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- Introduction & Monte Carlo Techniques
- Hard Processes at (Next-to-)Leading Order
- Parton Showers & Matching with Fixed Order
- Multiple Interactions, Hadronization & Hadron Decays
Disclaimer

These lectures will not cover:

- Heavy-ion collisions
- Specific aspects of heavy-flavour physics
- Beyond the Standard Model physics
- most of the details, derivations, ...

Recommended reading:

- Ellis, Stirling & Webber
  “QCD and Collider Physics”

- Sjöstrand, Mrenna & Skands
  “PYTHIA 6.4 Physics and Manual”
  JHEP 0605 (2006) 026

- ... papers cited on the way
Outline Lecture 1

Motivation/Introduction
- LHC challenges: highest energies & highest multiplicities
- A prototypical LHC event: the simulators point of view
- The case for Monte Carlo event generators

Basic Monte Carlo techniques
- The Hit-or-Miss method
  - importance sampling
  - the multi-channel approach
- The Veto Algorithm

Hard-Process generation
- The partonic hadron-hadron cross-section formula
- Multi-leg tree-level matrix elements
- One-loop matrix-element calculations
Hadron-collision experiments

identify & measure all particles coming from individual pp clashes

⇒ to be confronted with hypotheses for the underlying physics
Hadron-collision experiments

identify & measure all particles coming from individual pp clashes
The LHC challenge

Objectives for the LHC era

- reveal the mechanism of EWSB  
  [discovery of the Higgs?, alternatives?]
- search for physics beyond the SM: weak scale SUSY, ED, W' & Z', ...
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Higgs Hunting
- obvious production & decay channels buried in QCD backgrounds
- search for rare decays like $h \rightarrow \gamma\gamma$
- look for associated production modes, e.g. $hjj$, $W/Zh$, $t\bar{t}h$
- signals & backgrounds to be understood at high precision
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$\Rightarrow$ signals & backgrounds to be understood at high precision

Searching for New Physics

- large cross sections for producing new colored states
- subsequent decays into SM final states + $X$
- generic signals: $\#$-leptons $+$ $\#$-jets $+$ $E_T$

$\Rightarrow$ SM backgrounds: $V+$jets, $VV+$jets, $t\bar{t}+$jets, QCD jets

$\Rightarrow$ new physics encoded in energies, flavours, edges
The LHC challenge

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Typically searches in (rare) multi-particle / multi-jet final states!

Detailed understanding of QCD jet production indispensable!
Monte Carlo Event Generators

- **Hard interaction**
  - exact matrix elements $|\mathcal{M}|^2$

- **QCD bremsstrahlung**
  - parton showers in the initial and final state

- **Multiple Interactions**
  - beyond factorization: modelling

- **Hadronization**
  - non-perturbative QCD: modelling

- **Hadron Decays**
  - phase space or effective theories

⇒ stochastic simulation of pseudo data
⇒ fully exclusive hadronic final states
⇒ direct comparison with experimental data, e.g. ATLAS, CMS, LHCb, DØ, CDF

modullo detector simulation
The case for Monte Carlo generators

- theoretical & experimental studies of complex multi-particle physics
- large flexibility of physical quantities that can be assessed
- disseminate ideas from theorists to experimentalists

Applications I: theorists/experimentalists

- predict event rates and topologies \(\Rightarrow\) can estimate feasibility
- simulate possible backgrounds \(\Rightarrow\) can devise analysis strategies

Applications II: experimentalists [supplemented with detector simulation]

- study detector requirements \(\Rightarrow\) can optimize detector/trigger design
- study detector imperfections \(\Rightarrow\) can evaluate acceptance corrections
- study detector response \(\Rightarrow\) can unfold data to particle level
The case for Monte Carlo generators

**new analysis techniques**

**Searching for the boosted Higgs**


- consider $pp \rightarrow Vh$ with $h \rightarrow b\bar{b}$
- focus on Higgs with $p_T^h > 200$ GeV
- use subjet info to reduce QCD BG

$\Rightarrow pp \rightarrow Vh$ with $h \rightarrow b\bar{b}$ resurrected

**desperate new physics searches**

**New Physics in all-jet final states**

[Cilic, S., Son JHEP 0904 (2009) 128]

- consider $pp \rightarrow \tilde{\pi}_8 \tilde{\pi}_8 \rightarrow 4$jets
- fight QCD 4jet background
- mass window criterion for jet pairs

$\Rightarrow$ discovering color octets seems feasible
The case for Monte Carlo generators

The ATLAS simulation framework [arXiv:1005.4568]
LHC data is coming!

