Abstract

This paper analyzes the effects of trade liberalization on the political support for policies that redistribute income between workers in different sectors. We allow for worker heterogeneity and imperfect mobility of workers across sectors, giving rise to a trade-off between redistribution and the inefficiency of the labor allocation. We compare two environments, autarky and small open economy, and present three main findings. First, redistributive policies are more “likely” to arise in a small open than in a closed economy. Second, if a redistributive policy is adopted in both situations, its nominal level is higher in autarky than in the small open economy. Third, even though voters choose redistributive policies with lower nominal value in open economies, the actual extent of redistribution in equilibrium is larger in the open than in the closed economy. We discuss our results in the context of the debate about the effects of globalization on government activity.

Keywords: International trade, redistribution, political economy, factor mobility

J.E.L. classification: F1, H2
1 Introduction

A common source of inefficiency of redistributive policies stems from their heterogeneous impact on factor rewards across sectors and the resulting distortion in factor allocation between sectors (Saez (2004)). Subsidies to - or bailouts of - particular industries have this heterogeneity at their core. One of their typical aims, as far as their redistributive dimension is concerned, is to raise the wage or maintain employment of workers in declining sectors. Other policies, such as a progressive income tax, heterogeneous taxation of inputs or unemployment benefits (Wright (1986)) also distort factor allocation across sectors if these differ in their average wage, input mix¹ or unemployment risk respectively.

The present paper analyzes how trade liberalization changes the effect of and the political support for policies which redistribute income between workers in different sectors. Although theses policies are generally considered inefficient ((Acemoglu and Robinson (2001)), they remain an important channel through which governments across the world redistribute income or support employment (e.g. Ford and Suyker (1990), OECD (2010), Rickard (2012)). In developing countries, Rickard (2012) shows that their prevalence increased over the 1980s and 1990s and that globalization proved instrumental in driving this evolution. This may come as a surprise in light of the extensive academic literature and public debate, which stress that globalization imposes new constraints on governments’ ability to redistribute income or protect their citizens through the welfare state² (see Brady, Beckfield, and

¹Examples include among many others fuel, electricity, and water subsidies, the absence of kerosene tax, or differential taxation of capital and labor, which favor sectors intensive in energy, water or capital respectively.

²For example, Wilson (1987) shows that the higher mobility of the tax base in an open
Seeleib-Kaiser (2005) for a summary of the empirical literature). The present paper argues however that, for the case of cross-sectoral redistributive policies, trade openness reduces the inefficiency associated with redistribution and therefore makes these policies less costly to implement.

In parallel to this emphasis on the constraints arising from globalization, a second strand of the literature (Rodrik (1998)) has stressed that openness to trade raises the demand for the welfare state, as citizens wish to be protected against external risk. In the case of redistributive policies, we believe it is equally important to understand how globalization affects the demand for such policies by citizens, an exercise which is mostly absent from the literature. Since conflicts of interests are at the core of redistributive policies, we develop a voting model to determine under which conditions such policies will arise, and find that cross-sectoral redistribution is more likely to arise in an open than in a closed economy. We therefore contribute to the positive analysis of the role of trade in shaping government interventions in two ways. First, we focus on cross-sectoral redistribution, a type of policies which, though widely used, has largely been ignored by the literature. Second, we use a political economy analysis to determine how the support for cross-sectoral redistributive policies is affected by international trade.

Our theoretical framework offers a number of novel features which allow for a rich but tractable analysis. We assume that the economy consists of different sectors producing under perfect competition and using exclusively labor. The demand condition for each sector varies, thus setting the stage world limits the size of redistribution that a government can conduct, while Alesina and Perotti (1997) point to the negative effects of redistribution on a country’s competitiveness. Epifani and Gancia (2009) on the other hand argue that a terms of trade externality in the financing of public goods may create a positive effect of trade openness on the size of governments.
for redistribution towards workers in sectors with low demand. To capture
the inherent trade-off of cross-sectoral redistribution, the key novelty is our
parsimonious modeling of the imperfect mobility of workers between sectors,
which builds on recent insights of the trade literature on comparative advan-
tage\textsuperscript{3}. If workers were perfectly mobile, there would be no conflict of interest
as they would be made indifferent between sectors. If workers were tied to
a sector on the other hand (perfect immobility), a policy redistributing in-
come towards certain sectors would be very redistributive, but would carry
no inefficiency as it would not affect the sectoral allocation of workers.

Within this framework we assume that workers determine the level of
intersectoral redistribution by majority voting. This creates a conflict of
interest between workers in sectors with low demand, who benefit from re-
distribution, and workers in sectors with high demand, who lose. In this
setup, redistribution only arises in equilibrium if the majority of workers
choose to work in low-demand sectors, an outcome which depends among
others on the relative number of low-demand sectors in the economy. We
offer three main conclusions. First, we show that redistributive policies are
more likely to arise in a small open than in a closed economy. The relative
proportion of low-demand sectors necessary for a redistributive policy to arise
is smaller under an open than under a closed economy. Second, we show that
if a redistributive policy is adopted in both situations, its nominal level is
higher in autarky than in the small open economy. Taken together, these two
results imply that international trade (i) raises the set of parameter values

\textsuperscript{3}We apply the Eaton and Kortum (2002) framework on comparative advantage with
many countries to the case of workers, see section 2.2. In line with earlier work on factor
mobility (e.g. Grossman (1983)), we measure the degree of labor specificity by the per-
centage loss in productivity that workers incur when changing sector, which is a direct
determinant of the wage cost of switching sector.
for which redistributive policies are observed, but also that (ii) if there is redistribution, policies are smaller in an open economy. Third, we show that even though voters choose redistributive policies with lower nominal value in open economies, the actual extent of redistribution - which takes not only the policy rate but also the general equilibrium effects into account - is larger in open than in closed economies.

The economic mechanism behind our findings can be understood from the two effects that international trade has in our model. First, opening to trade increases the price elasticity of demand for all goods. In autarky, subsidizing a sector raises the supply of its product and reduces its equilibrium price, thereby limiting both the increase in that sector’s factor use (the distortive effect) and factor rewards (the redistributive effect). In a small open economy, since prices are exogenous, the dampening effect of price changes is absent. Any given policy is therefore both more redistributive and more distortive in an open economy⁴. These differences in redistributive effects explain both why voters in an open economy choose a redistributive policy with lower nominal value than in autarky and why this policy can be more redistributive than the one chosen in autarky. Second, the possibility to trade internationally relaxes the constraint that each good’s market must be in equilibrium at the national level. A deviation of factor allocation from the first-best does not affect relative prices in an open economy, and therefore does not distort the consumption patterns as it would in autarky. Policies distorting the sectoral factor allocation therefore cause less inefficiency in an open economy, making voters keener to support them. Furthermore, since redistributive policies make more workers move to the low demand sector

⁴Section 3 provides a formal definition of the redistributive strength and of the distortion induced by a policy.
in an open economy, the electoral basis in favor of redistribution is larger
in an open economy, such that redistribution is observed for a larger set of
parameters.

The present paper relates to two branches of the literature, in addition
to the one on globalization and the welfare state which we mentioned earlier.
First, we relate to the literature on the distributive effects of international
trade coming through a more elastic labor demand. Rodrik (1997) points out
that, by increasing competition in product and factor markets, globalization
may raise the price elasticity of labor demand, with potentially adverse con-
sequences for workers. Empirically, Slaughter (2001) finds strong evidence
that the elasticity of labor demand has increased between the 1970s and
1990s in the U.S., although he cannot identify a strong effect of globalization
on this pattern. Spector (2001) shows how changes in elasticity matters for
redistributive policies in a standard income taxation model à la Mirrlees.

Second, we relate to the literature considering the effects of international
trade when factors are imperfectly mobile between sectors or occupations.
Kambourov (2009) argues that a high mobility of workers between occupa-
tions is essential to reap the benefits of trade liberalization, which stem
from the reallocation of workers to activities where they are relatively more
productive. Artuc, Chaudhuri, and McLaren (2010) estimate high costs of
changing sectors for individual workers and draw the consequences of this ob-
servation for the welfare effects of trade shocks. Ohnsorge and Trefler (2007)
examine the patterns of trade resulting from a model in which workers are
imperfectly mobile between sectors.

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5Further empirical evidence has also been mixed: Krishna, Mitra, and Chinoy (2001)
do not find evidence of a link between globalization and labor demand elasticity for Turkey,
while Hasan, Mitra, and Ramaswamy (2007) do find such evidence for India.
Section 2 describes the setup of the model. Section 3 solves the model for a given redistributive policy, and describes the key differences in the effects of redistributive policies in a closed and in a small open economy. Section 4 introduces the political dimension of the model and endogenizes the choice of policy. Section 5 concludes and discusses an extension.

