Fiscal Policy in an Unemployment Crisis

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Abstract. This paper studies a model of equilibrium unemployment in which the efficacy of fiscal policy increases markedly in times of crises. A sudden rise in pessimism leads households to save rather than to spend, causing a fall in output and rising unemployment. But as a persistent rise in unemployment fuels pessimism, the economy is set on a downward spiral in which thrift reinforces thrift. The government can put this process to an end. An expansion in public spending bolsters demand and lowers the unemployment rate both in the present and in the future. Pessimism is replaced by optimism and the vicious cycle is turned into a virtuous. The marginal impact of government spending on output is negative during normal times. But in a severe recession the fiscal multiplier rises to about three, and expansionary fiscal policy is unambiguously Pareto improving.

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1. Introduction

The recent financial crisis marked the return of depression economics. Short term interest rates were cut to zero, and policy makers explored a plethora of alternative stabilization tools. However, not all of these actions passed without controversy. The American Reinvestment and Recovery Act of 2009 sparked a heated debate over the merits of expansionary fiscal policy, and critics expressed concern over several flaws underlying the theoretical reasoning. This paper aims to address some of the most pressing concerns by providing a novel answer to a classic question: What is the size of the fiscal multiplier?[1]

To answer this question I propose a simple model of equilibrium unemployment. Agents are rational and forward looking, markets clear, and Ricardian equivalence holds. Yet, I show that the potency of fiscal policy can be strikingly large during times of crises. The key reason behind this result stems from a novel interplay between two very basic ingredients. First, at a zero rate of nominal interest, output is largely determined by demand. If households wish to consume more, firms will also produce more. Second, the labor market is inertial. Any change in current unemployment will therefore persist into the future. Together these two mechanisms imply that an increase in government spending raises output and lowers the unemployment rate both in the present *and* in the future. But as rational economic actors desire to smooth consumption over time, the rise in future output feeds back to a further expansion in the present, and so on. This interplay between present- and future economic activity has the capacity to propagate the efficacy of demand-stimulating policies many time over, and the fiscal multiplier exceeds unity under a wide range of circumstances.[2]

I model a demand shock as the arrival of some disappointing news concerning future labor productivity.[3] The sudden rise in pessimism leads households to save rather than to spend, causing a decline in the nominal rate of interest. If news are sufficiently ominous, the interest rate is brought to zero and the economy falls into a liquidity trap. Savings materialize as excess cash holdings and nominal spending plummets.

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[1] *The fiscal multiplier* refers to the marginal change in output in response to a marginal change in *contemporaneous*, and *wasteful*, government purchases. I therefore abstract from possible cumulative effects on output, anticipation effects of future policy, public employment, and productive government investments.

[2] As Ricardian equivalence holds, there is no conflict between a tax- or debt-financed expansion in spending, and the balanced budget multiplier exceeds unity as well (Barro, 1974; Haavelmo, 1945).

[3] The idea that news about future fundamentals may be a key driver of business cycles originates back to Pigou (1927). Beaudry and Portier (2004) inspired a renewed interest in the topic (e.g. Jaimovich and Rebelo (2009); den Haan and Kaltenbrunner (2009); den Haan and Lozej (2010)), and recent empirical studies lend support to this view of macroeconomic fluctuations (e.g. Beaudry and Portier (2006); Barsky and Sims (2012)).

In a competitive market, a fall in demand takes the form of a fall in the price level. If nominal wages fail to adjust accordingly, falling prices reduces profits, discourages hiring, and provokes a marked decline in current economic activity. With rising, and *persistent*, unemployment the future appears even bleaker. Additional measures to smooth consumption only amplifies the initial decline in economic activity, raises unemployment, and further depresses the economic outlook. The economy is set on a downward spiral of self-reinforcing thrift which can have abysmal effects on economic activity *even in the absence* of any real shocks to contemporaneous productivity.

Where does this process end? I quantitatively assess these ideas by calibrating the model to the US economy. In the benchmark case, agents unexpectedly learn that future labor productivity will temporarily fall by 6.5 percent.[4] The economy responds in a Pigouvian fashion with a simultaneous fall in output, consumption, investment, and employment. The unemployment rate rises with around 3.5 percentage points on impact, and reaches 11.5 percent once the productivity shock materializes. Investments are driven to zero, and output immediately declines by 3.5 percent, with a total fall of 13 percent from peak to trough.

But the same mechanism which propels the economy downwards can also be turned to our advantage. An expansion in government spending reduces the unemployment rate both in the present and in the future, and pessimism is replaced by optimism. A temporary rise in spending equal to five percent of steady-state output attenuates the increase in unemployment to a mere 0.5 percentage points, and reduces the peak unemployment rate to about 8.5 percent. Investments rise, and the fall in output is cushioned to 0.5 percent on impact, and around 10 percent at the trough.

A simple back of the envelope calculation suggest that the immediate multiplier effect associated with this expansion is around 0.6.[5] This may appear small. But it would be misguided to pass too much judgement based on this metric. First, the calculation represents the *average* response to a relatively large expansion in government spending. The *marginal* response in output from a marginal rise in government spending is much higher, and reaches about *three*. Second, the expansion in output echoes for many periods into the future. The average multiplier is 0.6 on impact, and around 0.55 in the subsequent period. The

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[4]The standard deviation of HP-filtered labor productivity in post-war US data is around 0.02 log points (Shimer, 2005). A drop of 6.5 percent is roughly equivalent to a fall of three standard deviations. The only post-war recession which displays such a large drop in labor productivity occurred in the early 1980s. The unemployment rate rose from 5.6 percent in May 1979 to 10.8 percent in December 1982, which should be compared to a rise from 5 percent to 11.5 percent in the model presented here (see Figure 2, p. 20).

[5]The multiplier is calculate as the total change in output divided by the total change in government spending, which approximately equals $\frac{3.5-0.5}{5} = 0.6$.

cumulative response is therefore 1.15 only after two periods, and converges to 2.5 in the limit. Lastly, the expansion in government spending reduces the welfare cost associated with the demand crisis with almost 60 percent. Fiscal policy is unambiguously Pareto improving.

I further explore these nonlinearities by computing the response to a variety of different news shocks. During a deep recession with an unemployment rate exceeding the natural by two percentage points or more the marginal multiplier remains in the neighborhood of three. Optimal spending is about three percent of steady-state output, and welfare costs are reduced by around 35-60 percent, depending on the size of the crisis. The average multiplier tends to lie below unity on impact, but cumulates over time at a factor of around four and always exceeds one. In lesser recessions, the marginal multiplier is much smaller and optimal spending gravitates to zero. During normal times, the multiplier is negative.

Empirical research lend support to these findings. While estimates of the multiplier are generally very dispersed, a consensus view appears to have emerged in which the efficacy of fiscal policy varies crucially with the state of the economy.[6] Barro and Redlick (2011), for instance, find a multiplier of 0.7. But when they allow for interactions with the unemployment rate, the multiplier rises to unity.[7] Gordon and Krenn (2011) confine attention to the defense build-up associated with World War II. They argue that past estimates are attenuated by capacity constraints during the later stages of the war, as well as outright prohibitions on the production of civilian goods. By cutting their sample at the second-, instead of the fourth quarter, of 1941, they avoid such concerns and the estimated multiplier increases from 0.9 to 2.5. Recent evidence from cross-state studies further corroborates these findings. Nakamura and Steinsson (2011) and Shoag (2010) show that estimates of the multiplier may nearly double in periods in which the unemployment rate exceeds the sample mean. And using a structural VAR based approach, Auerbach and Gorodnichenko (2011; forthcoming) and Bachmann and Sims (2012) show that the multiplier is only moderate, or even negative, in expansions, while it exceeds two in periods of economic slack.[8]

Theoretical studies on fiscal policy have almost reached an equally wide range of conclusions. Within the standard flexible-price neoclassical framework, a rise in government spending reduces private wealth and stimulates labor supply. Real wages fall in response to

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[6]In a survey of the literature, Ramey (2011) concludes that an increase in government purchases stimulates the economy with a multiplier between 0.8 and 1.5, adding that "the data do not reject 0.5 or 2" (p. 673). Hall (2009) suggests a slightly smaller range of 0.7 to 1.0, but adds that "higher values are not ruled out" (p. 183). Parker (2011) provides a detailed discussion of the state-dependence of fiscal policy.

[7]Assuming an unemployment rate of 12 percent.

[8]Interestingly, Bachmann and Sims (2012) argue that the increase in policy effectiveness during periods of economic slack can be attributed to the rise in consumer confidence following an expansion in government spending. The rise in confidence is in turn associated with an improvement in future fundamentals.

clear the labor market, but the net effect on output is unambiguously positive. However, the same wealth-effect which instills a rise in output crowds out private consumption, and the multiplier always falls short of unity (e.g. Hall (1980), Barro (1981), and Barro and King (1984)).[9] Studies in the new Keynesian tradition suggest instead a countercyclical response in markups. As the rise in public spending stimulates labor supply, real wages increase instead of decrease, and cushion the aforementioned fall in private consumption. The response in economic activity, however, depends crucially on the conduct of monetary policy, and the fiscal multiplier remains below unity under a wide range of circumstances.[10]

Those circumstances do not extend to a situation of a liquidity trap. With nominal interest rates tied up against zero, monetary policy loses traction and the multiplier rises to unity (Krugman, 1998; Woodford, 2011). In recent and highly influential work, Christiano et al. (2011) and Eggertsson (2010) further explore these ideas. They show that when the economy is repeatedly hit by multiple liquidity spells, staggered pricing causes a deflationary spiral which raises the real interest rate and propels the economy into a deep recession. An expansion in government spending does not only rise output, but also temporarily eases some of the deflationary momentum. A committed long-lasting expansion in government spending may therefore set the economy on an inflationary path in which spending begets spending. The fiscal multiplier, they show, can under these circumstances be sizable and easily exceed one.

