

# Development of A4 Compton Polarimeter Chicane

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## Two main goals

- ▶ does not disturb the A4 main parity-violating experiment
- ▶ does handle the electron beam well to measure the electron polarization

## Outline

- ▶ A4 experiment and Compton polarimeter
- ▶ chicane optimization with the simulations and wire scanners
- ▶ two Synchrotron radiation effects
- ▶ summary and outlook

### A4 Parity-Violating Experiment

- ▶ the strangeness contribution to the proton ( $F_{1,2}^s$  or  $G_{E,M}^s$ ).
- ▶ the parity-violating asymmetry for elastic electron scattering off an unpolarized proton.

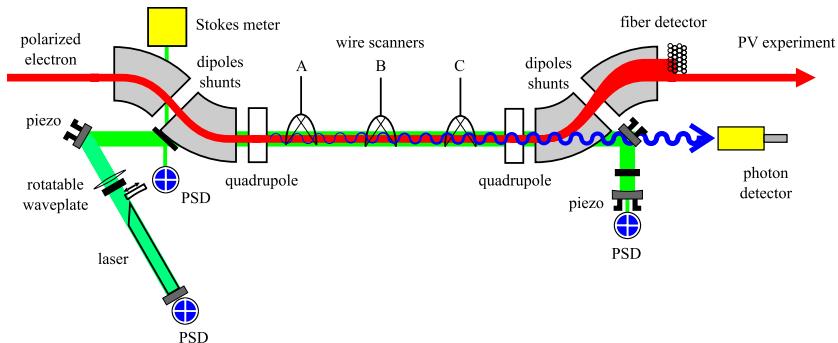
$$A_{\text{measured}} = \frac{N^+ - N^-}{N^+ + N^-} = \mathbf{P} \cdot \mathbf{A}_{\text{phys}}.$$

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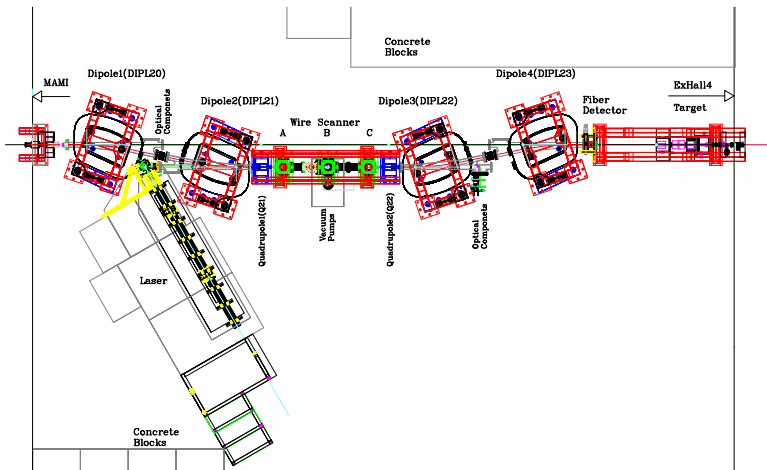
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## Compton Polarimeter



[Back](#)

# A4 Experiment and Compton Polarimeter



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- ▶ to make a good overlap between the electron beam and the laser

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- ▶ Wire scanner analysis  $\leftrightarrow$  TRANSPORT simulation

## Simulation

- ▶ TRANSPORT is based on a matrix formalism in order to design a static-magnetic beam transport systems. its web directory is <ftp://ftp.fnal.gov/pub/transport>
- ▶ How close to the real chicane is the simulation chicane?

## Simulation

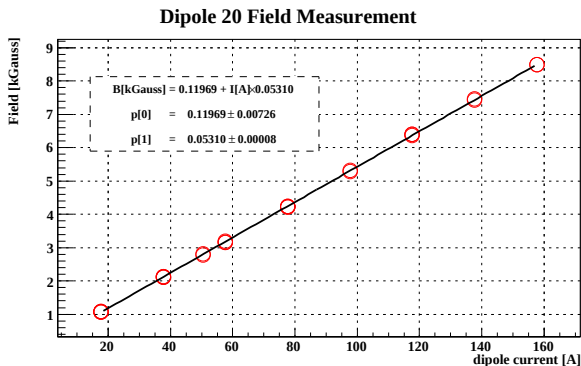
- ▶ TRANSPORT is based on a matrix formalism in order to design a static-magnetic beam transport systems. its web directory is `ftp://ftp.fnal.gov/pub/transport`
- ▶ How close to the real chicane is the simulation chicane?  
→ As possible as we can!

## dipole magnet - input

- ▶ effective length  $l_{\text{eff}}$  is used from the MIT collaborator
- ▶ TRANSPORT can handle the fringing field
  - ▶ measure the pole gap  $g$
  - ▶ measure the entrance and exit pole-face rotation angle
  - ▶ assume that the dipole to be a square-edged non-saturating magnet
- ▶ measure the magnetic field strengths as a function of the applied current

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  - ▶ assume that the dipole to be a square-edged non-saturating magnet
- ▶ measure the magnetic field strengths as a function of the applied current
- ▶ consider the current instabilities (below 3ppm)
  - magnetic field uncertainty per each dipole magnet
- ▶ shunt per each dipole is used to change magnetic field slightly

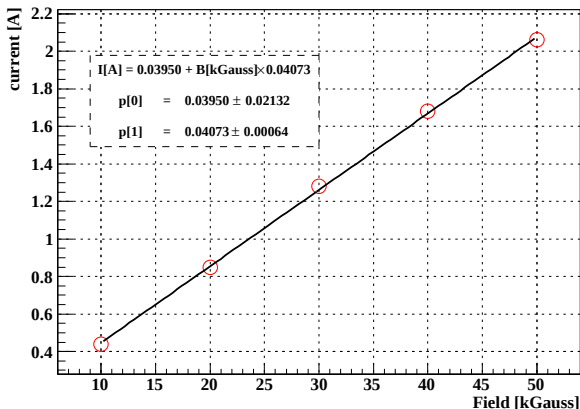
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Quadrupole 21 Field Measurement



## quadrupole magnet - input

- ▶ measure the magnetic field strengths as a function of the applied current (by J.Müller)
- ▶ measure the aperture radius  $a$
- ▶ measure the mechanical length  $l_m$
- ▶ effective length  $l_{\text{eff}}$

$$l_{\text{eff}} = l_m + c \cdot a$$

, where  $c$  is a constant varying between 0.8 and 1.1. we use the average value 0.95.

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- ▶ no device to measure the beam position, direction, and size at the beginning of the chicane.
- ▶ one solution about the initial beam position and direction
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→ with wire scanner measurement and simulation
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- ▶ no device to measure the beam position, direction, and size at the beginning of the chicane.
- ▶ one solution about the initial beam position and direction  
→ after wire scanner analysis
- ▶ one assumption about beam size  
→ the chicane has a periodic structure (best condition)

## periodic structure

- ▶ a region in phase space from the beginning to the end of the chicane with no first-order change in properties
- ▶ the same initial and final Twiss parameters  $(\beta, \alpha)$
- ▶ a waist condition in the middle of the chicane

## periodic structure

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→ the initial Twiss parameters
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## periodic structure

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- ▶ the same initial and final Twiss parameters  $(\beta, \alpha)$ 
  - the initial Twiss parameters
- ▶ a waist condition in the middle of the chicane
  - Rayleigh length of the electron beam

## initial Twiss parameters - periodic structure

The transfer matrix of the chicane by

$$M_{\text{chicane}} = M_{\text{dipole}_{23}} \cdot M_{\text{d}_4} \cdot M_{\text{dipole}_{22}} \cdot M_{\text{d}_3} \cdot M_{\text{quad}_{22}} \\ \cdot M_{\text{d}_2} \cdot M_{\text{quad}_{21}} \cdot M_{\text{d}_1} \cdot M_{\text{dipole}_{21}} \cdot M_{\text{d}_0} \cdot M_{\text{dipole}_{20}}$$

