



SINGLE SPIN ASYMMETRIES FROM THE MAINZ A4 EXPERIMENT

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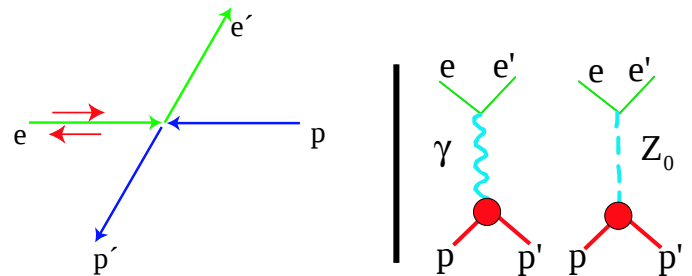
- Single Spin Asymmetries
- A4 Experimental setup
- Longitudinal spin: PV asymmetry
Measurements at $Q^2 = 0.23(\text{GeV}/c)^2$
and $Q^2 = 0.11(\text{GeV}/c)^2$
- Transverse spin - 2γ exchange
Measurements at $Q^2 = 0.23(\text{GeV}/c)^2$
and $Q^2 = 0.11(\text{GeV}/c)^2$



Single spin asymmetries in elastic scattering

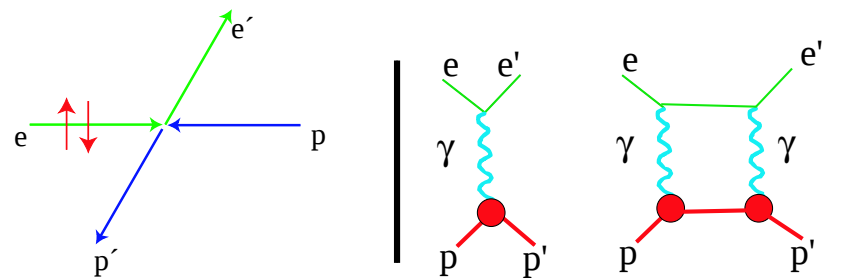
- Single Spin Asymmetries: e^- spin **longitudinal**,
parity violating, ϕ -symmetric

$$A_{PV} = 10^{-6}$$



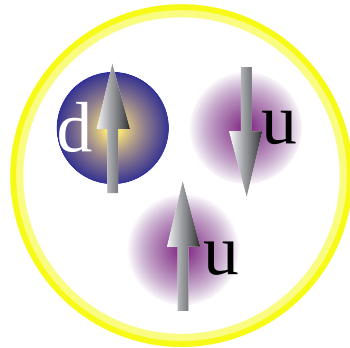
- Single-Spin-Asymmetries: e^- spin **transverse**,
Azimuthal dependency: $\sin(\phi)$,

$$A_{\perp}^{beam} = 10^{-5}$$



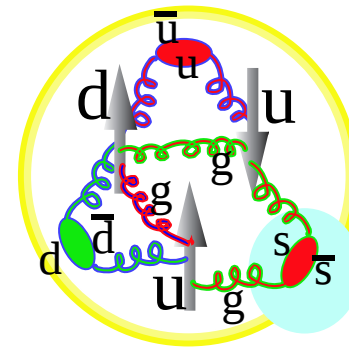


Strange quarks in the proton



Simple quarks model

$\Leftrightarrow$



Full QCD

πN scattering

$$\langle p | \bar{s} s | p \rangle$$

contribution to proton mass

DIS

$$\langle p | \bar{s} \gamma_\mu \gamma^5 s | p \rangle$$

contribution to spin

PV e^- scattering

$$\langle p | \bar{s} \gamma_\mu s | p \rangle$$

contribution to form factors



Electroweak interaction and parity violation

$$\sigma \sim \left| \begin{array}{c} e \quad e' \\ \gamma \\ p \quad p' \end{array} + \begin{array}{c} e \quad e' \\ Z_0 \\ p \quad p' \end{array} \right|^2$$

$$\sim \left| \begin{array}{c} e \quad e' \\ \gamma \\ p \quad p' \end{array} \right|^2 + \left| \begin{array}{c} e \quad e' \\ \gamma \\ p \quad p' \end{array} + \begin{array}{c} e \quad e' \\ Z_0 \\ p \quad p' \end{array} \right|^2 + \left| \begin{array}{c} e \quad e' \\ Z_0 \\ p \quad p' \end{array} \right|^2$$

$$\sigma \sim 1 + 10^{-5} + 10^{-10}$$

PV asymmetry $A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$



Extraction of strange quark form factor contribution

Flavour decomposition:

$$G_E^p = \frac{2}{3}G_E^{p,u} - \frac{1}{3}G_E^{p,d} - \frac{1}{3}G_E^{p,s}$$
$$G_M^p = \frac{2}{3}G_M^{p,u} - \frac{1}{3}G_M^{p,d} - \frac{1}{3}G_M^{p,s}$$

Strange quark form factors

Goal:

A_{PV} in terms of G_E^s , G_M^s and the known electromagnetic form factors

- Weak interaction
- Isospin symmetry



Extraction of strange quark form factor contribution

$$A_{PV} = A_V + A_A + A_S = A_0 + A_S$$

Asymmetry without strangeness

Strange quark contribution to asymmetry

$$A_V = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \left((1 - 4\sin^2 \hat{\Theta}_W) - \frac{\varepsilon G_E^p G_E^n + \tau G_M^p G_M^n}{\varepsilon (G_E^p)^2 + \tau (G_M^p)^2} \right)$$

$$A_A = \frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \frac{(1 - 4\sin^2 \hat{\Theta}_W) \sqrt{1 - \varepsilon^2} \sqrt{\tau(1 + \tau)} G_M^p \tilde{G}_A^p}{\varepsilon (G_E^p)^2 + \tau (G_M^p)^2}$$

$$A_S = \frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \frac{\varepsilon G_E^p G_E^s + \tau G_M^p G_M^s}{\varepsilon (G_E^p)^2 + \tau (G_M^p)^2}$$

$$\text{with } \tau = \frac{Q^2}{4M_p^2} \text{ and } \varepsilon = [1 + 2(1 + \tau) \tan^2 \frac{\Theta}{2}]^{-1}$$



Extraction of strange quark form factor contribution

Put in all known quantities... =>

- $E=854.3 \text{ MeV}, Q^2=0.23 \text{ (GeV/c)}^2$:

$$A_0 = (-6.30 \pm 0.43) \cdot 10^{-6}$$

$$A_V = -5.61 \cdot 10^{-6}, A_A = -0.69 \cdot 10^{-6}$$

- $E=70.4 \text{ MeV}, Q^2=0.11 \text{ (GeV/c)}^2$:

$$A_0 = (-2.10 \pm 0.14) \cdot 10^{-6}$$

$$A_V = -1.85 \cdot 10^{-6}, A_A = -0.25 \cdot 10^{-6}$$

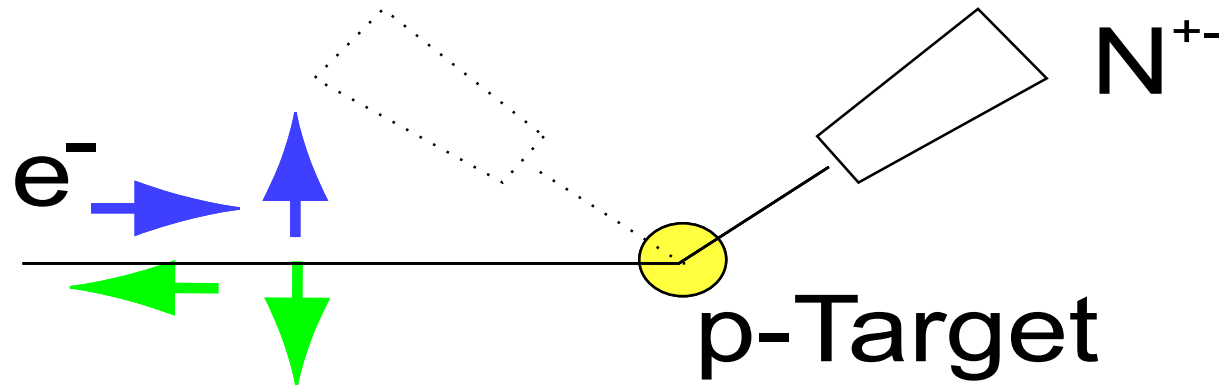
Experimental requirements:

- Large statistics: $\frac{\Delta A}{A} \approx \frac{1}{A\sqrt{N}} = 5\% \Rightarrow N \approx 10^{14}$ elastic counts
- Small systematics (beam fluctuations etc.)



A4 Experiment

A4 Measurement Principle

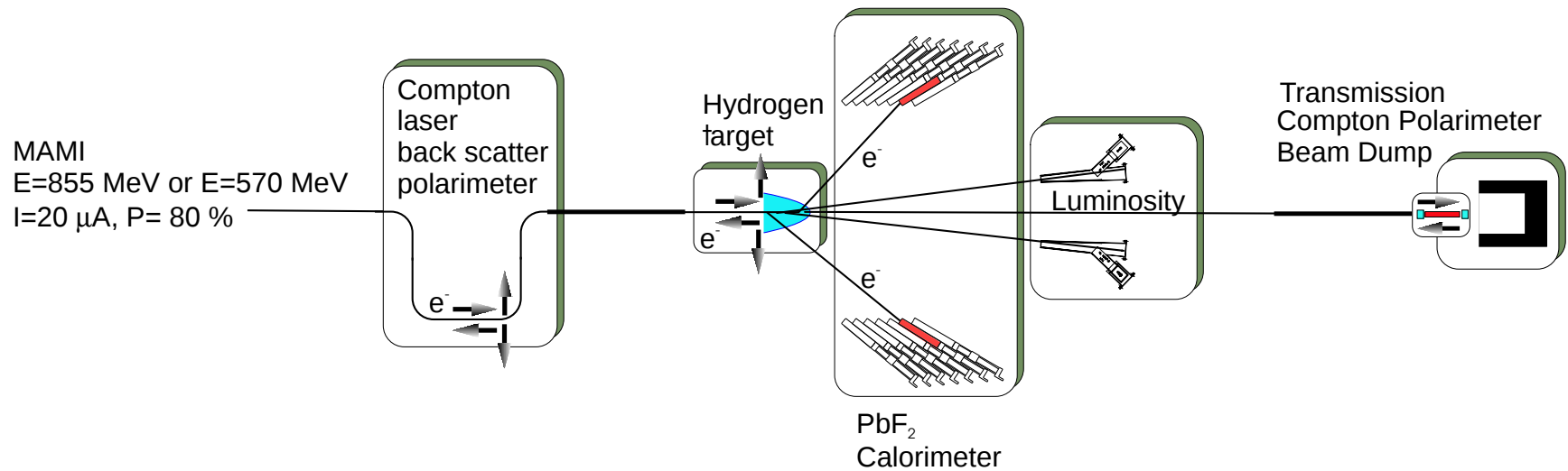


- Longitudinally or transversely polarised electrons
- Target: unpolarised protons
- Detector: scattering angle $35^\circ \pm 5^\circ$ or (in preparation) $145^\circ \pm 5^\circ$
- Counting experiment: $A = \frac{N^+ - N^-}{N^+ + N^-}$



A4 Experimental setup

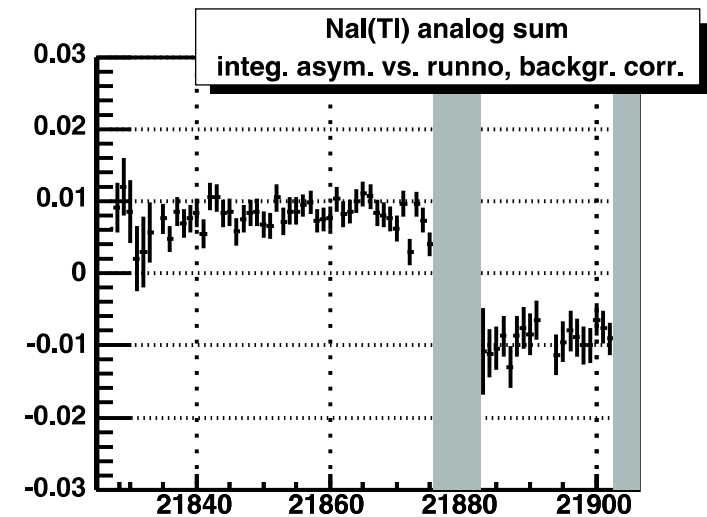
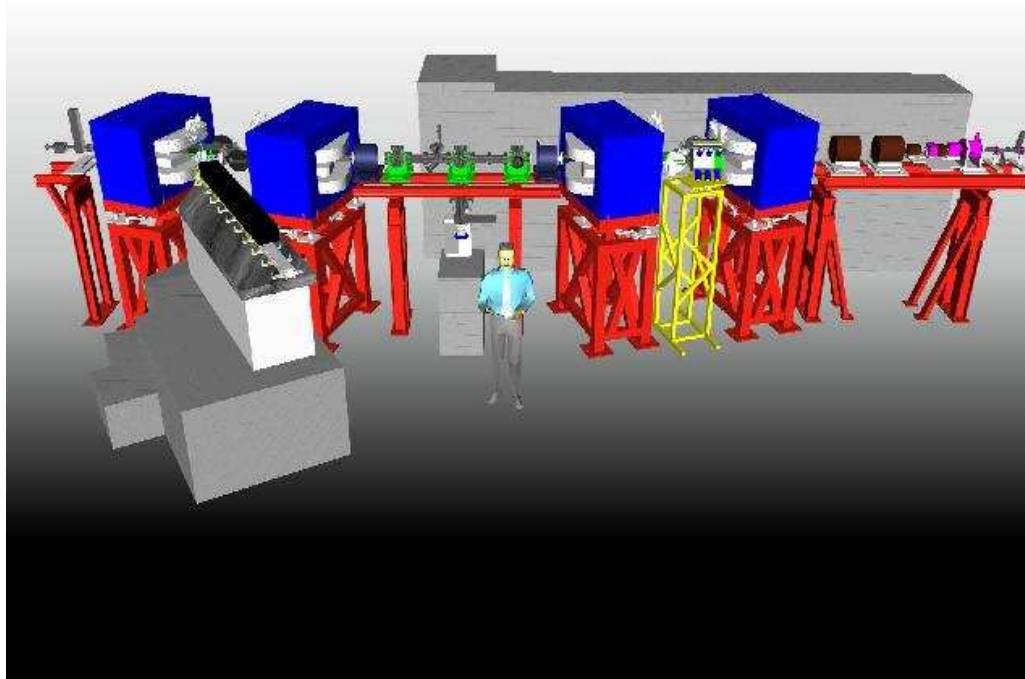
A4 Experimental setup