This paper departs from these studies along several important dimensions. Firstly, the main thrust in this paper is governed by labor *demand*, and not by supply. A rise in government spending bears no consequence on an agent's willingness to work, but instead affects a firm's willingness to hire.[11] At a positive rate of nominal interest, government spending raises the real interest rate, discourages hiring, and contracts output. During normal times the multiplier is therefore negative.

Secondly, Christiano et al. (2011) and Eggertsson (2010) analyze the countercyclical implications of a committed, long-lasting, expansion in fiscal outlays. I consider the implications of a temporary demand-stimulating burst in government spending. Letting the duration of the policy-expansion approach one attenuates the multiplier in Eggertsson (2010)

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[9]Fatas and Mihov (2001), Blanchard and Perotti (2002), Galí et al. (2007), Perotti (2008), Fisher and Peters (2009) find empirical support of a positive response in consumption. But studies such as Ramey and Shapiro (1998) and Ramey (2011) do not.

[10]See for instance, Galí et al. (2007), Monacelli and Perotti (2008), and Woodford (2011) for a detailed discussion. Hall (2009) provides an excellent survey.

[11]Michaillat (2012) explores a related framework in which firms ration jobs in recessions. An increase in public employment lowers the unemployment rate and raises output at a multiplier of around 0.5.

to unity, and Christiano et al. (2011) to 1.3.[12] It is therefore neither the presence of a liquidity trap nor the increase in contemporaneous spending *per se* which renders a large fiscal multiplier. Instead, it is the *combination* between a deep and prolonged recession with a long lasting, committed, fiscal expansion that provides a fertile ground for effective public spending.[13] These ideas contrast to this study in which a purely temporary increase in government spending may be highly effective even in a brief, albeit deep, downturn.

Lastly, the nonlinear properties of the model provide theoretical foundations to the state-dependent pattern of policy efficacy observed in several empirical studies.[14] The fiscal multiplier is low, or even negative, during normal times, but rises tremendously in times of crises. And in sharp contrast to most previous studies, the welfare consequences of (otherwise wasteful) fiscal stabilization policy is unambiguously positive (cf. Bilbiie et al. (2012)).

2. Model

The economy is populated by a government, a large number of potential firms, and a unit measure of households. The planning horizon is infinite, and time is discrete. There are two types of commodities in the economy. Cash, m_t, which is storable, but not edible. And output, y_t, which is edible, but not storable. Cash assumes the role of the numeraire, and output trades at relative price p_t. In order to abstract from any interaction effects with monetary policy, I assume that cash is in fixed supply, such that $m_t = m$ for all time periods, t. The output good, however, is repeatedly produced in each period using labor, n_t, and labor productivity, z_t, as the only factors of production. The labor market is frictional, and the precise nature of technology will be specified and discussed in the subsequent sections. There is no physical capital in the conventional sense, but there will be investments.

2.1. **Households.** Households initiate their lives in period zero. They supply labor inelastically and the time-endowment is normalized to one, $\ell_t = 1$. Employment is denoted n_t, and the unemployment rate is therefore given by the difference in labor supplied and labor demanded, $u_t = 1 - n_t$. In a frictional labor market employment is beyond the control

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[12]In this situation the real interest rate is constant and a unit multiplier follows straightforwardly from the analysis of Krugman (1998) and Woodford (2011). A multiplier of 1.3 in Christiano et al. (2011) follows from non-separable preferences in consumption and leisure as illustrated in Bilbiie (2009), and Monacelli and Perotti (2008).

[13]Woodford (2011), p. 24, summarizes these conclusions as "Eggertsson (2010) obtains a multiplier of 2.3, 1.0 of this is due to the increase in government purchases during the current quarter, while the other 1.3 is the effect of higher anticipated government purchases in the future". An analogous argument applies to Christiano et al.

[14]See, amongst others, Auerbach and Gorodnichenko (2011; forthcoming) and Bachmann and Sims (2012).

of the households and will, for the time being, be treated as given. The wage-rate in the economy is denoted $\tilde{w}_t$.

Each household own firm equity. I will let q_t^t denote the quantity of shares held in period t (subscript), and purchased in period t (superscript). As a fraction, λ, of firms will exit the market in each period it follows that $q_{t+1}^t = (1-\lambda)q_t^t$.[15] Equity dividends are denoted $\tilde{d}_t$.

Total income, or simply *income*, w_t, constitutes both total labor income, $n_t \times \tilde{w}_t$, as well as dividends, $q_t^t \times \tilde{d}_t$. There are complete insurance markets across households, so each household earns income w_t irrespective of whether she is employed or not. Following Lucas (1982), income is received by the very end of a period – i.e. after any consumption decisions – and is therefore *de facto* first disposable in the ensuing period.

A representative household enters period t with bonds b_t, and equity q_t^{t-1}. She receives income w_{t-1}, as well as any unspent cash from the preceding period. Bonds are nominally riskless and pay net return i_t. The price of equity in terms of the output good is denoted J_t, so the total nominal equity value is given by $p_t J_t q_t^{t-1}$. Out of these nominal resources, the household pays lump-sum taxes T_t, and may spend the remainder on consumption, $p_t c_t$, or on purchases of new assets.[16] Households are restricted to settle all consumption purchases in cash, such that $M_t \geq p_t c_t$. The sequence of budget constraints is therefore given by

$$b_t(1+i_t) + p_t J_t(q_t^{t-1} - q_t^t) + (M_{t-1} - p_{t-1}c_{t-1}) + w_{t-1} - T_t = M_t + b_{t+1} \tag{1}$$

$$M_t \geq p_t c_t, \qquad t = 0, 1, \ldots \tag{2}$$

where the term $(M_{t-1} - p_{t-1}c_{t-1})$ refers to unspent cash in the preceding period.

As *cash hoarding* plays an integral part in this paper, it is instructive to make the change of variables $x_{t+1} = M_t - p_t c_t$, and rewrite equations (1)-(2) as

$$b_t(1+i_t) + p_t J_t(q_t^{t-1} - q_t^t) + x_t + w_{t-1} - T_t = p_t c_t + x_{t+1} + b_{t+1} \tag{3}$$

$$x_{t+1} \geq 0, \qquad t = 0, 1, \ldots \tag{4}$$

The variable x_{t+1} will be referred to as *excess cash holdings* in period t.

Given a process of taxes, prices, and dividends, $\{T_t, p_t, J_t, i_t, \tilde{w}_t, \tilde{d}_t\}_{t=0}^{\infty}$, and given a process of employment $\{n_t\}_{t=0}^{\infty}$, the household decides on feasible consumption, excess cash, and asset plans, $\{c_t, x_{t+1}, b_{t+1}, q_t^t\}_{t=0}^{\infty}$, to maximize her expected net present value utility

$$V(\{c_t\}_{t=0}^{\infty}) = E \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{5}$$

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[15]Households therefore hold a diversified portfolio of otherwise identical firms/assets.

[16]As labor supply is entirely inelastic, lump-sum taxes are isomorphic to income taxes, and the distinction is meaningless from a substantive point of view.

The momentary utility function $u(\cdot)$ displays constant relative risk aversion, such that $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$. The parameter σ is commonly known as the coefficient of relative risk aversion, and its reciprocal represents the elasticity of intertemporal substitution. The expectations operator denotes the mathematical expectation with respect to future processes, conditional information available in period zero.

The first order conditions associated with the problem in (5) subject to (3)-(4) are given by the Euler equation for bond holdings, b_{t+1}

$$u'(c_t) = \beta E_t[(1 + i_{t+1})\frac{p_t}{p_{t+1}}u'(c_{t+1})] \tag{6}$$

the Euler equation for excess cash holdings, x_{t+1}

$$u'(c_t) - \mu_t = \beta E_t[\frac{p_t}{p_{t+1}}u'(c_{t+1})] \tag{7}$$

and the asset pricing equation for equity, q_t^t

$$J_t = \beta E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\left(\frac{\tilde{d}_t}{p_{t+1}} + (1-\lambda)J_{t+1}\right)\right] \tag{8}$$

The variable μ_t denotes the Lagrange multiplier associated with the cash in advance constraint (4). Thus, μ_t and x_{t+1} satisfy the complementary slackness conditions

$$\mu_t \geq 0, \quad x_{t+1} \geq 0, \quad \text{and} \quad \mu_t \times x_{t+1} = 0$$

As a consequence, excess cash holdings are strictly positive only if the nominal interest rate is zero – i.e. when bonds and cash act as perfect substitutes. This leads to the following definition of a *liquidity trap*.