## initial Twiss parameters - periodic structure

The transfer matrix of the chicane by

$$M_{\text{chicane}} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}_{\text{components}}$$

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$$\begin{aligned} M_{\text{chicane}} &= \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}_{\text{components}} \\ &\equiv \begin{pmatrix} \cos \mu + \alpha \sin \mu & \beta \sin \mu \\ -\gamma \sin \mu & \cos \mu - \alpha \sin \mu \end{pmatrix}_{\text{periodic}} \end{aligned}$$

## initial Twiss parameters - periodic structure

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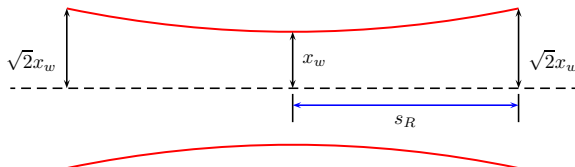
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Thus

$$\mu = \cos^{-1} \left( \frac{R_{11} + R_{22}}{2} \right), \quad \alpha = \frac{R_{11} - R_{22}}{2 \sin \mu}, \quad \beta = \frac{R_{12}}{\sin \mu}, \quad \text{and} \quad \gamma = -\frac{R_{21}}{\sin \mu}$$

With  $\alpha$  and  $\beta$ , TRANSPORT calculates the beam size and the divergence along the chicane.

## Rayleigh length - periodic structure



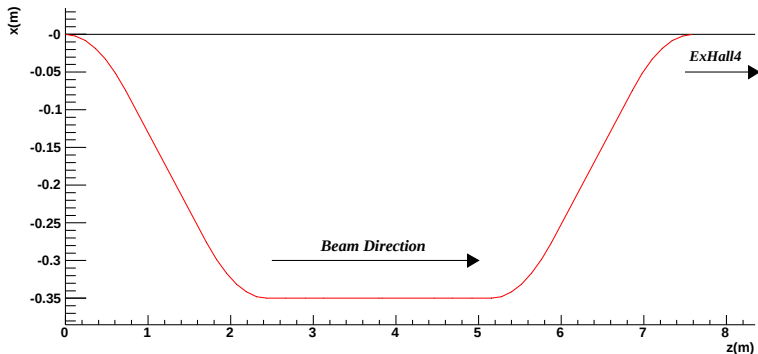
$$x(s_R) = \sqrt{2}x_w, \text{ where } x = \sqrt{\epsilon\beta} \quad \beta(s) = \beta_0 - 2\alpha_0 s + \gamma_0 s^2$$

$$\beta_R = 2\beta_w \quad \beta_R = \beta_w + \frac{s_R^2}{\beta_w}$$

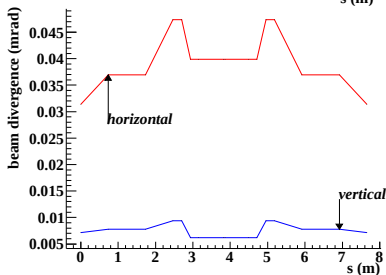
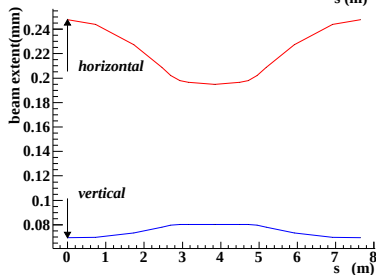
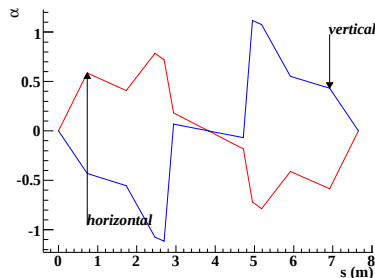
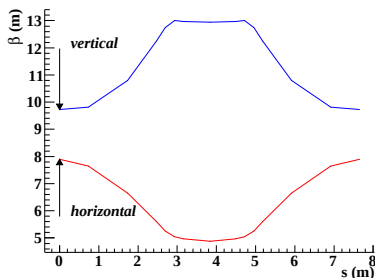
$$\therefore s_R = \pm\beta_w$$

## One result at 854.3 MeV

Ideal reference trajectory on the horizontal plane FL



## Twiss parameters - periodic structure



dispersion  $D$  and  $D'$ 

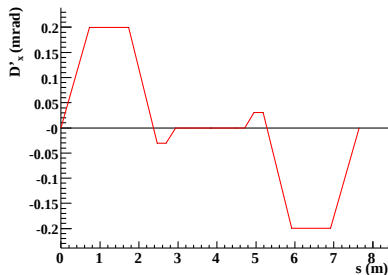
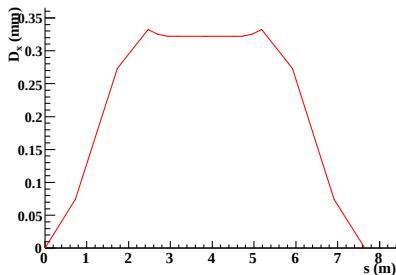
- ▶ Since the chicane has four dipole magnets, the position dispersion and the angular dispersion will be introduced.

$$\begin{pmatrix} x_f \\ x'_f \\ \delta \end{pmatrix}_{\text{end}} = \begin{pmatrix} R_{11} & R_{12} & \mathbf{D} \\ R_{21} & R_{22} & \mathbf{D}' \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ \delta \end{pmatrix}_{\text{begin}}$$

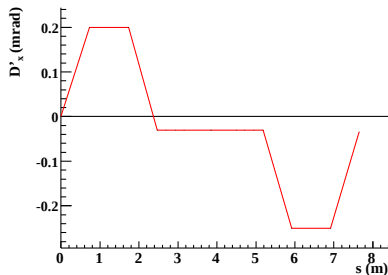
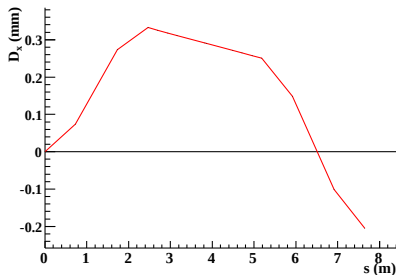
$$x_f = R_{11}x_0 + R_{12}x'_0 + \mathbf{D}\delta, \quad x'_f = R_{21}x_0 + R_{22}x'_0 + \mathbf{D}'\delta$$

- ▶ We should minimize  $\mathbf{D}$  and  $\mathbf{D}'$  by two quadrupoles
- ▶ Quadrupole field strengths are one of simulation results

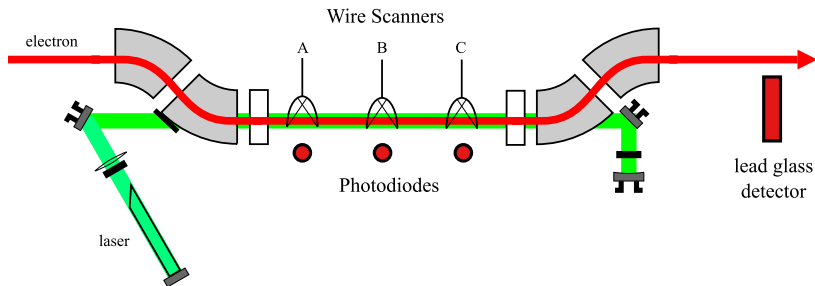
position and angular dispersion with      quadrupoles



position and angular dispersion without quadrupoles



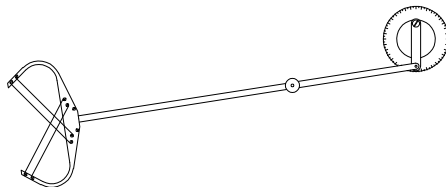
## Wire Scanner Analysis



- ▶ to make good overlap
- ▶ electron and laser position measurement at the same time