We are here!
Basic Monte Carlo techniques
Basic Monte Carlo techniques

“Spatial” problems: without memory

- What is the volume of a given body?
  Pick a random point, with equal probability in this area.

- What is the integrated cross section of a given process?
  Pick an event at random, according to the differential cross section.

“Temporal” problems: have memory

- Financial systems: What is the probability for a stock to have price $X$ at time $t$, given the price $Y$ at $t_0$?

- Parton shower: What is the probability for a parton to branch at a scale $Q$, given that it was created at a scale $Q_0$?

In particle physics often combined problems

What is the probability for a parton to branch at $Q$, with the daughters sharing the mother momentum in some specific way?
Basic Monte Carlo techniques: “Spatial” problems

Assume function $f(x)$ in range $x_{\text{min}} \leq x \leq x_{\text{max}}$, where $f(x) \geq 0$ everywhere (in practice $x$ is multi-dimensional).

Two standard tasks

1. Calculate (approximatively)

$$ I = \int_{x_{\text{min}}}^{x_{\text{max}}} f(x') \, dx' = (x_{\text{max}} - x_{\text{min}}) \langle f(x) \rangle $$

$$ \simeq I_N = (x_{\text{max}} - x_{\text{min}}) \frac{1}{N} \sum_{i=1}^{N} f(x_i) \pm (x_{\text{max}} - x_{\text{min}}) \sqrt{\frac{\langle f^2 \rangle - \langle f \rangle^2}{N}} $$

Example: integrated cross section from the differential one

2. Select $x$ at random according to $f(x)$

Example: given probability distribution from quantum mechanics
Basic Monte Carlo techniques: “Spatial” problems

Select $x$ according to $f(x)$

as $P(x) \sim \int_{0}^{f(x)} dy = f(x)$

equivalent to uniform selection of $(x, y)$ in area $x_{\text{min}} \leq x \leq x_{\text{max}} \& 0 \leq y \leq f(x)$

$\Rightarrow \int_{x_{\text{min}}}^{x_{\text{max}}} f(x') dx' = \# \int_{x_{\text{min}}}^{x_{\text{min}}} f(x') dx'$

Analytical solution

known primitive function $F(x)$ and inverse $F^{-1}$

$F(x) - F(x_{\text{min}}) = \#(F(x_{\text{max}}) - F(x_{\text{min}})) = \#A_{\text{tot}}$

$\Rightarrow x = F^{-1}(F(x_{\text{min}}) + \#A_{\text{tot}})$

Example: $f(x) = e^{-x}$ for $x \geq 0$

$F(x) = 1 - e^{-x}$

$1 - e^{-x} = \# \Rightarrow x = -\ln(\#)$

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Monte Carlo Event Generators for the LHC
Basic Monte Carlo techniques: “Spatial” problems

The Hit-or-Miss method: basics

\[ f(x) \leq f_{\text{max}} \text{ in } x_{\text{min}} \leq x \leq x_{\text{max}} \]

1. select \( x = x_{\text{min}} + \#_1(x_{\text{max}} - x_{\text{min}}) \)
2. select \( y = \#_2 f_{\text{max}} \)
3. if \( y \leq f(x) \) accept point \([N_{\text{acc}}++, N_{\text{try}}++]\)
   if \( y > f(x) \) reject point \([N_{\text{fail}}++, N_{\text{try}}++]\)