A4 Experimental setup

Compton backscatter polarimeter

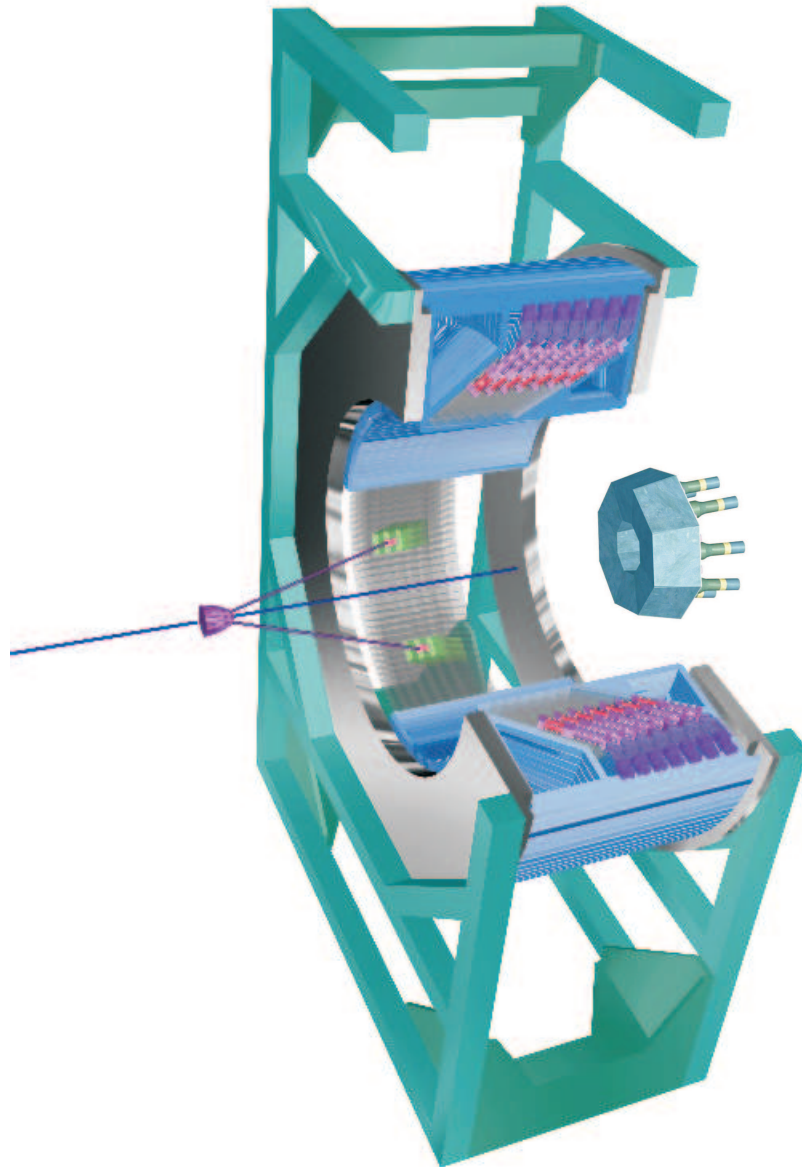


- Magnetic chicane
- Intracavity $e^- \gamma$ - scattering
- First asymmetries in june 2004



A4 Experimental setup

A4 Calorimeter and Luminosity Monitors



Calorimeter:

1022 PbF_2 crystals

146 frames, 7 rings

$\Theta = 30^\circ \dots 40^\circ$, $\Phi = 0 \dots 2\pi$

Luminosity monitors:

8 water cherenkov monitors

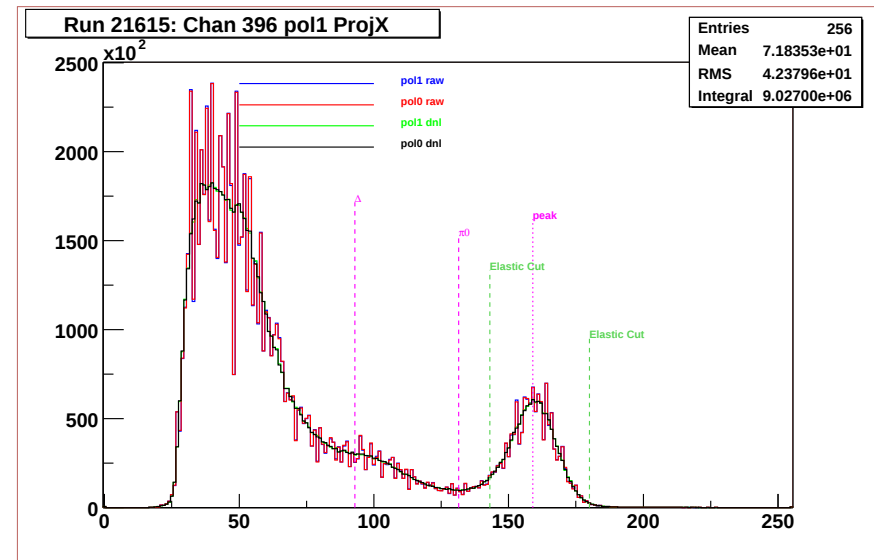
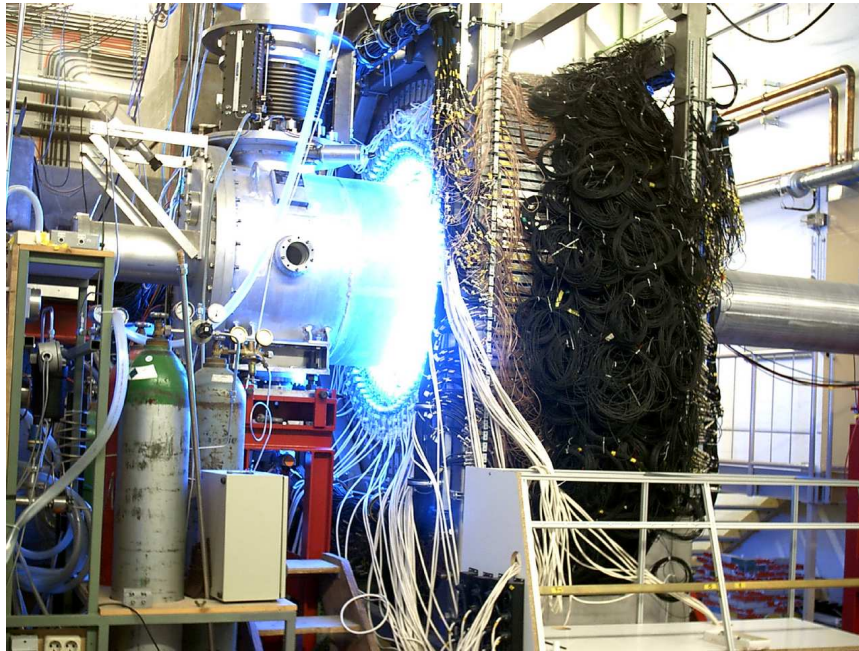
$\Theta = 4.4^\circ \dots 10^\circ$, $\Phi = 0 \dots 2\pi$

Full coverage of azimuthal range



A4 Experimental setup

The Calorimeter



$d\Omega = 0.64 \text{ sr}$, rates: 100 MHz total, 10 MHz elastic (@855 MeV)

Count Single events, separation of elastic from inelastic events

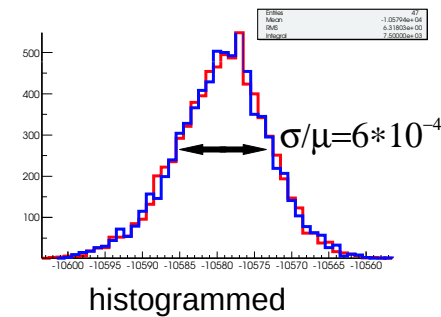
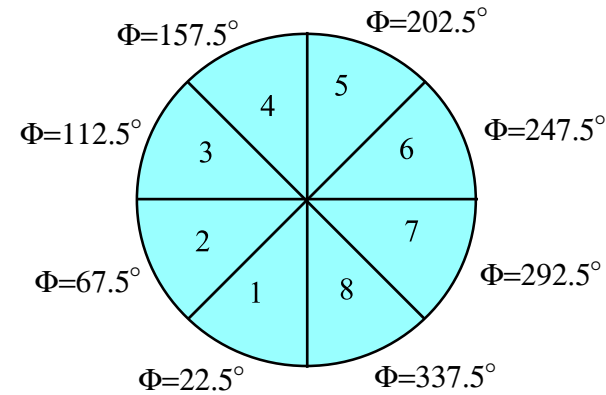
2044 energy histograms (+-) every 5 minutes

Extract N^+ and N^-



A4 Experimental setup

The luminosity monitors



$d\Omega = 80 \text{ msr}$, rates: 2.6 GHz elastic, 350 GHz Møller

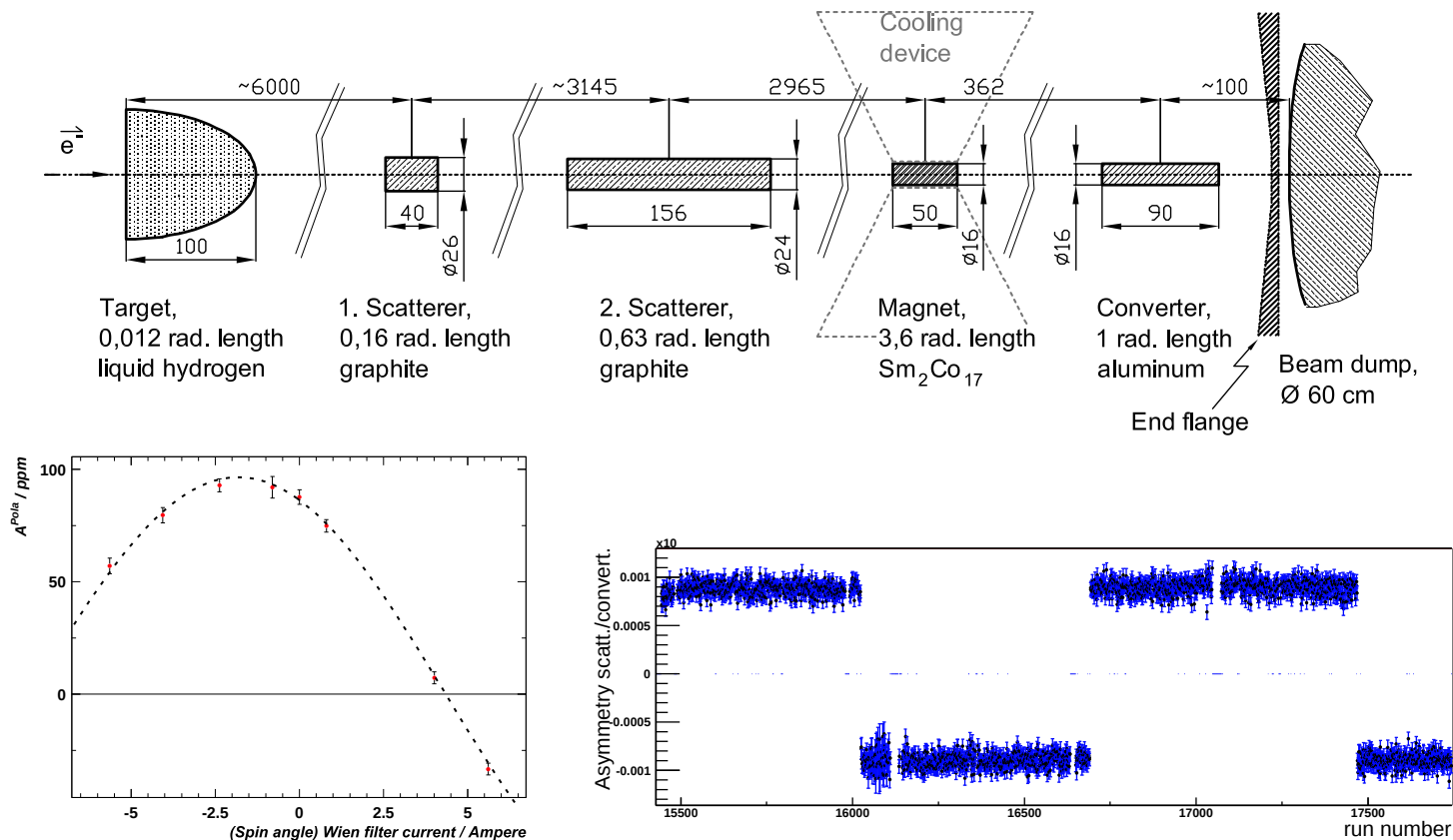
Integrating water cherenkov detectors, measurements every 20ms

Design optimised for Møller electrons



Experimental setup

Dump polarimeter

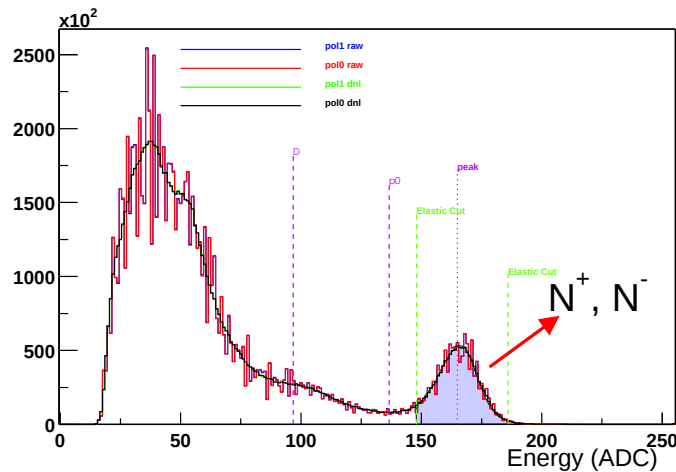


- Relative Measurement of polarization degree, sensitive to longitudinal spin
- Determination of spin angle
- Control of $\lambda/2$ -plate status