Definition 1. *The economy is in a liquidity trap in period t, if and only if $x_{t+1} > 0$.*

2.2. **Government.** Apart from lump-sum taxes, the government has access to two additional policy tools; government spending, G_t, and public debt, d_t. For the ease of exposition, they are all denominated in terms of the numeraire. A *fiscal plan* is a process of taxes, spending, and debt, $\{T_t, G_t, d_t\}_{t=0}^{\infty}$, which satisfies the sequence of budget constraints

$$T_t + d_{t+1} = G_t + (1 + i_t)d_t, \quad t = 0, 1, \dots \tag{9}$$

as well as the no-Ponzi condition

$$\lim_{t \to \infty} \frac{d_{t+1}/p_{t+1}}{\Pi_{n=0}^t (1 + i_{n+1})p_n/p_{n+1}} = 0 \tag{10}$$

The net present value of real government spending is therefore equal to the net present value of real tax revenues.

2.3. **Firms.** A potential firm opens up a vacancy at cost $k \geq 0$. The vacancy cost is denominated in terms of the output good. Conditional on having posted a vacancy, the firm will instantaneously meet a worker with probability ϕ_t. If not, the vacancy is void and the vacancy-cost, k, is sunk. A successfully matched firm-worker pair becomes immediately productive and produces z_t units of the output good in each period. The employment relation may last for perpetuity, but workers and firms separate at rate λ. Equity therefore pay nominal dividends $\tilde{d}_t = p_t z_t - \tilde{w}_t$.

A representative entrepreneur seek to maximize the share-value of the firm, J_t. As a consequence, a vacancy is posted in period t if and only if the expected benefits, $\phi_t J_t$, (weakly) exceed the associated cost; $\phi_t J_t \geq k$.

2.4. **Matching Market.** The labor market is frictional. Let $\hat{u}_t$ denote the *beginning of period* unemployment rate. That is, $\hat{u}_t = u_{t-1} + \lambda n_{t-1}$. And let $\hat{v}_t$ denote the measure of vacancies in period t. Following the ideas underlying the Diamond-Mortensen-Pissarides tradition (e.g. Diamond (1982); Mortensen and Pissarides (1994)), the measure of successful matches is given by the matching function

$$H_t = H(\hat{v}_t, \hat{u}_t)$$

The function $H(\cdot, \cdot)$ exhibits constant returns to scale, and a firm posting a vacancy will therefore find a worker with probability

$$\phi_t = \frac{H_t}{\hat{v}_t} = \phi(\theta_t), \quad \text{with} \quad \theta_t = \frac{\hat{v}_t}{\hat{u}_t}$$

As usual, θ_t, denotes the labor market tightness in period t. Analogously, a worker unemployed in the beginning of period t will find a job with probability

$$\rho_t = \frac{H_t}{\hat{u}_t} = \rho(\theta_t), \quad \text{with} \quad \rho_t = \theta_t \phi_t$$

The law of motion for employment is then given by

$$n_t = \hat{u}_t \rho_t + (1 - \lambda) n_{t-1}$$

2.4.1. *Wage bargaining.* Nominal wages, $\tilde{w}_t$, are determined by Nash-bargaining. Nash-bargaining seeks to maximize the Nash-product of each party's surplus associated with a match. As there are complete insurance markets across households, however, unemployment has no financial meaning in the current context and a household's surplus of a match is always zero. To circumvent this problem, I will follow the rhetorical device pioneered by den Haan et al. (2000) and later Gertler and Trigari (2009), in which workers are viewed as separate risk-neutral entities which are owned and traded by households. Thus there is

a price-tag attached to each worker. And the market price for an employed worker, V_t, is given by

$$V_t = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \left(\frac{\tilde{w}_t}{p_{t+1}} + \lambda U_{t+1} + (1-\lambda)V_{t+1} \right) \right]$$

While the market price for an unemployed worker, U_t, is given by

$$U_t = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \left(\frac{\tilde{b}_t}{p_{t+1}} + \rho_t V_{t+1} + (1-\rho_t)U_{t+1} \right) \right]$$

where $\tilde{b}_t$ denotes (nominal) unemployment benefits in period t. Unemployment benefits are financed through lump-sum taxes levied on households.

Wages are then set according to

$$\tilde{w}_t = \text{argmax} \left\{ J_t^\xi (V_t - U_t)^{1-\xi} \right\}$$

where the parameter $\xi \in (0,1)$ governs the firms' relative bargaining power over workers.

2.5. **Equilibrium.** We can now state the definition of a competitive equilibrium.

Definition 2. *Given a fiscal plan, a **competitive equilibrium** is a process of prices, $\{p_t, i_{t+1}, \tilde{w}_t, J_t\}_{t=0}^\infty$, and quantities, $\{c_t, q_t^t, b_{t+1}, x_{t+1}, \theta_t, y_t, n_t, I_t\}_{t=0}^\infty$, such that,*

(1) *Given prices, $\{c_t, q_t^t, b_{t+1}, x_{t+1}\}_{t=0}^\infty$ solves the households' problem.*

(2) *Labor market tightness, θ_t, satisfies the free-entry condition $k = \phi(\theta_t)J_t$.*

(3) *Employment, n_t, satisfies the law of motion*

$$n_t = (\lambda n_{t-1} + (1 - n_{t-1}))\rho(\theta_t) + (1-\lambda)n_{t-1}$$

(4) *Output, y_t, is given by $y_t = z_t n_t$.*

(5) *Investments, I_t, is given by $I_t = p_t v_t k$.*

(6) *Wages satisfy $\tilde{w}_t = \text{argmax} \left\{ J_t^\xi (V_t - U_t)^{1-\xi} \right\}$.*

(7) *Bond markets clear; $b_t = d_t$.*

(8) *Equity markets clear; $q_t^t = n_t$.*

(9) *Goods markets clear; $p_t y_t = p_t c_t + G_t + I_t$.*

2.5.1. *The equation of exchange.* Consolidating the households' and the government's budget constraints yields

$$(b_t - d_t)(1 + i_t) + x_t + p_t J_t(q_t^{t-1} - q_t^t) + w_{t-1} = p_t c_t + G_t + x_{t+1} + (b_{t+1} - d_{t+1}) \quad (11)$$

Bond market clearing infers that $b_t - d_t = 0$, and goods market clearing that $p_t c_t + G_t = p_t y_t - I_t$. Thus, the budget constraint in (11) can be rewritten as

$$x_t + p_t J_t(q_t^{t-1} - q_t^t) + w_{t-1} = p_t y_t - I_t + x_{t+1}$$

From the equilibrium conditions on the equity- and labor market it follows that the change in equity shares held between two consecutive periods, $(q_t^t - q_t^{t-1})$, must equal H_t; the measure of matches, or start-ups, in period t. Using the free-entry condition, it becomes immediate that

$$p_t J_t H_t = p_t \frac{k}{\phi_t} H_t$$

$$= p_t k v_t = I_t$$

Thus, the change in aggregate equity holdings between two consecutive periods must equal aggregate investments.

Equation (11) therefore simplifies to

$$x_t + w_{t-1} = p_t y_t + x_{t+1} \tag{12}$$

Suppose that for some period t, the left hand side of (12) is equal to m. As nominal output, $p_t y_t$, is equal to nominal income, w_t, the left hand side of (12) in period $t+1$ must also equal m. Thus as long as $x_0 + w_{-1} = m$, it follows that

$$m v_t = p_t y_t, \quad t = 0, 1, \ldots \tag{13}$$

where v_t is the *velocity of money*, $v_t = \frac{m - x_{t+1}}{m}$. Equation (13) is commonly known as the *equation of exchange*, and it relates the supply money and its velocity to the nominal level of output (Fisher, 1911).

Together with the households budget constraint (1), and the Euler equations (6) and (7), the equation of exchange reveals that Ricardian Equivalence holds (Barro, 1974). For instance, given a certain level of spending G_t, an increase in lump-sum taxes, T_t, followed by an equivalent decrease in government debt, d_{t+1}, will be met by an identical decrease in private savings, b_{t+1}, leaving both cash holdings, x_{t+1}, prices, and ultimately the path of consumption *entirely* unaltered. As a consequence, inasmuch as a debt-fueled increase in government spending may reverse a vicious circle, so may a contemporaneously tax financed expansion.

3. ANALYSIS

To analyze the mechanics of the model, I will proceed in two separate but complementary steps. First, I will impose some simplifying assumptions to derive analytical properties. While these assumptions are somewhat restrictive, it will provide a useful lens through which latter results can be viewed. Second, I will solve the complete model numerically.

In both approaches the economy will initially be at the steady-state. In period t the economy is hit by a negative demand shock. The demand shock is modeled as the arrival of some unexpected, pessimistic, news concerning labor productivity in $t+1$. The objective is

then to evaluate to which extent expansionary fiscal policy may alleviate the adverse effects brought on by the news. Throughout the analysis, *fiscal policy* will refer to a one-shot expansion in purely wasteful government spending at the time of the news shock; that is, at time t.

A key ingredient in the analysis concerns downward nominal wage rigidity. In particular, I assume that period t wages are negotiated before the arrival of the news shock. In practice this informational friction is tantamount to a *one*-period spell of downward nominal wage stickiness, as the future sequence of shocks is known. Nominal wage rigidity has received mixed empirical support. Bewley (1999) and Barattieri et al. (2010) argue that downward nominal wage rigidity is prevalent. Others argue that wages for *new hires* are much more flexible (Pissarides, 2009; Haefke et al., 2012). And as wages of new hires dictate the dynamics of unemployment in most search-theoretic frameworks, the economic consequences of wage stickiness may be called into question. Yet, wage stickiness appear important for the transmission of monetary policy (Olivei and Tenreyro, 2007, 2010), and there are reasons to believe that the extent and implications of nominal wage rigidity is not yet entirely understood.[17] Thus, following , Bordo et al. (2000), Gertler and Trigari (2009), den Haan and Lozej (2010), and many others, I will proceed under the assumption that nominal wages are downwardly rigid – at least for a short period of time.