As a byproduct the integral is given by:

\[
I = \int_{x_{\text{min}}}^{x_{\text{max}}} f(x) dx \simeq (x_{\text{max}} - x_{\text{min}}) f_{\text{max}} \frac{N_{\text{acc}}}{N_{\text{try}}} = A_{\text{tot}} \frac{N_{\text{acc}}}{N_{\text{try}}}
\]

with its relative variance approaching:

\[
\frac{\delta I}{I} \rightarrow \frac{1}{\sqrt{N_{\text{acc}}}}
\]
Basic Monte Carlo techniques: “Spatial” problems

Example: Battleships

Can we do better?
Basic Monte Carlo techniques: “Spatial” problems

The Hit-or-Miss method: importance sampling

$f(x) \leq g(x)$ in $x_{\text{min}} \leq x \leq x_{\text{max}}$

and $G(x) = \int_{x_{\text{min}}}^{x} g(x') \, dx'$ & $G^{-1}(y)$ are simple

1. select $x = \text{according to distribution } g(x)$
2. select $y = \#g(x)$
3. if $y \leq f(x)$ accept point $[N_{\text{acc}}++, N_{\text{try}}++]$
   if $y > f(x)$ reject point $[N_{\text{fail}}++, N_{\text{try}}++]$

Example: $f(x) = xe^{-x}$ and $g(x) = Ne^{-x/2}$

$$\frac{f(x)}{g(x)} = \frac{xe^{-x/2}}{N} \leq 1 \rightarrow \text{find maximum}$$

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{1}{N} \left(1 - \frac{x}{2}\right) e^{-x/2} \overset{!}{=} 0 \sim x = 2$$

normalize such that $g(2) = f(2) \sim N = 2/e$

from $G(x) \sim 1 - e^{-x/2} = \#_1$

$\sim x = -2 \ln(\#_1)$

$y = \#_2 g(x) = \#_2 2e^{-(1+x/2)}$
Basic Monte Carlo techniques: “Spatial” problems

The Hit-or-Miss method: multi-channel approach

\[ f(x) \leq g(x) = \sum_{i=1}^{N} \alpha_i g_i(x) \text{ in } x_{\text{min}} \leq x \leq x_{\text{max}}, \]

where all \( g_i \) are “nice”, \( \alpha_i \geq 0 \) & \( \sum_{i=1}^{N} \alpha_i = 1 \)

1. select \( i \) with relative probability \( \alpha_i \int_{x_{\text{min}}}^{x_{\text{max}}} g_i(x')dx' \)

2. select \( x \) according to \( g_i(x) \)

3. select \( y = \#g(x) = \# \sum_{i=1}^{N} \alpha_i g_i(x) \)

4. if \( y \leq f(x) \) accept point \([N_{\text{acc}}++ , N_{\text{try}}++]\]
   if \( y > f(x) \) reject point \([N_{\text{fail}}++ , N_{\text{try}}++]\]

5. adjust apriori weights \( \alpha_i \) to minimize variance
   \( \rightsquigarrow \) self-adaptive multi-channel Monte Carlo

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Monte Carlo Event Generators for the LHC
Basic Monte Carlo techniques: “Temporal” problems

The radioactive decay problem

- known probability $f(t)$ that *something will happen* at time $t$
  [nucleus decays, parton branches, transistor fails]
- *something happens* at $t$ *only* if it didn’t happen at $t' < t$

Define: $N(t)$: probability that *nothing* happen until $t$ [$N(0) = 1$]

$$P(t) = -dN(t)/dt = f(t)N(t): \text{ probability for decay at time } t$$

Solution:

$$N(t) = \exp \left\{- \int_0^t f(t')dt' \right\}$$

$$P(t) = f(t) \exp \left\{- \int_0^t f(t')dt' \right\}$$

- naïve answer $P(t) = f(t)$ modified by exponential suppression
- in parton-shower picture, $N(t)$ is called the Sudakov form factor
Basic Monte Carlo techniques: “Temporal” problems