Extracting experimental asymmetry A_{exp}

- Extract elastic counts N_i^+ , N_i^- from channel $i = 1..1022$



$$N^+ = \sum N_i^+, N^- = \sum N_i^-$$

- Normalisation to target density (L=luminosity, I=beam current)

$$A_{exp} = \frac{\frac{N^+}{\rho^+} - \frac{N^-}{\rho^-}}{\frac{N^+}{\rho^+} + \frac{N^-}{\rho^-}}, \quad \rho^\pm = \frac{L^\pm}{I^\pm}$$



Extraction of physical asymmetry A_{phys}

- Correction for helicity correlated false asymmetries and polarization

$$A_{exp} = P \cdot A_{phys} + \sum_{i=1}^6 a_i X_i$$

P : Beam polarisation

X_1 : A_I , Current asymmetry

X_2 : ΔX , Horizontal position difference

X_3 : ΔY , Vertical position difference

X_4 : $\Delta X'$, Horizontal angle difference

X_5 : $\Delta Y'$, Vertical angle difference

X_6 : ΔE , Energy difference

Determination of a_i via multiple linear regression

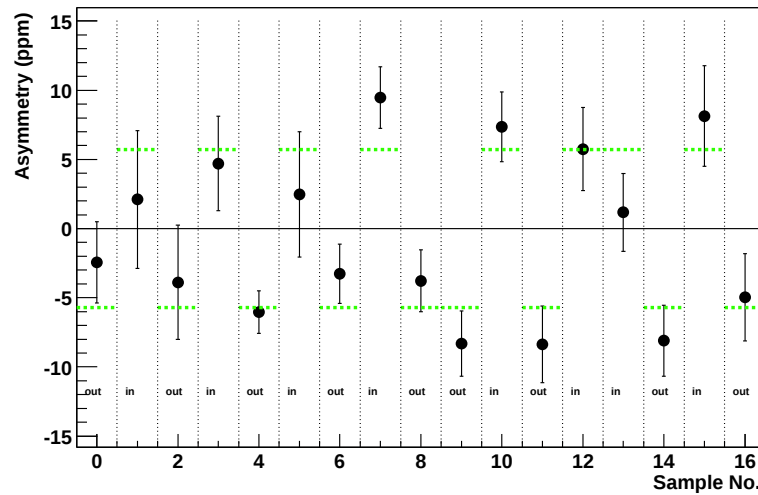
- Correction for dilution from quasielastic scattering off aluminium (target entrance/exit windows)
- Systematic check with $\lambda/2$ -plate flip



Physical asymmetries

$$Q^2 = 0.23(\text{GeV}/c)^2$$

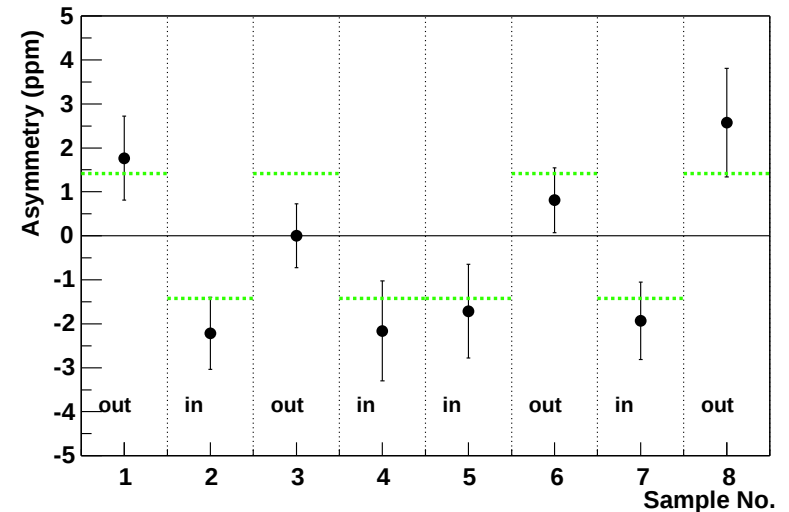
550 hours beam data



$$A_{phys} = (-5.44 \pm 0.54_{stat} \pm 0.26_{syst}) \cdot 10^{-6}$$

$$Q^2 = 0.11(\text{GeV}/c)^2$$

350 hours beam data

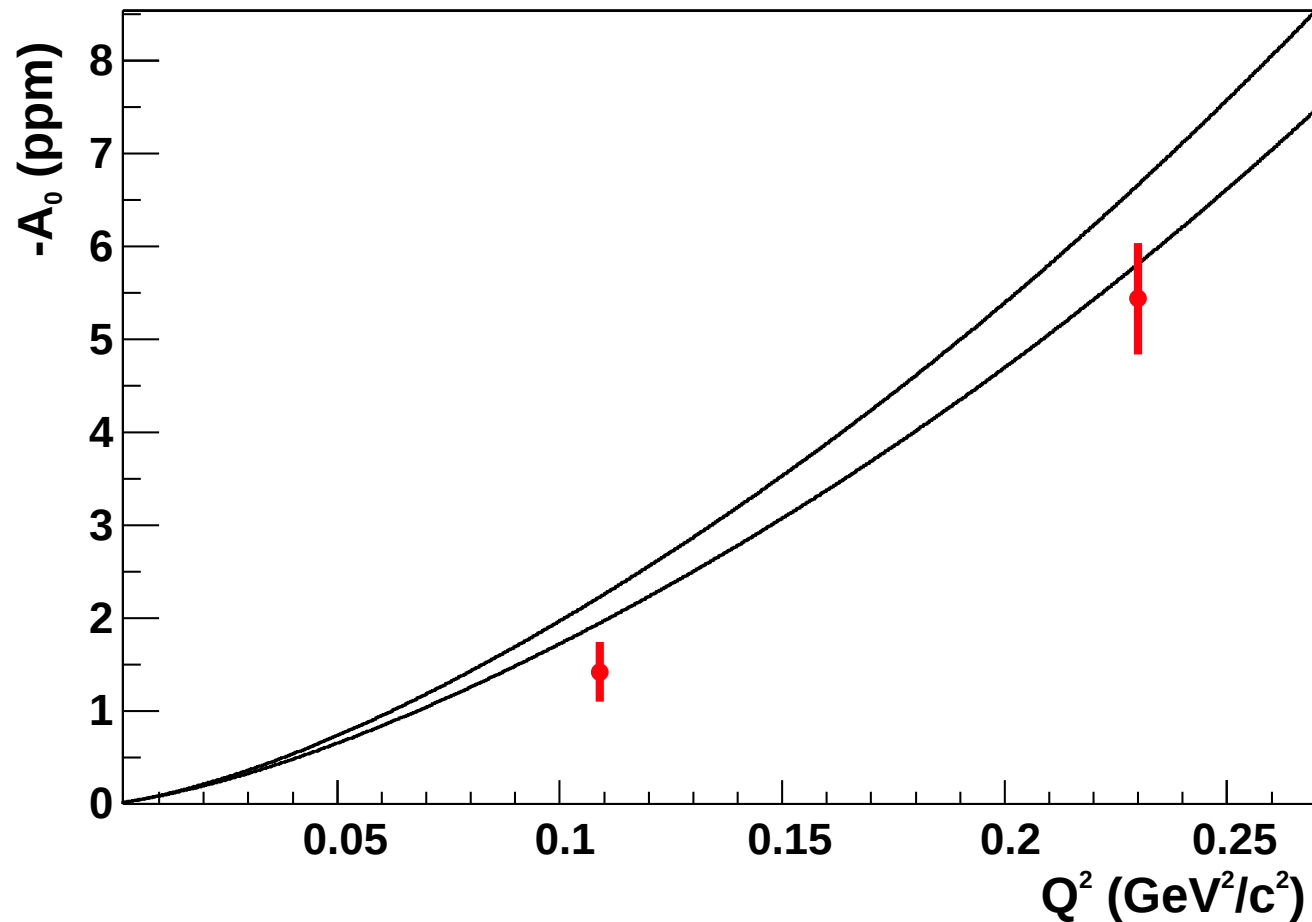


$$A_{phys} = (-1.42 \pm 0.29_{stat} \pm 0.12_{syst}) \cdot 10^{-6}$$



Physical asymmetries

Comparison of measured asymmetries with A_0

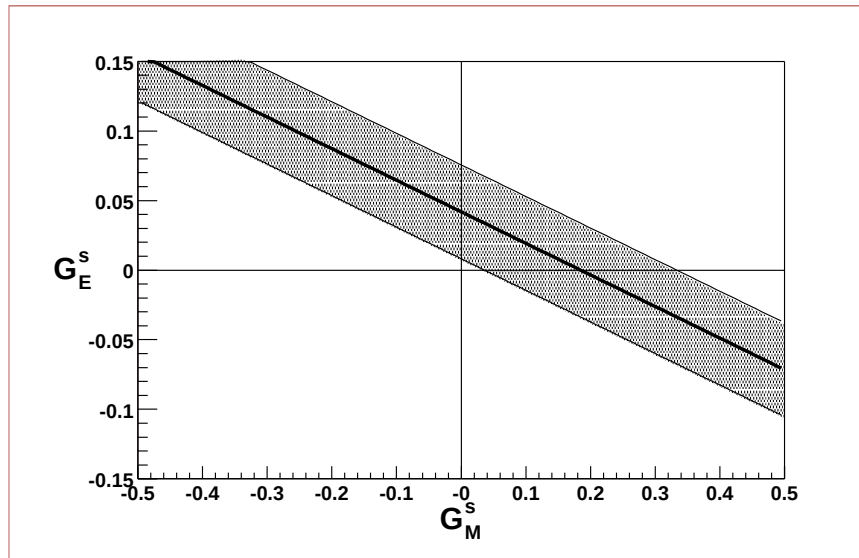




Strange form factors

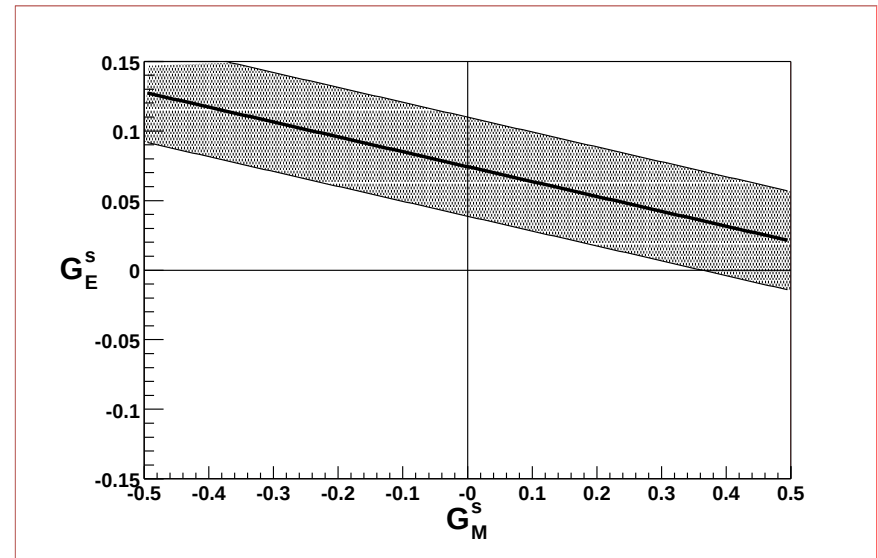
$$Q^2 = 0.23(\text{GeV}/c)^2$$

$$G_E^s + 0.225G_M^s = 0.039 \pm 0.034$$



$$Q^2 = 0.11(\text{GeV}/c)^2$$

$$G_E^s + 0.107G_M^s = 0.069 \pm 0.036$$

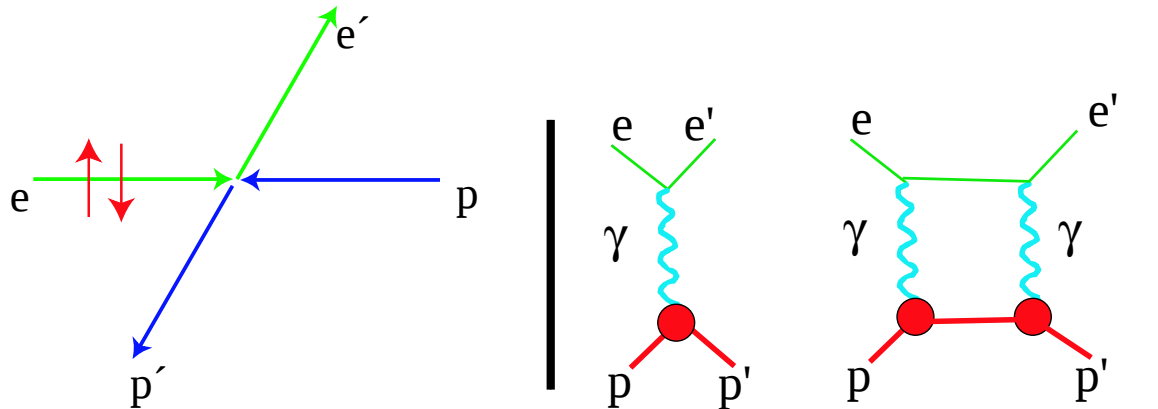


- Determination of G_E^s , G_M^s separately: Preparation of measurements under backward angle in preparation
- $Q^2 = 0.11(\text{GeV}/c)^2$: Combination with SAMPLE and HAPPEX results