3.1. **A disequilibrium analysis.** To simplify the model, I will dispense with the long-run nature of the firm's problem and instead consider firms that operate a static constant returns to scale technology. Inertia, or frictions, in the labor market exist but are imposed rather than derived, and the evolution of unemployment follows an exogenously specified law of motion. Despite these limitations, the resulting framework generates several insights which will prove useful in interpreting subsequent results. The basic structure of the model largely follows that of Krugman (1998), extended with labor market frictions.

Firms produce the output good using labor, n_t, and labor productivity, z_t, according to the technology

$$y_t = z_t n_t$$

As a consequence, the hiring decision of a price-taking and profit-maximizing firm observes

$$p_t z_t = \tilde{w}_t \tag{14}$$

Constant returns to scale implies that both the number of firms, as well as the measure of hired workers, are determined by the demand for goods.

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[17]Diamond (2011) argue that what matters for job creation is "reservation wages at the marginal vacancy", which is unlikely to observed in the data.

Proposition 1. *Suppose that $z_t = z > 0$ and $G_t/p_t = g$, with $g < z$, for $t = 0, 1, \ldots$ Then there exist a unique competitive equilibrium with prices $p_t = m/z$, $\tilde{w}_t = m$ and $i_{t+1} = 1/\beta - 1$.*

Proof. In Appendix A.   □

Now consider the following hypothetical scenario. The economy is in the steady-state in period $t - 1$. In period t agents receive some unexpected news that labor productivity in period $t + 1$ will decline and equal $\delta < z$. I will then analyze firstly the impact of news on output in period t, and secondly to what extent fiscal policy may backtrack any adverse consequences.

Government spending in period t is comprised by two components; a non-discretionary component, g_t, and a discretionary component, simply denoted g. Both components are denominated in terms of the output good. Spending in period $t + 1$ is given by g_{t+1}. For simplicity, non-discretionary spending is assumed to equal a constant fraction, γ, of output. The metric of policy effectiveness is given by the partial derivative $\partial y_t/\partial g$, or simply by the size of the fiscal multiplier.

To keep the analysis as simple as possible, I will assume that the economy is *not* in a liquidity trap in period $t + 1$. This assumption effectively transforms the economy to a two-period setting, in which there is no need to specify the future process of labor productivity nor government actions. The equation of exchange reveals that prices in $t + 1$ are given by $p_{t+1} = m/y_{t+1}$.

Nominal wages in period t are assumed to be downwardly rigid; $\tilde{w}_t \geq m$.[18] Downward nominal wage rigidity can give rise to a disequilibrium in the labor market in which labor supply may exceed labor demand. If, for instance, household wish to spend less nominal resources in period t – which is a fall in *nominal* demand – prices will fail to adjust leading to a shortfall in *real* demand. And as aggregate demand for labor is determined by the aggregate demand for goods, involuntary unemployment arises and the economy falls into a recession.

As a final ingredient, there are 'frictions' in the labor market. As both wages and prices flexibly adjust in any period beyond time t, one would normally expect labor demand to equal labor supply. This is indeed the working-hypothesis employed in, for instance, Krugman (1998), Shimer (2012) or Schmitt-Grohé and Uribe (2012), which all consider similar disequilibria frameworks albeit in different contexts. In contrast to these studies I will assume that the labor market outcome in period $t + 1$ is influenced by the disequilibrium

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[18]Notice that the firms' first order condition (14) infers that downward nominal wage rigidity implies downward price rigidity (and vice versa).

in period t. More precisely, the labor market outcome in period $t+1$ is given by the relation $n_{t+1} = n_t^\alpha$, with $\alpha \in [0, 1]$.

The parameter α dictates the degree of frictions in the labor market. If α is equal to zero there are no frictions and the labor market clears seamlessly in period $t+1$.[19] On the other hand, if α instead equals one, the economy exhibits unemployment hysteresis. Any rise in unemployment in period t will then persist into period $t+1$ in its entirety. Of course, for any intermediate value of α the economy will exhibit some persistence in unemployment and some reversion to the market clearing outcome.

Under these assumptions, the model is characterised by the following relations[20]

$$u'(y_t) = \beta(1 + i_{t+1})\frac{p_t}{m}\delta(y_t/z)^\alpha u'(\delta(y_t/z)^\alpha) \tag{15}$$

$$m - x_{t+1} = p_t y_t \tag{16}$$

$$x_{t+1} \geq 0, \quad i_{t+1} \geq 0, \quad x_{t+1} \times i_{t+1} = 0 \tag{17}$$

Define δ^* as the lowest possible productivity level in period $t+1$ which does *not* put the economy in a liquidity trap.[21] That is

$$u'(z) = \frac{\beta}{z}\delta^* u'(\delta^*)$$

With constant relative risk-aversion, δ^* is given by $z\beta^{\frac{1}{\sigma-1}}$, which is strictly less than z.

Proposition 2. *If $\delta \geq \delta^*$ there exist a unique equilibrium with $y_t = z$, $y_{t+1} = \delta$, and $p_t = m/z$, and the fiscal multiplier is zero.*

Proof. In Appendix A. □

Together with Proposition 2 above, the equation of exchange reveals that whenever $\delta \geq \delta^*$, $x_{t+1} = 0$ and the economy is not in a liquidity trap. Output in both period t and $t+1$ equal their potential values, z and δ, respectively, and the fiscal multiplier is zero. A dollar spent by the government is simply a dollar less spent by someone else and there is full crowding out.

Proposition 3. *If $\delta < \delta^*$ there exist a unique equilibrium such that*

$$y_t = z\left(\frac{\delta}{\delta^*}\right)^{\frac{\sigma-1}{\sigma-\alpha(\sigma-1)}} < z, \quad y_{t+1} = \delta(y_t/z)^\alpha < \delta, \quad \text{and} \quad p_t = m/z \tag{18}$$

Proof. In Appendix A. □

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[19]Setting α to zero nests the model of Krugman (1998), whose results can then be replicated in the current setting.

[20]As the permanent component of government spending, g_t and g_{t+1} is given by a fraction, γ, of output, consumption is given by $(1 - \gamma)y_t$, and the term $(1 - \gamma)$ washes out in the Euler equation.

[21]Propositions 2 and 3 verifies this interpretation of δ^*.

Again, the equation of exchange infers that $x_{t+1} > 0$, and for any $\delta < \delta^*$ the economy is therefore driven into a liquidity trap.

Under these circumstances, output in both period t and $t+1$ fall well below their potential values, z and δ, respectively. The reason is that a sufficiently severe fall in future labor productivity triggers a spur in savings which drives the nominal interest rate to zero. At zero interest, savings takes the form of excess cash holdings which drains the economy of liquidity. As wages are downwardly rigid, a fall in nominal demand lowers output and raises unemployment. When the unemployment rate is persistent, the future appears even bleaker, provoking a further rise in savings, weaker demand, and a larger rise in the unemployment rate, and so on.

Proposition 3 suggests that this intertemporal propagation mechanism can be profoundly compromising with respect to period t output. When the labor market displays hysteresis and the intertemporal elasticity of substitution approaches zero, the economy shuts down and the unemployment rate soars to 100 percent. Admittedly, of course, this draconian scenario hinges on quite unrealistic parameter values, and should not be taken literally.

Other features of Proposition 3 may come less as a surprise. Output tend to be lower the larger the decline in labor productivity, δ, as more disappointing news translate into larger savings; a steeper fall in both liquidity and economic activity; and ultimately a larger rise in the unemployment rate. In addition, a more persistent rise in unemployment – i.e. an α closer to unity – is associated with a larger decline in output, and a more pronounced rise in the unemployment rate. Clearly, a more persistent rise in unemployment yields an even more distressing outlook for the future, which in turn exasperates the private sector's willingness to save further.

3.1.1. *Fiscal Policy.* So to what extent may fiscal policy backtrack the downward spiral illustrated above? By borrowing – or taxing – unutilized cash and spending it, the government may turn a vicious circle around. The associated increase in aggregate demand raises output, lowers unemployment, and improves the outlook for the future. As a consequence, private savings fall, consumption rises, and the unemployment rate decreases further. Of course, inasmuch as an arbitrarily small elasticity of intertemporal substitution may have an abysmal effect on period-zero output in the wake of a liquidity trap, government spending may have an equally powerful impact in the opposite direction.

Under the hypotheses laid out in Proposition 3, the equilibrium conditions in (15)-(17) can be summarized by the Euler equation

$$u'(y_t - g_t - g) = \frac{\beta}{z} y_{t+1} u'(y_{t+1} - g_{t+1}) \tag{19}$$

with $y_{t+1} = \delta n_{t+1}$, and $n_{t+1} = n_t^\alpha$. I pay no attention to whether an increase in spending is debt- or tax financed, as this is inconsequential.