If $F(t)$ and $F^{-1}$ exist: Standard solution to find “decay” time $t$

$$\int_{0}^{t} P(t')dt' = N(0) - N(t) = 1 - \exp \{- (F(t) - F(0))\} = 1 - #$$

$$\Rightarrow \quad t = F^{-1}(F(0) - \ln(#))$$

The Veto Algorithm

if $f(t)$ has no simple $F(t)$ or $F^{-1}$: use “nice” $g(t) \geq f(t)$

1. start with $i = 0$ and $t_0 = 0$
2. increment $i$ and select $t_i = G^{-1}(G(t_{i-1}) - \ln(#_1))$
3. if $f(t_i)/g(t_i) \leq #_2$ go back to step 2
   otherwise, keep $t_i$ as “decay” time

$$\Rightarrow \text{next step only depends on the very previous one}$$
$$\Rightarrow \text{Markov chain process}$$
Basic Monte Carlo techniques: Summary

- use Monte Carlo to perform integrals and sample distributions
  - only need few points to estimate \( f \)
  - each additional point increases accuracy
- techniques generalize to many dimensions
  - typical LHC phase space \( \sim d^3 \vec{p} \times 100\text{'s particles} \)
    - error scales as \( 1/\sqrt{N} \) vs. \( 1/N^{2/d} \) or \( 1/N^{4/d} \) (Trapezoidal or Simpson’s Rule)
- suitable for complicated integration regions
  - kinematic cuts or detector cracks
- can sample distributions where exact solutions cannot be found
- Veto Algorithm applied to parton shower
Hard-Process generation
The partonic hadron-hadron cross section

- proton bound state of quarks and gluons
- universal probability distribution of finding parton a with momentum \( p_a = x_a P \)
  \[ f_a(x_a, \mu_F^2) \rightarrow \text{Parton Density Function} \]
  \( \leftarrow \text{partonic CM energy: } \hat{s} = x_a x_b s_{pp} \)
The partonic hadron-hadron cross section

- proton bound state of quarks and gluons
- universal probability distribution of finding parton $a$ with momentum $p_a = x_a P$ $f_a(x_a, \mu_R^2, \mu_F^2) \rightarrow$ Parton Density Function
- factorization in hard and soft component (resummed in PDFs)

\[
\sigma_{pp \rightarrow \text{x}_{\text{part}}}(s; \mu_R^2, \mu_F^2) \equiv \sum_{ab} \int d x_a d x_b \ f_a(x_a, \mu_R^2) f_b(x_b, \mu_F^2) \ d \hat{\sigma}_{ab \rightarrow \text{x}_{\text{part}}}(\hat{s}; \{ p_x \}, \mu_R^2, \mu_F^2)
\]

- soft/collinear QCD emissions summed in $f_a(x_a, \mu_R^2, \mu_F^2) \rightarrow \sigma$ is inclusive!
- hard production process described through

\[
d \hat{\sigma}_{ab \rightarrow \text{x}_{\text{part}}}(\hat{s}; \{ p_x \}, \mu_R^2, \mu_F^2) \equiv |\mathcal{M}_{ab \rightarrow \text{x}_{\text{part}}}(\hat{s}; \{ p_x \}, \mu_R^2, \mu_F^2)|^2 \ d \Phi_X
\]

\[\rightarrow \text{from model Lagrangian, e.g. } \mathcal{L}_{\text{SM}}, \mathcal{L}_{\text{MSSM}}, \ldots\]

\[\rightarrow \text{allows for probabilistic generation of partonic events } \{ p_X \}\]
\[\rightarrow \text{the center piece of each event}\]
\[\rightarrow \text{need to model transition of partons to hadrons: } \text{x}_{\text{part}} \rightarrow \text{x}_{\text{had}}\]
The Hard Process: setting the scene

\[ \sigma_{pp\to X_n} = \sum_{ab} \int dxa \, db \, f_a(x_a, \mu_F^2)f_b(x_b, \mu_F^2) \left| \mathcal{M}_{ab\to X_n} \right|^2 d\Phi_n \]
The Hard Process: setting the scene

\[ \sigma_{pp \to X_n} = \sum_{ab} \int d\mathbf{x}_a d\mathbf{x}_b \ f_a(x_a, \mu^2_F) f_b(x_b, \mu^2_F) |\mathcal{M}_{ab \to X_n}|^2 \ d\Phi_n \]

