or not. The likelihood for pions is calculated in the same way. The log likelihoods for kaons vs. pions in CEDAR 2 are shown in figure3. As expected the events in the kaon sample have a higher log likelihood for kaons while there is still a remaining pionic background. In the pion sample the likelihood for pions is higher. In the beam sample most of the events are in the pion region. But there are also a number of "clear" kaons with $\log L(K) \gg \log L(\pi)$. Additional likelihood distributions can be found in the appendix (figures 14ff).

E. Using likelihoods to identify particles

Having calculated the likelihoods they can be used for particle identification. To distinguish pions or kaons likelihood ratios are used. To improve the purity of the selection the ratio should be larger than one by a certain amount to decide for one of the cases. This translates to a certain difference in the log likelihood values

$$\log L(K) > \log L(\pi) + A \qquad \Rightarrow K \tag{5a}$$

$$\log L(\pi) > \log L(K) + B \qquad \Rightarrow \pi \tag{5b}$$

The likelihood differences A and B are tuned to obtain high purity and efficiency simultaneously (cf. section II C). For particles which do not fulfil any of these conditions no PID (indicated by "?" in the following) is given. As the CEDARs are used separately the obtained PIDs have to be combined afterwards as indicated in the table I.

C2 \ C1	?	π	K
?	?	π	K
π	π	π	?
K	K	?	K

TABLE I: Combination of the PIDs obtained from CEDAR 1 and CEDAR 2. If one CEDAR decides for Pion and the other CEDAR for Kaon the final PID is always set to "no ID".

II. PERFORMANCE OF THE METHOD

A. Additional Quality Cuts

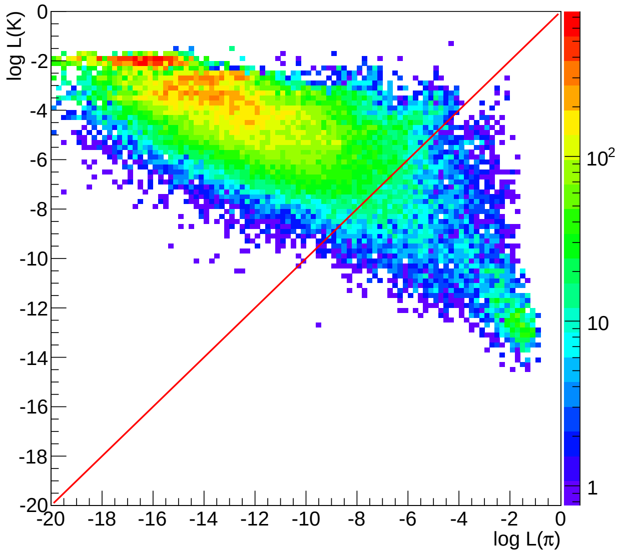
In the probabilitiy distributons for the single photomultipliers (figures 10ff) there are entries which are exactly 1 due to low statistics (based on only one event) or zero due to no events in that bin. These cases can especially be found for large divergences. Additional quality cuts reduce the influence of low statistics. Two possibilities will be discussed here:

1. Cut events which have a total divergence $r = \sqrt{\theta_x^2 + \theta_y^2}$ larger than 200×10^{-6}. (This will be referred to as "cut200" in the following.)
2. Give no ID to events which would use a bin with 0 or 1 entries in the probabilitiy distribution. (This will be referred to as "cut01" in the following.)

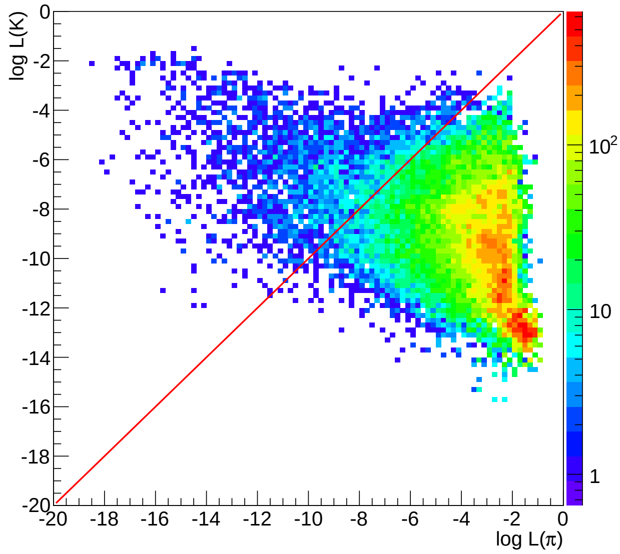
The two possibilities are depicted in figure 4, where the probabilitiy distribution for kaons for one of the photomultipliers (CEDAR 1, PMT 0) is shown with the two possible quality cuts. In the following these two possibilities for quality cuts will be compared with each other.

B. Application to Calibration Samples

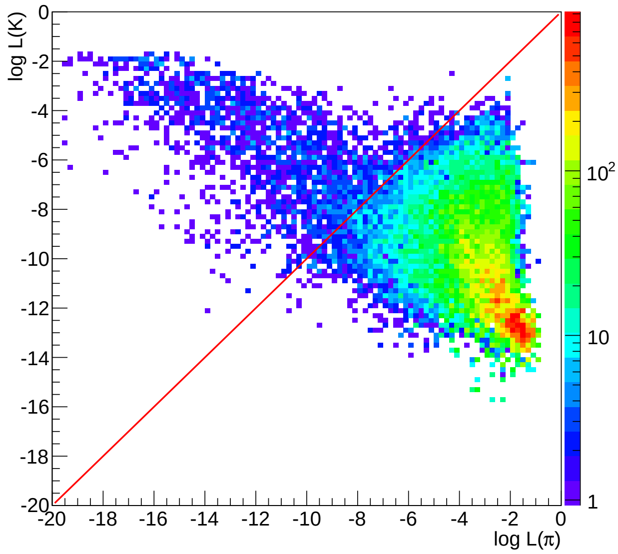
In a first step the new method is applied to the calibration samples and compared with the previously used multiplicity method.



(a) kaon sample



(b) pion sample



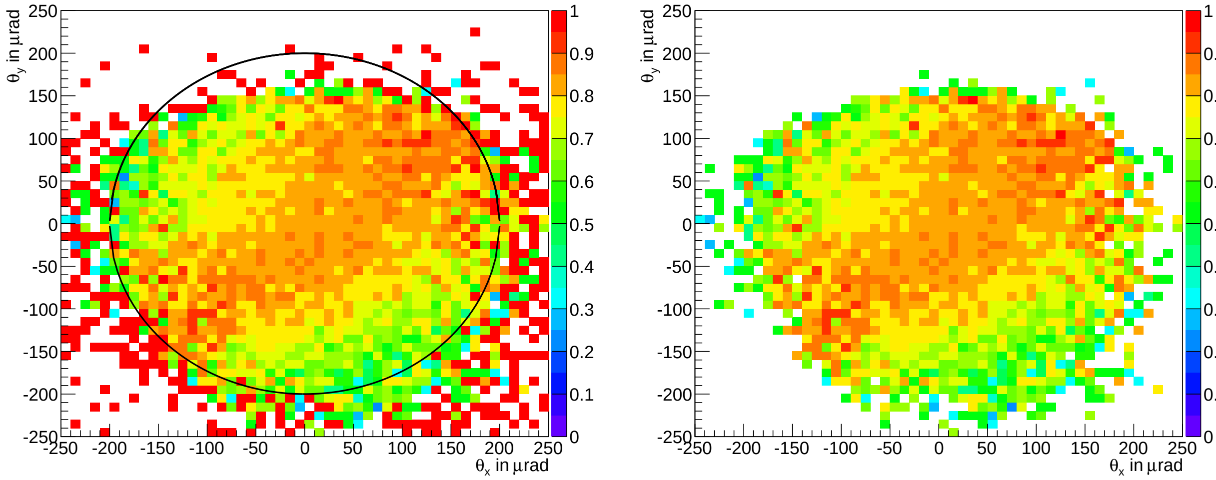
(c) beam sample

FIG. 3: Log likelihoods for different samples obtained from CEDAR 2. The red line indicates $\log L(\pi) = \log L(K)$. A cut on $r < 200 \times 10^{-6}$ is used, cf. section II A.