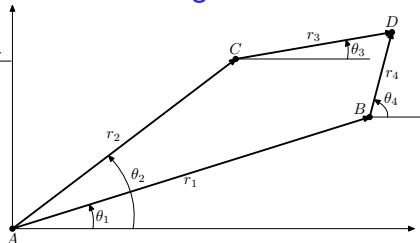
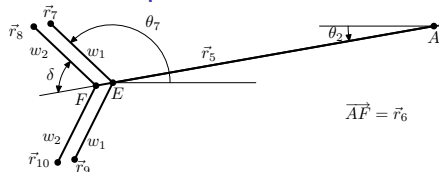
## MAMI wire scanner

- ▶ Made in Germany by MAMI
- ▶ old analysis based on the geometry



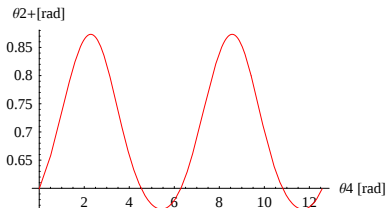
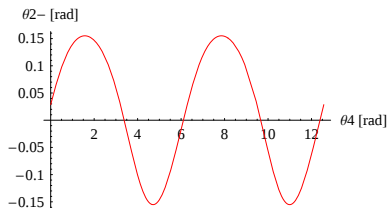
extended part

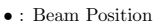
four-bar linkage



loop equation

From  $\vec{r}_2 + \vec{r}_3 = \vec{r}_1 + \vec{r}_4$ , we found  $\theta_2(\theta_4)$  function [Back](#)

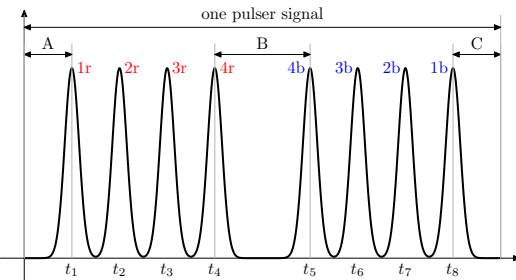




$$\theta_5 = \theta_2 + \pi$$

$$\beta = |\theta_2^r - \theta_2^b|/2$$

## Measured spectrum



- ▶  $\theta_4^{\text{raw}}[t_i] = A\omega \cdot t_i$ , where  $A\omega = 2\pi / |\text{pulser}_0 - \text{pulser}_1|$
- ▶  $A + C > B$  : up-motion  $\theta_2$   
 $t_1 \rightarrow 1r, t_2 \rightarrow 2r, t_3 \rightarrow 3r, t_4 \rightarrow 4r \dots$
- ▶  $A + C < B$  : down-motion  
 $t_1 \rightarrow 4b, t_2 \rightarrow 3b, t_3 \rightarrow 2b, t_4 \rightarrow 1b \dots$

How to know the  $\theta_4$ 

| wire  | 1     | 2     | 3     | 4     |
|-------|-------|-------|-------|-------|
| $t_a$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ |
| $t_b$ | $t_8$ | $t_7$ | $t_6$ | $t_5$ |

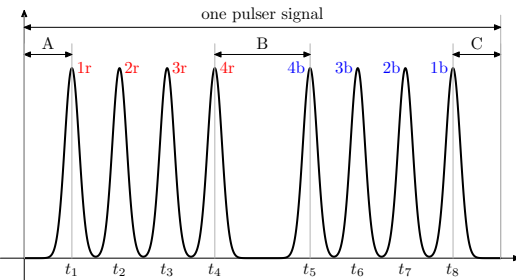
A wire goes into the beam twice with the same  $\theta_2$  and the different  $\theta_4$

$$\theta_4^{\text{raw}}[t_i] = \theta_4[t_i] + \theta_{\text{offset}}$$

$$\theta_4[t_b] = \theta_4[t_a] + \theta_4^{\text{raw}}[t_b] - \theta_4^{\text{raw}}[t_a]$$

$$\theta_2(\theta_4[t_a]) = \theta_2(\theta_4[t_a] + \theta_4^{\text{raw}}[t_b] - \theta_4^{\text{raw}}[t_a])$$

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$$\theta_2(\theta_4[t_a]) = \theta_2(\theta_4[t_a] + \theta_4^{\text{raw}}[t_b] - \theta_4^{\text{raw}}[t_a])$$

full motion of the wire scanner

## initial position and angle

$$x_0, x'_0, y_0, \text{ and } y'_0 = ?$$

Using matrix formalism with wire scanner measurement,

$$\begin{pmatrix} x_B \\ x'_B \end{pmatrix} = M_{BA} \cdot \begin{pmatrix} x_A \\ x'_A \end{pmatrix}$$
$$\begin{pmatrix} x_A \\ x'_A \end{pmatrix} = M_{A0} \cdot \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

## initial position and angle

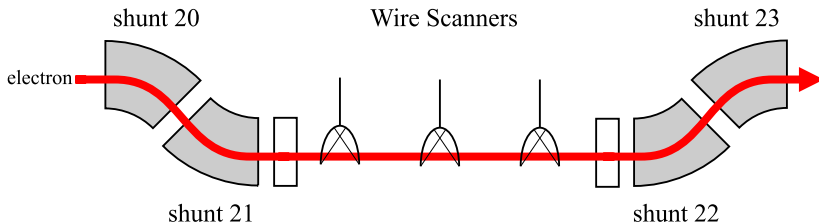
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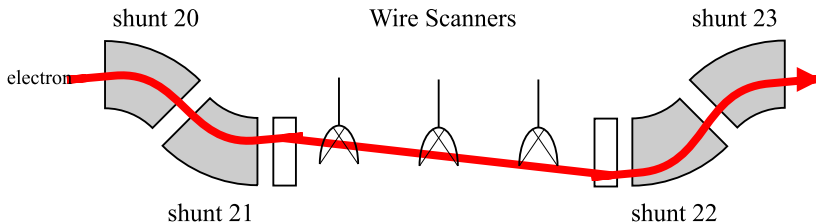
$x_0, x'_0, y_0, \text{ and } y'_0$  are the input parameters in the simulation

## rotating electron beam



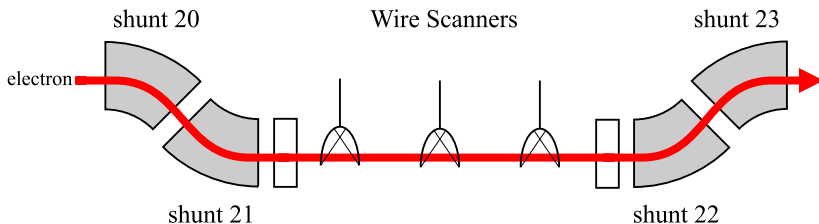
| shunt   | 20 | 21 | 22 | 23 |
|---------|----|----|----|----|
| percent | 50 | 50 | 50 | 50 |

## rotating electron beam



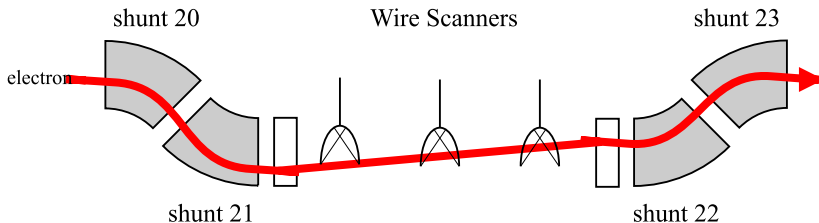
| shunt              | 20        | 21         | 22         | 23        |
|--------------------|-----------|------------|------------|-----------|
| percent            | 60        | 70         | 30         | 40        |
| $\curvearrowright$ | $+\alpha$ | $+2\alpha$ | $-2\alpha$ | $-\alpha$ |