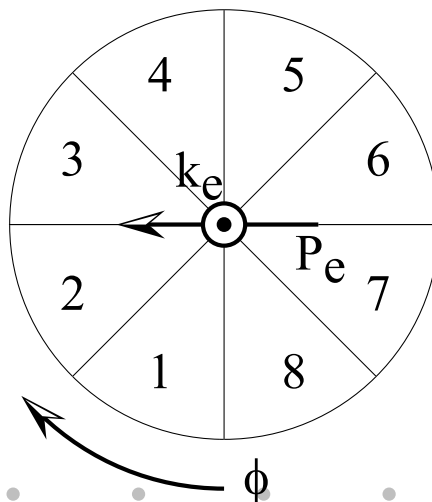


Transverse Spin

Single-Spin-Asymmetries: e^- spin **transverse**



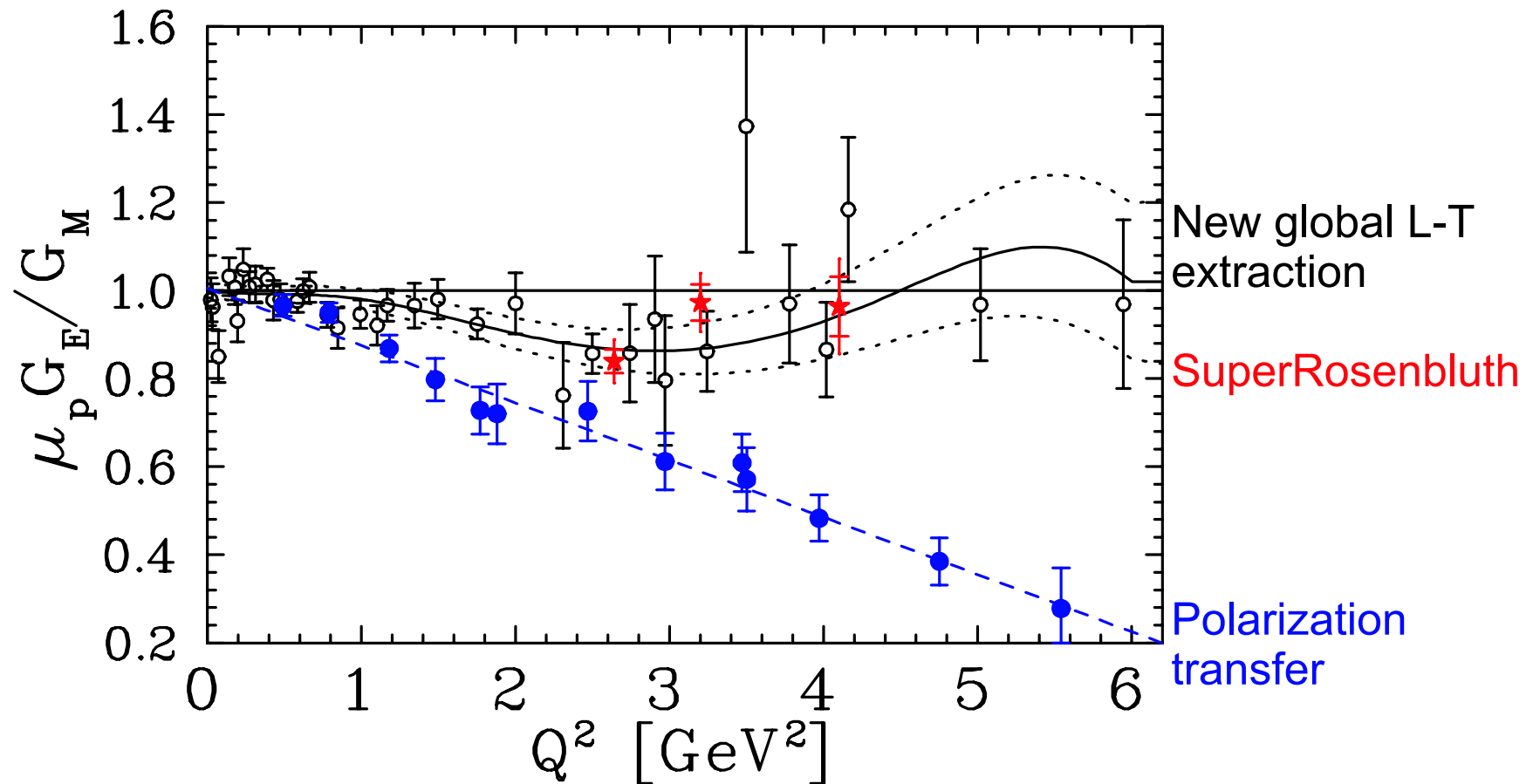
- Imaginary part of 2γ exchange amplitude
- πN intermediate states
- Azimuthal dependency: $A(\phi) = A_{\perp} \cos(\phi)$





Transverse Asymmetry: Why is it interesting?

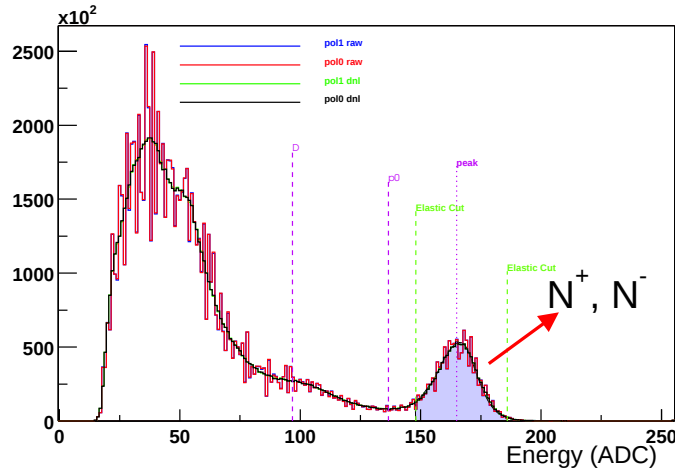
J. Arrington



$\mu_p G_E^p / G_M^p$ different in different experiments

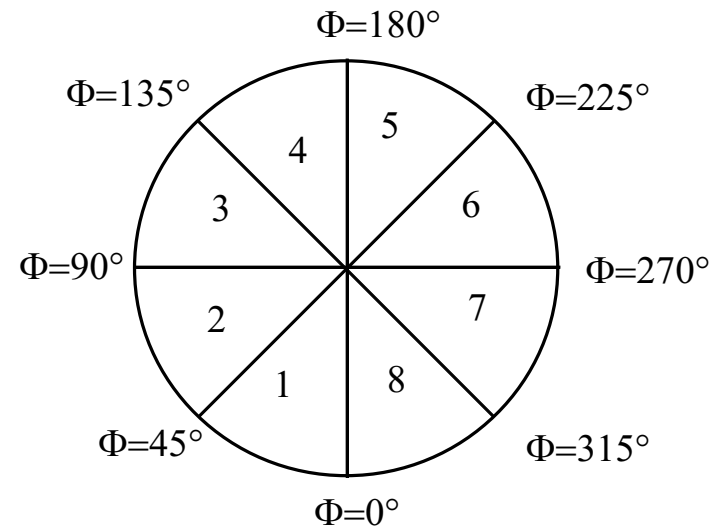


Extraction of physical asymmetry



Extract elastic counts N_i^+ , N_i^-
from channel $i = 1..1022$

Normalisation to target den-
sity (L=luminosity, I=beam
current



Combine the 1022 channels
to 8 sectors

$$N_{Sec}^+ = \sum N_i^+, N_{Sec}^- = \sum N_i^-$$

$$A_{exp}^{Sec} \equiv A_{exp}(\Phi_i) = \frac{\frac{N_{Sec}^+}{\rho^-} - \frac{N_{Sec}^-}{\rho^-}}{\frac{N^+}{\rho^-} + \frac{N^-}{\rho^-}}, \quad \rho^\pm = \frac{L^\pm}{I^\pm}$$



Normalisation to target density

$$A_{exp} = \frac{\frac{N^+}{\rho^-} - \frac{N^-}{\rho^-}}{\frac{N^+}{\rho^-} + \frac{N^-}{\rho^-}}, \quad \rho^\pm = \frac{L^\pm}{I^\pm}$$

with N = Elastic counts, L = Luminosity, I = Beam current



Normalisation to target density

$$A_{exp} = \frac{\frac{N^+}{\rho^-} - \frac{N^-}{\rho^-}}{\frac{N^+}{\rho^-} + \frac{N^-}{\rho^-}}, \quad \rho^\pm = \frac{L^\pm}{I^\pm}$$

with N = Elastic counts, L = Luminosity, I = Beam current

$$A_{exp} \approx \frac{N^+ - N^-}{N^+ + N^-} - \frac{L^+ - L^-}{L^+ + L^-} + \frac{I^+ - I^-}{I^+ + I^-}$$



Normalisation to target density

$$A_{exp} = \frac{\frac{N^+}{\rho^-} - \frac{N^-}{\rho^-}}{\frac{N^+}{\rho^-} + \frac{N^-}{\rho^-}}, \quad \rho^\pm = \frac{L^\pm}{I^\pm}$$

with N = Elastic counts, L = Luminosity, I = Beam current

$$A_{exp} \approx \frac{N^+ - N^-}{N^+ + N^-} - \frac{L^+ - L^-}{L^+ + L^-} + \frac{I^+ - I^-}{I^+ + I^-}$$

=> Requirement: $A_L \ll A_{phys}$

=> In PV case A_L an order of magnitude smaller than PV asymmetry

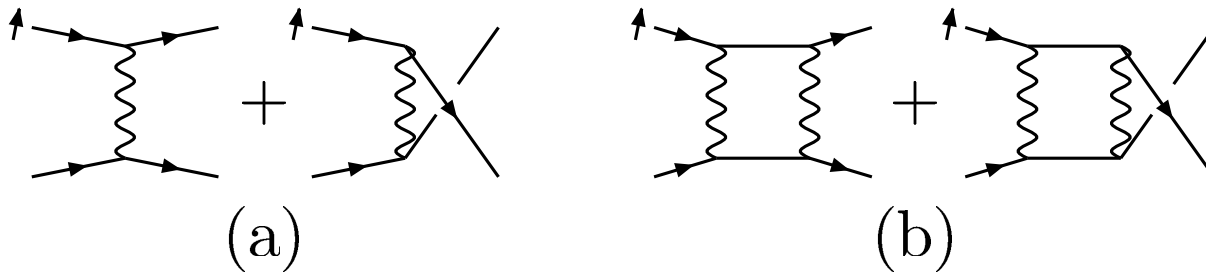
=> But with transverse spin: Two photon exchange...



• Møller Scattering

Single Spin Asymmetry in Møller Scattering

Dixon, Schreiber, hep-ph/0402221: *Azimuthal Asymmetry in Transversely Polarized Møller Scattering*



- (a) Tree diagrams
- (b) Box diagrams
- => Leading term in cross section containing azimuthal asymmetry arises at order α^3 from interference of diagrams in (a) and (b)



• Møller Scattering

Leading Order Single Spin Asymmetry in Møller Scattering

$$\frac{d\sigma^{Born}}{d\Omega} = \frac{\alpha^2}{2s} \left[\frac{t^2 + tu + u^2}{tu} \right]^2$$

$$\begin{aligned} \frac{d\sigma^\Phi}{d\Omega} = & -\frac{\alpha^3}{8} \frac{m_e}{\sqrt{s}} \sin \Theta_{CM} \sin \Phi \frac{1}{t^2 u^2} \cdot \\ & [3s[t(u-s) \ln(\frac{-t}{s}) - u(t-s) \ln(\frac{-u}{s})] - 2(t-u)tu] \end{aligned}$$

with

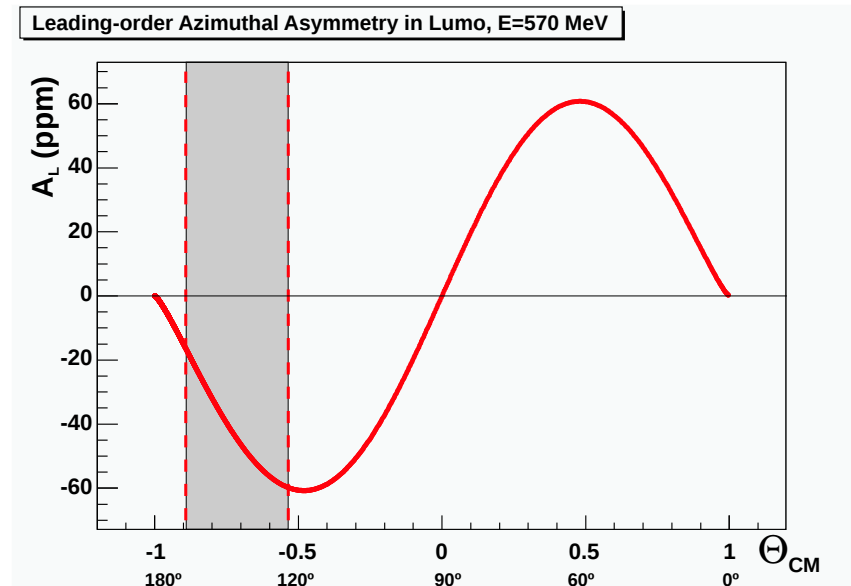
$$s = 2m_e E, t = -\frac{s}{2}(1 - \cos \Theta_{CM}), u = -\frac{s}{2}(1 + \cos \Theta_{CM})$$

$$E' = \frac{E}{2}(1 + \cos \Theta_{CM}), \cos \Theta_{lab} = 1 - \frac{m_e}{E} \frac{1 - \cos \Theta_{CM}}{1 + \cos \Theta_{CM}}$$



Møller Scattering

Single Spin Asymmetry in Møller Scattering



- Asymmetry seen by a detector is sensitive to the acceptance
- Symmetric acceptance in Θ_{CM} would lead to zero asymmetry
- Luminosity monitors A4: backward angles in CM frame
- => Nonvanishing azimuthal asymmetry in luminosity monitors