Proposition 4. *Under the hypotheses laid out in Proposition 3, the fiscal multiplier is given by*

$$\frac{\partial y_t}{\partial g} = \frac{1}{1 - \alpha\left(1 - \frac{1}{\hat{\sigma}}\right)}, \qquad \text{with} \quad \hat{\sigma} = \frac{\sigma}{1 - \gamma}$$

Proof. In Appendix A. □

To get an intuitive grasp of how an expansion in government spending trickles through the economy, it is illustrative to decompose the total effect on output into several successive rounds. For the moment I will ignore the case in which the non-discretionary component of government spending is positive, such that $\gamma = 0$.

First of all, as the aggregate supply relation – the firms' first order condition in (14) – is horizontal, a marginal increase in government spending translates to an immediate one-to-one response in output. That one is easy.

Second, however, an increase in contemporaneous output lowers unemployment both in the present and in the future, which in turn raises current output further, and so on. Thus, to understand the impact of any successive rounds beyond the immediate, it is imperative to understand how changes in current output translates to changes in future output and vice versa.

Employment evolves according to $n_{t+1} = n_t^\alpha$. If period t output equals y_t, employment is straightforwardly given by y_t/z. And as period $t+1$ output is given by δn_{t+1}, it follows that $y_{t+1} = \delta(y_t/z)^\alpha$. Thus, the elasticity of future output with respect to current output is given by α.

To find the reverse effect – i.e. the elasticity of current output with respect to future output – implicit differentiation of the Euler equation in (19) reveals that

$$\frac{\partial \ln y_t}{\partial \ln y_{t+1}} = 1 - 1/\sigma \tag{20}$$

The reason is straightforward. Following the Euler equation, a unit percentage increase in future consumption yields a *ceteris paribus* one-to-one percentage increase in current consumption. But this is not a ceteris paribus world. An increase in future output is deflationary, and the associated substitution effect offsets the initial response by the elasticity of intertemporal substitution, $1/\sigma$.[22] The total net effect is therefore $1 - 1/\sigma$.

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[22]Following the equation of exchange, inflation is simply "too much money chasing too few goods", and an increase in output reduces prices.

Combining the these two elasticities reveals a striking relation

$$\frac{\partial y_{t+1}}{\partial y_t} \times \frac{\partial y_t}{\partial y_{t+1}} = \alpha \left(1 - \frac{1}{\sigma}\right) \tag{21}$$

That is, a marginal increase in current output propagates by way of a persistent labor market *and* a brighter future to a further marginal increase of $\alpha(1 - 1/\sigma)$. As a consequence, an increase in government spending carries a first round impact on output of *one*, a second round impact of $\alpha(1 - 1/\sigma)$, a third round of $(\alpha(1 - 1/\sigma))^2$, and so on. The fiscal multiplier is given by the sum of the successive rounds. That is

$$\frac{\partial y_0}{\partial g} = 1 + \alpha \left(1 - \frac{1}{\sigma}\right) + \left(\alpha \left(1 - \frac{1}{\sigma}\right)\right)^2 + \ldots = \frac{1}{1 - \alpha \left(1 - \frac{1}{\sigma}\right)}$$

which replicates the result in Proposition 4 with γ set to zero.

Consider the case in which there are no labor market frictions, i.e. $\alpha = 0$. Proposition 4 then reveals that the fiscal multiplier is equal to unity. This result corroborates the findings of Krugman (1998), Eggertsson (2010), and Christiano et al. (2011), and suggests that it is not the presence of a liquidity trap *per se* which is the main driving force behind a potentially large multiplier.[23]

In the polar-, but arguably more realistic, scenario in which unemployment displays hysteresis, α is equal to one, and the fiscal multiplier is instead given by the parameter σ; the inverse of the elasticity of intertemporal substitution. While estimates of either σ or its reciprocal are both unreliable and controversial, I believe few economists would outrightly reject an elasticity of around one-half or smaller. From this perspective, labor market frictions in conjunction with low nominal interest rates appear incredibly important for the effectiveness of fiscal policy.

Lastly, for any intermediate case, i.e. $\alpha \in (0, 1)$, the multiplier varies but *always* exceeds unity. As the elasticity of intertemporal substitution approaches zero, the multiplier peaks at $1/(1 - \alpha)$.

The above discussion intentionally ignores the amplifying effects of non-discretionary spending, g_t. Proposition 4, however, reveals that the mere size of the public sector, γ, may be of importance. The reason is straightforward. One of the key components of the fiscal multiplier is given by the elasticity of current output with respect to future output. In the absence of non-discretionary spending, this elasticity is given by $1 - 1/\sigma$. However, as the fiscal multiplier is related to how *output*, and not *consumption*, responds to government

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[23]Christiano et al. (2011) and Eggertsson (2010) study the effect of multiple liquidity spells with associated multiple spending shocks. Setting the duration of the liquidity trap to one yields a multipler of unity.

spending, the relevant measure of intertemporal substitution is given by

$$-\frac{\partial \ln(y_{t+1}/y_t)}{\partial \ln(u'(c_{t+1})/u'(c_t))} = \frac{1-\gamma}{\sigma} = \frac{1}{\hat{\sigma}}$$

As a consequence, in the presence of non-discretionary spending, the elasticity of current output with respect to future output equals $1 - 1/\hat{\sigma}$, which thus further intensifies the potency of discretionary fiscal policy.

3.2. Numerical analysis.

3.2.1. *Calibration.* The model is calibrated to target the US economy at a monthly frequency. The discount factor, β, is set to $0.95^{1/12}$, and the separation rate, λ, to 0.034 (Hall, 2005; Shimer, 2005). The matching function is given by

$$H(\hat{v}_t, \hat{u}_t) = \hat{v}_t(1 - e^{-\frac{\eta}{\theta_t}})$$

which exhibits constant returns to scale, with $\phi(\theta)$ and $\rho(\theta) \in [0,1]$ for all $\theta \in \mathbb{R}_+$ (Petrongolo and Pissarides, 2001). The parameter η is known as the index of mismatch.[24] The steady-state level of labor productivity, z, is normalized to unity, and cash, m, is set equal to the steady-state employment rate, n. As a consequence, the steady-state price level, p, equals one. Unemployment benefits are set to target a 66% replacement rate (Meyer, 1990), and I consider a symmetric Nash-bargaining solution with $\xi = 0.5$ (Hall, 2005). The resulting steady-state wage is equal to 0.971.[25]

Under these parameter values, the steady-state asset value of the firm, J, is equal to 0.77. Given a labor market tightness of 0.45, the parameter η in the matching function is set such that the steady-state unemployment rate, u, equals five precent.[26] Thus, the cost of posting a vacancy k is set to $\phi(0.45)J$. The elasticity of intertemporal substitution (EIS) is set to 1/3, and real non-discretionary government spending equals 20 percent of steady-state output.

The calibrated parameter values are summarized in Table 1, and the details of the computational procedure is outlined in Appendix B.

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[24]Petrongolo and Pissarides (2001), p. 402, interpret η as "the fraction of workers who are suitable employees for a randomly selected vacancy".

[25]A value of 0.971 should be compared to 0.965 in Hall (2005), 0.98 in Pissarides (2009) and 0.993 in Shimer (2005).

[26]A labor market tightness of 0.45 corresponds to the US average in the years 2001-2012 according to JOLTS data.

TABLE 1. Calibrated parameters (monthly frequency)

	Interpretation	Value	Source/Steady-state target
σ	Inverse of EIS	3	Convention
β	Discount factor	0.996	Annual real interest rate of 5%
η	Index of mismatch	0.98	Unemployment rate of 5%
λ	Separation rate	0.034	JOLTS, 2001-2012
ξ	Firms bargaining power	0.5	Hall (2005)
$\tilde{b}$	Unemployment benefits	0.66	Meyer (1990)
k	Vacancy posting cost	0.68	Labor market tightness of 0.45
$\bar{g}$	Steady-state fiscal spending	0.2	Convention/NIPA

3.2.2. *Experiment.* The economy is in the steady-state at time $t-1$. In period t agents
are notified that labor productivity in $t+1$ will decline. The fall in labor productivity is
temporary and reverts back to its steady-state value in period $t+2$ onwards.

Nominal wages are, again, downwardly rigid. In contrast to the preceding analysis,
however, wage stickiness does not infer a disequilibrium in the labor market. Instead, the
negative demand shock is deflationary, and a fall in the general price level reduces nominal
revenues while leaving nominal costs unchanged. With falling quasi-rents, asset prices fall,
and unemployment rises. The implied outcome is therefore very similar.

Lastly, non-discretionary spending, g_t, will remain constant throughout the experiment
and is equal to its steady state value, $\bar{g}$.

3.2.3. *Impulse Responses.* The black line in Figure 1 illustrates the response of the economy
to an anticipated 6.5 percent decline in labor productivity.[27] Time is given on the x-axis,
and the y-axis illustrates the percent deviation of a variable from its steady-state value.
The associated dynamics displays what is colloquially referred to as a Pigouvian cycle, in
which output, consumption, investment and asset prices all fall together.