1. Kaon sample

The obtained IDs for an arbitrary choice of the likelihood differences $A = B = 1$ are given in table II.

The number of particles with no ID is much higher for cut01 than for cut200. This is connected with the creation of the kaon sample. To create the sample the kaontrigger has to be used. This trigger (as it is based on a multiplicity cut) does not identify many kaons with large divergences. Thus only few particles in the sample have a divergence larger than $r = 200 \times 10^{-6}$ and miss the cut200. Bins with low statistics also appear at smaller divergences and thus more particles miss the cut01.



(a) Cut on $r < 200 \times 10^{-6}$

(b) Cut out low statistics

FIG. 4: Probability distribution for the two methods of quality cuts: On the left each particle with a divergence outside of the black circle obtains no ID. On the right particles in bins which are empty don't get an ID.

PID	cut200	cut01
no ID	4.1%	23.1%
π	2.1%	1.6%
K	98.8%	75.3%

TABLE II: PIDs obtained for the kaon sample for likelihood differences $A = B = 1$

2. *Pion sample*

For $A = B = 1$ the obtained IDs for the pion sample are given in table III.

PID	cut200	cut01
no ID	20.3%	42.8%
π	78.4%	56.1%
K	1.3%	1.1%

TABLE III: PIDs obtained for the pion sample for likelihood differences $A = B = 1$

As one can see the number of no ID events for cut01 is nearly as large as the number of identified pions. This can be explained as follows: Using the cut01 a beamtrack obtains no ID even if only one bin in one of the 32 probabilitiy distributions (2×8 PMTs for each pion and kaon) is equal to zero or one. This occurs also for a large number of particles with $r < 200 \times 10^{-6}$. Thus using cut01 significantly decreases the statistics of the sample and should not be the first choice for an analysis.

C. Efficiency and Purity

1. *Purity*

For the calculation of purity the particle in the CEDARs has to be identified without using the CEDARs themselves. Thus a reaction is used where the beam particle ID is determined by the final state. One possible reaction is

$$h^- p \to h'^- K_S^0 p$$

Due to conservation of strangeness the incoming particle is tagged by the outgoing particle. The correlation of the incoming hadron h^- and the outgoing hadron h'^- is given in table IV. To identify the outgoing hadron RICH

h'^-	h^-
K^-	π^-
π^-	K^-

TABLE IV: Correlation between incoming and outgoing hadrons in $h^-p \to h'^- K_S^0 p$

information is used. The purity of the CEDAR selection is obtained by selecting pions or kaons in the CEDAR and looking at the given RICH ID for the outgoing hadron. The impurity of the CEDAR selection is then given by the number of hadrons identified in the RICH which don't fulfil the correlation given in table IV. Those have to be produced by a misidentified incoming particle.
The purity of the RICH selection (especially for kaon identification) gives rise to a systematic error which is considered to be small and thus neglected.

Pion Purity

In figure 5 the base-2 logarithm of the ratio of kaon and pion probability in the RICH over the beam momentum is depicted. Every entry above zero is a RICH identified kaon, every entry below zero a RICH identified pion.

For a pure pion beam – according to table IV – only kaons should be found in the RICH, therefore every entry in figure 5 should be above zero. All entries below zero (RICH identified pions) thus arise from an impurity of the CEDAR selection. To calculate the impurity the number of this "wrong" entries $N_{\mathrm{RICH}}(\pi)$ is divided by the total number of particles in the RICH:

$$I_{\mathrm{CEDAR}}(\pi) = \frac{N_{\mathrm{RICH}}(\pi)}{N_{\mathrm{RICH}}}$$

The two bins around a logarithmic ratio of zero (a probabilitiy ratio of one) are excluded from the calculation as this particles are not identified strong enough.

A cut on the particle momentum $8\,\mathrm{GeV}/cp < 50\,\mathrm{GeV/c}$ is performed selecting the region where pions and kaons can be separated sufficiently. The resulting impurity as a function of the likelihood difference B for both quality cuts is shown in figure 6.

The resulting impurity is nearly independent of B and has a value of around 13%. This corresponds to a purity of $\approx 87\%$ whereas the multiplicity method (black line) has a purity of only 78%.

Kaon Purity

For the purity of the kaon selection in the CEDAR the same method is used switching the CEDAR selection and the RICH cut (only entries below zero – RICH identified pions – are expected in figure 7a). The impurity is now given by

$$I_{\mathrm{CEDAR}}(K) = \frac{N_{\mathrm{RICH}}(K)}{N_{\mathrm{RICH}}} \ .$$

The RICH probabilities can be found in figure 7a, the corresponding impurity as a function of A in figure 7b.

The resulting purity is around 86% for the whole range in A and compatible with the multiplicity method. In addition – just as expected for a pure statistical method – the purity for kaons is comparable with the one for pions.

2. Efficiency

To calculate the efficiency the obtained number of kaons and pions is compared with the expected number of kaons and pions in the beam. According to Lau Gatignon the composition of the beam at the T6 target is the following:

$$\pi^- : 94.76\%$$
$$K^- : 4.594\%$$
$$\bar{p} : 0.7\%$$

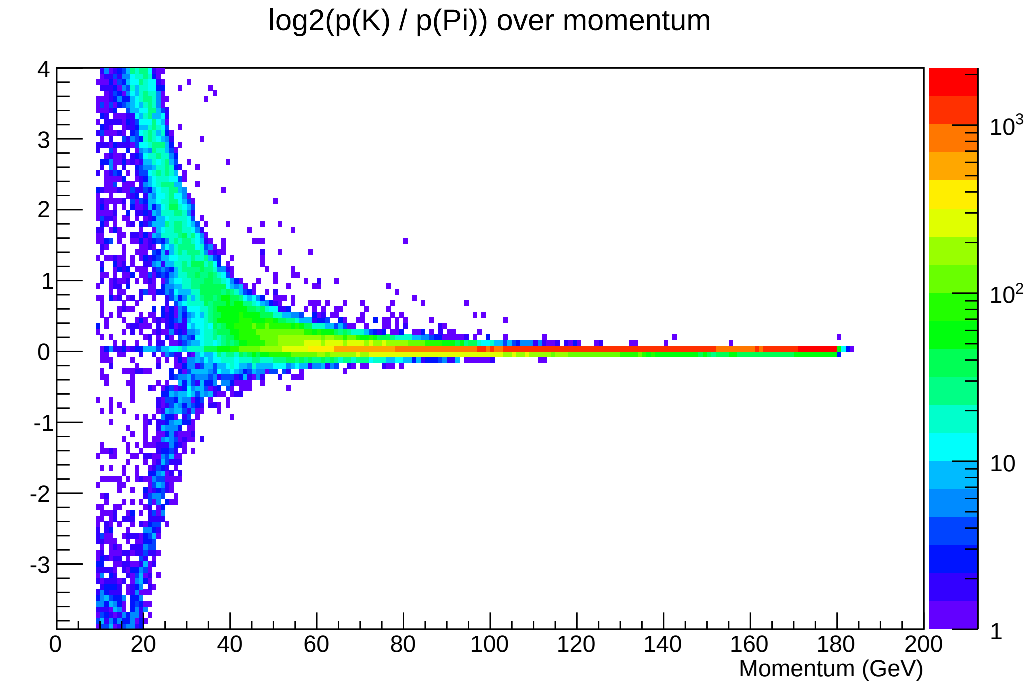


FIG. 5: $\log_2 \left(p(K)/p(\pi)\right)$ for CEDAR selected pions over beam momentum. One would only expect kaons (values above zero) in the RICH in this case.

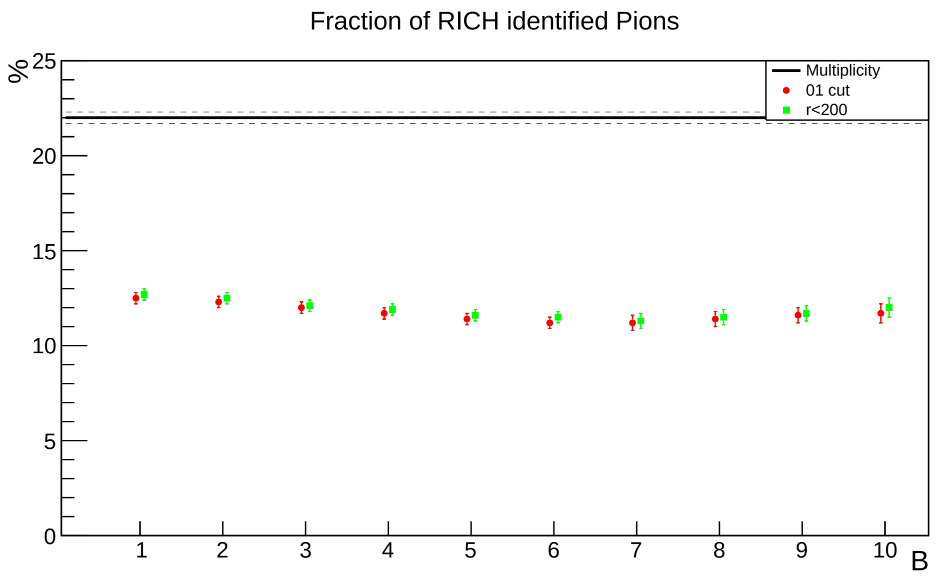
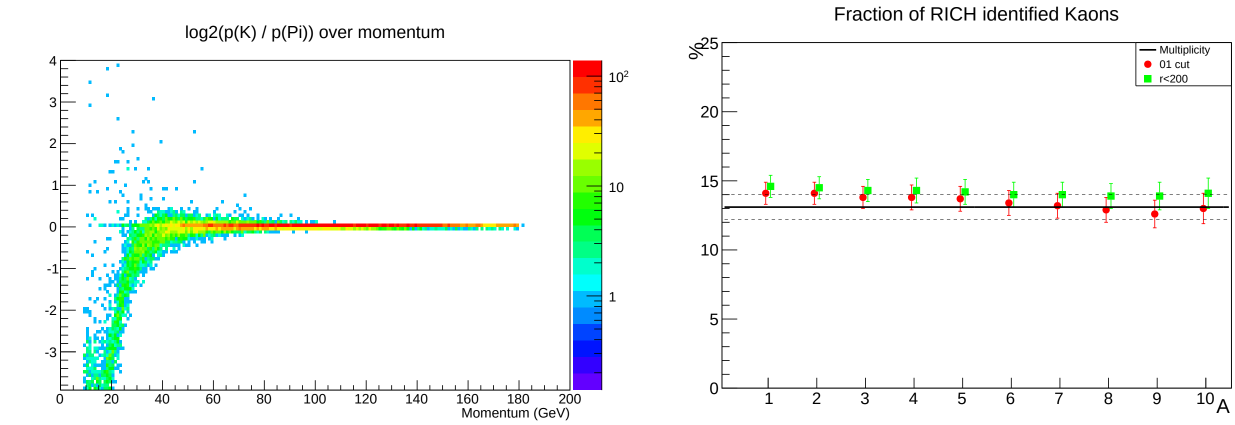


FIG. 6: Impurity for pion identification in the CEDAR for different quality cuts and multiplicity.

with a relative error of 2% for pions and antiprotons and 3% for kaons, respectively. Propagating these fractions 1079.7 m to the CEDAR region with a momentum of 190 GeV/c assuming that the decay particles leave the beam the following beam composition at the CEDARs is obtained:

$$\pi^- : (96.77 \pm 4.33)\%$$
$$K^- : (2.44 \pm 0.12)\%$$
$$\bar{p} : (0.79 \pm 0.04)\%$$



(a) $\log_2\left(p(K)/p(\pi)\right)$ for CEDAR selected kaons. One would only expect pions (values below zero) in the RICH in this case.

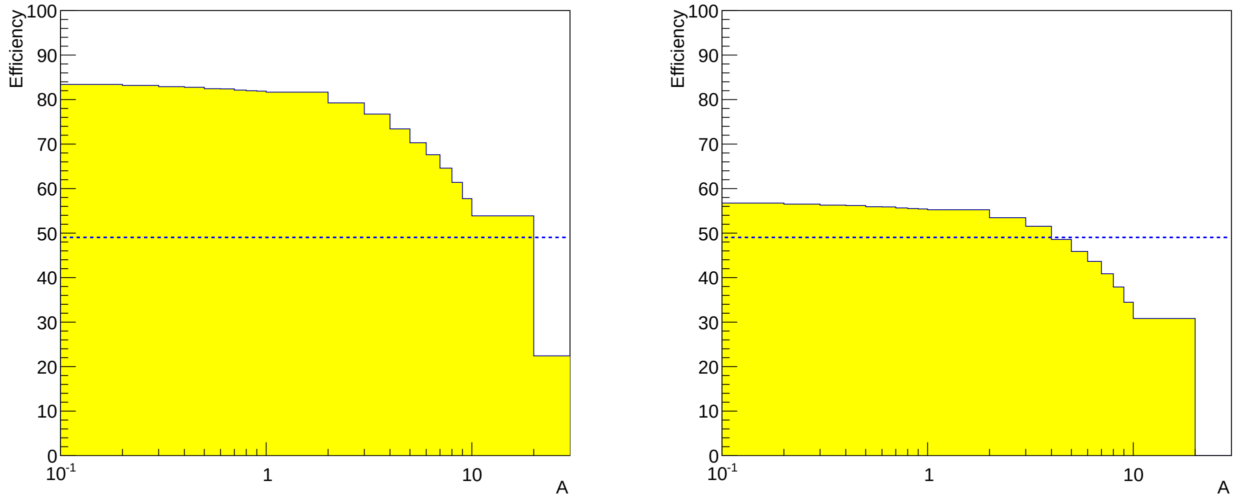
(b) Impurity for kaon identification in the CEDAR for different quality cuts and multiplicity.