# 2 The setup

## 2.1 Overview

The economy consists of a mass one of individuals who share the same Cobb-Douglas utility function over $N + 1$ goods, indexed from 0 to $N$:

$$U = \prod_{n=0}^{N} q_n^{\alpha_n}$$  \hspace{1cm} (1)

where $q_n$ denotes the consumption of good $n$ and $\sum_{n=0}^{N} \alpha_n = 1$. Individuals, indexed by $j$, maximize utility subject to their income. Defining the economy-wide income as $I$ and the price of good $n$ as $p_n$, the aggregate demand for good $n$ is:

$$q_n^D = \alpha_n \frac{I}{p_n}.$$  \hspace{1cm} (2)

Each good is produced in a separate sector under conditions of perfect competition. Labor is the only factor of production in the economy, and all individuals in the model are workers, who supply inelastically one unit of labor. The productivity of a unit of labor is specific to a worker-sector pair: each worker independently draws a productivity parameter for each sector. The distribution of productivity draws determines the typical loss of productivity incurred by workers when changing sector, and indexes the degree to which workers are sector-specific. The government can subsidize
sectors in which equilibrium wages are relatively low due to low demand parameters \((\alpha_n)\).\(^6\) We do not explicitly model how and why demand is low in some sectors. In a more general model, the demand parameters could be the result of the realization of a stochastic process. In section 3 we analyze the effects of a given sectoral subsidy vector, while in section 4 we assume that workers decide by majority voting on the size of these subsidies. We characterize and compare the political-economy equilibria for this economy when it is in autarky (all prices are endogenous) and when it is a small open economy (output prices are given from the world market).

The timing of the model can be summarized as follows. At time \(t_0\), each individual observes his vector of productivity draws for each sector \(n \in N\). At \(t_1\), individuals decide by majority voting on the level of redistribution towards sectors with low demand. At \(t_2\), workers decide on the sectors in which they want to work.

We now turn in detail to each of the three steps of the model.

### 2.2 Worker heterogeneity

Workers differ in their labor productivity, which is sector-specific. This makes for a realistic situation and leads to heterogenous interests when voting over sectoral subsidies takes place. At time \(t_0\), each worker independently draws

\(^6\)With a Cobb-Douglas utility, differences in sector-wide productivity would not affect the share of total income spent on a particular sector. We concentrate on the Cobb-Douglas case and on demand heterogeneity for simplicity. All results in the model hold with a CES utility when sectors have heterogeneous productivity parameters. Redistribution would in this case take place towards sectors with a low productivity parameter.
a productivity parameter \( z \) for each sector from a Fréchet distribution\(^7\):

\[
F(z) = \exp(-z^{-\nu}).
\] (3)

Worker \( j \) obtains a vector of productivity draws, \( \{z_{jn}\}_{n=0}^N \), which he observes\(^8\). \( z_{jn} \) denotes the number of efficiency units of labor that worker \( j \) provides if he works in sector \( n \). The parameter \( \nu > 0 \) affects the heterogeneity of productivity draws between sectors and provides a parsimonious way of capturing the degree of sector-specificity of workers. If \( \nu \) is low, the heterogeneity of draws between sectors is large, and the percentage loss in productivity incurred by a worker changing sector is large.\(^9\) The parameter \( \nu \) captures both technological and regulatory reasons for the sector specificity of workers\(^10\).

### 2.3 Production and redistributive policies

Each sector consists of a large number of firms which behave in a perfectly competitive manner both on the product and on the labor market. Production in a sector equals the number of effective units of labor employed by the

\(^7\)For the sake of simplicity, we assume that the Fréchet distribution has only one parameter, \( \nu \). We could generalize the analysis and allow the average \( z \) to be sector-specific. The extension is straightforward and does not convey much additional insight.

\(^8\)The assumption that workers observe their sector-specific productivity is realistic as long as the number of sectors is not too large, see Sattinger (1993).

\(^9\)This interpretation is the counterpart to that of comparative advantage made by Eaton and Kortum (2002) in a trade context. Artuc, Chaudhuri, and McLaren (2010) use the Fréchet distribution to model idiosyncratic shocks to the benefits of working in a particular sector.

\(^10\)In a dynamic perspective, the sector specificity of workers is similar to the concept of mobility of workers between sectors.
sector (Λ) times a sector-specific productivity parameter (φ):  

\[ y_n = \varphi_n \Lambda_n. \]  

(4)  

To redistribute income towards workers in particular sectors, the government can use two types of policies at the sectoral level. First, it can impose a sector-specific sales tax or subsidy. Thus profits in sector \( n \) are given by:  

\[ \pi_n = \left[ (1 - \tau_n)p_n \varphi_n - c_n \right] \Lambda_n \]  

(5)  

where \( c_n \) denotes the wage paid per unit of effective labor in sector \( n \). Anticipating the equilibrium solution of the model, the zero profit condition is given by:  

\[ c_n = (1 - \tau_n)p_n \varphi_n. \]  

(6)  

Sector specific taxes (\( \tau_n > 0 \)) or subsidies (\( \tau_n < 0 \)) thus affect the gross wage per unit of effective labor in \( n \). Second, the government can use a proportional income tax imposed on workers, the rate of which may be conditional on the sector of employment. In this case, workers choosing to work in sector \( n \) receive a net wage of \( (1 - t_n)c_n = (1 - t_n)(1 - \tau_n)p_n \varphi_n \) per unit of effective labor. For simplicity, and unless otherwise specified, we will refer to \( c_n \) as the “wage” in sector \( n \) in the rest of the analysis, which should be understood as the gross wage per unit of effective labor in that industry. Similarly, we will denote the net wage per unit of effective labor in \( n \) \( ((1 - t_n)c_n) \) simply as the “net wage” in sector \( n \).

Before proceeding, it is worth discussing the nature of taxation and redistribution in our model. As argued in the introduction, intersectoral redistribution is a general phenomenon. Often, support to specific sectors is directly done, for example through price subsidies, bailouts or guarantees. Agriculture, coal mining, or the car industry are typical recipients of such
policies. Sometimes subsidies are more hidden and are not directly targeted to particular sectors, but are in practice when they are tied to characteristics of the production process (such as R&D, capital or energy intensity), which vary across sectors. The sales tax $\tau_n$ in our model comes close to mimicking a price subsidy. The second tax instrument ($t_n$) is a sector-specific income tax on workers’ income. While most income tax systems do not differentiate explicitly by sector, various tax provisions effectively amount to such differentiation. For instance, in some countries, like Germany, income earned during night and weekend-shifts is tax favored. The work pattern, such as night- and weekend-shifts, is not uniform across sectors so that workers in different sectors are de facto subject to differential taxation. Note that none of our results requires the existence of both types of taxes at the same time.

We define $\beta_n$ as the policy parameters applying to sector $n$ relative to those applying to sector 0:

$$
\beta_n \equiv \frac{(1 - t_n)(1 - \tau_n)}{d_0}
$$

where: $d_0 \equiv (1 - t_0)(1 - \tau_0)$. (7)

By definition, the value for sector 0 is one, $\beta_0 = 1$. If $\beta_n > 1$, sector $n$ is taxed relatively less (or subsidized relatively more) than sector 0. Note that for given $d_0$ there are infinitely many combinations of sales tax and income tax in sector $n$ that lead to the same value of $\beta_n$.

We define a policy as a vector $\beta = \{\beta_n\}_{n=1,...,N}$, and denote $\beta = 1$ as a vector of ones, which correspond to the case with no redistributive policy and $p_n(\beta)$ as the equilibrium price of good $n$ under policy $\beta$. The set of policies that we consider in the analysis are those which redistribute income from sectors with a high to sectors with a low demand parameter.

Specifically, we characterize redistribution by comparing the relative net wages between sectors with and without redistributive policy. Note that, from (6) and (7), the net wage in sector $n$ under policy $\beta$ is $c_n = (1 - t_n)$.
A redistributive policy is a vector $\beta$ such that, for any two sectors $n$ and $n'$ with $\varphi_n p_n(1) / \varphi_{n'} p_{n'}(1) \leq 1$:

1. $\frac{\varphi_n \beta_n p_n(\beta)}{\varphi_{n'} \beta_{n'} p_{n'}(\beta)} \geq \frac{\varphi_n p_n(1)}{\varphi_{n'} p_{n'}(1)}$

2. $\frac{\varphi_n \beta_n p_n(\beta)}{\varphi_{n'} \beta_{n'} p_{n'}(\beta)} \leq 1$

We adopt a relatively strong criterion for redistribution as it not only requires that some index of intersectoral inequality decreases, but that for any pair of sectors, the policy reduces the difference between their net wages. A redistributive policy thus compresses the whole distribution of net wages across sectors (the first condition in the definition). The second condition implies that we restrict attention to policies that do not reverse the pre-tax ranking of sectors.