The causal mechanism runs as follows. When news arrive, asset prices fall and the vacancy
to unemployment ratio declines. Money dominates equity in return, and agents favor excess
cash holdings to investments. The associated decline in velocity puts deflationary pressure
on the aggregate price level which, as nominal wages are sticky, provokes a further drop
in the share value of a firm. With rising and persistent unemployment (see Figure 2), the

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[27]The standard deviation of detrended labor productivity in post-war US data is around 0.02 log points
(Shimer, 2005). A drop of 6.5 percent is roughly equivalent to a fall of three standard deviations. The only
post-war recession which displays such a large drop in labor productivity occurred in the early 1980s. The
unemployment rate rose from 5.6 percent in May 1979 to 10.8 percent in December 1982, which should be
compared to a rise from 5 percent to 11.4 percent in the current setting (Figure 2).
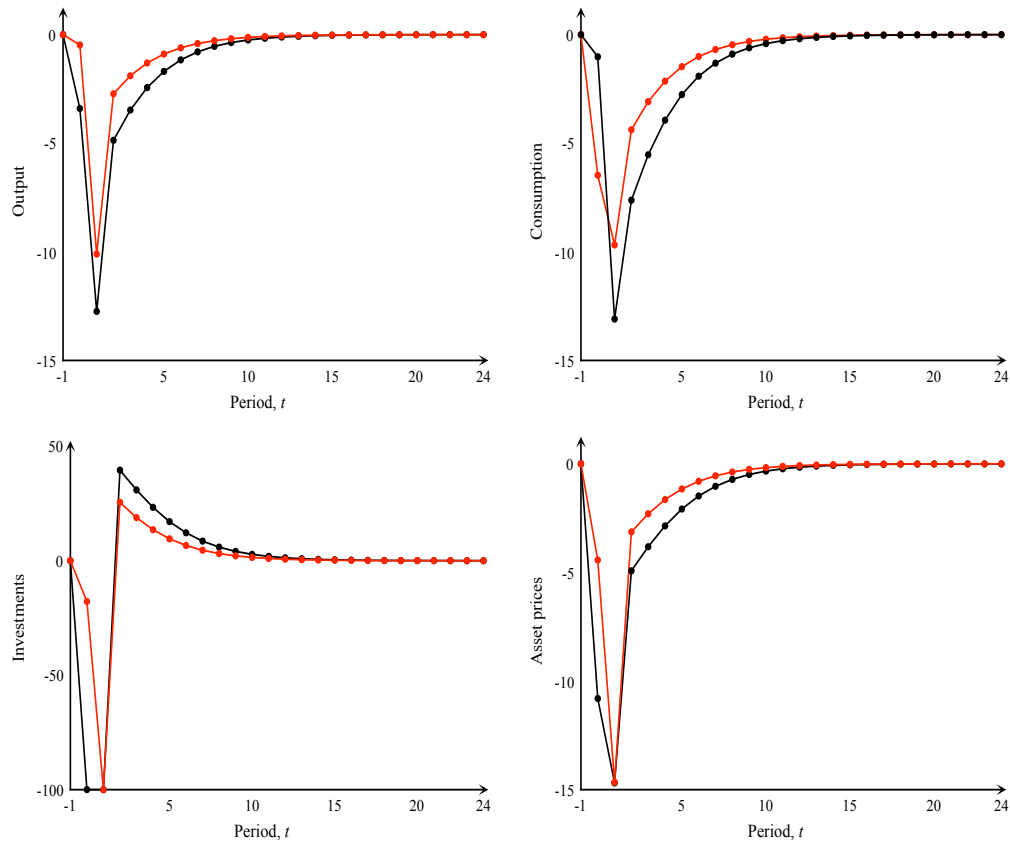
FIGURE 1. Impulse responses of macroeconomic aggregates to a news shock.

future looks even bleaker. Excess cash holdings takes yet another leap, the velocity of money takes another fall, and there is a further rise in the level of unemployment, and so on.

Where does this process end? As can be seen in Figure 1, asset prices fall with around 11 percent in period t, and investments are driven to zero. Output consequently declines with around 3.4 percent.[28] Consumption falls slightly less than output, as the reduction in investments help to buffer the shortfall in income. And the velocity of money (not graphed) declines in total with around 23 percent. The dire news concerning future labor productivity causes a sharp slump already today.

The red line in Figure 1 illustrates the response to an identical shock, but now under the hypothesis that there is a one-shot expansion in discretionary government spending. The expansion in fiscal outlays is set to equal five percent of steady-state output. The increase in spending soaks up excess cash holdings and bolsters demand. In a competitive world a rise in demand is a rise in the price for output goods. With rigid nominal wages the share value

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[28]The fall in output is equal to the rise in unemployment, which, as investment falls to zero, must equal the separation rate, λ, of 3.4 percent.

of firms rises and investments increase. More vacancies are posted, employment prospects improve, the unemployment rate falls, and output goes up. The rise in output, however, is not sufficiently large to offset the increase in investment spending, and consumption is partly crowded out.
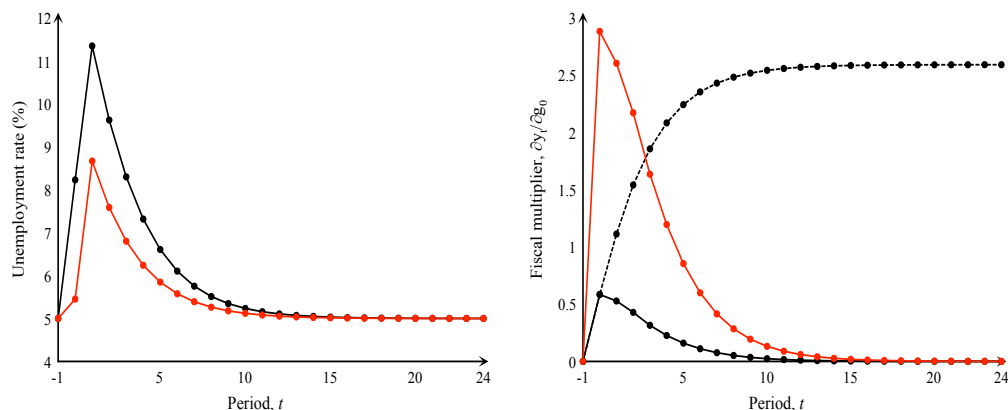

FIGURE 2. Impulse response of unemployment (left graph), and the marginal, average, and cumulative multiplier (right graph).

The left graph in Figure 2 illustrates the response in unemployment in the presence and absence of fiscal action, given by the red and black line respectively. The expansion in government spending reduces the unemployment rate on impact from around 8.2 percent to 5.4 percent. But much more importantly, the effect on impact tends to linger. The unemployment rate in period $t+1$ is almost *three* percentage points lower than it would be if there were no fiscal expansion in period t. And the unemployment rate in period $t+4$ is still *one* percentage point lower than in the absence of fiscal policy. Thus, an expansion in government spending does not only boost output and employment in the present, but also in the future. And as argued, it is precisely this interplay between the present and the future that has the capacity to substantially raise the efficacy of fiscal policy.

So how powerful is this mechanism? The solid black line in Figure 2 (right graph) illustrates the *average multiplier*. That is, the total change in output divided by the total change in government spending. As can be seen from the graph, the average multiplier on impact is small, around 0.55.[29] But it would be highly misleading to conclude that fiscal policy is inefficacious based on this metric. For the same reasons as there is a lingering effect of government spending on unemployment, there is also a multiplier effect which echoes for

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[29]Reading straight of Figure 1 reveals that the this multiplier, on impact, must be around $\frac{94.6-91.8}{5} = 0.56$, which indeed is the result reproduced in Figure 2.

several periods into the future, even though spending only takes place in period t. And this effect accumulates over time.

Thus, the dotted line in Figure 2 plots the *cumulative multiplier*, which is equal to the sum of the average multipliers over time. The long-run cumulative effect of a fiscal expansion in period t on output is substantial, and reaches 2.5 or thereabout. And while government spending crowds out private consumption on impact, the cumulative response (the *consumption multiplier*) reaches about 1.5, and there are reason to believe that the net effect on welfare can well be positive.

Lastly, the red line in Figure 2 (right graph) illustrates the *marginal multiplier*, or, simply, *the* fiscal multiplier. A marginal increase in output over a marginal increase in government spending is around 2.9 on impact, and fades out slowly over time. The cumulative marginal response reaches a striking 13 (see Table 2).

3.2.4. *Multipliers, Welfare and Debt.* The second and third row of Table 2 provides the marginal- and cumulative marginal multiplier for a range of possible news shocks. The marginal multiplier is very large for news shocks that provoke a substantial rise in unemployment (around seven percent and above), but swiftly decays and eventually dies off at milder recessions. The cumulative multiplier exceeds the marginal by around a factor of four. In the absence of a shock to the economy, i.e. when z_{t+1} equals one, there is no recession and the multiplier, whether marginal or cumulative, is *negative*. The reason behind

TABLE 2. The impact of government spending

	Unemployment rate, u_t							
	8.23%	8.0%	7.5%	7.0%	6.5%	6.0%	5.5%	5.0%
News shock, z_{t+1}	0.935	0.9353	0.936	0.9367	0.939	0.944	0.978	1.0
Marginal multiplier	2.88	2.94	3.0	1.4	0.53	0.27	0.2	-0.26
Cumulative marginal	13.25	13.45	13.53	6.23	2.3	1.2	0.7	-0.8
Optimal spending	3.23%	3.18%	3%	2.8%	2.2%	0.8%	0%	0%
Average multiplier	0.8	0.74	0.6	0.45	0.335	0.25	N/A	N/A
Cumulative average	3.55	3.3	2.64	2	1.46	1.08	N/A	N/A
Welfare (1)	27%	25%	19%	12%	5%	0.5%	N/A	N/A
Welfare (2)	58%	55%	46%	34%	17%	2.2%	N/A	N/A
Public debt	12%	10%	7%	4%	1.2%	0%	0%	0%

Notes: The marginal and cumulative marginal multipliers are calculated as $\partial y_t/\partial g$ and $\sum_{s=0}^{T} \partial y_{t+s}/\partial g$, respectively. The average and cumulative average multipliers are analogously calculated but with differences replacing partial derivatives. The calculations of welfare measures and debt are described in the main text.

this latter result follows a familiar logic. First, in the absence of news, the economy does

not fall into a liquidity trap and a dollar spent by the government is a dollar less spent by the someone else (cf. Proposition 2). Second, however, a rise in government expenditures raises the real interest rate and crowds out private investments. Less jobs are therefore created, and output falls.