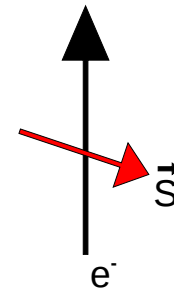
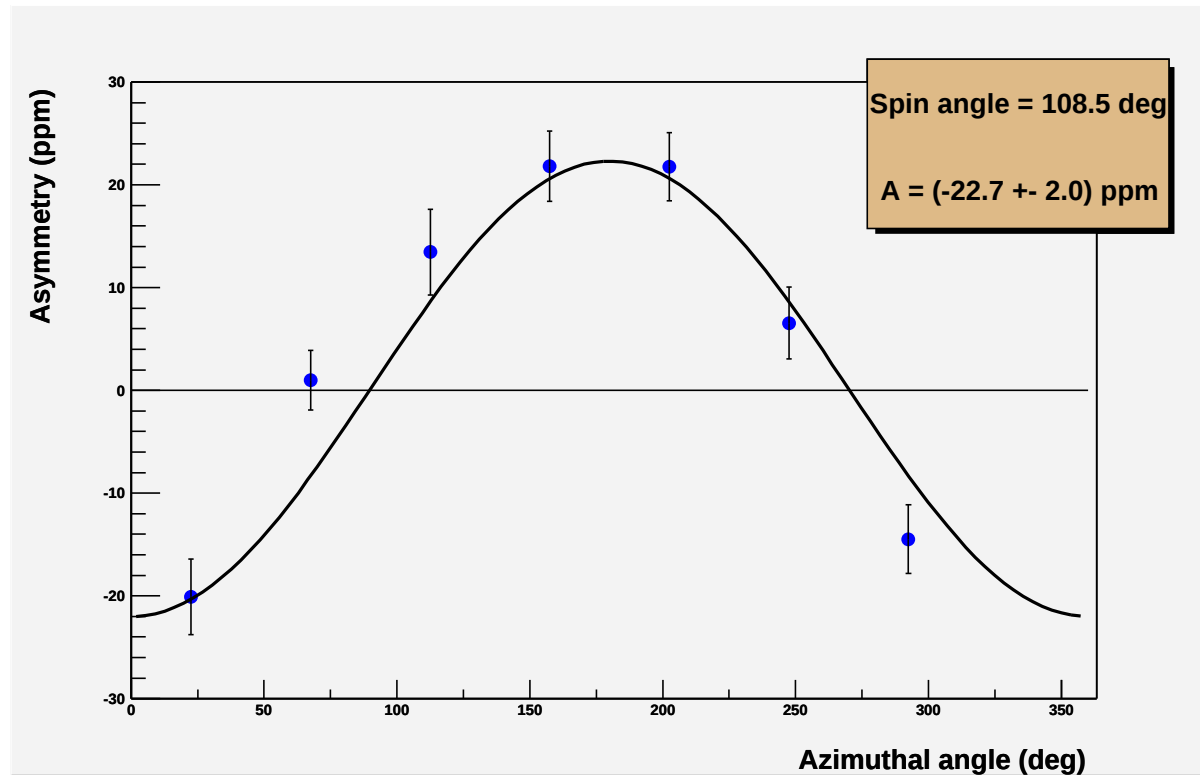
$$A_{Lumi}(\Phi_i) = A_L \cdot \sin \Phi_i$$



Møller Scattering

Luminosity Monitors

Spin Angle $\phi_s = 108.5^\circ$

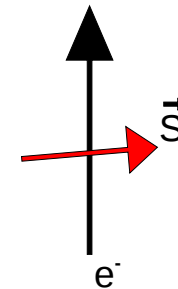
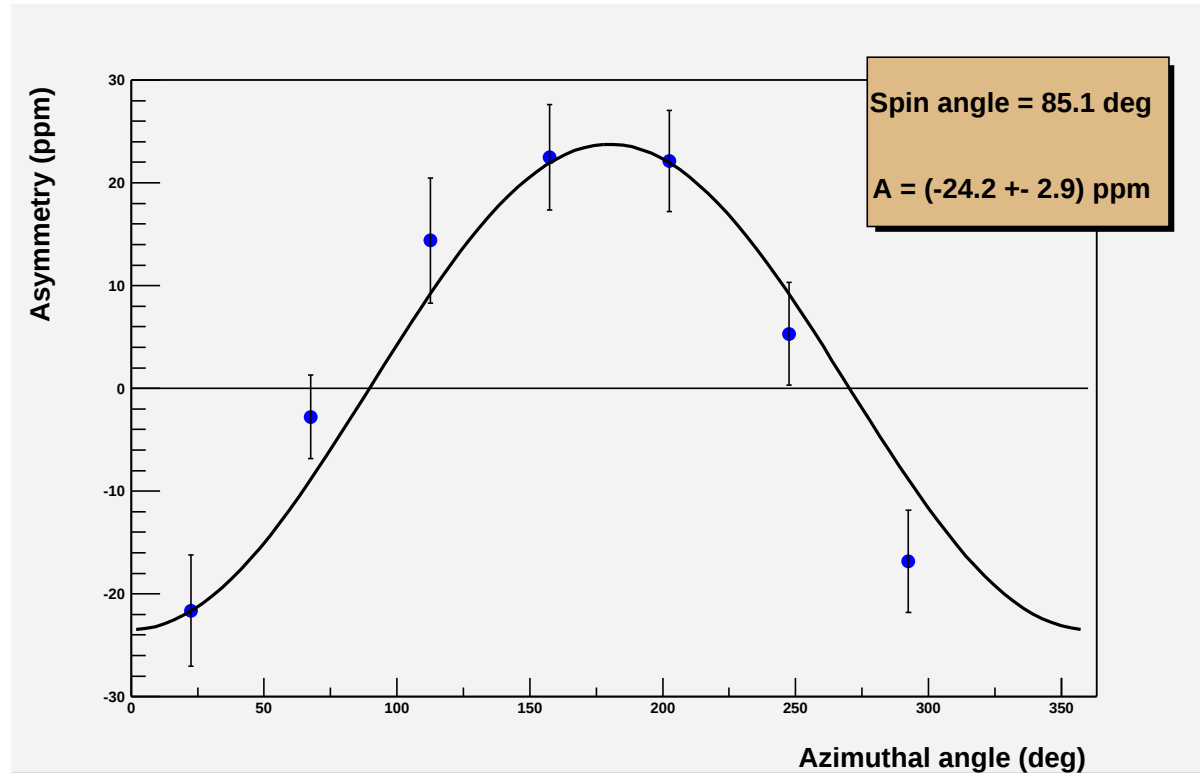




Møller Scattering

Luminosity Monitors

Spin Angle $\phi_s = 85.1^\circ$

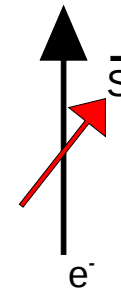
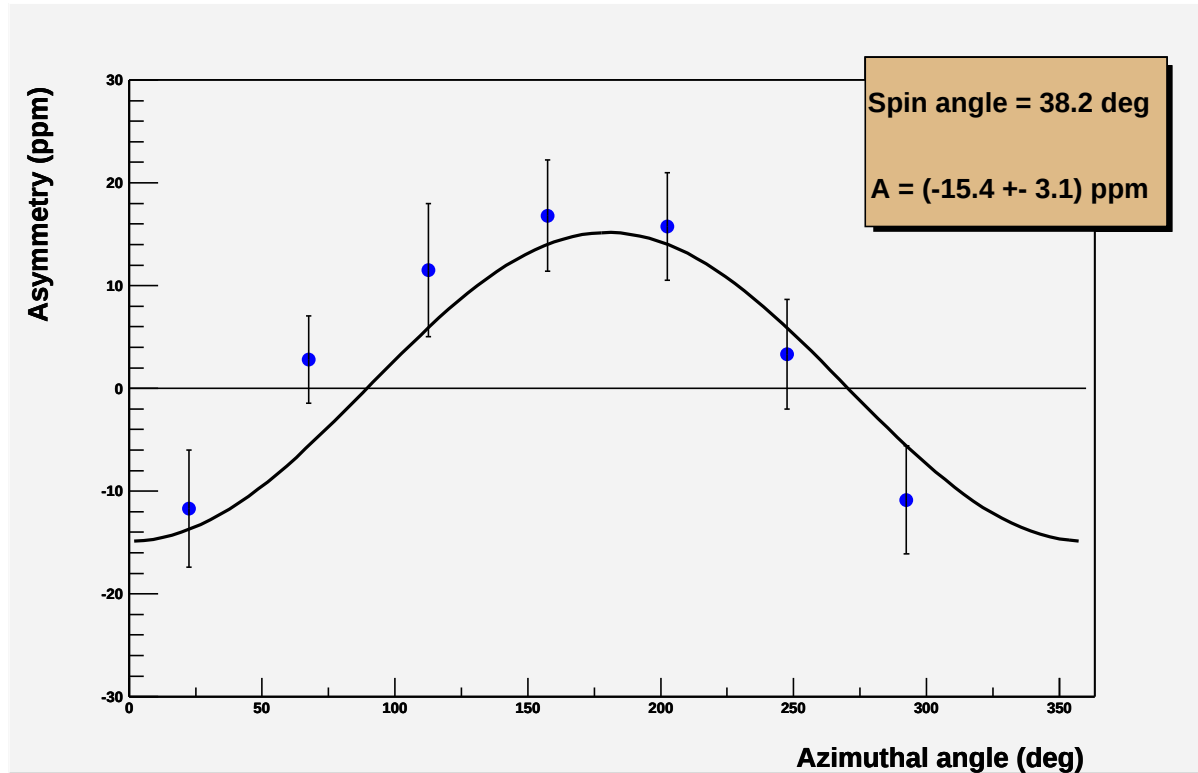




Møller Scattering

Luminosity Monitors

Spin Angle $\phi_s = 38.2^\circ$

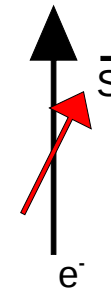
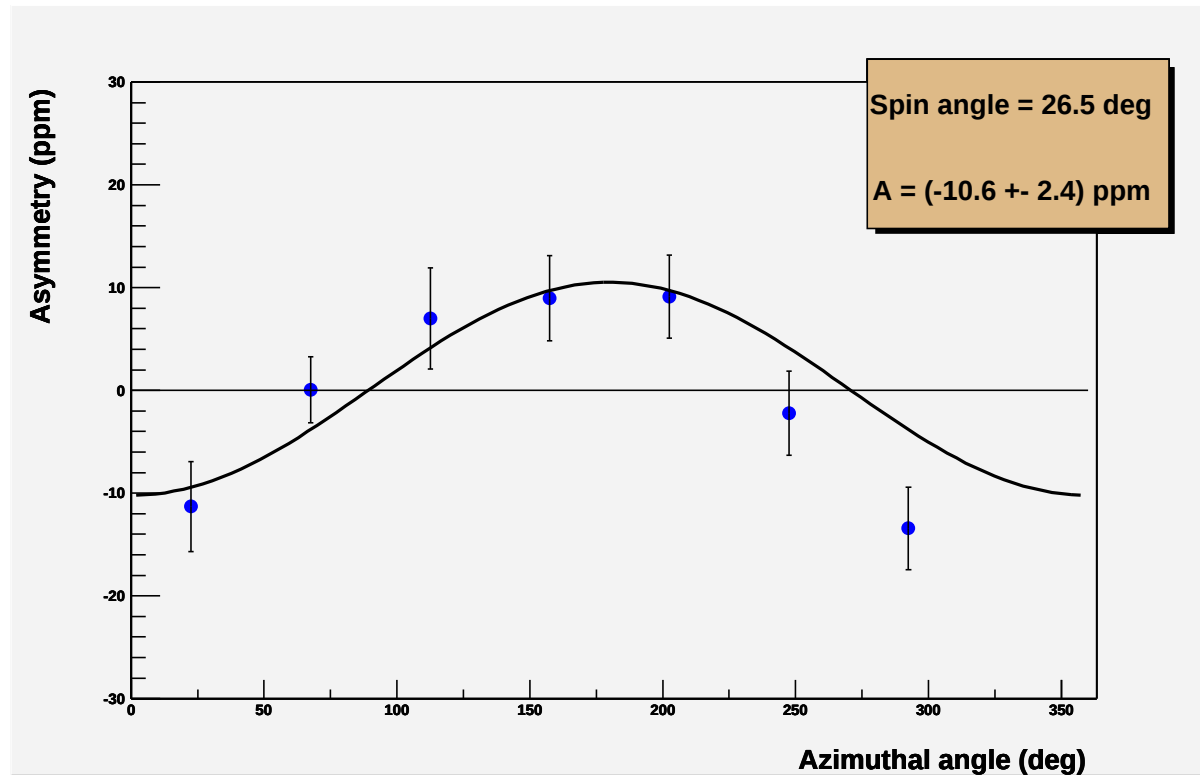