A policy $\beta$ fixes the ratio of $(1 - t_n)(1 - \tau_n)/(1 - t_{n'})(1 - \tau_{n'})$ for any pair of sectors $n, n'$, but does not fix the level of taxation, which depends on $d_0$. The latter is defined by the requirement that the policy be feasible, in the sense that the government runs a balanced budget:

$$\sum_{i=0}^{N} t_i c_i \Lambda_i + \sum_{i=0}^{N} \tau_i p_i \varphi_i \Lambda_i = 0 \iff \sum_{i=0}^{N} \beta_i p_i y_i = \frac{I}{d_0} \tag{8}$$

where the second equation uses the zero profit condition (6). In words, revenues from the sectoral income and sales tax must add up to zero.

### 2.4 Sectoral choice of workers

At $t_2$, individuals decide in which sector to work. They observe their idiosyncratic vector of sector-specific productivities $\{z_{jn}\}_{n=0}^{N}$, the vector of income
taxes \( \{t_n\}_{n=0}^N \), and the vector of sectoral wages \( \{c_n\}_{n=0}^N \). Worker \( j \) chooses to work in the sector which gives him the highest net income, which is the product of the net wage per effective unit of labor in the sector times the worker-sector specific productivity \( (1 - t_n)z_{jn}c_n \). As shown in the appendix 6.1, the fraction of individuals deciding to work in sector \( n \) is:

\[
L_n = \frac{((1 - t_n)c_n)^\nu}{\sum_{i=0}^N((1 - t_i)c_i)^\nu},
\]

which is also the supply of labor as the number of individuals is normalized to one. \( L_n \) is increasing in the net wage paid in sector \( n \). On the other hand, a higher net wage in other sectors makes employment in \( n \) relatively less attractive and reduces \( L_n \). The parameter \( \nu \) represents a measure of the sector-specificity of labor and determines the sensitivity of employment to relative differences in net wages between sectors. If \( \nu \) is large, the productivity parameters drawn by individuals for different sectors are similar, making the choice of sector very dependent on the relative net wages. The degree of sector specificity of workers is in this case very low. If \( \nu \to 0 \), on the other hand, workers are fully sector-specific and each sector employs \( 1/(N + 1) \) of the labor force regardless of differences in net wages.

Using the sectoral supply of labor, we show in the appendix 6.1 that the supply of good \( n \) as given by (4) is equal to:

\[
y_n = \varphi_n \Delta((1 - t_n)c_n)^{\nu - 1}\left(\sum_{i=0}^N((1 - t_i)c_i)^\nu\right)^{\frac{1 - \nu}{\nu}},
\]

where \( \Delta \equiv \Gamma(1 - 1/\nu) \) and \( \Gamma() \) denotes the gamma function. Sectors which pay higher net wages have a higher supply curve since they attract more workers. For \( y_n \) to be defined, we assume in the rest of the analysis that \( \nu > 1 \). If \( N \) is large, \( \nu - 1 \) is the elasticity of the number of effective units of labor employed in a sector with respect to the wage paid in that sector.\(^{11}\)

\(^{11}\)Note that, as \( \nu \to 1 \), the allocation of labor remains sensitive to \( 1 - t_n \) or \( c_n \) (see (9))
Solving for $c_n$ in (10) shows that the wage in $n$ is increasing in $y_n$ and that the total costs of production in sector $n$ are convex. To expand, sector $n$ needs to attract workers who may be relatively more productive in other sectors, and who therefore need to be paid a higher wage to accept working in sector $n$.

2.5 A More general model

Our base model has only one factor of production. In this short section we want to outline a model with two factors of production. A more general model has the advantage of being more realistic and also allows us to model other tax instruments. Assume that output is produced with labor and capital, and profits are taxed with a sector-specific profits tax. When capital costs are not fully deductible, which is consistent with many tax systems around the world, a sector-specific profits tax has a redistributive character and works similar to the sales tax in the present framework. A sectoral profits tax lends itself to a simple interpretation when sectors differ in terms of firm size. In sectors where firms are large, it is more attractive for them to incorporate, while sectors with typically low firms may avoid the additional costs of incorporation. The sectoral tax on large firms can thus be understood as a corporation tax, while the tax on smaller firms represents the taxation of profits under the personal income tax.

In this formulation of the model, firms in sector $n$ have profits of:

$$\pi_n = (1 - \tau_n)(p_n y_n - c_n \Lambda_n) - \delta_n r K_n$$ (11)

where $K_n$ is capital in sector $n$, $\tau_n$ is a sectoral profit tax and $\delta_n$ indexes but the amount of effective labor is not. For $\nu \to 1$, the differences in $z$ are such that the workers who join sector $n$ following an increase in $c_n$ are infinitely less productive than the average worker already in $n$, thereby increasing the production of $n$ by zero percent.
the deductibility of capital. Solving the model for this profit function under a CES utility function with elasticity of substitution larger than one gives qualitatively very similar results, but is more complex and less accessible. For this reason we prefer the simple model in the remainder of the paper.

3 Economic equilibrium

In this section, we characterize the economic equilibrium for a given policy vector $\beta$, first under autarky and then in a small open economy. Although we assume that the vector $\beta$ is exogenous, the variable $d_0$, which pins down the level of $(1-t_n)(1-\tau_n)$ for each sector, is endogenous and ensures the feasibility of a particular policy $\beta$. The main contribution of this section is twofold. First, we characterize and decompose the effect of a given nominal policy vector $\beta$ on individual utility. For this end we use the fact that for our specific utility function in Eq. (1), indirect utility can be written as the product of i) individual j’s sector $n$-specific productivity $z_{jn}$, ii) the redistribution effect represented by the ratio of the net wage in sector $n$ to total income, and iii) total real income in the economy. Note that iii) is common to all workers in the economy, and ii) is common to all workers in a particular sector $n$. As i) is exogenously given and enters utility multiplicatively, our welfare analysis can thus focus on the effects ii) and iii). We do so by showing that the allocative distortion of the policy $\beta$ can be decomposed into a misallocation effect (Lemma 1 below), which describes the output vector under the policy relative to the case $\beta = 1$, and an inefficiency effect (Lemma 2 below), which describes how a given misallocation in terms of output vector translates into real income and thus into iii). Lemma 3 below provides a characterization of the pure redistribution effect by showing how policy $\beta$ affects sectoral net
wages and thus captures effect ii).

Our second contribution in this section is to compare the economic effects of a given redistribution policy in a closed and small open economy. We show that in terms of net wages (effect ii), a given policy under the small open economy leads to more redistribution than under a closed economy. The comparison between the two situations is made meaningful by assuming that the price distribution in the open economy is identical to the one under autarky with no redistribution ($\beta = 1$).

### 3.1 The autarkic equilibrium

In autarky, the market for each good must be in equilibrium, i.e. $y_n = q_n^D$ for all sectors $n$. Using (2) and (10), the goods market equilibrium implies\(^{12}\):

$$\beta_n p_n^A \varphi_n = (\alpha_n \beta_n)^{\frac{1}{2}} \frac{I^A}{\Delta} \left( \sum_{i=0}^{N} \alpha_i \beta_i \right)^{\frac{\nu}{\nu - 1}}$$

(12)

$$y_n^A = \varphi_n \Delta (\alpha_n \beta_n)^{\frac{\nu - 1}{\nu}} \left( \sum_{i=0}^{N} \alpha_i \beta_i \right)^{\frac{1}{\nu - 1}}$$

(13)

where $p_n^A$ and $y_n^A$ are the price and the production in sector $n$ in the autarkic equilibrium. Equation (12) shows the net wage - scaled by $d_0^{-1}$ - obtained by workers in sector $n$. The net wage is a key parameter in our subsequent welfare analysis in section 3. This wage and the production in a sector $n$ are increasing in the demand parameter ($\alpha_n$) of a sector and in the redistributive policy towards it. The degree of worker mobility ($\nu$) indexes the relative extent to which these parameters affect wages or quantities produced.

\(^{12}\)To obtain (12), we set $q_n^D$ of (2) and $y_n$ of (10) equal. We then solve for $((1 - t_n)c_n)^\nu$ and add up over all $n$, which gives a solution for $\sum_{j=1}^{N}(1 - t_i)c_i)^\nu$. Plugging this solution back in the condition $q_n^D = y_n$ and rearranging gives (12).
From (12) and (13) the balanced budget constraint (8) in autarky can be rewritten as:

$$d_0 = \frac{1}{\sum_{i=0}^{N} \alpha_i \beta_i}$$

(14)

This condition, combined with (12) and (13), pins down the vector of prices and of production in autarky for any given $I^A$.