**generic features**

- high-dimensional phase space \( \text{dim}[\Phi_n] = 3n - 4 \)

\[ d\Phi_n = \delta^4 \left( p_a + p_b - \sum_{i=1}^n p_i \right) \prod_{i=1}^n \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} \]

\[ \mapsto \text{subject to non-trivial cuts} \]
The Hard Process: setting the scene

\[ \sigma_{pp \to X_n} = \sum_{ab} \int d{x}_a d{x}_b \ f_a(x_a, \mu_F^2) f_b(x_b, \mu_F^2) \left| M_{ab \to X_n} \right|^2 d\Phi_n \]

generic features

- high-dimensional phase space \( \text{dim}[\Phi_n] = 3n - 4 \)
- \( \left| M_{ab \to X_n} \right|^2 \) wildly fluctuating over \( \Phi_n \)
  - competing resonances [Breit-Wigner]
  - enhanced phase-space regions
  - momentum correlations
  - number of diagrams \( \sim n! \) [costly]
The Hard Process: setting the scene

\[ \sigma_{pp \rightarrow X_n} = \sum_{ab} \int dx_a dx_b \ f_a(x_a, \mu_F^2)f_b(x_b, \mu_F^2) \ |M_{ab \rightarrow X_n}|^2 \ d\Phi_n \]

**generic features**

- high-dimensional phase space \( \text{dim}[\Phi_n] = 3n - 4 \)
- \( |M_{ab \rightarrow X_n}|^2 \) wildly fluctuating over \( \Phi_n \)
- steep parton densities [parametrization]

![Graph showing the distribution of xf(x,Q^2) for u-quark, \( \bar{u} \)-quark, and gluon (\(*0.05*)\)]

- MRST2002NLO
- \( Q^2 = (100 \text{ GeV})^2 \)
The Hard Process: setting the scene

\[ \sigma_{pp \rightarrow X_n} = \sum_{ab} \int d{x_a} d{x_b} \ f_a({x_a}, \mu_F^2) f_b({x_b}, \mu_F^2) \ |M_{ab \rightarrow X_n}|^2 \ d\Phi_n \]

generic features
- high-dimensional phase space \( \text{dim}[\Phi_n] = 3n - 4 \)
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state-of-the-art
- traditionally based on LO \( 2 \rightarrow 2 \) only
- now dedicated \( 2 \rightarrow n \) tree-level codes
  - extract Feynman rules from Lagrangian \( \mathcal{L} \)
  - generate compact expressions for \( |M|^2 \)
  - flexible phase-space integrators
- \( \xrightarrow{\text{ALPGEN, AMEGIC, COMIX, HELAC, MADGRAPH}} \)
- automation of one-loop corrections on the way

\[ \mathcal{L}^{(B)\text{SM}} \]

\[ \downarrow \]

\[ f \]

\[ \downarrow \]

\[ f \]

\[ Z_\mu' \]

\[ \downarrow \]

\[ |M_n|^2 \ d\Phi_n \]
Phase-Space integration: a look into the tool box

build-up suitable channels during amplitude generation

\[ |\mathcal{M}|^2 \equiv \left| \sum_{i=1}^{k} \mathcal{A}_i \right|^2 = \sum_{i=1}^{k} |\mathcal{A}_i|^2 + \sum_{i \neq j} |\mathcal{A}_i \mathcal{A}_j^\dagger| \]

- phase-space map for each topology \(|\mathcal{A}_i|^2\)

\[ \begin{align*}
  d\Phi_4 & \propto d\phi_{12,34} d\cos \theta_{12,34} \\
  & \times ds_{12} d\phi_{1,2} d\cos \theta_{1,2} \\
  & \times ds_{34} d\phi_{3,4} d\cos \theta_{3,4} \\
  \text{with} \ s_{ij} = (p_i + p_j)^2
\end{align*} \]

\[ \Rightarrow \text{use suitable distributions} \]