## rotating electron beam



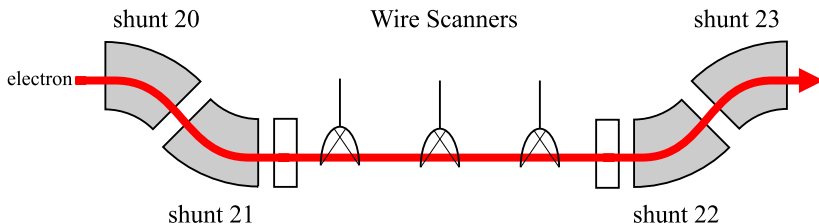
| shunt              | 20        | 21         | 22         | 23        |
|--------------------|-----------|------------|------------|-----------|
| percent            | 50        | 50         | 50         | 50        |
| $\curvearrowright$ | $-\alpha$ | $-2\alpha$ | $+2\alpha$ | $+\alpha$ |

## rotating electron beam



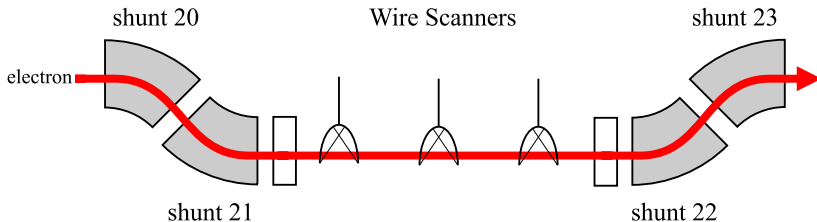
| shunt              | 20        | 21         | 22         | 23        |
|--------------------|-----------|------------|------------|-----------|
| percent            | 40        | 30         | 70         | 60        |
| $\curvearrowright$ | $-\alpha$ | $-2\alpha$ | $+2\alpha$ | $+\alpha$ |

## rotating electron beam



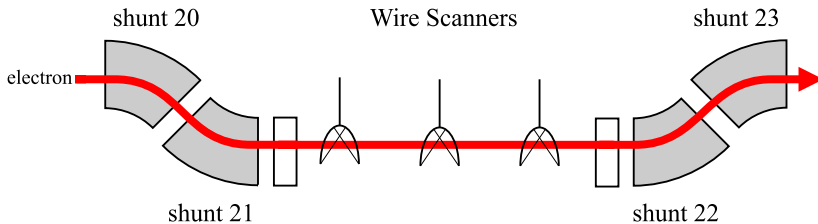
| shunt              | 20        | 21         | 22         | 23        |
|--------------------|-----------|------------|------------|-----------|
| percent            | 50        | 50         | 50         | 50        |
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## shifting electron beam



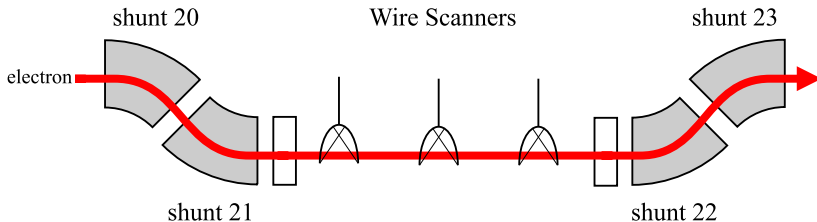
| shunt   | 20 | 21 | 22 | 23 |
|---------|----|----|----|----|
| percent | 50 | 50 | 50 | 50 |

## shifting electron beam



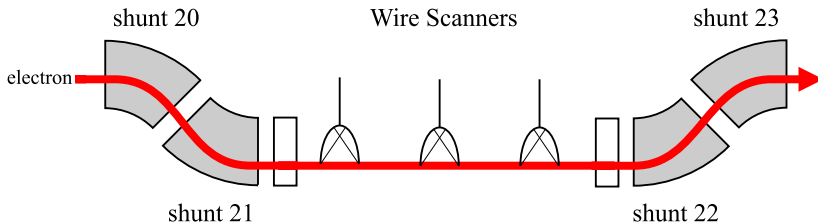
| shunt   | 20        | 21        | 22        | 23        |
|---------|-----------|-----------|-----------|-----------|
| percent | 60        | 60        | 60        | 60        |
| ↑       | $+\alpha$ | $+\alpha$ | $+\alpha$ | $+\alpha$ |

## shifting electron beam



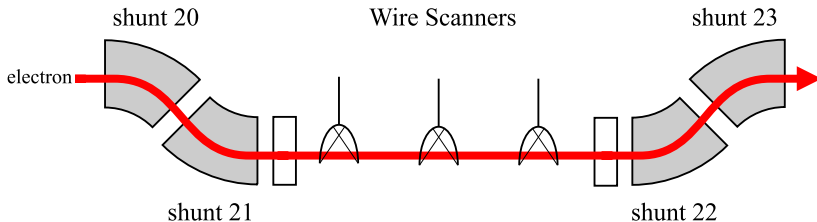
| shunt   | 20        | 21        | 22        | 23        |
|---------|-----------|-----------|-----------|-----------|
| percent | 50        | 50        | 50        | 50        |
| ↓       | $-\alpha$ | $-\alpha$ | $-\alpha$ | $-\alpha$ |

## shifting electron beam



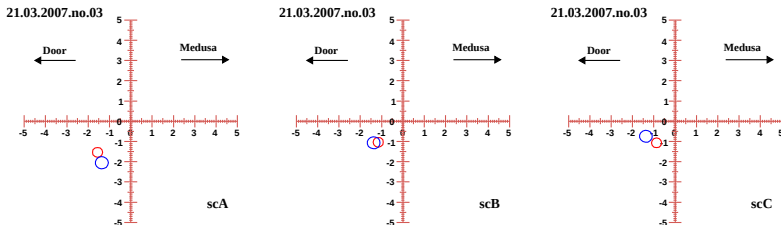
| shunt   | 20        | 21        | 22        | 23        |
|---------|-----------|-----------|-----------|-----------|
| percent | 40        | 40        | 40        | 40        |
| ↓       | $-\alpha$ | $-\alpha$ | $-\alpha$ | $-\alpha$ |

## shifting electron beam



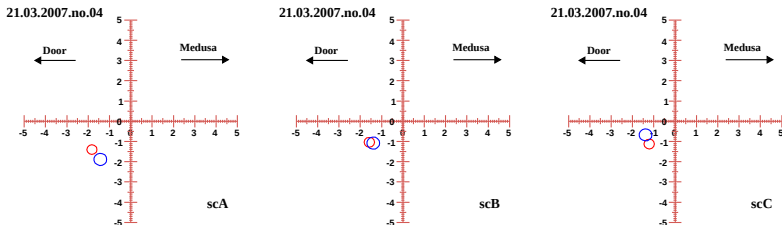
| shunt   | 20        | 21        | 22        | 23        |
|---------|-----------|-----------|-----------|-----------|
| percent | 50        | 50        | 50        | 50        |
| ↑       | $+\alpha$ | $+\alpha$ | $+\alpha$ | $+\alpha$ |

## Spatial overlap



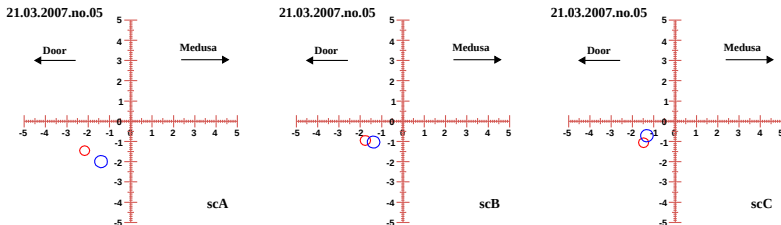
- ▶ decide the good overlap or not
- ▶ TRANSPORT simulation (input:  $I_{\text{main}}$ ,  $I_{\text{shunt}}$ , scA, scB)
- ▶ shunt and current optimization → ask MAMI operator