### 3.2 The equilibrium in a small open economy

We now consider a small open economy facing a vector of exogenous prices \( \{p^T_n\}_{n=0}^N \) set on the world market. We define \( I^T \) as the total nominal income of this economy. Since prices are now fixed on the world market, the net wage in sector \( n \) is simply given by: \( \beta_n p^T_n \varphi_n \). In a small open economy, domestic supply and demand of a good are not necessarily equal. The equilibrium production of a sector is determined by the supply equation (10) where the net wage is \( \beta_n p^T_n \varphi_n \), i.e.:

$$y^T_n = \Delta \varphi_n (\beta_n p^T_n \varphi_n)^{\nu-1} \left( \sum_{i=0}^{N} (\beta_i p^T_i \varphi_i)^\nu \right)^{\frac{1-\nu}{\nu}}.$$

(15)

Using (15) and rearranging, we can express the total nominal income of the small open economy \( I^T \) and the balanced budget condition of the government (8) respectively as:

$$I^T = \sum_{i=0}^{N} p^T_i y^T_i = \Delta \left( \sum_{i=0}^{N} \beta_i^{\nu-1} (p^T_i \varphi_i)^\nu \right) \left( \sum_{i=0}^{N} (\beta_i p^T_i \varphi_i)^\nu \right)^{\frac{1-\nu}{\nu}}$$

(16)

$$d_0 = \left( \sum_{i=0}^{N} \beta_i^{\nu-1} (p^T_i \varphi_i)^\nu \right) \left( \sum_{i=0}^{N} (\beta_i p^T_i \varphi_i)^\nu \right)^{-1}.$$

(17)

This completes the description of the small open economy.

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13 Since we have not fixed any numeraire, total income - which is the sum of wages paid to all workers - can take any value.
3.3 Redistribution and Distortion

The indirect utility of a worker $j$ in sector $n$ can, from (1) and (2), be rewritten as:

$$V_{jn}^S(\beta) = z_{jn}u_n^S(\beta) = z_{jn}D_n^S(\beta)R_n^S(\beta) \quad S \in \{A, T\}$$  \hspace{1cm} (18)

with:

$$D_n^S(\beta) \equiv \frac{d_0^\beta \varphi_n p_n^S(\beta)}{I_S^S(\beta)} \quad S \in \{A, T\}$$  \hspace{1cm} (19)

$$R^S(\beta) \equiv I_S^S(\beta) \prod_{i=0}^N \alpha_i^\alpha (p_i^S(\beta))^{-\alpha_i} \quad S \in \{A, T\}$$  \hspace{1cm} (20)

where $S \in \{A, T\}$ indexes whether we are considering the autarkic or small open economy case (“trade”). From the equilibrium derived in the previous sections, $p^A(\beta)$ and $I^T(\beta)$ are functions of the policy vector $\beta$.

Equation (18) decomposes the indirect utility of worker $j$ in sector $n$ between a common component to all workers in sector $n$, $u_n^S(\beta)$, and an idiosyncratic parameter representing the productivity draw of $j$ in $n$, $z_{jn}$. The common component $u_n^S(\beta)$ can further be decomposed into two parts. The first, $D_n^S(\beta)$, is the net wage in sector $n$ as a fraction of total income. The impact of the policy vector on $D_n^S(\beta)$, which captures the extent to which workers in $n$ benefit from the policy relative to others, is the redistributive effect of the policy. The second component, $R^S(\beta)$, is the total real income in the economy. The impact of the policy vector on this second component captures the distortive effect of the policy.

We now express the total real income of the economy as a function of exogenous parameters and of the policy vectors, which are considered exogenous in the present section. In autarky and in a small open economy, these
are respectively:

\[ R^A(y^A(\beta)) = \prod_{i=0}^{N} (y^A_i(\beta))^\alpha_i \] (21)

\[ R^T(y^T(\beta)) = \left( \sum_{i=0}^{N} p^T_i y^T_i(\beta) \right) \left( \prod_{i=0}^{N} \alpha_i (p^T_i)^{-\alpha_i} \right) . \] (22)

In both cases, there is no other distortion than the redistributive policy. The total real income is therefore maximized\(^{14}\) when \( \beta = 1 \). Any deviation from this efficient allocation induces a misallocation of resources.

To conduct a meaningful comparison between the small open and the closed economy cases, we assume that the world price distribution is identical to the one which prevails in autarky with no redistribution \((\beta = 1)\), i.e.:

\[ p^T_n = \frac{I}{\Delta \varphi_n} \alpha_n^{\frac{1}{\Delta}} \forall n. \] (23)

This assumption means that trade is “unbiased” in the sense that (a) the efficient allocation of resources is identical in the closed and open economy, (b) the total real income with no redistribution is the same under the closed and the open economy and (c) if \( \beta = 1 \) in the open economy, no international trade takes place.

To disentangle the effects of redistributive policies on welfare, we decompose the distortive effect of the redistributive policy into two parts. First, we denote the discrepancy between the first-best sectoral allocation of effective labor and that obtaining with policy \( \beta \) as the misallocation induced by the redistributive policy. Second, we map the sectoral allocation of effective labor to real income using the functions \( R^A \) in autarky and \( R^T \) in the small open economy. We denote the loss of real income due to misallocation - i.e. by how much is real income under the misallocation lower than under

\(^{14}\) Appendix 6.2 gives a formal proof of this result.
the efficient sectoral allocation of labor - as the \textit{inefficiency} of a given misallocation. While we define misallocation purely in terms of production vectors, the inefficiency converts a level of misallocation to the corresponding loss of real income.

\textbf{Definition 2 Misallocation}

Consider two production vectors $y^G$ and $y^H$ and call $y^*$ the vector of output which maximizes real income in the economy. There is more misallocation of resources with $y^G$ than with $y^H$ if, for all pairs $n, n'$:

$$\left| \log \left( \frac{y^G_n}{y^*_n} \right) - \log \left( \frac{y^G_{n'}}{y^*_n} \right) \right| > \left| \log \left( \frac{y^H_n}{y^*_n} \right) - \log \left( \frac{y^H_{n'}}{y^*_n} \right) \right|. \quad (24)$$

The above definition establishes a restrictive characterization\textsuperscript{15} of misallocation. If we order the sectors by the ratio of produced output to efficient output, Definition 2 requires for misallocation to increase that the sectors with a comparatively large ratio become larger and those with a comparatively low ratio become smaller. As shown in the appendix 6.3, this definition directly relates to the concept of second order stochastic dominance of production vectors.

The misallocation reflects the fact that redistributive policies, by subsidizing sectors with a low laissez-faire price, make the size of sectors ($y_n$) relatively more symmetric than would be efficient. The strength of this effect differs between the autarkic and the open economy case, as can be seen from

\textsuperscript{15}Definition 2 characterizes a misallocation of resources based solely on a comparison of production vectors. It is a priori more restrictive than a comparison based on real income as there are production vectors which Definition 2 cannot order in terms of misallocation but which can be ordered according to the real income they imply.
From (25) and Definitions 1 and 2, it is immediate that redistributive policies cause a misallocation of resources. The misallocation generated by a redistributive policy is stronger the larger the mobility of workers (the higher the \( \nu \)), since the output distortion depends on the ability of workers to move. If \( \nu \to 1 \) on the other hand, effective labor is immobile between sectors and redistributive policies do not cause any intersectoral misallocation of resources.

**Lemma 1 Misallocation**

Any feasible redistributive policy causes a stronger misallocation of resources under an open economy than under autarky. The level of misallocation that policy \( \beta \) causes in autarky is equal to that of policy \( \beta^{1/2} \) in an open economy.

Both in a small open economy and in autarky, a subsidy to sector \( n \) causes more workers to work for that sector. In autarky, the increased supply of good \( n \) puts a downward pressure on its price - and therefore on the wage in \( n \) - thereby limiting the inflow of workers in \( n \). This dampening effect of prices does however not occur in a small open economy, inducing more workers to work in \( n \) than in autarky.

The following two equations map the vector of sectoral output to the total real income in the economy. Comparing these to the real income under the efficient allocation of effective labor allows to quantify the inefficiency
induced by a given misallocation of effective labor in autarky and in a small open economy. From (13), (15), (21) and (22):

\[
R^A(y) = \left( \prod_{i=0}^{N} (y_i(1))^{\alpha_i} \right) \left( \prod_{i=0}^{N} \left( \frac{y_i}{y_i(1)} \right)^{\alpha_i} \right)
\]

(26)

\*

\*

\[
R^T(y) = \left( \prod_{i=0}^{N} (y_i(1))^{\alpha_i} \right) \left( \sum_{i=0}^{N} \alpha_i \left( \frac{y_i}{y_i(1)} \right) \right).
\]

(27)

**Lemma 2 Inefficiency**

1. The inefficiency of a given misallocation is larger in autarky than in a small open economy.

2. The percentage change in inefficiency caused by a stronger misallocation is larger in autarky than under a small open economy.