- use adaptive multi-channel technique to combine channels
Example: Breit-Wigner propagator – particle with mass $M$ and width $\Gamma$

mapping

\[
s = M\Gamma \tan \rho + M^2 \\
\sim \sim \ dl = M\Gamma \sec^2 \rho \, d\rho
\]
yields

\[
I = \int_{s_{\text{min}}}^{s_{\text{max}}} \frac{dl}{(s-M^2)^2 + M^2\Gamma^2}
\]

\[
= \int_{\rho_{\text{min}}}^{\rho_{\text{max}}} \frac{M\Gamma \sec^2 \rho \, d\rho}{M^2\Gamma^2 \tan^2 \rho + M^2\Gamma^2}
\]

\[
= \frac{1}{M\Gamma} \int_{\rho_{\text{min}}}^{\rho_{\text{max}}} \, d\rho \sim \sim \delta I = 0
\]
Matrix-Element generation: a look into the tool box

Helicity amplitudes [e.g. Hagiwara, Zeppenfeld Nucl. Phys. B 274 (1986) 1]
- express amplitudes $A_i$ in terms of spinor products $\bar{u}(p_i, \lambda_i)u(p_j, \lambda_j)$
- for assigned helicities & momenta $A_i$’s just complex numbers
- can perform the sum of amplitudes before squaring

Feynman-diagram based
- factor out common pieces
- $2 \rightarrow 6$ is feasible

Recursive approaches
- exponential growth of complexity
- $2 \rightarrow 8 - 10$ doable

$J_\mu = \sum_{i=2}^{n-1} \frac{1}{i+2} V_3 + \sum_{j=2}^{n-2} \frac{1}{j+2} V_4$
Tree-level matrix elements: state-of-the-art

LHC cross sections \([\sqrt{s} = 14 \text{ TeV}, \text{generic cuts}]\) [Gleisberg, Höche JHEP 0812 (2008) 039]

<table>
<thead>
<tr>
<th>Process + n QCD jets</th>
<th>Order</th>
<th>Number of jets</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^+ \nu_e ) [pb]</td>
<td>(\alpha^2_{EM} \alpha^n_S)</td>
<td>5434(5)</td>
</tr>
<tr>
<td>(e^+ e^- ) [pb]</td>
<td>(\alpha^2_{EM} \alpha^n_S)</td>
<td>723.5(4)</td>
</tr>
<tr>
<td>(e^+ e^- + b\bar{b} ) [pb]</td>
<td>(\alpha^2_{EM} \alpha^{n+2}_S)</td>
<td>18.86(3)</td>
</tr>
<tr>
<td>(t\bar{t} ) [pb]</td>
<td>(\alpha^{n+2}_S)</td>
<td>754.8(8)</td>
</tr>
<tr>
<td>(b\bar{b} ) [(\mu b)]</td>
<td>(\alpha^{n+2}_S)</td>
<td>471.2(5)</td>
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<tr>
<td>(\text{pure jets} ) [(\mu b)]</td>
<td>(\alpha^n_S)</td>
<td>–</td>
</tr>
</tbody>
</table>

**parton-level event generation**

- fully differential \([\{\rho_{\text{part}}\} \rightarrow \text{any distribution can be studied}]\)
- full momentum/spin correlations
- theory/pheno analyses
- limited in final-state multiplicity \([\text{factorial growth of Feynman diagrams / phase-space maps}]\)
- not directly observable / measurable
- Leading-Order dependence on \(\mu_F^2 \& \mu_R^2\)

\(\Rightarrow\) NLO QCD precision desirable
The need for higher precision

- derive exclusion bounds from a given measurement
- non-observation of a given hypothesis, e.g. light Higgs, BSM model
- crucial to know the corresponding production cross sections precisely
- derive limits on particle masses, model parameters
- counting experiments to extract certain parameters, couplings

stop mass bounds from Tevatron

[from the early days, T. Plehn private communication]