## Spatial overlap



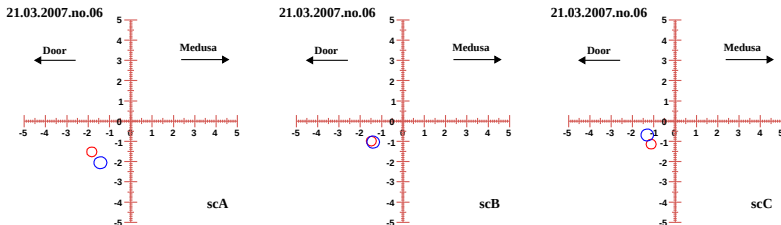
- ▶ decide the good overlap or not
- ▶ TRANSPORT simulation (input:  $I_{\text{main}}$ ,  $I_{\text{shunt}}$ , scA, scB)
- ▶ shunt and current optimization → ask MAMI operator

## Spatial overlap



- ▶ decide the good overlap or not
- ▶ TRANSPORT simulation (input:  $I_{\text{main}}$ ,  $I_{\text{shunt}}$ , scA, scB)
- ▶ shunt and current optimization → ask MAMI operator

## Spatial overlap



- ▶ the good overlap
- ▶ TRANSPORT simulation (input:  $I_{\text{main}}^{\text{opt}}$ ,  $I_{\text{shunt}}^{\text{opt}}$ , scA, scB)
- ▶ two quadrupole field strengths  $\rightarrow$  ask MAMI operator

## Synchrotron Radiation

- ▶ motivation, aim, and assumptions
- ▶ overview of SR
- ▶ energy loss inside the chicane
- ▶ SR on the photon detector

## Motivation

- ▶ One bending magnet is the most simply radiation source. And four bending magnets define the geometry of the electron chicane.
- ▶ A viewpoint of electron: How much will electron energy loss be inside the chicane? Is there some effect of the energy loss on PV experiment?
- ▶ A viewpoint of radiation: We estimated Synchrotron Radiation(SR) effect could be ignored or be too small. It would be better to know SR effect on the photon detector exactly.

## Aim and Assumption

- ▶ To calculate the electron energy loss inside the Compton polarimeter chicane
- ▶ To determine that the effect on the photon detector will be crucial or not.
- ▶ no quadrupole effect, perfect vacuum condition, and midplane symmetry inside four dipole magnets.

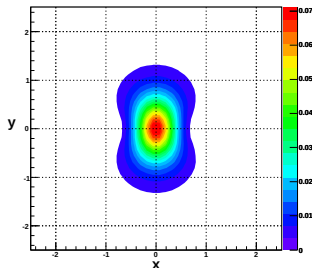
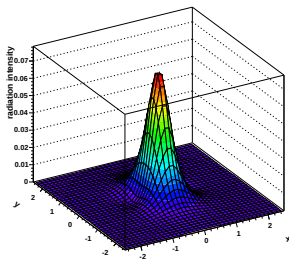
Power radiated of an electron  $P_0$ 

$$P_0 = \frac{2}{3} r_e m c^3 \frac{\gamma^4 \beta^4}{\rho^2}$$

Spatial Distribution  $\frac{dP}{d\Omega}$ 

$$x = \gamma \sin \theta_\gamma \cos \phi$$

$$y = \gamma \sin \theta_\gamma \sin \phi$$



## Critical Photon Energy $\varepsilon_c$

- ▶  $\varepsilon_c = \hbar\omega_c = \hbar\frac{3c\gamma^3}{2\rho}$
- ▶ the upper bound for the synchrotron radiation spectrum

## Photon or Quantum Distribution Function $\dot{n}(\varepsilon)$

- ▶ Number of quanta emitted per unit time at energy  $\varepsilon$ .
- ▶  $\dot{n}(\varepsilon) = \frac{P_0}{\varepsilon_c^2} F(\varepsilon/\varepsilon_c) = \frac{P_0}{\varepsilon\varepsilon_c} S(\varepsilon/\varepsilon_c)$ .
- ▶  $S(\xi) = \frac{9\sqrt{3}}{8\pi} \xi \int_{\xi}^{\infty} K_{5/3}(s) ds$ , where  $K_{5/3}(s)$  is a modified Bessel function of the second kind.

## energy loss of an electron per a magnet

$$U_0 = \int P_0 dt \approx \frac{C_0 E^4}{2\pi \rho^2} L_{\text{eff}}$$

,where  $L_{\text{eff}}$  is the effective length of the dipole magnet and  $C_0$  Sand's radiation constant.

## energy loss of a chicane

energy loss due to SR is below 2 keV

| Electron energy (MeV) | Energy loss of a dipole magnet (keV) | Energy loss of four dipole magnets (keV) | After chicane electron energy (MeV) |
|-----------------------|--------------------------------------|------------------------------------------|-------------------------------------|
| 854.30(16)            | 0.42129(39)                          | 1.6852(13)                               | 854.29958(16000)                    |
| 570.30(16)            | 0.083667(94)                         | 0.33467(38)                              | 570.29992(16000)                    |
| 315.13(16)            | 0.007800(16)                         | 0.0312000(63)                            | 315.12999(16000)                    |

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| 570.30(16)            | 0.083667(94)                         | 0.33467(38)                              | 570.30(16)                          |
| 315.13(16)            | 0.007800(16)                         | 0.0312000(63)                            | 315.13(16)                          |

four-momentum transfer  $Q^2$ 

$$Q^2 = 4EE' \sin^2 \frac{\theta}{2}, \quad \delta Q^2 = Q^2 \sqrt{\left(\frac{\delta E}{E}\right)^2 + \left(\frac{\delta E'}{E'}\right)^2}, \quad \text{and} \quad \delta E' = \frac{E'^2}{E^2} \delta E$$

|                            |              |              |
|----------------------------|--------------|--------------|
| $E$ (GeV)                  | 0.85430(16)  | 0.85430(16)  |
| Energy Loss                | NO           | YES          |
| $\theta_M$ (Ring 1)        | 39.22°       |              |
| $E'$ (GeV)                 | 0.70890(11)  | 0.70889(11)  |
| $Q^2$ (GeV/c) <sup>2</sup> | 0.272859(66) | 0.272858(66) |

all  $Q^2$

four-momentum transfer  $Q^2$ 

$$Q^2 = 4EE' \sin^2 \frac{\theta}{2}, \quad \delta Q^2 = Q^2 \sqrt{\left(\frac{\delta E}{E}\right)^2 + \left(\frac{\delta E'}{E'}\right)^2}, \quad \text{and} \quad \delta E' = \frac{E'^2}{E^2} \delta E$$

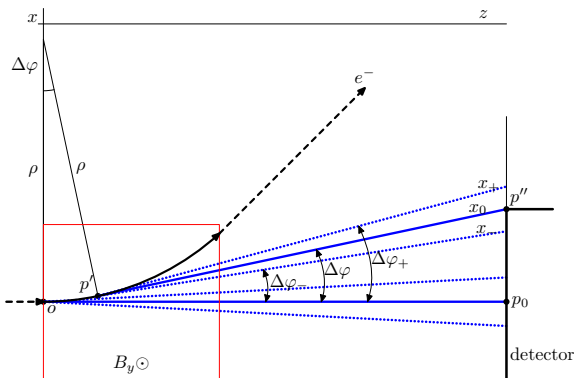
- ▶ However, after averaging  $Q^2$  with two significant figures  
→ the same  $Q^2$  and the same  $\delta Q^2$

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- ▶ However, after averaging  $Q^2$  with two significant figures  
→ the same  $Q^2$  and the same  $\delta Q^2$
- ▶ We can ignore the SR effect from the viewpoint of PV experiment.