**Proof.** see Appendix

The rationale behind Lemma 2 can readily be seen from (26) and (27). A change in the vector of production only distorts the supply side in the open economy while it also distorts the demand side in the closed economy. In autarky, equation (26) shows that distortions enter welfare in a Cobb Douglas form (see the second large bracket in (26)) as they distort the prices observed by consumers and therefore their consumption patterns. In the open economy, on the other hand, prices are exogenously set on the world markets. Redistributive policies affect the income of the country but not its relative consumption of different goods. This effect can best be seen by taking an extreme example. Assume that the labor allocation is such that no worker produces good \( n \). In autarky, it implies that consumers cannot buy good \( n \), driving their utility to zero. In an open economy on the other hand, consumers can still buy good \( n \) at the world market price, ensuring that their utility remains positive.
We finally turn to the redistributive impact of the policy and consider the share of total income that a worker $j$ providing one unit of effective labor in sector $n$ obtains in autarky and in a small open economy respectively:

$$D^A_n(\beta) = \Delta^{-\frac{\nu}{\nu+1}} \left( \frac{y^A_n(\beta)}{\varphi_n} \right)^{\frac{1}{\nu+1}} \Delta^{-\frac{\nu}{\nu+1}}$$

(28)

$$D^T_n(\beta) = \Delta^{-\frac{\nu}{\nu+1}} \alpha_n^\nu \beta_n \left( \sum_{i=0}^{N} \alpha_i / \beta_i^\nu \right)^{-\frac{\nu}{\nu+1}} = \left( \frac{y^T_n(\beta)}{\varphi_n} \right)^{\frac{1}{\nu+1}} \Delta^{-\frac{\nu}{\nu+1}}$$

(29)

A higher $\beta_n$ raises the share of total income accruing to workers in sector $n$, which is the redistributive effect of the policy. Workers in sectors with a high $\beta_n$ effectively benefit from a transfer from the rest of the population. To make precise statements about the equilibrium redistribution taking place in an economy, we define the extent of redistribution by relating the distribution of net wages across sectors under policy $\beta$, $E_n = d_0 \beta_n p_n \varphi_n$, to the situation with no policy intervention.

**Definition 3 Redistribution**

Consider two vectors of net wages, $E^G$ and $E^H$ and call $E^*$ the vector of net wages with no redistributive policy. The extent of redistribution is larger with $E^G$ than with $E^H$ if and only if, for each pair of sectors $n$ and $n'$ such that $E^*_n > E^*_n'$:

$$\log \left( \frac{E^G_n}{E^*_n} \right) - \log \left( \frac{E^G_{n'}}{E^*_n} \right) < \log \left( \frac{E^H_n}{E^*_n} \right) - \log \left( \frac{E^H_{n'}}{E^*_n} \right).$$

(30)

We are now in a position to compare the cross-sectoral redistributive effect of policy $\beta$ under autarky and the small open economy. The comparison depends on the degree of sector-specificity of labor, $\nu$. Consider autarky first. From (28), a higher $\nu$ reduces the extent of redistribution in autarky. An increase in $\beta_n$ induces more workers to choose sector $n$, since it increases the net wage perceived by workers in that sector. If $\nu$ goes to infinity, the size
of the sector increases and the equilibrium price $p^A_n$ decreases in such a way that net wages remain constant, thereby making redistribution impossible. If, on the other hand, mobility is very limited, the small inflow of workers in $n$ does not affect $p^A_n$ much and guarantees that workers in sector $n$ see their net wage increase. Hence, the incidence of redistribution under autarky is crucially tied to the sector-specificity of labor. Next, consider the small open economy. $\nu$ does not affect the extent of redistribution due to the fact that the price $p^T_n$ is exogenous. Lower taxes (or higher subsidies) in $n$ are therefore fully transmitted to the net wage of sector $n$ workers, which is similar to the limit case $\nu \to 1$ under autarky. Since $\nu > 1$, Lemma 3 immediately follows.

**Lemma 3** Redistirbution

*A nominal policy $\beta$ gives rise to more effective redistribution under an open than under a closed economy. The extent of redistribution in autarky with policy $\beta$ is equal to that of policy $\beta^\frac{1}{\nu}$ in an open economy.*

The Lemmas 1 to 3 summarize the three main effects of redistributive policies.

## 4 Political Equilibrium

In the previous section we characterized the economic effects of government intervention via the nominal policy vector $\beta$. In this section, we endogenize the choice of policy via a political process (and require economic equilibrium given policy choice). This is non-trivial in so far as workers are heterogenous after obtaining their distribution of sectoral productivity draws (even though the draws come from the same (Fréchet) distribution). We assume that all individuals in the economy vote on the policy $\beta$. The voting takes place at time $t_1$, which is prior to the sectoral work choice by individuals.
Without further assumptions there would be little hope that a voting equilibrium exists, simply because the policy vector is multidimensional and voters have different productivity profiles at the time of voting. To overcome this problem, we will assume two types of sectors only. Still, this leaves a high degree of heterogeneity, as workers who would join the same sector have typically different productivity draws and thus incomes. We will argue below that the latter aspect of heterogeneity does not map into different policy preferences, conditional on working in the same sector. A remaining problem, however, is that with different policy vectors the distribution of workers across sectors changes, and even workers who have the same wage in a given sector typically have different opportunities when moving to another sector.

In order to state the political equilibrium properly we summarize the conditions for an economic equilibrium (given policy $\beta$) : i) individuals make optimal consumption choices given prices and income (as captured in (2)), ii) individuals as workers choose the sector with the highest net income given policy and prices, iii) firms maximize profits given technology and prices (which leads to the zero profit condition (6)), iv) sectoral choices by individuals give rise to the sectoral labor and output supply condition (9) and (10), which are consistent with all markets clearing (goods market equilibrium conditions imply (11) and (12) under autarky, and (14) under the small open economy and correspondingly the net wage $d_0\beta_n p_n^T \varphi_n$), and v) the government budget is balanced (as in (8)).

We are now in position to formulate the politico-economic equilibrium of the economy. For every feasible policy vector $\beta$ there exists a corresponding economic equilibrium, as summarized above. Each individual considers his utility that is corresponding to this economic equilibrium. A majority voting
equilibrium is a policy vector $\beta^*$ such that there is no other policy vector $\beta'$ under which more than half of the population would experience a higher utility in the corresponding economic equilibrium than under $\beta^*$. The definition entails that each worker anticipates correctly that a deviation from the policy $\beta$ to some other policy $\beta'$ involves a change of worker allocation across sectors including his or her own choice.

To make use of the median voter theorem, we restrict attention to the case with only two types of sectors. We denote $X_L$ and $X_H$ as the sets of sectors with respective demand parameters $\alpha_L$ and $\alpha_H$, where $\alpha_H > \alpha_L$, and respective productivity parameters $\varphi_L \leq \varphi_H$. We assume that $X_L$ consists of $x_L$ sectors while $X_H$ consists of $x_H$ sectors. By assumption: $x_L\alpha_L + x_H\alpha_H = 1$.

By definition of a redistributive policy (see Definition 1), all sectors with the same demand parameter must have the same policy parameter. Using sectors with high demand shocks as the numeraire, choosing a policy boils down to choosing a parameter $\beta = (1 - t_L)(1 - \tau_L)/((1 - t_H)(1 - \tau_H)) \geq 1$, where the subscripts $L$ and $H$ refer to the policies applying to sectors in $X_L$ and $X_H$ respectively. The larger is $\beta$, the more redistribution there is towards sectors with low demand parameters. With a slight abuse of notation, we use the subscripts $H$ or $L$ to denote sectors in $X_H$ and $X_L$ respectively. The policy choice problem has thus been reduced to a one-dimensional policy problem. Note that we restrict the policy space from below by assuming $\beta \geq 1$. Hence we exclude the case that redistribution takes place from low to high demand sectors. At the upper end, we assume based on Definition 1 that the net wage in the low demand sector cannot exceed the counterpart in the high demand sector.
4.1 The median voter equilibrium

Our first step is to establish that voter preferences satisfy the single-crossing condition, which then allows us to apply the median voter theorem. Each worker chooses to work in the sector which gives him the highest level of utility. In autarky or in a small open economy, worker $j$ therefore has utility:

$$V^S_j = \max_n \{ \{ z_{jn}u^S_L(\beta) \}_{n \in X_L}, \{ z_{jn}u^S_H(\beta) \}_{n \in X_H} \} \quad S \in \{A, T\}. \quad (31)$$

Define $z^M_{jk} = \max \{ z_{jn} : n \in X_k \}$ as the highest draw of productivity of individual $j$ among all sectors in $X_k$, $k \in \{L, H\}$, and $Z_j = z^M_{jL}/z^M_{jH}$ as the ratio between the highest productivity draw in $X_L$ and the highest productivity draw in $X_H$. Since all sectors within $X_k$ are symmetric, an individual $j$ effectively only needs to decide whether to work in the sector with $z^M_{jL}$ or that with $z^M_{jH}$. The preferences can therefore be rewritten as$^{16}$:

$$V^S (z^M_{jH}, Z_j, \beta) = z^M_{jH} \max \{ Z_j u^S_L(\beta), u^S_H(\beta) \} \quad S \in \{A, T\} \quad (32)$$