FIG. 7: Purity calculation for CEDAR selected kaons

The number of identified kaons and pions in the beamtrigger sample is compared with the values given above.

Kaon efficiency

Figure 8 shows the efficiency for the CEDAR kaon selection over the likelihood difference A. As a comparison also the value obtained by the multiplicity method is shown.
For CEDAR kaons the efficiency for cut200 is around 80% for $A = 1$ and decreases for larger A as expected. For cut01 the efficiency is much worse (around 55% for $A = 1$) but still slightly better than for the multiplicity method (49%). Using cut200 for $A = 1 \approx 60\%$ more kaons are obtained than using the multiplicity method.



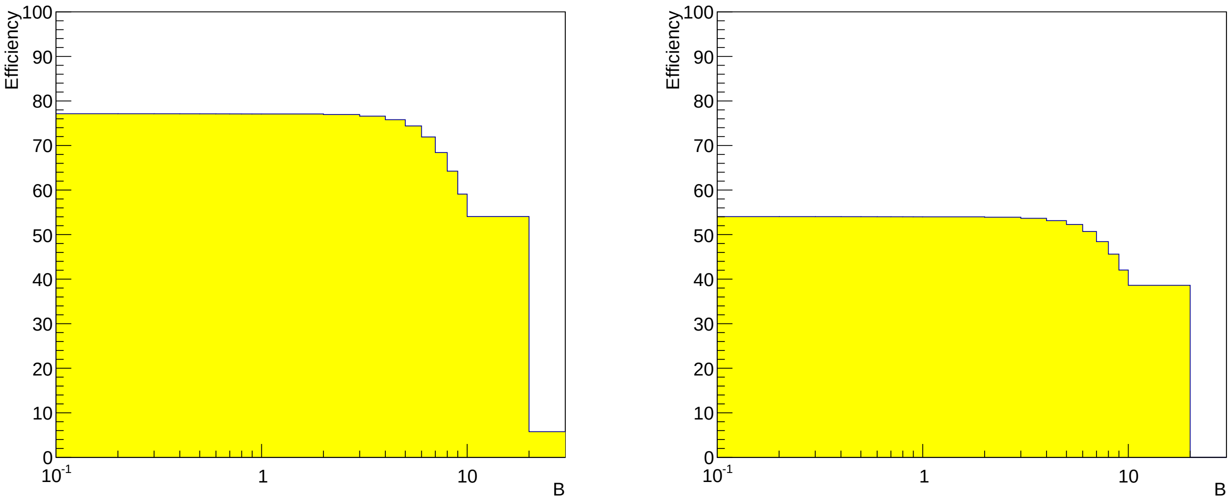
(a) cut200

(b) cut01

FIG. 8: Efficiency calculation for CEDAR selected kaons for different quality cuts. As a comparison the multiplicity method is indicated by the dotted line.

Pion efficiency

Figure 9 shows the efficiency for the CEDAR pion selection over the likelihood difference B. Here no value for multiplicity is given due to the fact that the multiplicity method is not valid to positively identify pions. It can only identify kaons, every particle not identified as a kaon is then assumed to be a pion. For CEDAR pions the efficiency for $A = 1$ is 77% for cut200 and 52% for cut01, respectively. This is comparable to CEDAR kaons as expected.



(a) cut200 (b) cut01

FIG. 9: Efficiency calculation for CEDAR selected pions for different quality cuts.

III. CONCLUSIONS

Given the fact that the purities for kaons as well as for pions are independent of the likelihood cut (A or B) the method can be tuned in means of efficiency. Thus a proposed starting point for an analysis would be the choice $A = B = 1$ and the use of cut200. The resulting values for efficiency and purity (in comparison with multiplicity) are given in table V. Thus the new likelihood method provides a considerable improvement in kaon efficiency and pion

	Likelihood	Multiplicity
Kaon Efficiency	$(80.3 \pm 0.4)\%$	$(48.4 \pm 0.2)\%$
Kaon Purity	$(85.4 \pm 0.9)\%$	$(86.9 \pm 0.9)\%$
Pion Efficiency	$(77.1 \pm 0.3)\%$	X
Pion Purity	$(87.5 \pm 0.3)\%$	$(78.0 \pm 0.3)\%$

TABLE V: Efficiencies and purities for the likelihood method ($A = B = 1$, cut200) in comparison with the multiplicity method

purity and is compatible with multiplicity in means of kaon purity.

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[1] Prometeusz Jasinski. *Analysis of diffractively dissociation of K^- into $K^-\pi^+\pi^-$ on a liquid hydrogen target at the COMPASS spectrometer*. PhD thesis, Johannes Gutenberg-Universität Mainz, 01 2012.
[2] Jan Friedrich. Cedar performance 2009. COMPASS Note 2010-15, Dezember 2010.
[3] C. Bovet et al. The cedar counters for particle identification in the sps secondary beams: a description and an operation manual. *CERN SPS*, 82-13, 1982.

Appendix A: User's Manual for Bayesian Likelihoods in the CEDAR Helper

All methods described above are implemented in the current version of the CEDAR helper (`http://wwwcompass.cern.ch/tw`

1. Getting the Likelihoods

To get the likelihoods for Pions and Kaons the functions

```
double CEDAR::GetLikes_Pion(const PaEvent& e, const int iv, int cedar)
double CEDAR::GetLikes_Kaon(const PaEvent& e, const int iv, int cedar)
```

are used where `const PaEvent& e` is the current event and `const int iv` the index of the primary vertex. The value `int cedar` can be chosen as

- `cedar=1`: CEDAR 1 (8 photomultipliers)
- `cedar=2`: CEDAR 2 (8 photomultipliers)
- `cedar=3`: CEDARs combined (16 photomultipliers)

2. Getting the PID

To get the PID

```
int CEDAR::LikeID_bayes(const PaEvent& e, const int iv, double A, double B)
```

is called, which determines the likelihood for a combination of the CEDARs as described in I E. The values for `A` and `B` are set to default values of $A = B = 1$ and thus do not have to be specified. PID information of a single CEDAR or the combination of all 16 photomultipliers can be obtained through

```
int CEDAR::LikeID_bayes(const PaEvent& e, const int iv, int cedar, double A, double B)
```

where `int cedar` has to be chosen as before and the values of `A` and `B` have to be specified.

In both cases the PID uses the following convention (same as RICH):

value	PID
-1	no ID
0	π
1	K

TABLE VI: Convention for Likelihood PID

3. Tools for further studies

Using

```
double CEDAR::GetTheta_X(const PaEvent& e, const int iv, int cedar)
double CEDAR::GetTheta_Y(const PaEvent& e, const int iv, int cedar)
```

the corrected values for the beam divergence – θ_x and θ_y –for both CEDARs can be accessed.

```
bool CEDAR::Hit_in_PMT(const PaEvent& e, const int iv, int cedar, int pmt)
```

will be `true` if photomultiplier `pmt` $(0-7)$ in CEDAR `cedar` $(1, 2)$ has a hit.