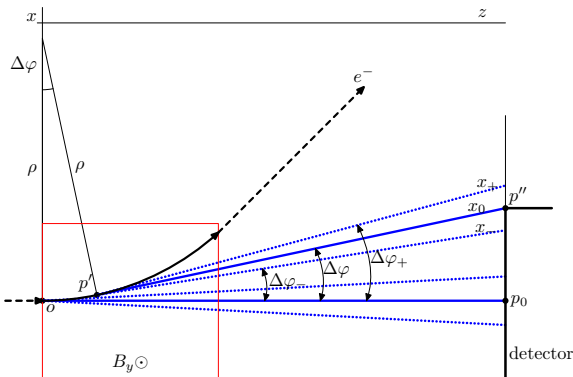
angle coverage  $\Delta\varphi$



$$x_0 = z_0 \tan \Delta\varphi^{\max} + \rho (1 - \sec \Delta\varphi^{\max})$$

$$x_{\pm} = z_0 \tan \Delta\varphi_{\pm}^{\max} + \rho (1 - \cos \theta_{\gamma} \sec \Delta\varphi_{\pm}^{\max})$$

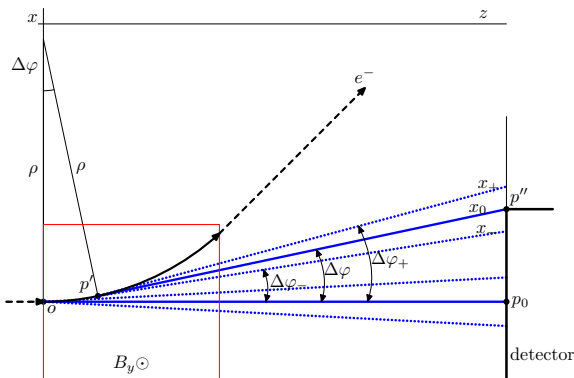
angle coverage  $\Delta\varphi$



$$x_0 = z_0 \tan \Delta\varphi^{max} + \rho (1 - \sec \Delta\varphi^{max})$$

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angle coverage  $\Delta\varphi$



$$x_0 = z_0 \tan \Delta\varphi^{max} + \rho (1 - \sec \Delta\varphi^{max})$$

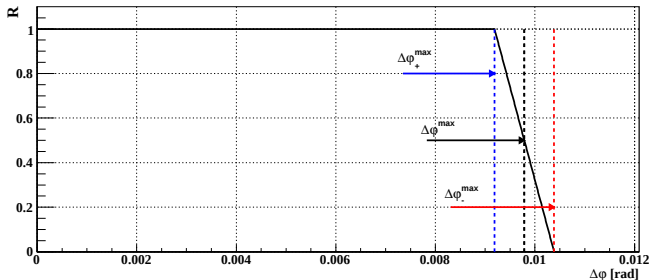
$$x_{\pm} = z_0 \tan \Delta\varphi_{\pm}^{max} + \rho (1 - \cos \theta_{\gamma} \sec \Delta\varphi_{\pm}^{max})$$

| Energy (GeV) | $\Delta\varphi_{+}^{max}$ (mrad) | $\Delta\varphi^{max}$ (mrad) | $\Delta\varphi_{-}^{max}$ (mrad) |
|--------------|----------------------------------|------------------------------|----------------------------------|
| 0.85430      | 9.18775                          | 9.78613                      | 10.38405                         |
| 0.57030      | 8.88958                          | 9.78613                      | 10.68162                         |
| 0.31513      | 8.16285                          | 9.78613                      | 11.40595                         |

$$R = \frac{\mathcal{A}}{|x_+ - x_-|}$$

,where

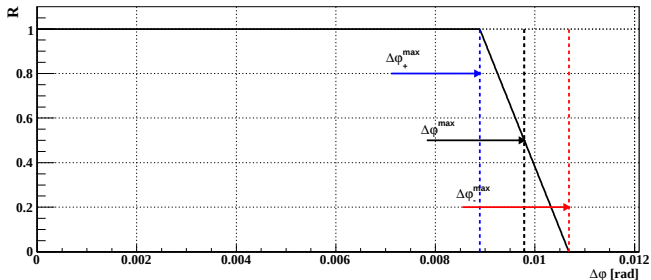
$$\mathcal{A} = \begin{cases} |x_+ - x_-| & 0 \leq \Delta\varphi < \Delta\varphi_+^{\max} \\ |x_0 - x_-| & \Delta\varphi_+^{\max} \leq \Delta\varphi < \Delta\varphi_-^{\max} \\ 0 & \Delta\varphi_-^{\max} \leq \Delta\varphi \end{cases}$$



$$R = \frac{\mathcal{A}}{|x_+ - x_-|}$$

,where

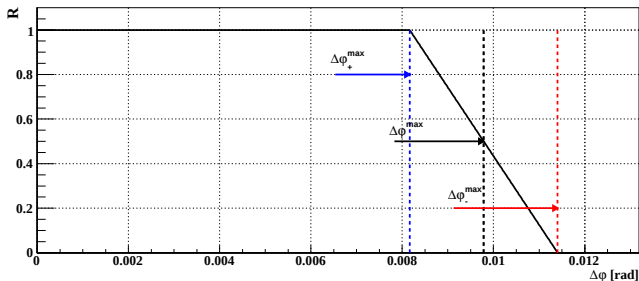
$$\mathcal{A} = \begin{cases} |x_+ - x_-| & 0 \leq \Delta\varphi < \Delta\varphi_+^{\max} \\ |x_0 - x_-| & \Delta\varphi_+^{\max} \leq \Delta\varphi < \Delta\varphi_-^{\max} \\ 0 & \Delta\varphi_-^{\max} \leq \Delta\varphi \end{cases}$$



$$R = \frac{\mathcal{A}}{|x_+ - x_-|}$$

,where

$$\mathcal{A} = \begin{cases} |x_+ - x_-| & 0 \leq \Delta\varphi < \Delta\varphi_+^{\max} \\ |x_0 - x_-| & \Delta\varphi_+^{\max} \leq \Delta\varphi < \Delta\varphi_-^{\max} \\ 0 & \Delta\varphi_-^{\max} \leq \Delta\varphi \end{cases}$$



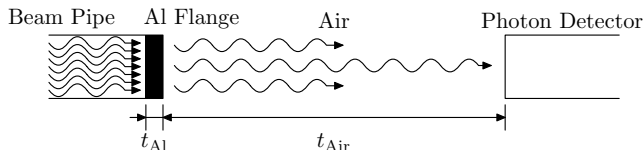
## total emitted photon number

After integration over all SR photon energy,

$$\dot{N}_{\varphi}^e = \int_0^{\infty} \dot{n}_{\varphi}^e(\varepsilon) d\varepsilon = \frac{15\sqrt{3}}{16\pi} \frac{P_0}{\varepsilon_c} \frac{I}{e} \Delta\varphi^{max}$$

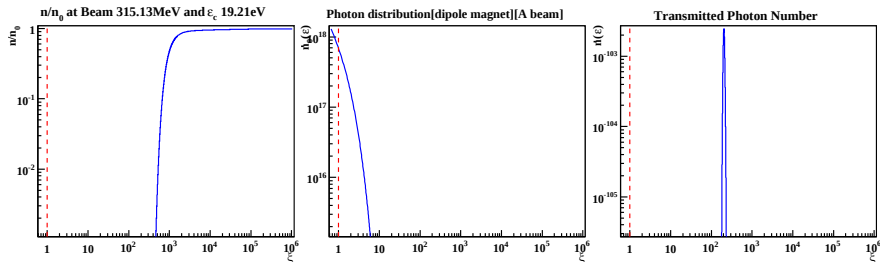
| Beam [MeV] | $\varepsilon_c$ (eV) | $\dot{N}_{\varphi}^e$ (rad/s) |
|------------|----------------------|-------------------------------|
| 854.30     | 382.74763            | $2.84211 \times 10^{20}$      |
| 570.30     | 113.86530            | $1.89729 \times 10^{20}$      |
| 315.13     | 19.21102             | $1.04838 \times 10^{20}$      |

transmitted photon number with a certain energy  $\epsilon$

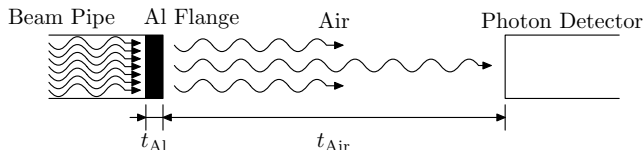


$$\dot{n}_{\phi}^e(\epsilon) = \dot{n}_{\phi 0}^e(\epsilon) \exp \left[ \frac{\mu}{\rho}(\epsilon)_{Al} \cdot x_{Al} + \frac{\mu}{\rho}(\epsilon)_{Air} \cdot x_{Air} \right]$$

where  $x = \rho t$  is the mass thickness and  $\mu/\rho$  a mass attenuation coefficient.