Higgs coupling extraction

- $\sigma(gg \to h \to \gamma\gamma) \sim \frac{\Gamma_g \Gamma_\gamma}{\Gamma}$
- $\sigma(gg \to h \to WW^*) \sim \frac{\Gamma_g \Gamma_W}{\Gamma}$
- $\sigma(qq \to qqh, h \to \tau\tau) \sim \frac{\Gamma_W \Gamma_\tau}{\Gamma}$
- $\sim \Gamma_i \sim g_{ihi}, \Gamma_g \sim g_{thh}$
- theoretical uncertainties dominate
- $\sim$ systematics reduced for $\sigma$ ratios
\[ \sigma_{2 \rightarrow n}^{NLO} = \int_{n+1} d(d) \sigma^R + \int_{n} d(d) \sigma^V + \int_{n} d(4) \sigma^B \]

- real-emission \( \sigma^R \) \( \sim \) IR divergent
- (UV renormalized) virtual-corrections \( \sigma^V \) \( \sim \) IR divergent

\( \sim \) for IR safe observables sum is finite
Hard Processes at Next-to-Leading Order QCD

Anatomy of NLO QCD calculations

\[ \sigma_{2 \to n}^{NLO} = \int_{n+1} d^{(d)} \sigma^R + \int_n d^{(d)} \sigma^V + \int_n d^{(4)} \sigma^B \]

- real-emission \( \sigma^R \sim IR \) divergent
- (UV renormalized) virtual-corrections \( \sigma^V \sim IR \) divergent
  \( \sim \) for IR safe observables sum is finite


\[ \sigma_{2 \to n}^{NLO} = \int_{n+1} \left[ d^{(4)} \sigma^R - d^{(4)} \sigma^A \right] + \int_n \left[ d^{(4)} \sigma^B + \int_{\text{loop}} d^{(d)} \sigma^V + \int_1 d^{(d)} \sigma^A \right]_{\epsilon=0} \]

- subtraction terms yield local approximation for the real emission process
- describe the amplitude in the soft & collinear limits \([1/\epsilon \ \text{and} \ 1/\epsilon^2 \ \text{poles}]\)

\[ \int_{n+1} d^{(d)} \sigma^A = \sum \int_n d^{(d)} \sigma^B \otimes \int_1 d^{(d)} V_{\text{dipole}} \]

\[ \xrightarrow{\text{spin- & color correlations}} \]

universal dipole terms

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Monte Carlo Event Generators for the LHC
The emerging picture

\[
\sigma_{2 \to n}^{\text{NLO}} = \int_{n+1} [d^{(4)} \sigma^R - d^{(4)} \sigma^A] + \int_n [d^{(4)} \sigma^B + \int_{\text{loop}} d^{(d)} \sigma^V + \int_1 d^{(d)} \sigma^A] \epsilon = 0
\]

Monte-Carlo codes
- all the tree-level bits
- subtraction of singularities*
- efficient phase-space integration

One-Loop codes
- Loop amplitudes, i.e. \(2\Re(A_V A_B^\dagger)\)
- Loop integration
\[\sim 1/\epsilon, 1/\epsilon^2\] coefficients & finite terms

* automated
Catani-Seymour subtraction in:
- MadDipole/MadGraph [Frederix, Gehrmann, Greiner JHEP 1006 (2010) 086]
- Helac [Czakon, Papdopoulos, Worek JHEP 0908 (2009) 085]

- MadFKS/MadGraph [Frederix et al. JHEP 0910 (2009) 003]
Example: $W + 3\text{jets} \ @ \ NLO$

- **real emission corrections:**

- **one-loop corrections:**
Hard Processes at Next-to-Leading Order QCD

Example: $W + 3\text{jets} @ \text{NLO}$

- real emission corrections:

- one-loop corrections:

recently calculated using BLACKHAT+SHERPA


- BLACKHAT: on-shell methods for one-loop amplitudes [arXiv:0808.0941]

- SHERPA:
  - real-emission processes
  - Catani–Seymour subtraction terms
  - phase-space integration

⇒ also completed: $Z/\gamma^* + 3\text{jets}$ [arXiv:1004.1659] & $W + 4\text{jets}$ [arXiv:1009.2338]
NLO QCD calculations: $W + 3\text{jets}$


- consider $W \rightarrow e\nu$ and SISCONE jets with $E_T^{\text{th-jet}} > 25$ GeV & $R=0.4$