4. How to produce your own calibration files

If you want to use your own calibration files for the likelihoods you can use `UserEvent493677` to produce them. Just do the following

1. Produce a kaon and/or pion sample with a minimal amount of background.
2. Run `UserEvent493677` on your sample.
3. Answer the question concerning the type of sample.
4. Put the produced `probKxy.db` and/or `probPxy.db` into the `CEDAR_DB` folder. (For 2009/2012 Primakoff files put `probKxy_2009.db` etc.)
5. Compile PHAST and you are done.

5. How to adapt the method to Primakoff data

For Primakoff data you have to do only two things have to be done:

1. Take care for the zero correction of the beam divergence in the two CEDARs in `double x_cent_20XX[2]` and `double y_cent_20XX[2]` (where `XX=09/12`) in `CEDAR_Helper.cc`.
2. Produce new calibration files as described above.
3. Be sure to tune the method according to your needs.

Appendix B: Additional Figures

1. Probability distributions

In the following figures the probability distributions for pions and kaons in the single photomultipliers are shown. They clearly show the expected behaviour:

For kaons the probability to have a hit in a photomultiplier is large for particles with small divergence. For pions the probability is small for small divergences. If the divergence moves the pion ring to one direction the photomultipliers in the opposite direction start to see signals as the ring moves into their acceptance.

Also performance effects of single photomultipliers can be observed. Especially the PMTs in the upper right of CEDAR 1 (but also some others) show a lack of efficiency wich can be clearly seen in figure 11.

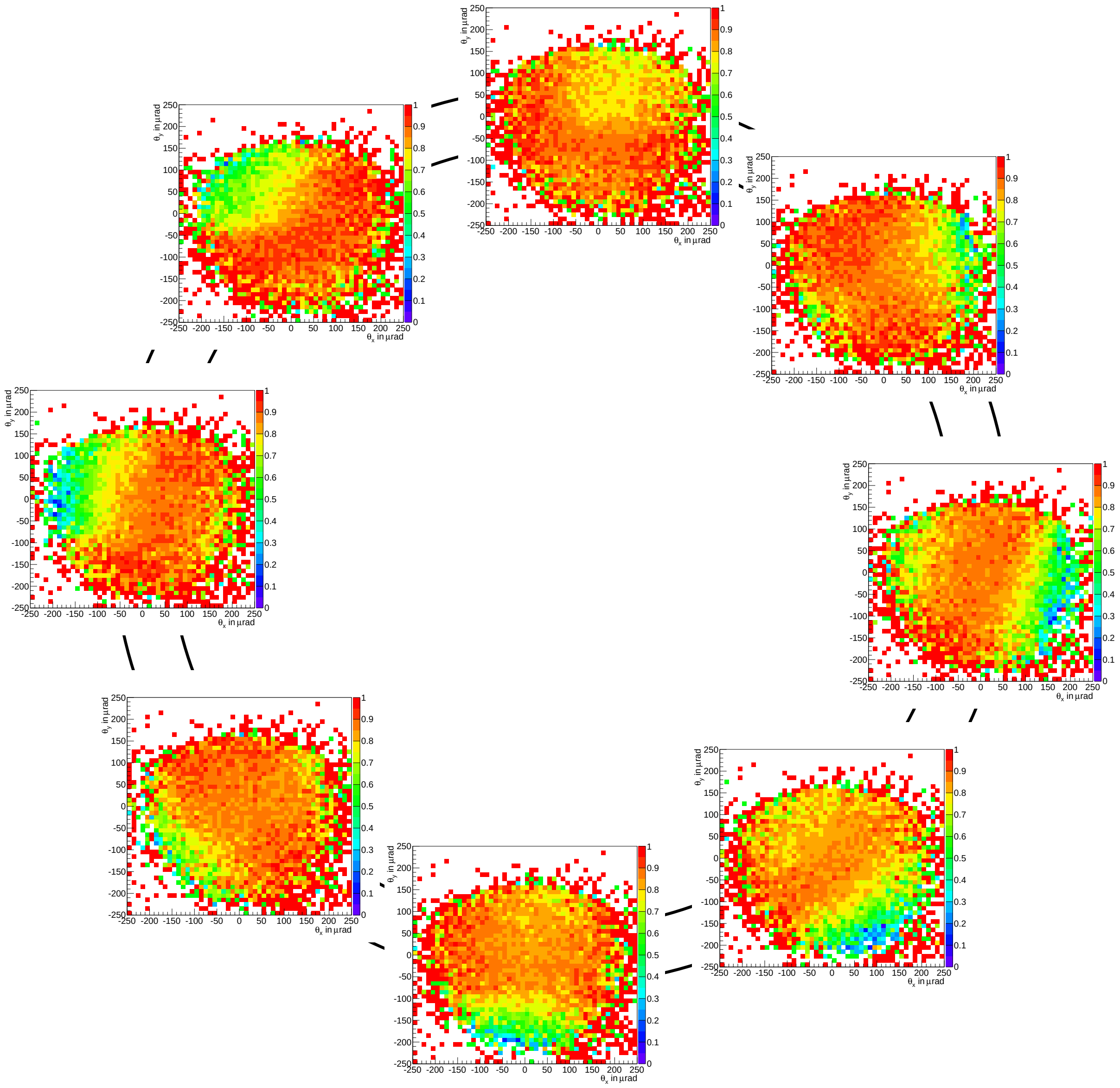


FIG. 10: Probability distribution for hits in CEDAR 1 for kaons arranged according to CEDAR geometry