**Lemma 4** Assume that $Z' > Z$ and $\beta' > \beta$. If $V^S(z^M_{jH}, Z, \beta') > V^S(z^M_{jH}, Z, \beta)$, then

$$V^S(z^M_{jH}, Z', \beta') > V^S(z^M_{jH}, Z, \beta) \quad S \in \{A, T\}. \quad (33)$$

**Proof:** See Appendix □

As illustrated in Figure 1, the preferences of individual workers are not single-peaked in general. Lemma 4, however, guarantees that the preferences of voters satisfy the single-crossing property in $(Z, \beta)$, ensuring that there

$^{16}$The decomposition of the function $V$ between two parameters, $z^M_{jH}$ and $Z_j$ is similar to the treatment in Ohnsorge and Trefler (2007). As in their model, we will show that the sorting of workers between sectors relies on the “comparative advantage” parameter ($Z_j$ in our case) and not on the “absolute advantage” parameter ($z^M_{jH}$).
exists an equilibrium policy which coincides with the bliss point of the median $Z_j$ in the economy (Gans and Smart (1996)).

As evident from (32), the preferred level of $\beta$ chosen by an individual $j$ is either $\beta^S_L = \arg\max u^S_L(\beta)$ or $\beta^S_H = \arg\max u^S_H(\beta)$, where $\beta^S_L$ is preferred to $\beta^S_H$ by worker $j$ if and only if $Z_j u^S_L(\beta^S_L) > u^S_H(\beta^S_H)$. The policy chosen by majority voting is the policy preferred by the voter with the median $Z_j$ and is characterized by the following Proposition.

**Proposition 1** The policy $\beta^S$ chosen by majority voting is:

$$
\beta^S = \begin{cases} 
\beta^S_L & \text{if } x_L(u^S_L(\beta^S_L))^\nu > x_H(u^S_H(\beta^S_H))^\nu \\
\beta^S_H & \text{otherwise}
\end{cases}
$$

The parameter $\nu$ plays an essential role in the above condition. It determines the extent to which the winning policy depends on the relative number of sectors in $X_L$ and $X_H$ ($x_L/x_H$) or on differences in sector-specific utility components $u^S_L(\beta)/u^S_H(\beta)$. If $\nu$ is small, idiosyncratic productivity draws are very heterogeneous between sectors. The preferred policy of a worker is in this case typically determined by whether he receives his highest productivity in a high or in a low demand sector, the likelihood of which depends on $x_L$ and $x_H$. In the extreme case of no mobility ($\nu = 0$), which we do not allow here, all workers with highest productivity draw in a low (high) demand sector would vote for $\beta^S_L$ ($\beta^S_H$) so that the winning policy would be solely determined by $x_L$ and $x_H$. If, by contrast, $\nu$ is large, idiosyncratic productivity draws are typically similar in all sectors and differences in sector specific

---

\footnote{We show in the appendix 6.6 and 6.8 that $u^S_L(\beta)$ and $u^S_H(\beta)$ are single-peaked and that $\beta^S_L$ and $\beta^S_H$ are unique for $S = \{A, T\}$.}
utilities play a central role in policy choices. In the limit, with $\nu \to \infty$, workers are perfectly mobile and only the sector-specific utility components matter.

### 4.2 Policy Characterization under Autarky

With two types of sectors, a higher $\beta$ means larger subsidies to sectors with low demand and higher tax rates on sectors with high demand. If $\beta > 1$, workers in high demand sectors are net contributors to the redistributive policy and also lose from its distortive effect. They therefore oppose redistributive policies and choose the lower bound of the feasible range, i.e., $\beta_H^A = 1$. Workers choosing to work in low demand sectors on the other hand benefit from the redistributive effect of the policy, although they lose from its distortive effect. For $\beta$ close to 1, the marginal redistribution is first-order while the distortive effect is second-order, meaning that workers in low demand sectors want at least some redistribution and thus $\beta_L^A > 1$. Two factors limit the size of the redistributive policy from the perspective of workers in low demand sectors. First, the distortive effect of redistribution is convex in $\beta$, limiting the redistribution that workers in sectors $L$ wish to implement. Second, by definition of a redistributive policy (Definition 1), the net effective wage in low demand sectors should not surpass that in high demand sectors, as discussed above. This effectively limits the size of the policy to: $\beta \leq \alpha_H/\alpha_L$. As shown in the appendix 6.6, the policy maximizing $u_L^A(\beta)$ is:

$$
\beta_L^A = \min \left( 1 + \frac{1}{(\nu - 1)\alpha_L x_L}, \frac{\alpha_H}{\alpha_L} \right). \tag{34}
$$

18If $\nu$ is large, many workers with highest productivity draw in high demand sectors ($z_{jH}^M > z_{jL}^M$) may prefer working in a low demand sector with policy $\beta_L^S$ and be better off than in a high demand sector with $\beta_H^S$. Recognizing this, these workers would vote for $\beta_L^S$ over $\beta_H^S$. 

29
The above equation highlights three characteristics of the preferred redistributive policy for workers in low demand sectors. First, they want strictly positive redistribution as long as \( \nu < \infty \). Under imperfect sectoral mobility, individuals working in low demand sectors gain from the redistribution while the marginal distortion to the economy is close to zero for \( \beta \approx 1 \). Second, redistribution is less attractive the higher the mobility of labor (as long as the first term in (34) is smaller than the second term). The reason is twofold and is in line with the analysis in section 3: (i) the resource misallocation is stronger the larger the labor mobility and (ii) the redistribution is less strong under higher mobility as the inflow of workers into subsidized sectors drives wages down in these sectors. Third, \( \beta^A_L \) is weakly decreasing in \( \alpha_L x_L \).

From the perspective of workers in \( X_L \), the redistributive gain of the policy decreases in the number of workers in \( X_L \), as the fraction of net contributors to the policy decreases.

As shown in the appendix, combining Proposition 1 and (34) shows that policy \( \beta^A_L \) beats policy \( \beta^A_H = 1 \) if and only if:

\[
x_L \alpha_L (\beta^A_L)^{\alpha_L(\nu-1)+1} (x_L \alpha_L \beta^A_L + \alpha_H x_H)^{-\nu} > x_H \alpha_H.
\]

(35)

It is useful to define \( \chi_A \in (0, 1/2) \) as the unique value of \( \alpha_L x_L \) for which (35) holds with equality.

**Proposition 2** Equilibrium Policy in Autarky

The equilibrium policy \( \beta^A \) in autarky is:

\[
\beta^A = \begin{cases} 
1, & \text{if } \alpha_L x_L < \chi_A \\
\beta^A_L, & \text{if } \alpha_L x_L \geq \chi_A 
\end{cases}
\]

(36)

where \( \beta^A_L \) is defined in (34).

30
Proof: See Appendix ■

The product $\alpha_L x_L$ is a direct determinant of the equilibrium size of sectors with low demand and of the policy chosen by majority voting. A redistributive policy can only win if many workers prefer working in a sector with low demand. In this case, the chosen policy is characterized by $\beta^A_L$.

4.3 Policy characterization in a small open economy

The analysis of the small open economy is qualitatively similar to that of the autarkic case. As in autarky, workers in high demand sectors would lose from redistributive policies and choose to vote for policy $\beta^T_H = \beta^A_H = 1$. Similarly, workers in low demand sectors want some redistribution, which is limited by the same condition as in autarky ($1 < \beta^T_L \leq \alpha_H/\alpha_L$). As shown in Appendix 6.8, the preferred policy of workers choosing to work in low demand sectors is implicitly given by the following equation if it is interior, and by $\alpha_H/\alpha_L$ otherwise:

$$x_H \alpha_H + x_L \alpha_L (\beta^T_L)^\nu \left(\beta^T_L (1 - \nu) + \nu\right) = 0.$$  \hspace{1cm} (37)

The following Proposition characterizes the equilibrium in the open economy. We define $\chi_T$ as the unique value of $\alpha_L x_L$ for which $x_L (u^T_L (\beta^T_L))^\nu = x_H (u^T_H(1))^\nu$.

**Proposition 3** Equilibrium Policy in a Small Open Economy

The equilibrium policy under a small open economy is given by

$$\beta^T = \begin{cases} 1, & \text{if } \alpha_L x_L < \chi_T \\ \beta^T_L, & \text{if } \alpha_L x_L \geq \chi_T \end{cases}$$  \hspace{1cm} (38)

where $\beta^T_L$ is implicitly defined by (37).