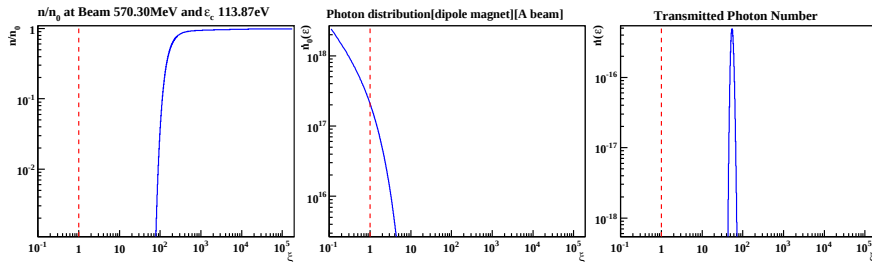


transmitted photon number with a certain energy  $\epsilon$

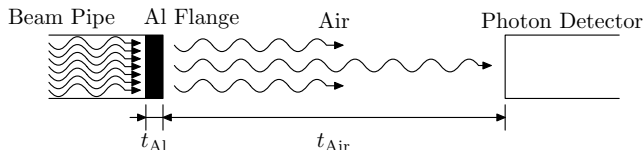


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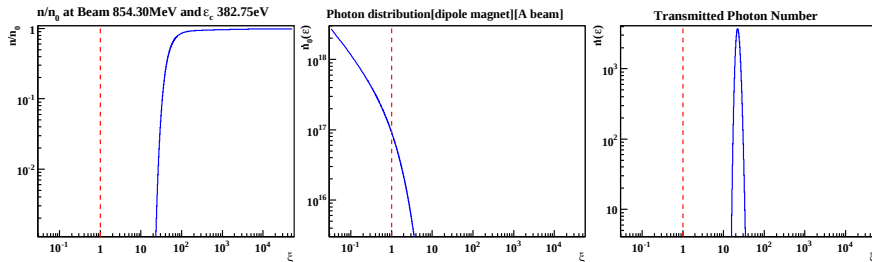


transmitted photon number with a certain energy  $\epsilon$



$$\dot{n}_{\varphi}^e(\epsilon) = \dot{n}_{\varphi 0}^e(\epsilon) \exp \left[ \frac{\mu}{\rho}(\epsilon)_{Al} \cdot x_{Al} + \frac{\mu}{\rho}(\epsilon)_{Air} \cdot x_{Air} \right]$$

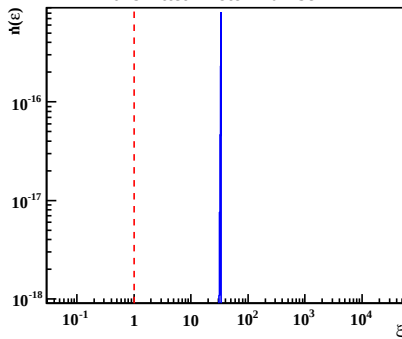
where  $x = \rho t$  is the mass thickness and  $\mu/\rho$  a mass attenuation coefficient.



|                               |                 |                 |                 |                 |                 |                 |
|-------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Al Thickness (mm)             | 0               | 1               | 2               | 3               | 4               | 10              |
| $\dot{n}_\varphi^e$ (maximum) | $\sim 10^3$     | $\sim 10^0$     | $\sim 10^{-2}$  | $\sim 10^{-4}$  | $\sim 10^{-5}$  | $\sim 10^{-10}$ |
| Pb Thickness (mm)             | 0.5             | 1               | 1.5             | 2               | 2.5             | 10              |
| $\dot{n}_\varphi^e$ (maximum) | $\sim 10^{-15}$ | $\sim 10^{-31}$ | $\sim 10^{-39}$ | $\sim 10^{-43}$ | $\sim 10^{-47}$ | $\sim 10^{-76}$ |

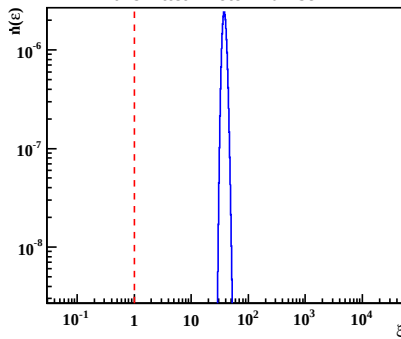
0.5mm pb

Transmitted Photon Number



5 mm aluminum

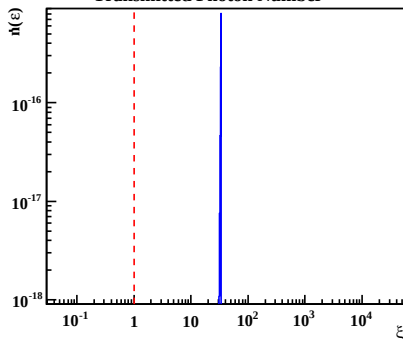
Transmitted Photon Number



|                               |                 |                 |                 |                 |                 |                 |
|-------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Al Thickness (mm)             | 0               | 1               | 2               | 3               | 4               | 10              |
| $\dot{n}_\varphi^e$ (maximum) | $\sim 10^3$     | $\sim 10^0$     | $\sim 10^{-2}$  | $\sim 10^{-4}$  | $\sim 10^{-5}$  | $\sim 10^{-10}$ |
| Pb Thickness (mm)             | 0.5             | 1               | 1.5             | 2               | 2.5             | 10              |
| $\dot{n}_\varphi^e$ (maximum) | $\sim 10^{-15}$ | $\sim 10^{-31}$ | $\sim 10^{-39}$ | $\sim 10^{-43}$ | $\sim 10^{-47}$ | $\sim 10^{-76}$ |

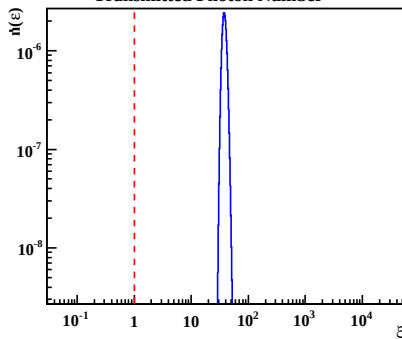
0.5mm pb

Transmitted Photon Number



5 mm aluminum

Transmitted Photon Number



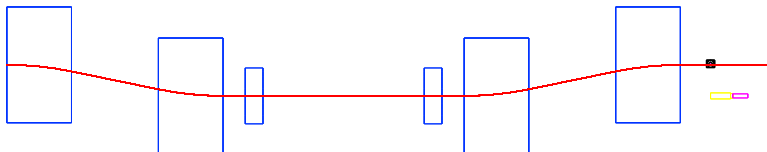
We can ignore the SR effect on the photon detector.