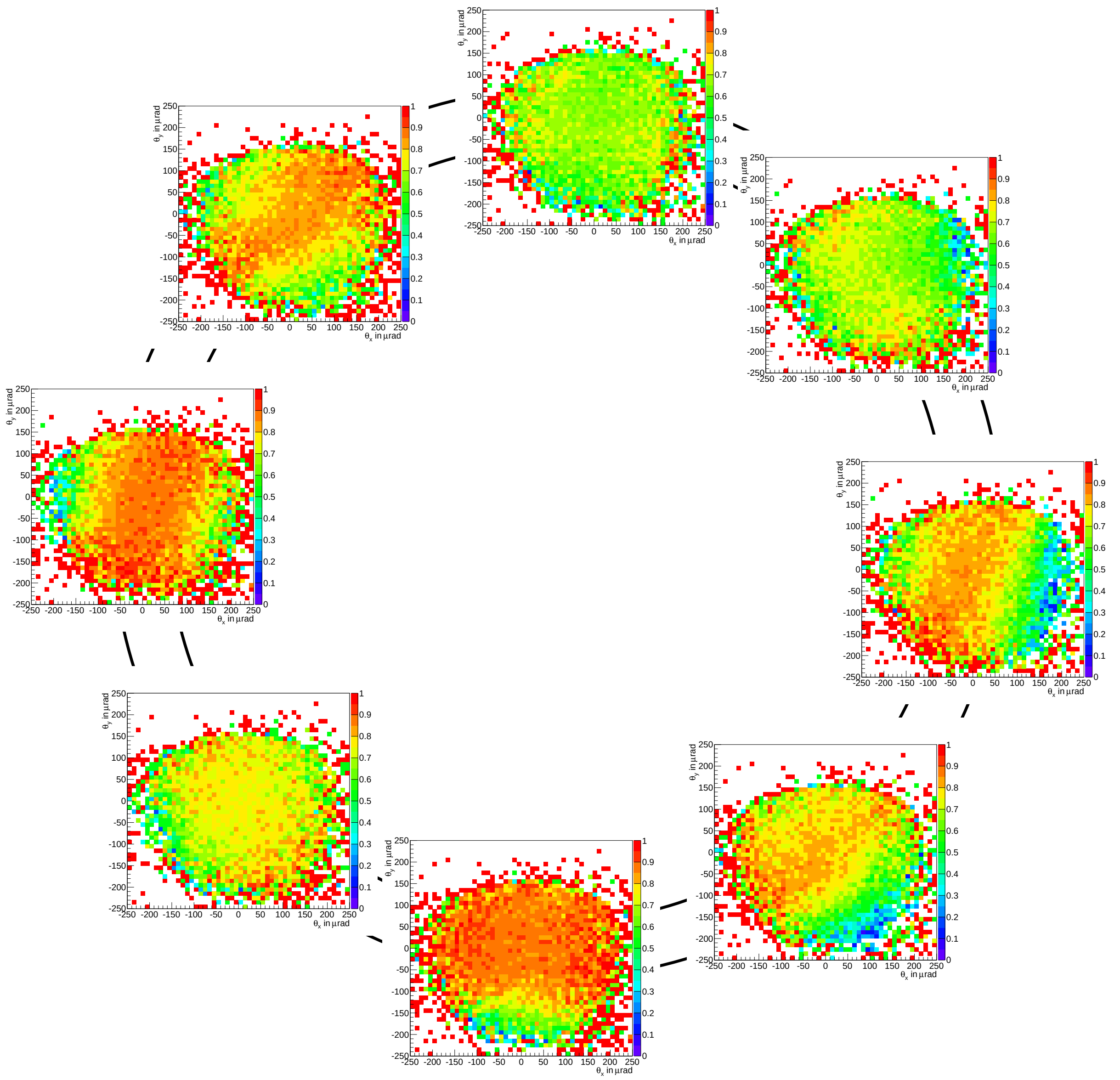


FIG. 11: Probability distribution for hits in CEDAR 1 for kaons arranged according to CEDAR geometry

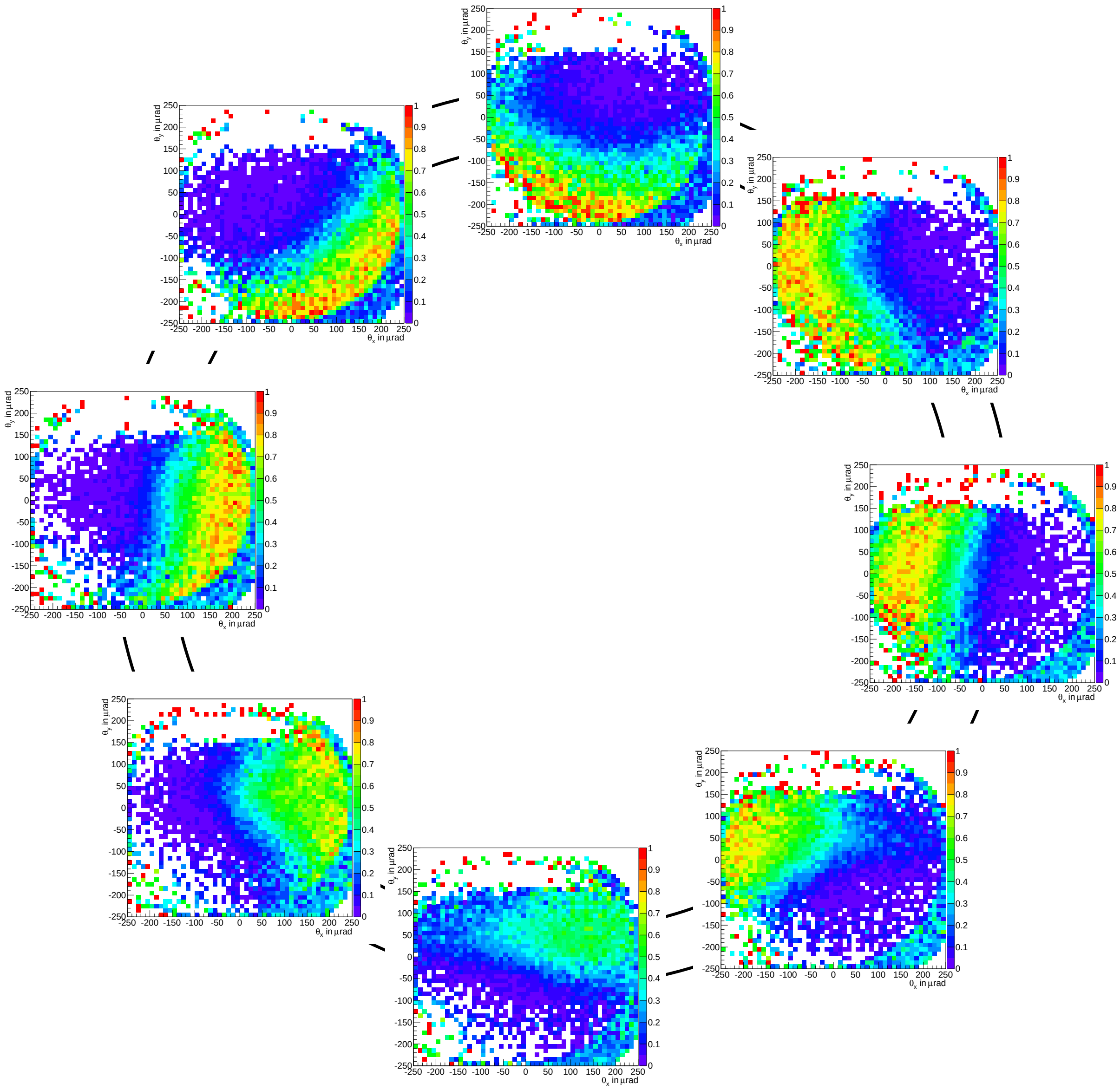


FIG. 12: Probability distribution for hits in CEDAR 1 for pions arranged according to CEDAR geometry