Proof: See Appendix ■
The Proposition shows that the equilibrium policy in a small open economy has a similar structure to that in autarky. A redistributive policy can only win by majority voting if there are enough workers who would benefit from its adoption, i.e. if $\alpha_L x_L$ is large enough. On the other hand, there is no redistribution in equilibrium if the majority of voters would lose from redistribution.

4.4 Globalization and Redistribution in Political Equilibrium

We now come to the comparison of the two situations.

Proposition 4 Comparison of policies under autarky and small open economy.

1. For any $\nu$, $\chi_T < \chi_A$, that is, redistribution is more “likely” to occur in the small open economy;

2. Conditional on redistribution taking place under autarky and in the open economy, nominal redistribution under autarky is (weakly) higher: Specifically, $\beta_L^T < \beta_L^A \leq \beta_L^A$. The second inequality is strict as long as $\beta_L^T < \alpha_H / \alpha_L$.

Proof: See Appendix

The two parts of Proposition 4 are the key results of the present paper. Part 1 states that the range of $\alpha_L x_L$ for which a redistributive policy is adopted is larger in an open economy than in autarky. Part 2 on the other hand shows that, given that redistribution takes place both in autarky and in the small open economy ($\alpha_L x_L \geq \chi_A$), the level of redistributive policy
chosen by voters is lower in an open than in a closed economy ($\beta_T \leq \beta_L$). Taken together, these results imply that international trade (i) raises the set of parameter values for which redistributive policies are observed but also that (ii) if there is redistribution, policies are smaller in an open economy. Figure 2 provides a numerical illustration of these results by plotting the equilibrium policies in autarky and in a small open economy as a function of the number of sectors with low demand in the economy.

Part 2 of Proposition 4 adds an important insight. Although the observed nominal level of policy is lower in a small open economy, the extent of effective redistribution and of misallocation arising in equilibrium is larger in an open economy than in autarky ($\beta^4_L < \beta_L$ and Lemmas 1 and 3).

From Lemmas 1 and 3, a given policy $\beta$ in a small open economy has the same redistributive and misallocative effects as a policy $\beta'$ in autarky. This comes from the higher elasticity of labor demand in open economies, which magnifies the effects of a given redistributive policy. If this was the sole difference between the closed and open economy cases, voters would choose under a small open economy a policy with a lower nominal value than in autarky but with the same extent of redistribution in equilibrium. As shown in Lemma 2, however, opening to trade also reduces the inefficiency caused by a given misallocation. This is due to the fact that opening up to trade relaxes the constraint that goods market should be in equilibrium at the national level. Consumer prices are therefore not distorted by redistributive policies in an open economy, whereas they are in autarky. Since redistributive policies are less inefficient in an open economy, workers choose a stronger equilibrium redistribution than in autarky, causing a stronger inflow of workers to low demand sectors. As the number of workers choosing sectors with low demand increases, the support for redistributive policies widens and these win by
majority voting under a wider parameter range.

5 Conclusion

In this paper we have adopted a general equilibrium perspective on sectoral redistribution policies, which are prevalent in many countries and situations. The key theoretical tool was the modelling of partial labor mobility or sectoral specificity, which means that heterogeneous workers respond to wage differences across sectors, but not completely and not in the same way. Redistribution policies thus favor some workers, but at a cost of allocational inefficiency, which workers need to take into account when they vote over policies. This flexible framework allows us to compare policies at two extreme ends of openness to international trade: no trade and the small open economy with given world market prices. We expect that results for an open economy with some pricing power would lie between these two situations, although this claim would have to be proven in future research.

Our main findings make clear that the relationship between openness and redistribution is complex and multi-dimensional. We show that in political equilibrium redistribution is more likely to occur in the open economy in the sense that it takes fewer sectors with low demand parameters to make redistribution attractive compared to no trade. Yet, when redistribution takes place in both situations, the nominal or face value in the small open economy is smaller, even though the redistributive effect - once general equilibrium effects are taken into account - is larger. International trade increases the elasticity of labor demand, which magnifies the effects of a given redistribution policy, and reduces the inefficiency of a given labor misallocation because consumer prices are not (or less) distorted.
References


6 Appendix

6.1 Derivation of \( L_n \) and \( y_n \) in (9) and (10)

If worker \( j \) receives a productivity draw \( z \) in sector \( n \), the probability that it is best to work in \( n \) is the probability that the draws of \( z_i \) in all other sectors are lower than \( (1 - t_n) c_n z / ((1 - t_i) c_i) \). This probability is given by:

\[
G \left( \frac{(1 - t_n) c_n z}{(1 - t_i) c_i} \right) = \prod_{i \neq n} F \left( \frac{(1 - t_n) c_n z}{(1 - t_i) c_i} \right) = \exp \left( -((1 - t_n) c_n z)^{-\nu} \left( \sum_{i \neq n} ((1 - t_i) c_i)^{-\nu} \right) \right).
\]

(39)
The supply of workers in sector $n$ is given by the integral over all $z$ of the probability that a draw of $z$ makes it optimal to work in $n$, i.e.:

$$L_n = \int_0^\infty G \left( \frac{(1 - t_n) c_n z}{(1 - t_n) c_n} \right) dF(z) = \frac{(1 - t_n) c_n^\nu}{\sum_{n=1}^N (1 - t_n) c_i^\nu}. \quad (40)$$

The supply of goods from sector $i$ is given by the total effective labor in $i$, i.e.:

$$Q^S_i = \int_0^\infty z G \left( \frac{(1 - t_n) c_n z}{(1 - t_i) c_i} \right) dF(z) \quad (41)$$

which yields (10).

### 6.2 Proof that total real income is maximized when $\beta = 1$

In the autarkic case, total real income is given by:

$$I^A \left( \prod_{i=0}^N \alpha_i^{\alpha_i (p_i^A)^{-\alpha_i}} \right) = \Delta \left( \prod_{i=0}^N (\alpha_i \beta_i)^{\alpha_i \frac{\nu - 1}{\nu}} \varphi_i^{\alpha_i} \right) \quad (42)$$

It is immediate that total real income is maximized when $\sum_{i=0}^N \alpha_i \log(\beta_i)$ is maximized. By Jensen’s inequality:

$$\sum_{i=0}^N \alpha_i \log(\beta_i) \leq \log \left( \sum_{i=0}^N \alpha_i \beta_i \right) \quad (43)$$

so that $\beta = 1$ is the only vector such that (43) holds with equality. It must therefore maximize total real income in autarky.

From (22), it is immediate that maximizing total real income in the small open economy is equivalent to maximizing $I^T$. We are here deriving the vector $\beta$ which maximizes total real income subject to the budget constraint of the government (17):

$$\max_{\{\beta_i\}_{i=0}^N} \sum_{i=0}^N (p_i^T \varphi_i \beta_i)^\nu \quad \text{s.t.} \quad \sum_{i=0}^N (p_i^T \varphi_i)^\nu \beta_i^{\nu-1} = \sum_{i=0}^N (p_i^T \varphi_i \beta_i)^\nu \quad (44)$$
The first order condition for this problem is given by:

\[ \nu \beta_n^\nu (p_n^T \varphi_n)^\nu (1 - \lambda) - \lambda (\nu - 1) \beta_n^{\nu - 1} (p_n^T \varphi_n)^{\nu - 1} = 0 \quad \forall n \in N \]  

(45)

Summing up over all \( n \) gives and using (17) gives:

\[(1 - \lambda) \nu = \lambda (\nu - 1) \]  

(46)

Solving (45) for \( \beta_n \) and using (46) shows that \( \beta_n = 1 \) for all \( n \).

### 6.3 Proof of Lemma 2

- **Part 1**

  The proof that, for any output vector which differs from the efficient output, total real income is strictly lower in autarky than in a small open economy is a direct consequence of Jensen’s inequality:

\[
\sum_{i=0}^{N} \alpha_i \log \left( \frac{y_i}{y_i(1)} \right) \leq \log \left( \sum_{i=0}^{N} \alpha_i \frac{y_i}{y_i(1)} \right)
\]  

(47)

which ensures that \( R_T(y) > R_A(y) \) as long as \( y \neq y(1) \).