But the fiscal multiplier is a very crude metric of policy efficacy. The multiplier measures how many dollars of extra output is gained from one dollar additional spending. But it does not account for potential changes in the composition of output. In the situation depicted in Figure 1, for instance, consumption falls on impact which, ceteris paribus, reduces welfare. On the other hand, consumption in subsequent periods rise, which instead improves welfare. Which of these contrasting forces dominates?

To answer this question, I follow the ideas developed in Lucas (1985) and define welfare in terms of consumption equivalents.[30] If V_a denotes the households' *value* associated with some event or policy a, and V_b the value associated with some other event or policy b, the welfare cost of b relative to a is given by

$$\lambda_{a,b} = \left(\frac{V_a}{V_b}\right)^{\frac{1}{1-\sigma}} - 1$$

Exploiting this definition suggests that the expansion in government spending illustrated in Figure 1 gives rise to an increase in welfare of about *one-twentieth* consumption equivalents. The net gain to welfare is therefore small, but positive. However, the small gain to countercyclical policy derives from the small cost of business cycles in general. The cost associated with the entire recession in Figure 1, for instance, is equal to a bare one-fifth consumption equivalents. And the cost associated with the *demand component* of the recession is one-tenth consumption equivalents.[31] Thus, by taking a more relative view on welfare, the expansion in government spending can be said to alleviate around one-fourth of the entire cost of the crisis, or around one-half of the demand component.

Given this metric of welfare, the fourth row of Table 2 reports the optimal expansion in government spending, expressed as percent of steady-state output. The seventh and eighth row reports the associated welfare gains. Welfare gains are denoted in relative terms, and express to which extent the expansion in government spending may alleviate the cost of the crisis. As can be seen from the table, optimal spending tends to be large whenever the fiscal multiplier is large. During a severe recession with an unemployment rate exceeding seven percent or more, an expansion in government spending of around three percent of

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[30]A rise of *one* consumption equivalent should be interpreted as an increase in welfare equal to a *one* percent perpetual rise in consumption.

[31]The welfare cost of the demand component of the recession is defined as the welfare cost of the recession relative to the counterfactual hypothesis that the decline in period $t + 1$ labor productivity is entirely unanticipated, and not known in period t.

output may mitigate half of the demand component of the crisis, and around 20 percent of the total costs.[32] Given these relatively large stimulus packages, the associated average multipliers are quite low, and always falls short of unity. The cumulative response, however, always exceed one. Interestingly, the decay in policy efficacy that occurs with respect to the severity of the recession observed in rows two and three of Table 2 is much less pronounced. In situations where the marginal efficacy of policy is relatively low, the amount dollar spent is adjusted accordingly such that the average effect still remains reasonably high.

Some may fear that a large expansion in public spending must come at the price of ramping up debt. This is not necessarily a rational fear. Ricardian equivalance infers that a contemporaneously tax financed expansion in public spending will yield a similar kickback to that of a debt fueled expansion, and the model is entirely silent with respect to the mode of financing. Thus in order to say anything meaningful with respect to public debt, some additional structure must be imposed.

To this end, I assume that the debt-to-output ratio in period $t-1$ is equal to 0.5, and that taxes are set to keep the steady-state deficit at zero. This implies an income tax-rate of about 20.2 percent. Keeping the tax-rate constant, I iterate on the government's budget constraint in equation (9) to compute the entire trajectory of nominal debt from period t and two years forward. Following the benchmark situation analyzed in Figure 1, public debt rises from 50 percent of output in period $t-1$ to around 101 percent two years later. With an optimal expansion in public spending of around 3.2 percent of steady-state output, debt rises instead to around 89 percent of output. Thus, while the debt-to-output ratio increases both in the presence, and in the absence of fiscal policy, the rise is more pronounced in its absence. The log difference, about 12 percent, is reported in first column of the last row of Table 2. And as can be seen from the table, an optimal expansion in government spending always improves the debt position relative to the case without any policy intervention. This result is not, however, unique to this study, but echoes the message conveyed in recent work by DeLong and Summers (2012) and Denes et al. (2012).

3.2.5. *Monetary policy.* Although this paper puts an emphasis on the question of fiscal policy efficacy, it appears reasonable to provide a brief discussion of the possible merits of monetary policy. Within the benchmark scenario portrayed in Figure 1 (or the analytical example outlined in Proposition 3), consider the following thought-experiment. The monetary authority injects a certain amount of cash in period t, arbitrary in size and through whichever means. The injection is revoked in period $t+1$. Now suppose that households decide to hoard the entire monetary expansion. All first-order conditions are left intact, prices are unaffected, and so are all real quantities. *Any* expansion in the monetary base

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[32]These numbers are given in rows eight ('Welfare (2)') and seven ('Welfare (1)'), respectively.

will be met by an identical expansion in excess cash holdings. Monetary policy is 'pushing on a string'.

Now suppose instead that the monetary authority commits *not* to revoke the expansion in period $t+1$. As the economy is not in a liquidity trap, the equation of exchange reveals that the price level will rise in proportion with the expansion of money. Forward-looking households will of course anticipate this rise, and the real return on cash holdings fall. Now the first-order conditions *are* violated. Excess money holdings drop, and private spendings rise. Crisis averted.

To verify this logic, I recompute the experiment in Section 3.2.3 under the hypothesis that the monetary authority can credibly commit to a future rise in the price level. Welfare increases monotonically with the expansion of the monetary base, but eventually levels off. Expanding the monetary base with nine percent or more alleviates 100 percent of the demand component of the crisis, and around 50 percent of the entire cost. By 'credibly committing to act irresponsibly' the monetary authority may stave off a potentially deep recession at a zero rate of interest (Krugman, 1998; Eggertsson and Woodford, 2003).

4. Concluding Remarks

This paper has studied a model in which the efficacy of fiscal policy rises markedly in periods of low nominal interest rates and high, persistent, unemployment. At the core of the analysis lies a novel intertemporal propagation mechanism in which the labor market plays a pivotal role. With persistent unemployment, any increase in current demand translates to an associated rise in future supply. But as rational economic actors desire to smooth consumption over time, the increase in future supply feeds back to a further rise in current demand.

These reinforcing mechanisms amplify the effectiveness of fiscal policy many times over. In a stylized framework displaying unemployment hysteresis the fiscal multiplier equals the reciprocal of the elasticity of intertemporal substitution. In a more realistic setting, the effect is somewhat dampened and displays significant nonlinearities both with respect to the output-gap and the size of the stimulus package. But in a severe recession with an unemployment rate exceeding the natural by two percentage points or more, the marginal impact of government spending on output is around three.

But the same mechanism that may cause a large multiplier also infer some restrictions on the conduct of fiscal policy. Government spending must create jobs. Real jobs. Letting idle workers dig a hole only to fill it up again is not a viable option as it is unlikely to allow for a *persistent* decline in the unemployment rate. Indeed, within the framework analyzed in this paper, a hole-digging policy is isomorphic to a tax-financed tax-cut, which, from

a representative agent's perspective, is a wash. Spending must therefore take the form of *purchases* of goods or services which would *normally* be provided in the economy even in the absence of fiscal intervention.

In addition, while the analysis is centred around a one-sector framework, it is, to a certain extent, straightforward to extrapolate results to a more realistic setting. Spending must target sectors which exhibits spare capacity. Outbidding potential buyers at a Sotheby's auction is likely to yield a multiplier of zero or less. But investing in infrastructure during a housing crisis may plausibly carry a much larger kickback. Perhaps paradoxically then, while government purchases ought to be directed towards sectors where private demand is temporarily slack, public goods must not substitute for private consumption. If the private enjoyment of publicly purchased goods substitute for that of privately purchased goods, the stimulative properties of government spending vanish (cf. Eggertsson (2010)).

Nevertheless, the main point of this paper still remains. At low levels of nominal interest and high, persistent unemployment, accurately targeted fiscal policy may be a potent tool in combatting a deep, demand driven, recession.

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Appendix A. Proofs

A.1. **Proof of Proposition 1.** In the steady state, $y_t = z$ and $c_t = y_t(1 - \gamma)$. As a consequence an equilibrium allocation of prices and quantities must satisfy the following sequence of equations for all time-periods t

$$\beta(1 + i_{t+1})\frac{p_t}{p_{t+1}} = 1 \tag{A1}$$

$$m - p_t z \geq 0 \tag{A2}$$

$$i_{t+1} \geq 0 \tag{A3}$$

$$(m - p_t z)i_{t+1} = 0 \tag{A4}$$

The transversality condition with respect to x_{t+1} is given by

$$\lim_{n \to \infty} \beta^n \frac{x_{t+n+1}}{p_{t+n}} = 0 \tag{A5}$$

Let us first verify that $p_t = m/z$ and $i_{t+1} = 1/\beta - 1$ is indeed a solution. Obviously $p_t = m/z$ and $(1 + i_{t+1})\beta = 1$ satisfy equation (A1), as well as equation (A2) with equality. As a consequence, $i_{t+1} = 1/\beta - 1$ satisfies the inequality in equation (A3). Since, $p_t = m/z$, equation (A4) follows. Since $x_{t+1} = m - p_t z_t$, the transversality condition in (A5) is satisfied with equality. Thus $p_t = m/z$ and $i_{t+1} = 1/\beta - 1$ is indeed a sequence of equilibrium prices.