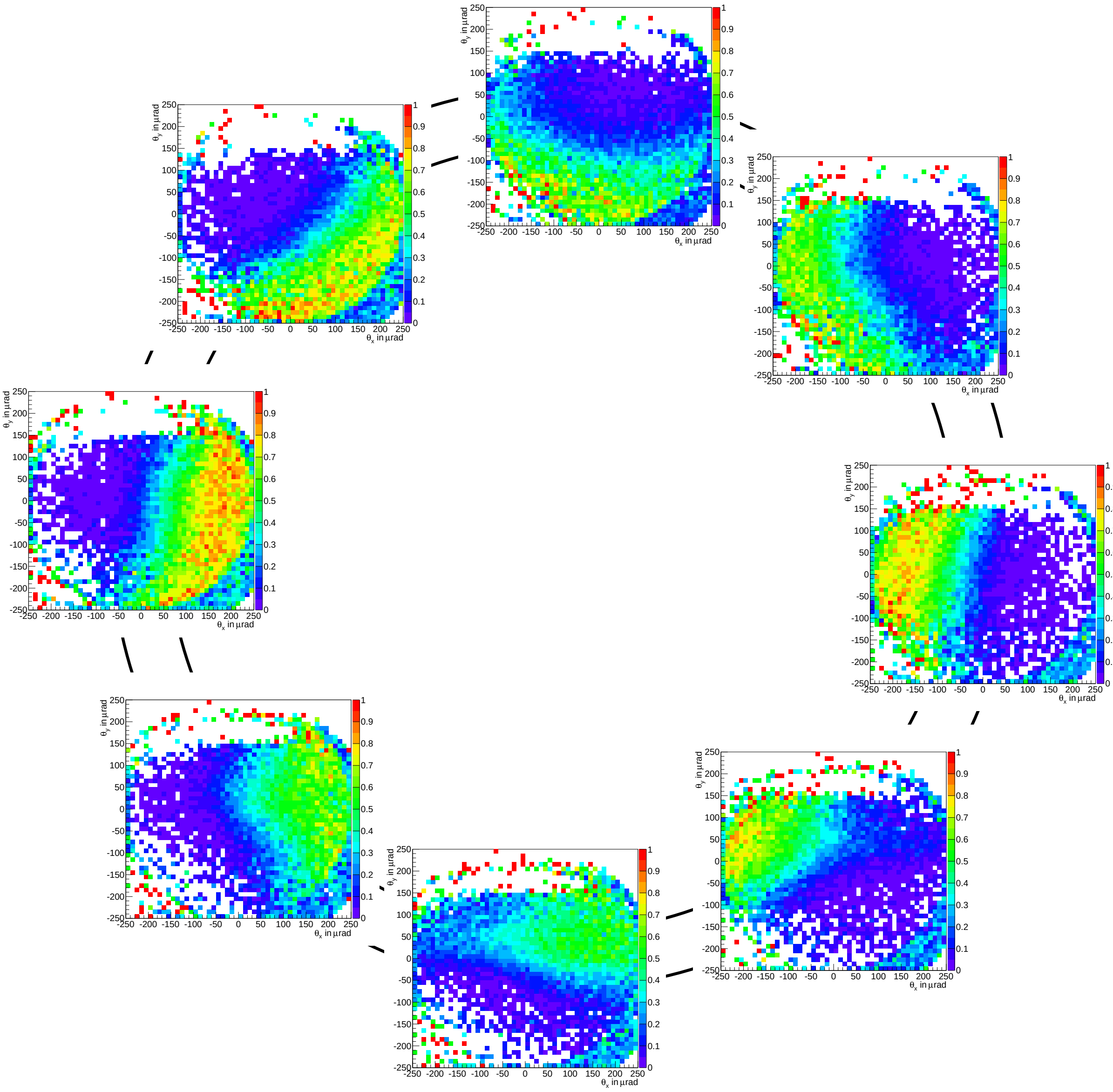
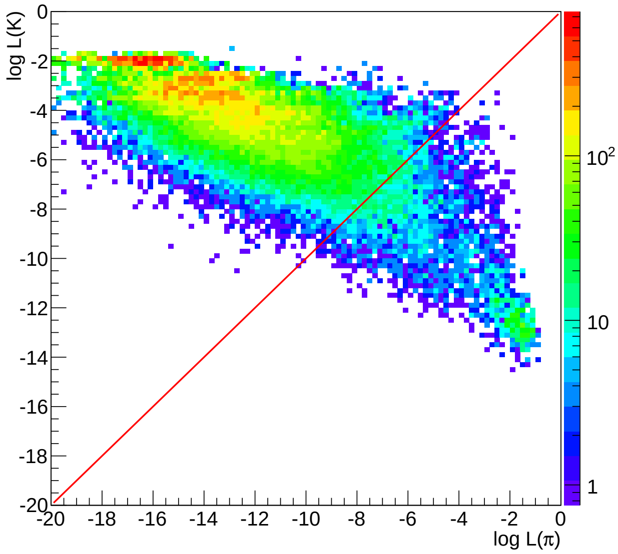


FIG. 13: Probability distribution for hits in CEDAR 1 for pions arranged according to CEDAR geometry

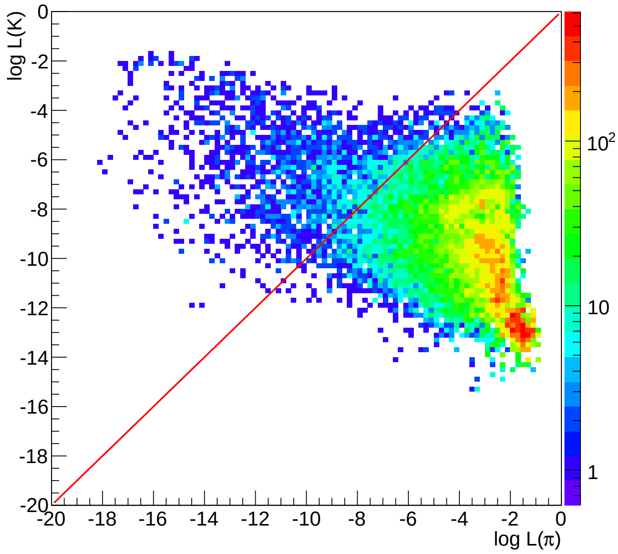
2. Likelihoods

Here the additional likelihood distributions are shown. All distributions show the same behaviour described in figure 3:

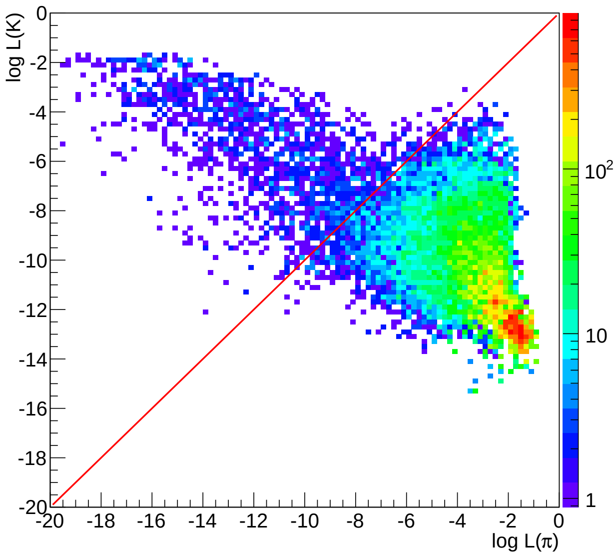
As expected the events in the kaon sample have a higher log likelihood for kaons where there is still a remaining pionic background. In the pion sample the likelihood for pions is higher. In the beam sample most of the events are in the pion region. But there are also a number of “clear” kaons with $\log L(K) \gg \log L(\pi)$.