## Summary

- ▶ more physically realistic beam line simulation
- ▶ wire scanner analysis to overlap two beams
- ▶ synchrotron radiation effects on the beam line and the detector

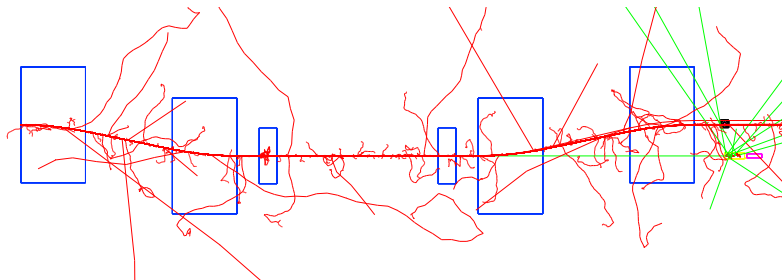
## Outlook

- ▶ improvement realistic beam line simulation with TRANSPORT  
→ more fancy way to tilt and shift the electron beam
- ▶ Using the results of TRANSPORT  
→ beam line and detector simulation with GEANT4

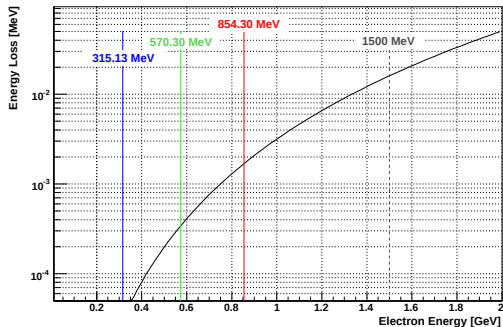


## Outlook

- ▶ improvement realistic beam line simulation with TRANSPORT  
→ more fancy way to tilt and shift the electron beam
- ▶ Using the results of TRANSPORT  
→ beam line and detector simulation with GEANT4



## Energy loss



| Electron energy (MeV) | Energy loss of a dipole magnet (keV) | Energy loss of four dipole magnets (keV) | After chicane electron energy (MeV) |
|-----------------------|--------------------------------------|------------------------------------------|-------------------------------------|
| 1500.00(16)           | 4.0041(17)                           | 16.0165(68)                              | 1499.99600(16000)                   |
| 854.30(16)            | 0.42129(39)                          | 1.6852(13)                               | 854.29958(16000)                    |
| 570.30(16)            | 0.083667(94)                         | 0.33467(38)                              | 570.29992(16000)                    |
| 315.13(16)            | 0.007800(16)                         | 0.0312000(63)                            | 315.12999(16000)                    |

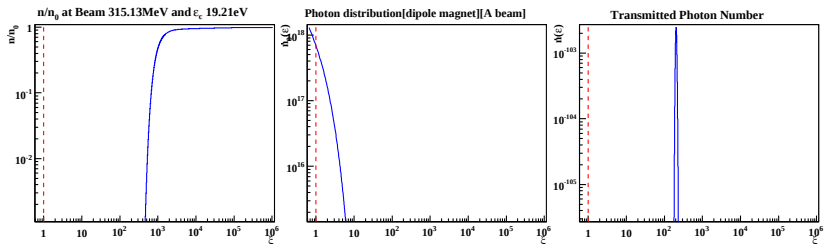
Q<sup>2</sup>

Back

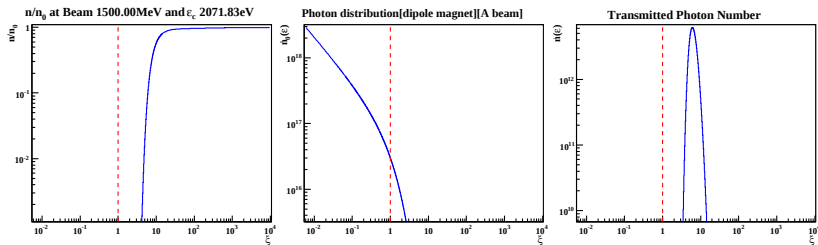
|                            |              |              |              |              |              |              |
|----------------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| E (GeV)                    | 0.85430(16)  | 0.85430(16)  | 0.57030(16)  | 0.57030(16)  | 0.31313(16)  | 0.31513(16)  |
| Energy Loss                | NO           | YES          | NO           | YES          | NO           | YES          |
| $\theta_M$ (Ring 1)        | 39.22°       |              | 39.22°       |              | 140.78°      |              |
| $E'$ (GeV)                 | 0.70890(11)  | 0.70889(11)  | 0.50162(12)  | 0.50162(12)  | 0.197442(63) | 0.197442(63) |
| $Q^2$ (GeV/c) <sup>2</sup> | 0.272859(66) | 0.272858(66) | 0.128890(48) | 0.128890(48) | 0.22085(13)  | 0.22085(13)  |
| $\theta_M$ (Ring 2)        | 37.69°       |              | 37.69°       |              | 142.31°      |              |
| $E'$ (GeV)                 | 0.71790(11)  | 0.71790(11)  | 0.50611(12)  | 0.50611(12)  | 0.196755(62) | 0.196755(62) |
| $Q^2$ (GeV/c) <sup>2</sup> | 0.255956(62) | 0.255956(62) | 0.120458(45) | 0.120458(45) | 0.22214(13)  | 0.22214(13)  |
| $\theta_M$ (Ring 3)        | 36.20°       |              | 36.20°       |              | 143.80°      |              |
| $E'$ (GeV)                 | 0.72659(12)  | 0.72659(12)  | 0.51041(13)  | 0.51041(13)  | 0.196111(62) | 0.196111(62) |
| $Q^2$ (GeV/c) <sup>2</sup> | 0.239650(59) | 0.239650(59) | 0.112383(42) | 0.112383(42) | 0.22334(13)  | 0.22334(13)  |
| $\theta_M$ (Ring 4)        | 34.77°       |              | 34.77°       |              | 145.23°      |              |
| $E'$ (GeV)                 | 0.73483(12)  | 0.73483(12)  | 0.51446(13)  | 0.51446(13)  | 0.195520(62) | 0.195520(62) |
| $Q^2$ (GeV/c) <sup>2</sup> | 0.224179(55) | 0.224176(55) | 0.104774(40) | 0.104774(40) | 0.22445(13)  | 0.22445(13)  |
| $\theta_M$ (Ring 5)        | 33.39°       |              | 33.39°       |              | 146.61°      |              |
| $E'$ (GeV)                 | 0.74269(12)  | 0.74269(12)  | 0.51830(13)  | 0.51830(13)  | 0.194971(61) | 0.194971(61) |
| $Q^2$ (GeV/c) <sup>2</sup> | 0.209448(52) | 0.209448(52) | 0.097577(37) | 0.097577(37) | 0.22548(13)  | 0.22548(13)  |
| $\theta_M$ (Ring 6)        | 32.06°       |              | 32.06°       |              | 147.94°      |              |
| $E'$ (GeV)                 | 0.75014(12)  | 0.75014(12)  | 0.52192(13)  | 0.52192(13)  | 0.194464(61) | 0.194464(61) |
| $Q^2$ (GeV/c) <sup>2</sup> | 0.195466(49) | 0.195466(49) | 0.090788(35) | 0.090788(35) | 0.22643(14)  | 0.22643(14)  |
| $\theta_M$ (Ring 7)        | 30.77°       |              | 30.77°       |              | 149.23°      |              |
| $E'$ (GeV)                 | 0.75724(13)  | 0.75724(13)  | 0.52535(14)  | 0.52535(14)  | 0.193993(61) | 0.193993(61) |
| $Q^2$ (GeV/c) <sup>2</sup> | 0.182134(46) | 0.182134(46) | 0.084352(32) | 0.084352(32) | 0.22732(14)  | 0.22732(14)  |

## Additional Slides

### 854.3 MeV



### 1.5 GeV



## Current Stability Test from the manufacturer

$$I_{\text{dipole}} = I_0 \pm \frac{I_0 \delta I_{\text{FWHM}}}{2 \sqrt{2 \ln 2}} = I_0 \pm \delta I_0.$$

|                                | design value | 3 mins | 30 mins | 8 hours |
|--------------------------------|--------------|--------|---------|---------|
| $\delta I_{\text{FWHM}}$ (ppm) | 3            | 0.5    | 1       | 2       |

## Beam distribution and phase space with GEANT4