Now suppose there exist some other equilibrium allocation with $0 \leq p_t < m/z$ for some t.[33] Then by equation (A4), $i_{t+1} = 0$. By equation (A1), $p_{t+1} = \beta p_t$, and thus $i_{t+2} = 0$, and so on. As a consequence, $p_{t+n} = \beta^n p_t$. Inserting into the transversality condition reveals that

$$\lim_{n \to \infty} \beta^n \frac{x_{t+n+1}}{p_{t+n}} = \lim_{n \to \infty} \frac{\beta^t}{p_t}(m - z\beta^n p_t) > 0$$

As a consequence, $p_t < m/z$ for some t cannot be an equilibrium. Thus there exist a unique steady-state equilibrium with prices $p_t = m/z$ and $i_{t+1} = 1/\beta - 1$. $\qquad \square$

A.2. **Proof of Proposition 2.** First, notice that $y_t > z$ is not a possible equilibrium as it would violate the time-endowment of unity. As a consequence, $y_t \leq z$. Suppose that the inequality is strict. The aggregate budget constraint, $m = p_t y_t + x_{t+1}$, then infers that either $p_t > m/z$, or $x_{t+1} > 0$ (or both). Under this hypothesis there is involuntary unemployment and wages must fall until $p_t = m/z$. The Euler equation is therefore given by

$$u'(y_t) = \frac{\beta}{z}\delta(y_t/z)^\alpha u'(\delta(y_t/z)^\alpha)$$

Using the parametric forms $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ reveals that there are two solutions associated with the above equation

$$y_t = z\left(\frac{\delta}{\delta^*}\right)^{\frac{1-\sigma}{\sigma - \alpha(\sigma-1)}} \quad \text{or} \quad y_t = 0$$

In the first case, y_t must (weakly) exceed z as, $\delta \geq \delta^*$, and $\frac{1-\sigma}{\sigma - \alpha(\sigma-1)} > 0$. This is a contradiction. In the second case, $y_t = 0$ indeed solves the Euler equation, but the not the households' optimization problem: If $y_t = 0$, a representative household allocates all her nominal resources towards period $t + 1$, in which prices are infinite, and resources useless. Given such prices, the household would be better off by spending some initial nominal resources in period zero, ruling out $y_t = 0$ as a possible equilibrium.

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[33]Notice that $p_t > m/z$ would imply that $x_{t+1} < 0$ which is an impossibility.

It remains to be verified that $y_t = z$ is indeed an equilibrium. Due to downward nominal wage rigidity $p_t \geq m/z$. Thus, according to the aggregate budget constraint together with the condition $x_{t+1} \geq 0$, it follows that $p_t = m/z$ and $x_{t+1} = 0$. As a consequence, there exist a unique $i_{t+1} \geq 0$ such that

$$u'(z) = \beta(1 + i_{t+1})\frac{\delta}{z}u'(\delta)$$

Lastly, consider the effect of fiscal policy. For any $g > 0$, there exist an $i_{t+1} > 0$ such that

$$u'(z(1 - \gamma) - g) = \beta(1 + i_{t+1})\frac{\delta}{z}u'(\delta(1 - \gamma))$$

As a consequence, there is full crowding-out and the multiplier is zero.   □

A.3. **Proof of Proposition 3.** First, suppose that $y_t = z$. Then using the arguments in the proof of Proposition 2 it is immediate that $p_t = m/z$ and that $x_{t+1} = 0$. However, as $\delta < \delta^*$, the interest rate which satisfies

$$u'(z) = \beta(1 + i_{t+1})\frac{\delta}{z}u'(\delta)$$

must be negative, which violates the zero lower bound. As a consequence, $y_t < z$, $p_t = m/z$, $x_{t+1} > 0$ and $i_{t+1} = 0$. The Euler equation is thus given by

$$u'(y_t) = \frac{\beta}{z}\delta(y_t/z)^\alpha u'(\delta(y_t/z)^\alpha)$$

Using the aforementioned parametric form we have

$$y_t = z\left(\frac{\delta}{\delta^*}\right)^{\frac{1-\sigma}{\sigma-\alpha(\sigma-1)}} \quad \text{or} \quad y_t = 0$$

Again, $y_t = 0$ can be ruled out by repeating the previous arguments.   □

A.4. **Proof of Proposition 4.** The Euler equation is now given by

$$u'(y_t - g_t - g) = \frac{\beta}{z}\delta h(y_t/z)u'(\delta h(y_t/z) - g_{t+1})$$

where $h(x) = x^\alpha$.

Applying the implicit function theorem evaluated at $g_t = \gamma y_t$, $g_{t+1} = \gamma y_{t+1}$, and $g = 0$ yields

$$\frac{\partial y_t}{\partial g} = -\frac{u''(y_t(1-\gamma))}{\frac{\beta}{z}\delta h'(y_t/z)\frac{1}{z}u'(\delta h(y_t/z)(1-\gamma)) + \frac{\beta}{z}\delta h(y_t/z)u''(\delta h(y_t/z)(1-\gamma))\delta h'(y_t/z)\frac{1}{z} - u''(y_t(1-\gamma))}$$

Using the Euler equation together with the following relations

$$-\frac{yu''(y(1-\gamma))}{u'(y(1-\gamma))} = \frac{\sigma}{1-\gamma} = \hat{\sigma} \quad \text{and} \quad h'(y/z)\frac{y}{zh(y/z)} = \alpha$$

leaves us with

$$\frac{\partial y_t}{\partial g} = -\frac{u''(y_t(1-\gamma))}{\frac{\alpha}{y_t}u'(y_t(1-\gamma)) - \hat{\sigma}\frac{\alpha}{y_t}u'(y_t(1-\gamma)) + \hat{\sigma}\frac{1}{y_t}u'(y_t(1-\gamma))}$$

$$= \frac{1}{1 - \alpha\left(1 - \frac{1}{\hat{\sigma}}\right)}$$

                    □

Appendix B. Computational Details

The model can be rewritten in terms of ten equations,

$$J_t = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \left(\frac{z_t - \tilde{w}_t}{p_{t+1}} + (1-\lambda)J_{t+1} \right) \right] \tag{A6}$$

$$V_t = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \left(\frac{\tilde{w}_t}{p_{t+1}} + \lambda U_{t+1} + (1-\lambda)V_{t+1} \right) \right] \tag{A7}$$

$$U_t = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \left(\frac{\tilde{b}_t}{p_{t+1}} + \rho_t V_{t+1} + (1-\rho_t)U_{t+1} \right) \right] \tag{A8}$$

$$\tilde{w}_t = \mathrm{argmax}\{J_t^\xi (V_t - U_t)^{1-\xi}\} \tag{A9}$$

$$n_t = (1 - n_{t_1} + \lambda n_{t-1})\rho_t + (1-\lambda)n_{t-1} \tag{A10}$$

$$m = p_t y_t + x_{t+1} \tag{A11}$$

$$p_t y_t = p_t c_t + I_t + G_t \tag{A12}$$

$$y_t = n_t z_t \tag{A13}$$

$$k = \phi_t J_t \tag{A14}$$

$$u'(c_t) = \beta E_t \left[(1 + i_{t+1}) \frac{p_t}{p_{t+1}} u'(c_{t+1}) \right] \tag{A15}$$

with the complementary slackness conditions $x_{t+1} \geq 0$, $i_{t+1} \geq 0$ and $x_{t+1} \times i_{t+1} = 0$. The state-variables are z_t and n_{t-1}. I start by setting $z_t = 1$ for all t and construct a linearly spaced grid in n_{t-1} comprised of 1000 nodes between 0.88 and 0.95. I subsequently guess for the policy functions $X_0(n_{-1}) = \{J_0(n_{-1}), V_0(n_{-1}), U_0(n_{-1}), p_0(n_{-1}), c_0(n_{-1})\}$ and use linear interpolation to evaluate these functions in-between grid nodes. By repeatedly solving the system (A6)-(A15) and updating the policy functions, I iterate until convergence; $X_0(n_{-1}) \rightarrow X(n_{-1})$. This gives me the solution to the simulation in periods $t + 2$ onwards.

To solve the problem in period $t + 1$, I insert $X(n_{-1})$ into the right hand side of equations (A6)-(A15), and set z_{t+1} to the value guided by the news shock. I then simply re-solve system (A6)-(A15) to find the policy functions in period $t + 1$, $X^{t+1}(n_t)$.

Lastly, to solve the problem in period t I proceed as in the above paragraph and insert $X^{t+1}(n_t)$ into the right hand side of the system of equations. I set z_t equal to one, and re-solve the system to find the policy functions in period t, $X^t(n_{t-1})$. Using $X^t(n_{t-1})$, $X^{t+1}(n_t)$, and $X(n_{-1})$ together with equations (A6)-(A15) allows me to perform all the calculations provided in the text.