(a) kaon sample

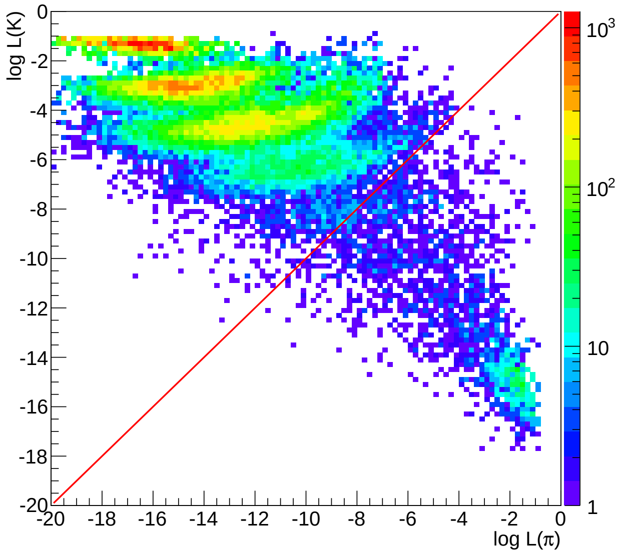


(b) pion sample

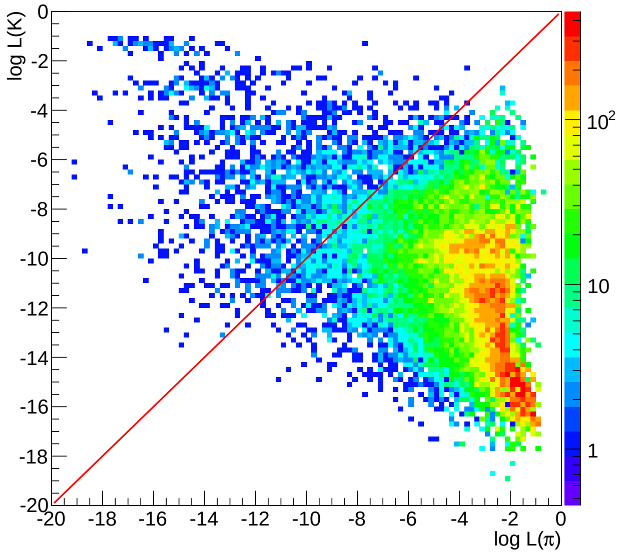


(c) beam sample

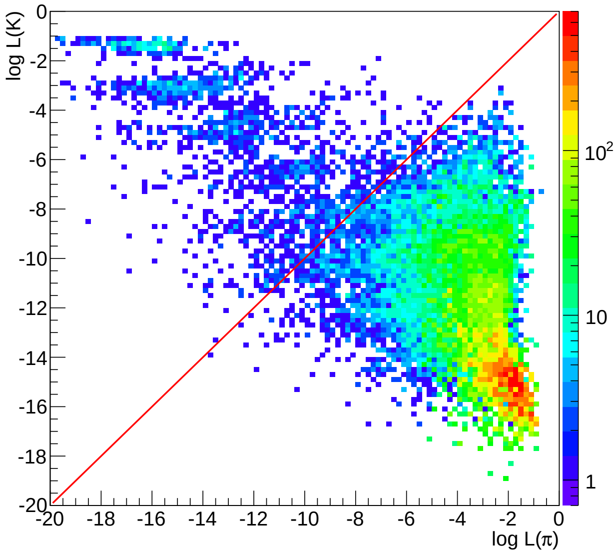
FIG. 14: Log likelihoods for the three samples obtained from CEDAR 2 with cut01



(a) kaon sample

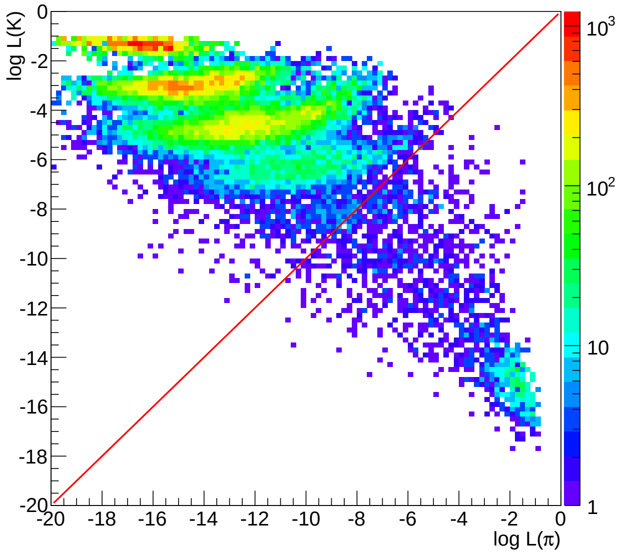


(b) pion sample

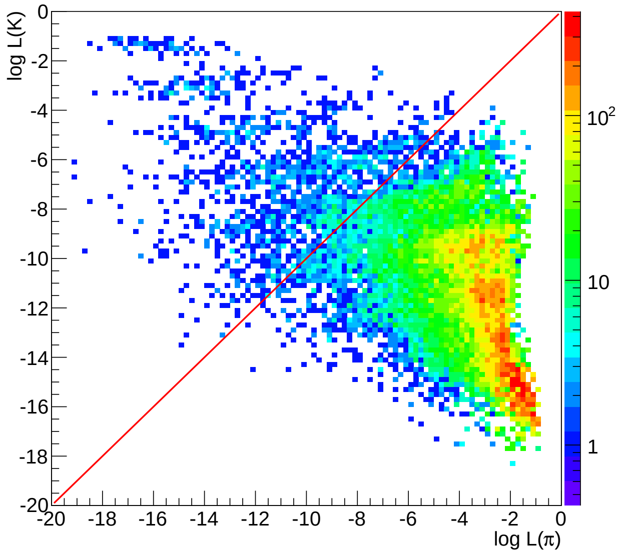


(c) beam sample

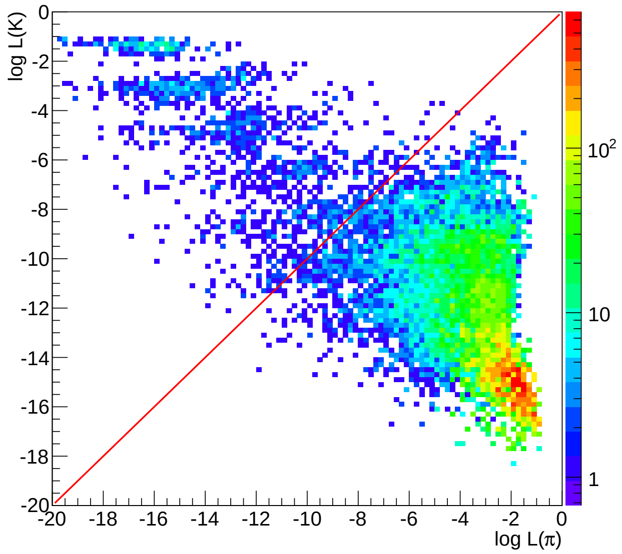
FIG. 15: Log likelihoods for the three samples obtained from CEDAR 1 with cut200



(a) kaon sample



(b) pion sample



(c) beam sample

FIG. 16: Log likelihoods for the three samples obtained from CEDAR 1 with cut01