Møller Scattering

Luminosity Monitors

Spin Angle $\phi_s = 26.5^\circ$

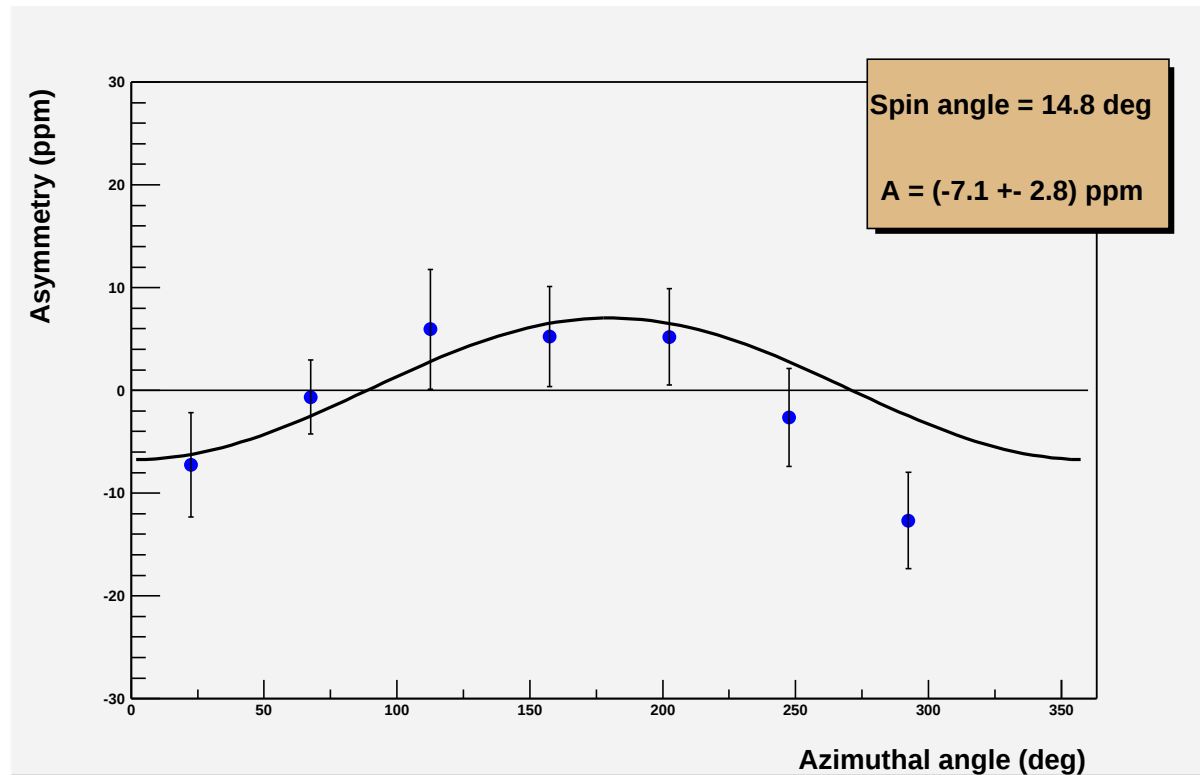




Møller Scattering

Luminosity Monitors

Spin Angle $\phi_s = 14.8^\circ$

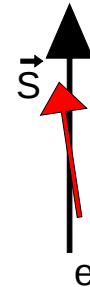
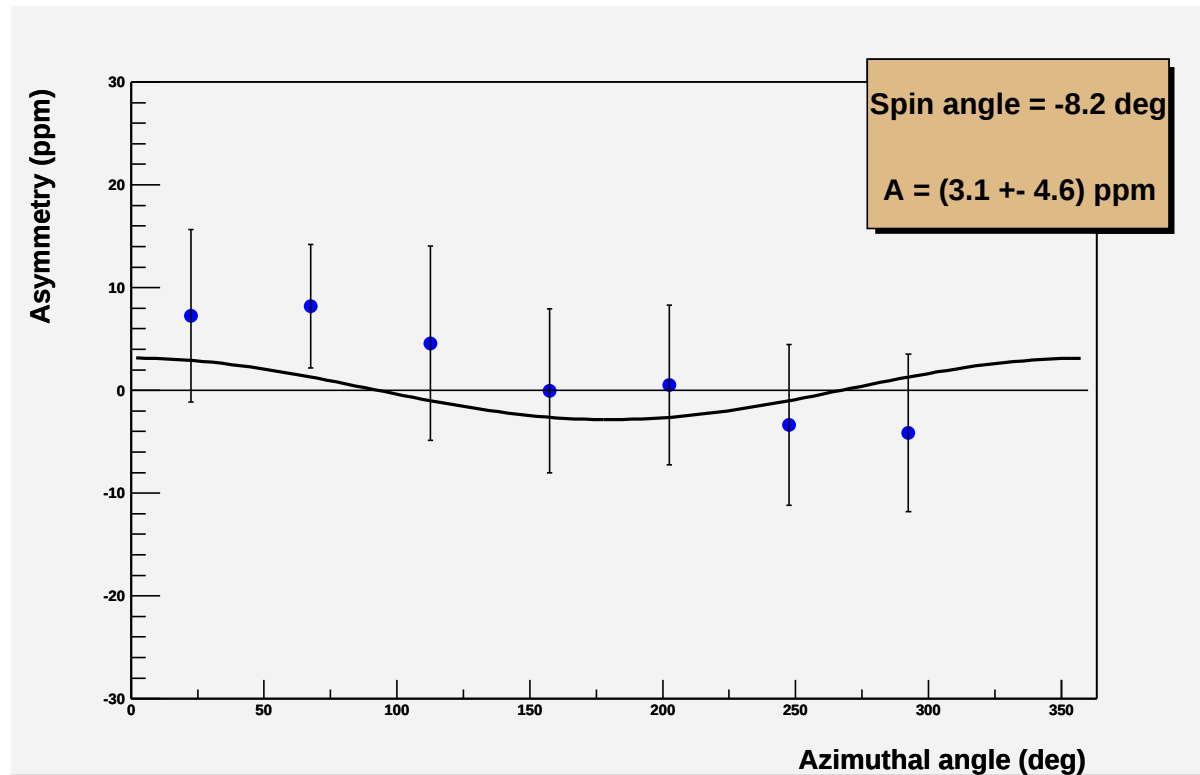




Møller Scattering

Luminosity Monitors

Spin Angle $\phi_s = -8.6^\circ$

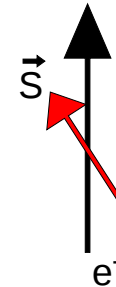
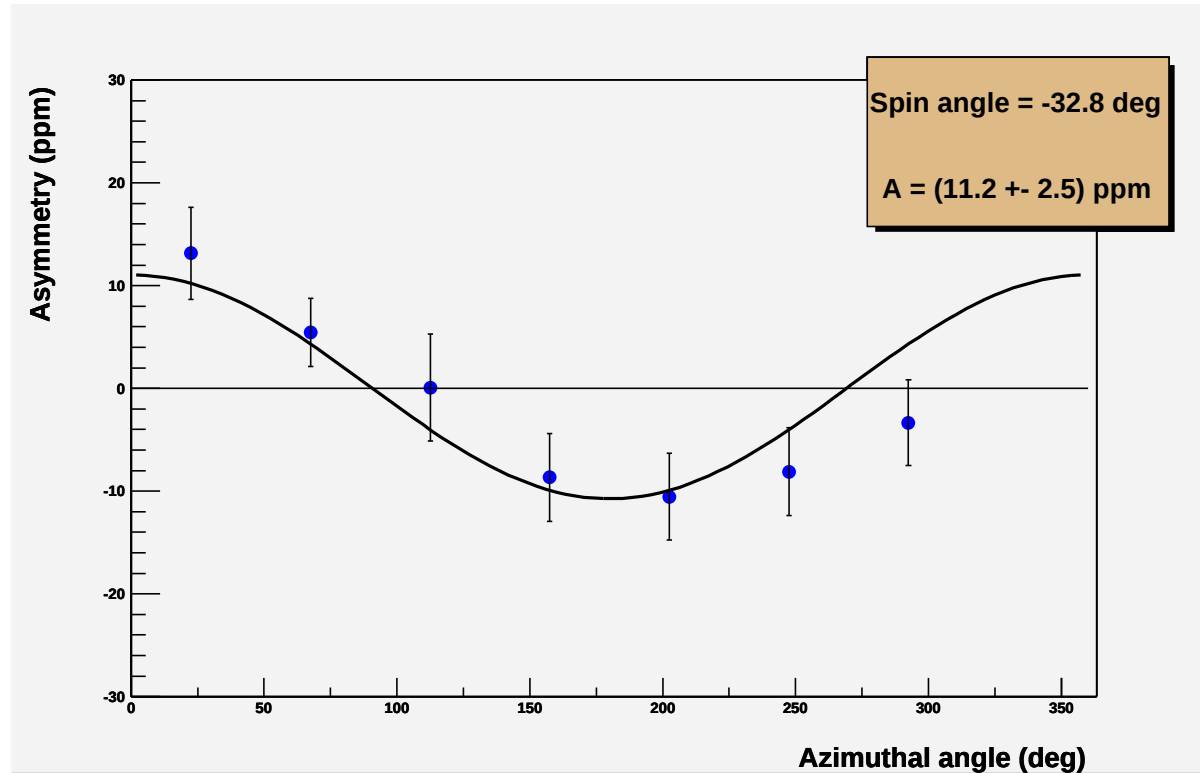




Møller Scattering

Luminosity Monitors

Spin Angle $\phi_s = -32.8^\circ$

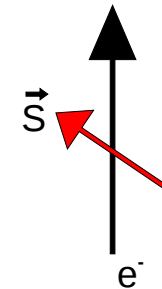
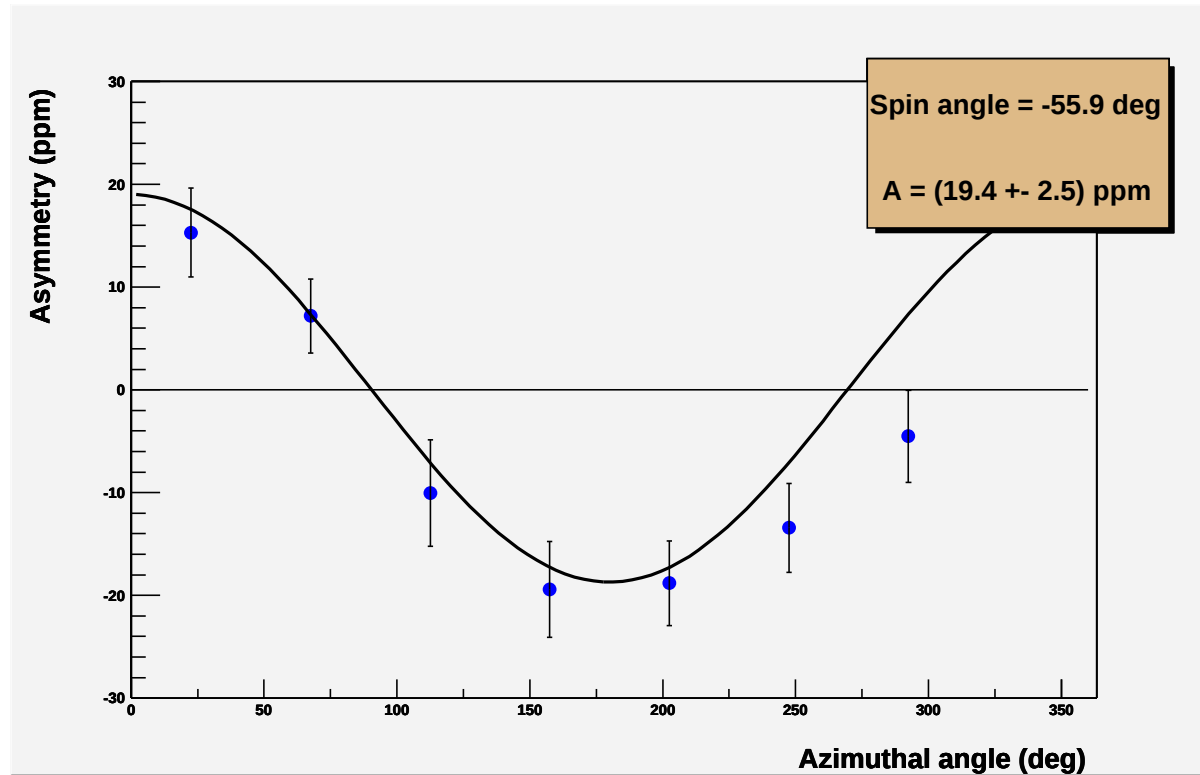




Møller Scattering

Luminosity Monitors

Spin Angle $\phi_s = -55.9^\circ$



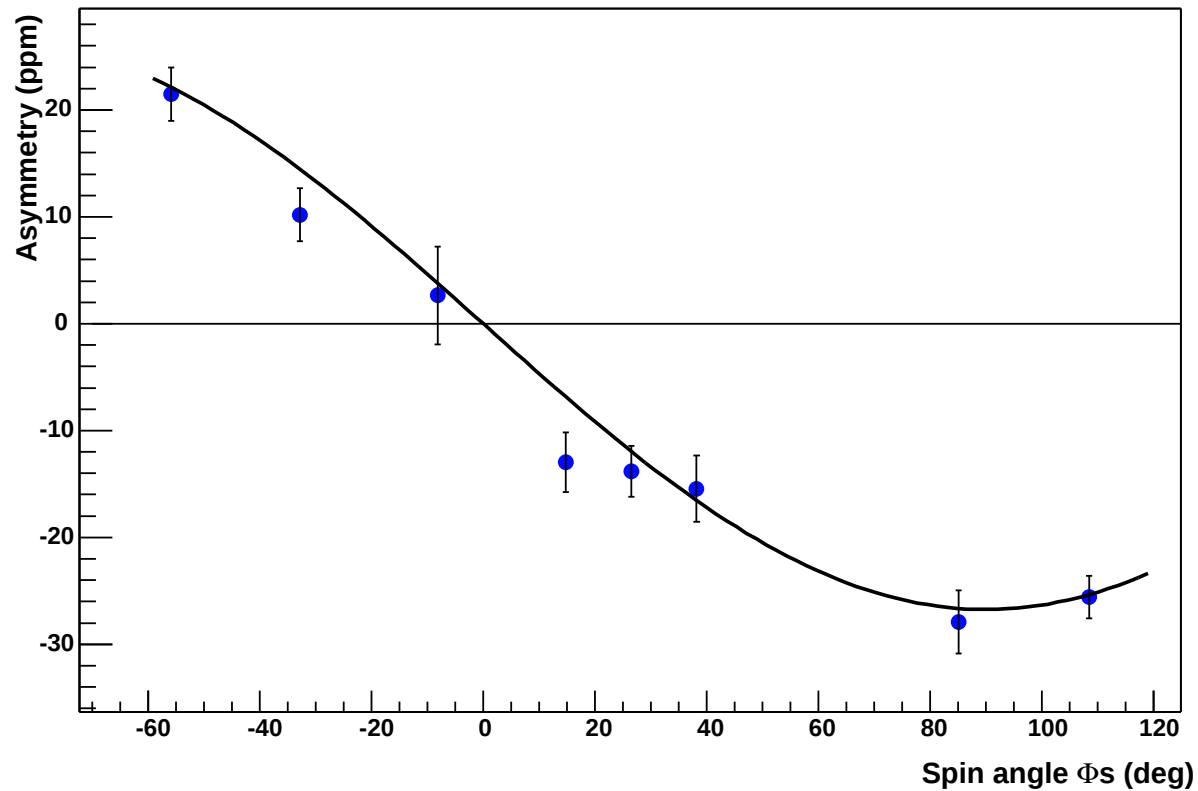
$\Rightarrow$ 8 Spin Angles ϕ_s , 8 Amplitudes $A_L(\phi_s)$



Møller Scattering

Spin angle dependency of A_L

$A_L(\phi_s)$ seen by luminosity monitors:



Sensitive to **transverse** spin



Møller Scattering

Single Spin Asymmetry in Møller Scattering, Results

Energy	A_{Lumi} Experiment (ppm)	A_{Lumi} NLO calculation (ppm)
570 MeV	-26.7 ± 1.3	-29.6
855 MeV	-23.3 ± 4.3	-16.6

- We agree in sign and magnitude with QED calculations
- Asymmetries in LuMos not negligible
- Φ -symmetric summation: cancellation of asymmetry

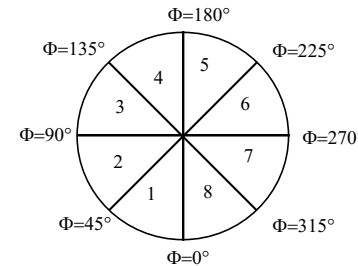


Determination of $A_{\perp}$

Geometrical aspects

Azimuthal range of each sector

$$d\Phi \approx \pi/4$$



$$P \cdot A_{Phys}(\Phi) = \frac{\int_{Sec} A_{\perp} \cdot \cos \varphi d\varphi}{\int_{Sec} d\varphi} \equiv F^{Sec} \cdot A_{\perp}$$

Sec	Φ_0	Φ_{min}	Φ_{max}	F^{Sec}
1	22.19°	0°	44.38°	0.903
2	66.58°	44.38°	88.77°	0.388
3	112.19°	88.77°	135.62°	-0.367
4	157.81°	135.62°	180°	-0.903
5	202.19°	180°	224.38°	-0.903
6	246.58°	224.38°	268.77°	-0.388
7	292.19°	268.77°	315.62°	0.367
8	337.18°	315.62°	360°	0.903



Determination of $A_{\perp}$

$$E = 855.2 \text{ MeV}, Q^2 = 0.23 (\text{GeV}/c)^2$$

$\approx 50h$ beam data

Data taking in January 03:

Sectors 1, 2, 5, 6, 8 of PbF_2 calorimeter equipped

Total number of elastic counts: $N_{total} = 6.3 \cdot 10^{11}$

$$\Phi_S = 99.9^\circ \pm 1^\circ$$

$$E = 569.3 \text{ MeV}, Q^2 = 0.11 (\text{GeV}/c)^2$$

$\approx 50h$ beam data

Data taking in September 03:

Calorimeter fully equipped

Total number of elastic counts: $N_{total} = 3.9 \cdot 10^{12}$

$$\Phi_S = 96.8^\circ \pm 0.5^\circ$$



Determination of $A_{\perp}$

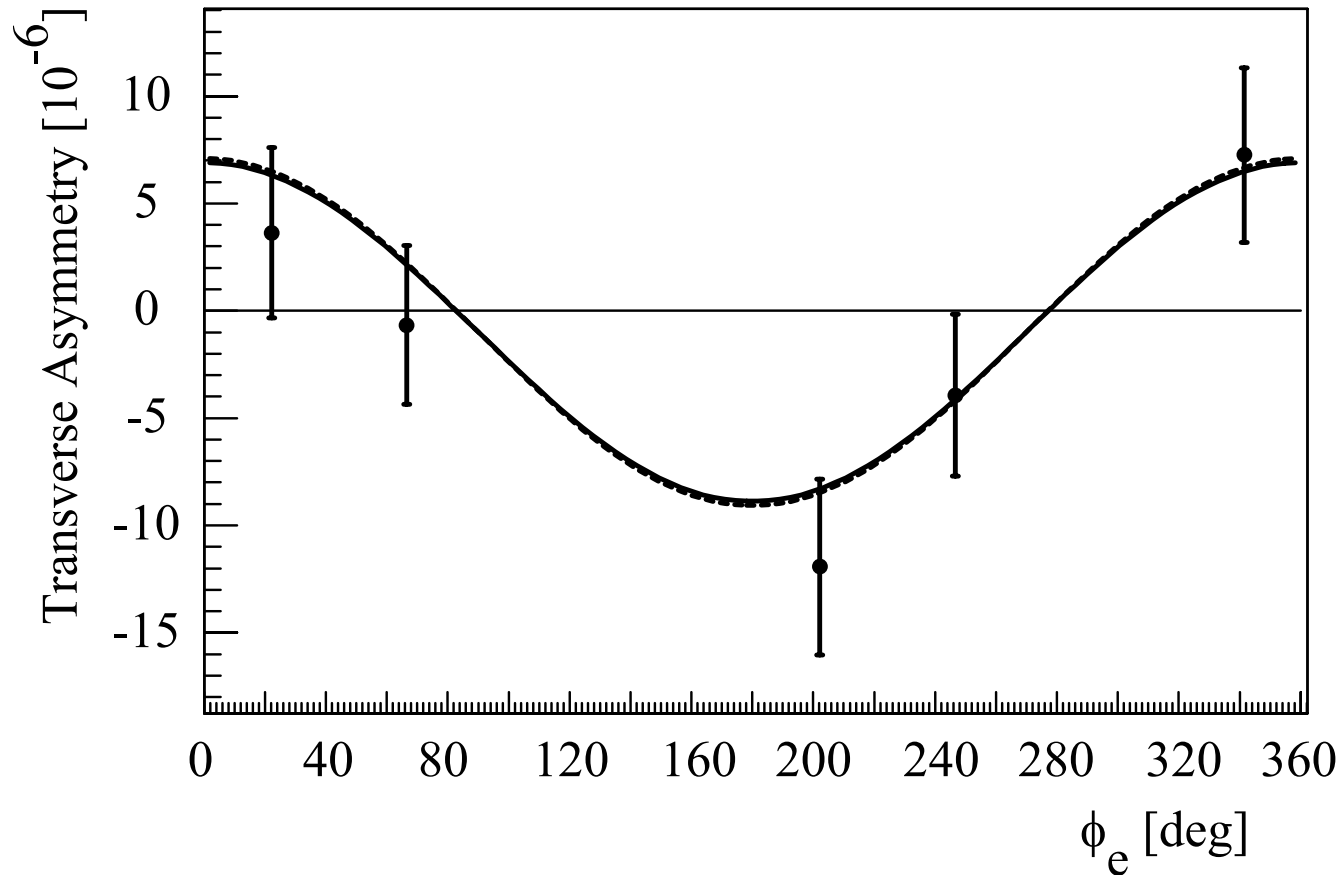
Beam parameter

Parameter	0.23 (GeV/c)^2	0.11 (GeV/c)^2
A_I	$(2.5 \pm 0.6) \text{ ppm}$	$(0.4 \pm 1.4) \text{ ppm}$
Δx	$(15.1 \pm 3.4) \text{ nm}$	$(157.0 \pm 150.0) \text{ nm}$
Δy	$(-126.7 \pm 18.0) \text{ nm}$	$(-504.3 \pm 43.7) \text{ nm}$
$\Delta x'$	$(5.4 \pm 0.8) \text{ nrad}$	$(20.5 \pm 12.9) \text{ nrad}$
$\Delta y'$	$(21.5 \pm 3.0) \text{ nrad}$	$(45.4 \pm 3.9) \text{ nrad}$
ΔE	$(0.2 \pm 0.3) \text{ eV}$	$(41.4 \pm 2.6) \text{ eV}$