<table>
<thead>
<tr>
<th># of jets</th>
<th>CDF</th>
<th>LO</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$53.5 \pm 5.6$</td>
<td>$41.40(0.02)^{+7.59}_{-5.94}$</td>
<td>$57.83(0.12)^{+4.36}_{-4.00}$</td>
</tr>
<tr>
<td>2</td>
<td>$6.8 \pm 1.1$</td>
<td>$6.159(0.004)^{+2.41}_{-1.58}$</td>
<td>$7.62(0.04)^{+0.62}_{-0.86}$</td>
</tr>
<tr>
<td>3</td>
<td>$0.84 \pm 0.24$</td>
<td>$0.796(0.001)^{+0.488}_{-0.276}$</td>
<td>$0.882(0.005)^{+0.057}_{-0.138}$</td>
</tr>
</tbody>
</table>

$\sqrt{s} = 1.96$ TeV

$\mu_R = \mu_F = E_T^W$

$E_T > 20$ GeV, $|\eta| < 2$

$E_T > 20$ GeV, $|\eta| < 1.1$

$E_T > 30$ GeV, $M_T^W > 20$ GeV

$R = 0.4$ [siscone]
NLO QCD calculations: $W + n$-jets

**BLACKHAT + SHERPA:** LHC predictions for $\sqrt{s} = 7$ TeV [arXiv:1009.2338]

<table>
<thead>
<tr>
<th>$\sigma_{pp \rightarrow e^- \nu_e + n-jets}$</th>
<th>Number of jets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{LO}$ [pb]</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_{NLO}$ [pb]</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma_{NLO}$ [pb]</td>
<td>3</td>
</tr>
<tr>
<td>$\sigma_{NLO}$ [pb]</td>
<td>4</td>
</tr>
</tbody>
</table>

| $\sigma_{LO}$ [pb]                      | 264.4(0.2)$^{+22.6}_{-21.4}$ |
| $\sigma_{NLO}$ [pb]                     | 331(1)$^{+15}_{-12}$         |
| $\sigma_{LO}$ [pb]                      | 73.14(0.09)$^{+20.81}_{-14.92}$ |
| $\sigma_{NLO}$ [pb]                     | 78.1(0.5)$^{+1.5}_{-4.1}$   |
| $\sigma_{LO}$ [pb]                      | 17.22(0.03)$^{+8.07}_{-4.95}$ |
| $\sigma_{NLO}$ [pb]                     | 16.9(0.1)$^{+0.2}_{-1.3}$   |
| $\sigma_{LO}$ [pb]                      | 3.81(0.01)$^{+2.44}_{-1.34}$ |
| $\sigma_{NLO}$ [pb]                     | 3.56(0.03)$^{+0.08}_{-0.30}$ |

### Graphical Representation

- **$W^-$ + 4 jets + X**
- **$\sqrt{s} = 7$ TeV**

- $p_T^{jet} > 25$ GeV, $|\eta^{jet}| < 3$
- $E_T^e > 20$ GeV, $|\eta^e| < 2.5$
- $E_T^\nu > 20$ GeV, $M_W > 20$ GeV
- $R = 0.5$ [anti-$k_T$]

- **BlackHat + Sherpa**
- **LO / NLO**
- **NLO scale dependence**
- **LO scale dependence**

**Steffen Schumann**
Monte Carlo Event Generators for the LHC
Automated tools to generate and evaluate tree-level amplitudes
- compact matrix-element expressions
- efficient phase-space integrators

Enormous progress in automating NLO calculations
- modularity of the problem
- automated subtraction terms + improved loop methods
- $W/Z + 3\text{jets}, \ W + 4\text{jets}$ with BLACKHAT+SHERPA
- $t\bar{t} + 2\text{jets}$ with HELAC-NLO [Bevilacqua et al. Phys. Rev. Lett. 104 (2010) 162002]
- $t\bar{t}b\bar{b}$ with HELAC-NLO [Bevilacqua et al. JHEP 0909 (2009) 109]
- $e^+e^- \rightarrow 5\text{jets}$ with MadFKS/MadGraph [Frederix et al. arXiv:1008.5313 [hep-ph]]
Wanna reality check???