Determination of $A_{\perp}$

$$E = 855.2 \text{ MeV}, Q^2 = 0.23 (\text{GeV}/c)^2$$

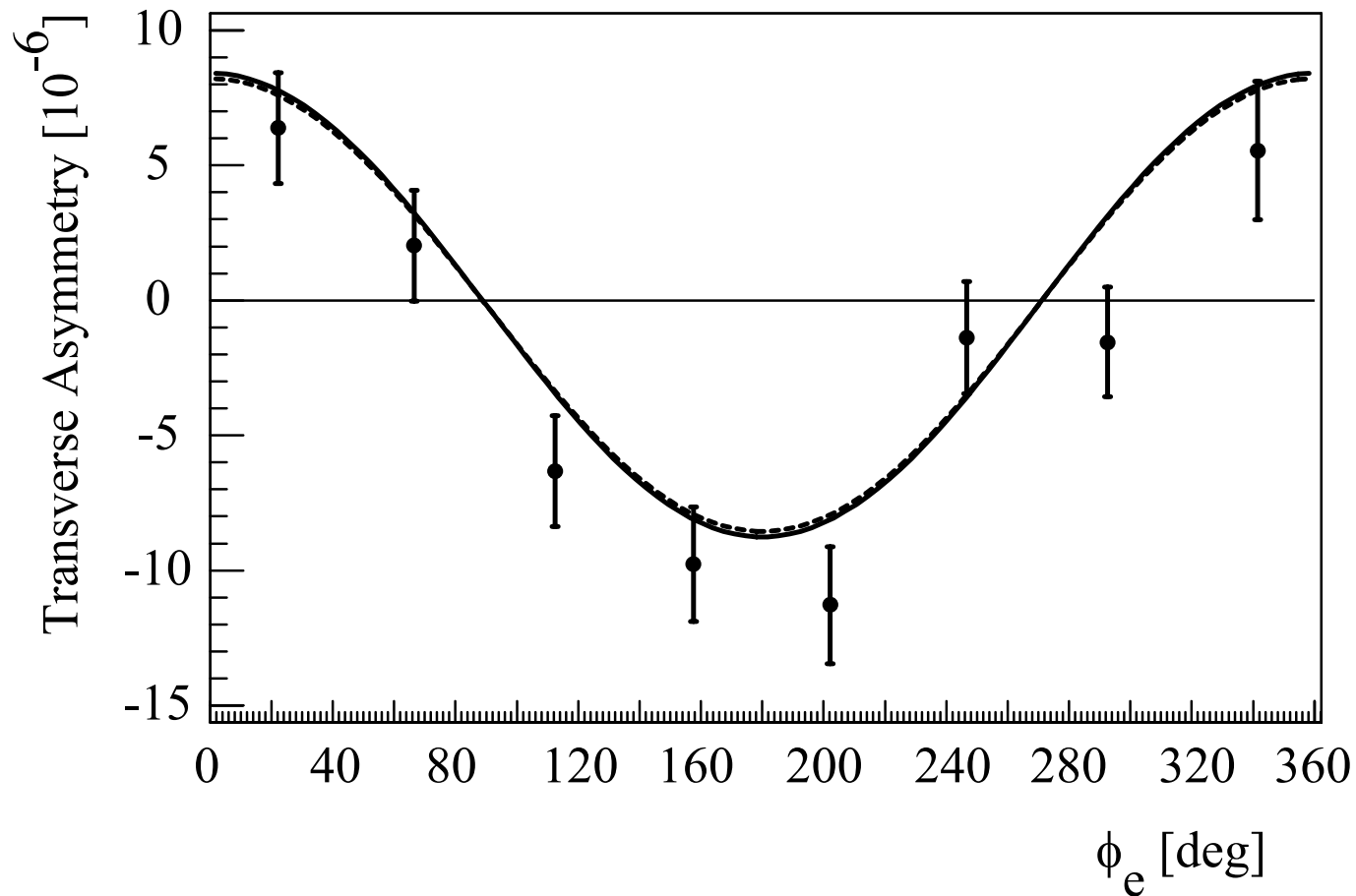


$$A_{\perp} = (-8.52 \pm 2.31_{stat} \pm 0.87_{syst}) \text{ ppm}$$



Determination of $A_{\perp}$

$$E = 569.3 \text{ MeV}, Q^2 = 0.11 (\text{GeV}/c)^2$$

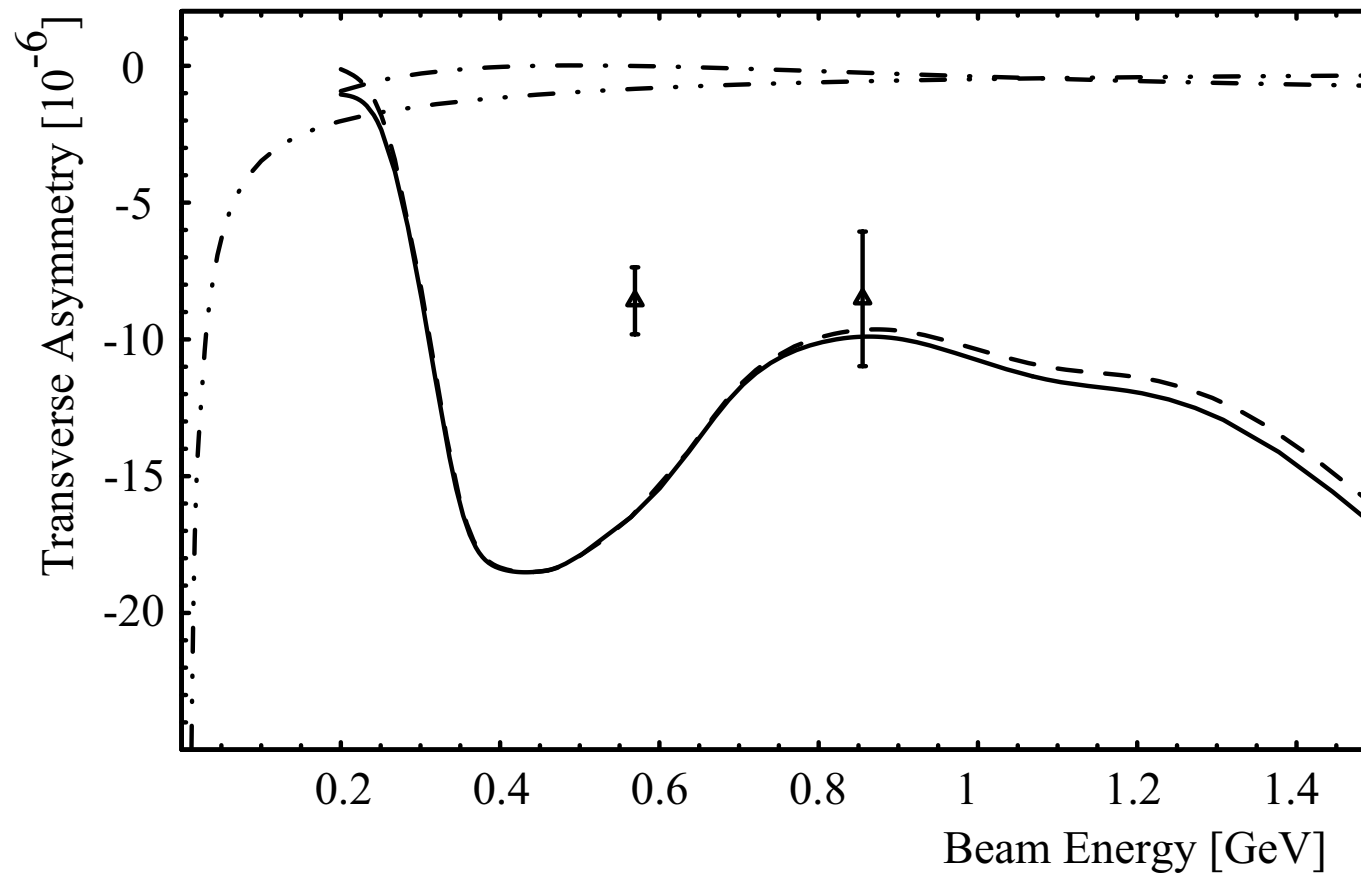


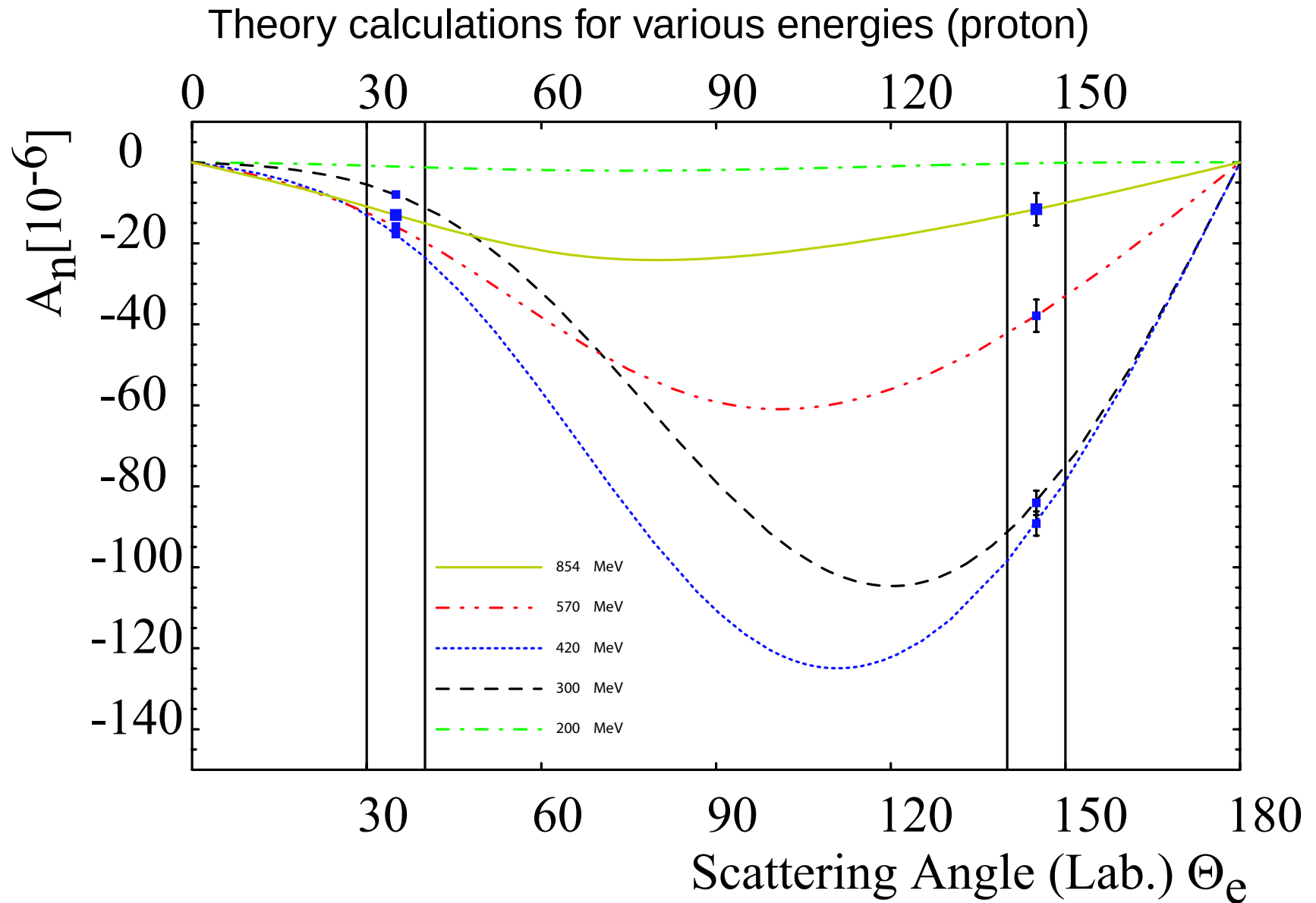
$$A_{\perp} = (-8.59 \pm 0.93_{\text{stat}} \pm 0.75_{\text{syst}}) \text{ ppm}$$



Comparison of $A_{\perp}$ with theory

Intermediate state: proton, πN states (MAID)
(B. Pasquini)







Projected future measurements

E_e [MeV]	θ_e [°]	$\delta A_{\perp}^p$ [ppm]	hours proton	$\delta A_{\perp}^D$ [ppm]	hours deuteron	$\delta A_{\perp}^n$ [ppm] (extracted)
300	$(35 \pm 5)^\circ$	0.5	20	0.5	20	20
420	$(35 \pm 5)^\circ$	0.5	40	0.5	35	11
570	$(35 \pm 5)^\circ$	0.5	90	0.5	70	7
854	$(35 \pm 5)^\circ$	0.5	300	0.5	220	4
1200	$(35 \pm 5)^\circ$	1	260	1	180	6
1500	$(35 \pm 5)^\circ$	2	180	2	120	11
300	$(145 \pm 5)^\circ$	3	90	2	130	10
420	$(145 \pm 5)^\circ$	3	230	2	320	10
570	$(145 \pm 5)^\circ$	4	370	3	390	13
854	$(145 \pm 5)^\circ$	8	490	7	380	28

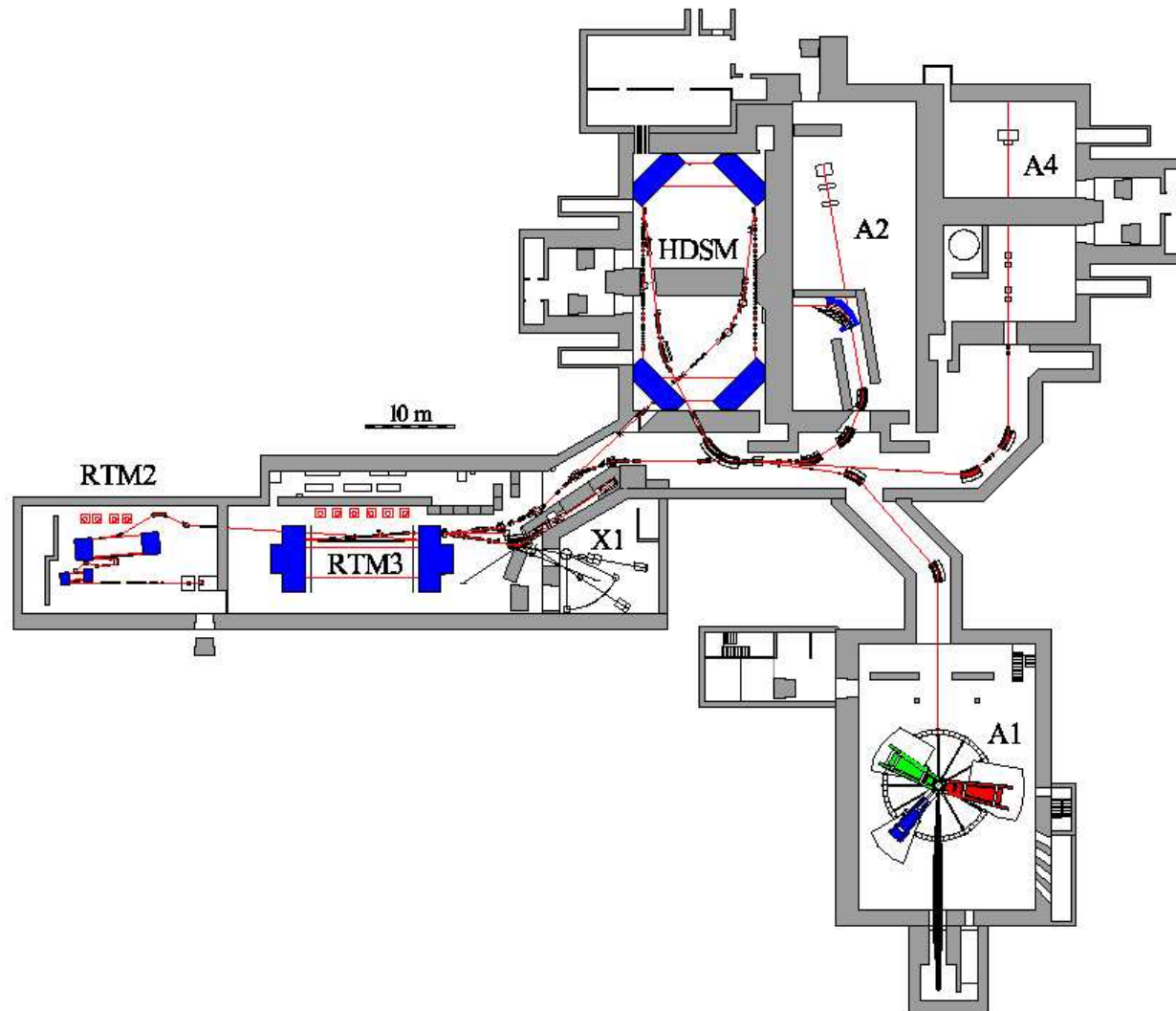


Summary

- Single spin asymmetries in elastic ep scattering
- Longitudinal spin: Contribution of strange quarks to form factors
- Determination of $G_E^s + xG_M^s$ for two momentum transfers
- Measurements with backward kinematics in preparation
- Transverse spin: Imaginary part of 2γ exchange amplitude
- Substantial contribution of πN intermediate states
- Measurements at various energies at forward and backward angles, proton and deuteron projected



MAMI Accelerator





$$E = 855.2 \text{ MeV}, Q^2 = 0.23 (\text{GeV}/c)^2$$

Contributions to corrections and systematic error

Quantity	Correction (ppm)	Systematic Error (ppm)
Current asymmetry A_I	-1.14	0.30
Horizontal pos. diff. Δx	-0.47	0.43
Vertical pos. diff. Δy	0.02	0.14
Horizontal angle diff. $\Delta x'$	-0.09	0.39
Vertical angle diff. $\Delta y'$	0.02	0.18
Energy diff. ΔE	0.09	0.07
Pile-Up correction	-0.11	0.26
LuMo Nonlinearity	0.39	0.01
Target density (current)	0.00	0.12
Target density (LuMo)	0.18	0.03
Spin angle deviation δ	0.46	0.06
Polarisation	1.90	0.30



$$E = 569.3 \text{ MeV}, Q^2 = 0.11 (\text{GeV}/c)^2$$

Contributions to corrections and systematic error

Quantity	Correction (ppm)	Systematic Error (ppm)
Current asymmetry A_I	-0.16	0.11
Horizontal pos. diff. Δx	-0.55	0.14
Vertical pos. diff. Δy	0.29	0.17
Horizontal angle diff. $\Delta x'$	-0.52	0.14
Vertical angle diff. $\Delta y'$	0.42	0.17
Energy diff. ΔE	-0.39	0.04
Pile-Up correction	-1.31	0.29
LuMo Nonlinearity	-0.30	0.00
Target density (current)	0.12	0.05
Target density (LuMo)	-0.91	0.03
Spin angle deviation δ	-0.44	0.01
Polarisation	1.49	0.33



Linear Regression

Covariance matrix:

$$\mathbf{S} = \begin{pmatrix} s_{00} & s_{01} & s_{02} & \dots & s_{06} \\ s_{10} & s_{11} & s_{12} & \dots & s_{16} \\ \dots & \dots & \dots & \dots & \dots \\ s_{60} & s_{61} & s_{62} & \dots & s_{66} \end{pmatrix}$$

with

$$s_{jk}^2 = \frac{\frac{1}{N-1} \sum_{i=1}^N \left[\frac{1}{\delta(X_i^0)^2} (X_i^j - \overline{X^j})(X_i^k - \overline{X^k}) \right]}{\frac{1}{N} \sum_{i=1}^N \frac{1}{\delta(X_i^0)^2}}$$

and

$$\overline{X^j} = \frac{1}{N} \sum_{i=1}^N X_i^j$$



Linear Regression

Correlation matrix:

$$\mathbf{R} = \begin{pmatrix} r_{00} & r_{01} & r_{02} & \dots & r_{06} \\ r_{10} & r_{11} & r_{12} & \dots & r_{16} \\ \dots & \dots & \dots & \dots & \dots \\ r_{60} & r_{61} & r_{62} & \dots & r_{66} \end{pmatrix}$$

with

$$r_{jk}^2 = \frac{s_{jk}^2}{s_{jj}s_{kk}}$$

Correlation coefficients:

$$a^j = \frac{s_{00}}{s_{jj}} \sum_{k=1}^6 r_{k0} r_{j0}^{-1}$$



Linear Regression

Error estimation:

$$\delta a^j = \sqrt{\frac{\frac{1}{N-1} \frac{1}{s_{jj}^2} r_{jj}^{-1}}{\frac{1}{N} \sum_{i=1}^N \frac{1}{(\delta X_i^0)^2}}}$$

$$\delta A^{phys} = \sqrt{\frac{\frac{1}{N} + \frac{1}{N-1} \sum_{j=1}^6 \sum_{k=1}^6 (\overline{X^j X^k} \frac{1}{s_{jj}} \frac{1}{s_{kk}} r_{jk}^{-1})}{\frac{1}{N} \sum_{i=1}^N \frac{1}{(\delta X_i^0)^2}}}$$