- **Part 2**

  Consider two output vectors \( y^G \) and \( y^H \) corresponding to two possible equilibria. These equilibria may differ either because the policy vector \( (\beta) \) differs or because one equilibrium is in a small open economy while the other is in autarky. Assume that \( y^G \) corresponds to more misallocation than \( y^H \) according to Definition 2. We first prove that \( y^H \) second-order stochastically dominates \( y^G \).
From (13) and (15), we can rewrite the ratio of produced to efficient output in sector $i$ in autarky and in a small open economy as:

$$\frac{y^A_i(\beta)}{y^A_i(1)} = \beta^{1-\nu} \left( \frac{\sum_{i=0}^{N} \alpha_i \beta_i^\nu}{\sum_{i=0}^{N} \alpha_i \beta_i^\nu} \right)^{\frac{1-\nu}{\nu}} \quad (48)$$

$$\frac{y^T_i(\beta)}{y^T_i(1)} = \beta^{1-\nu} \left( \frac{\sum_{i=0}^{N} \alpha_i \beta_i^\nu}{\sum_{i=0}^{N} \alpha_i \beta_i^\nu} \right)^{\frac{1-\nu}{\nu}} \quad (49)$$

Rearranging and summing up over all $i$, we obtain that both in autarky and in a small open economy, for any policy $\beta$:

$$\sum_i \alpha_i \left( \frac{y^S_i(\beta)}{y_i(1)} \right)^{\frac{\nu}{1-\nu}} = 1 \quad S \in \{A, T\} \quad (50)$$

Define $m_i^G$ and $m_i^H$ as the ratios $m_i^G = (y_i^G/y_i(1))$ and $m_i^H = (y_i^H/y_i(1))$ which are consistent with the two equilibria. Without loss of generality, we order the sectors $i$ from those with the highest $\alpha_i$ to those with the lowest $\alpha_i$, implying the orderings $m_0^G < m_1^G < \ldots < m_{N-1}^G < m_N^G$ and $m_0^H < m_1^H < \ldots < m_{N-1}^H < m_N^H$. From the above equation, it must be the case that $\sum_{i=0}^{N} \alpha_i m_i^G = \sum_{i=0}^{N} \alpha_i m_i^H = 1$ and therefore that:

$$\sum_{i=0}^{N} \alpha_i \left( m_i^H - m_i^G \right) = 0 \quad (51)$$

From Theorem 5 in Levy (1992), $m_i^{\frac{\nu}{1-\nu}}$ second order stochastically dominates $m_i^{\frac{\nu}{1-\nu}}$ if, for all $k < N$:

$$\sum_{i=0}^{k} \alpha_i \left( m_i^{\frac{\nu}{1-\nu}} - m_i^{\frac{\nu}{1-\nu}} \right) > 0 \quad (52)$$

It is sufficient for the above equation to hold that the difference $m_i^{\frac{\nu}{1-\nu}} - m_i^{\frac{\nu}{1-\nu}}$ be decreasing in $i$. From the definition of misallocation and since $y^G$
is associated with more misallocation than \(y^H\), it must be the case that:

\[
\frac{y^G_i}{y_{i-1}} > \frac{y^H_i}{y_{i-1}} \iff \frac{G_{i-1}^y}{G_{i-1}^y_{i-1}} > \frac{G_{i-1}^y}{G_{i-1}^y_{i-1}} \quad \forall i > 0 \tag{53}
\]

\[
\frac{m_{i-1}^G}{m_{i-1}^G} - \frac{m_{i-1}^H}{m_{i-1}^H} > \frac{m_{i-1}^G - m_{i-1}^H}{m_{i-1}^G} \tag{54}
\]

Since \(m_{i-1}^H\) is increasing in \(i\), it implies that \(m_{i-1}^G - m_{i-1}^H > m_{i-1}^G - m_{i-1}^H\) for any \(i > 0\). This proves that \(m_i^H\) second order stochastically dominates \(m_i^G\), and since second order stochastic dominance is preserved by increasing and concave transformations, \(m_i^H\) second order stochastically dominates \(m_i\). The concept of misallocation in the model is thus closely linked to that of second order stochastic dominance of the sectoral deviations from efficient output. Since in \(R^A\) and \(R^T\) are concave functions of \(m\), a stronger misallocation raises the inefficiency in the economy.

We now turn to the proof of the second part of Lemma 2. We need to show that, when going from \(y^H\) to \(y^G\) (i.e. increasing misallocation),

\[
\log(R^A(y^G)) - \log(R^A(y^H)) < \log(R^T(y^G)) - \log(R^T(y^H)) < 0, \text{ i.e.:}
\]

\[
\sum_{i=0}^{N} \alpha_i \log \left( \frac{m_i^G}{m_i^H} \right) < \log \left( \sum_{i=0}^{N} \alpha_i m_i^G \right) \tag{55}
\]

By Jensen’s inequality:

\[
\sum_{i=0}^{N} \alpha_i \log \left( \frac{m_i^G}{m_i^H} \right) < \log \left( \sum_{i=0}^{N} \alpha_i m_i^G \right) \tag{56}
\]

To show that (55) holds, it is thus sufficient to show that:

\[
\left( \sum_{i=0}^{N} \alpha_i m_i^G \right) > \left( \sum_{i=0}^{N} \alpha_i m_i^H \right) \left( \sum_{i=0}^{N} \alpha_i m_i^H \right) \tag{57}
\]

We conduct a reasoning by induction. For any \(k \leq N\), define:

\[
S_k = \left( \sum_{i=0}^{N} \alpha_i \right) \left( \sum_{i=0}^{N} \alpha_i m_i^G \right) - \left( \sum_{i=0}^{N} \alpha_i m_i^H \right) \left( \sum_{i=0}^{N} \alpha_i m_i^H \right) \tag{58}
\]
Rearranging gives:

\[ S_k = S_{k-1} + \alpha_k m_k^G \left[ \sum_{i=0}^{N} \alpha_i \left( 1 - \frac{m_i^H}{m_k^H} \right) \left( 1 - \frac{m_i^G m_k^H}{m_k^G m_i^H} \right) \right]. \]  

Ordering the sectors by increasing \( m_i \) as described earlier, it is immediate that \((1 - m_i^H/m_k^H) > 0\). Furthermore, since there is more misallocation under \( m^G \) than under \( m^H \), it must be by Definition 2 that \( m_k^G/m_k^H > m_i^G/m_i^H \) for any \( k > i \). This proves that \( S_k \geq S_{k-1} \). For \( k = 1 \), it can easily be seen that \( S_{k-1} = 0 \), implying that \( S_k > 0 \) for any \( k \) and in particular for \( k = N \). The inequality (55) therefore holds.

\[ \blacksquare \]

### 6.4 Proof of Lemma 4

Using (32), equation (33) is equivalent to:

\[ \max\{Z'u_L^S(\beta'), u_H^S(\beta')\} > \max\{Z'u_L^S(\beta), u_H^S(\beta)\}. \]  

(60)

Note that from (32), and since \( u_H^S(\beta) \) is strictly decreasing in \( \beta \), the assumption that \( V^S(z_{jH}^M, Z, \beta') > V^S(z_{jH}^M, Z, \beta) \), is equivalent to assuming that \( Zu_L^S(\beta') > Zu_L^S(\beta) \) and that \( Zu_L^S(\beta') > u_H^S(\beta) \). It must therefore be the case that \( Z'u_L^S(\beta') > Z'u_L^S(\beta) \) and, since \( Z' > Z \), that \( Z'u_L^S(\beta') > u_H^S(\beta) \), which ensures that (33) holds.

\[ \blacksquare \]

### 6.5 Proof of Proposition 1

The median voter has the median ratio of \( Z_j \equiv z_L^M/z_H^M \). Since \( z_{ji} \) is drawn independently from a Fréchet distribution, the distribution of the maximum draw in the set of sectors with high or low shocks \( z_k^M, k \in \{L, H\} \) also follows a Fréchet distribution and is given by:

\[ \text{Prob}(z_k^M \leq b) = \exp(-x_k z^{-\nu}). \]  

(61)
Following Theorem 1.3 of Nadarajah and Kotz (2006), the distribution of $Z_j$ is given by:

$$\text{Prob}(Z_j < y) = \frac{x_H y^\nu}{x_H + x_L y^\nu}. \quad (62)$$

Setting the above equal to 1/2, the worker with the median $Z_j$ is the one such that

$$Z_j = (x_L/x_H)^{\frac{1}{\nu}}. \quad (63)$$

A worker votes for policy $\beta_L^S$ over policy $\beta_H^S$ if and only if:

$$u_{L}^S(\beta_L^S) < Z_j u_{L}^S(\beta_L^S). \quad (64)$$

From (63), the equilibrium policy is chosen by voting is $\beta_L^S$ if and only if the above equation holds for the median $Z_j$, i.e. if:

$$x_L^{\frac{1}{\nu}} u_{L}^S(\beta_L^S) > x_H^{\frac{1}{\nu}} u_{H}^S(\beta_H^S). \quad (65)$$

### 6.6 The autarkic case with two types of sectors

In autarky, $u_{L}^A(\beta)$ and $u_{H}^A(\beta)$ are given by (18), (19) and (20) and by the solutions of the autarkic equilibrium in section 3.1:

$$u_{L}^A(\beta) = \left( \frac{\alpha_{L} \beta}{x_L \alpha_L \beta + x_H \alpha_H} \right)^{\frac{1}{\nu}} \frac{D_{L}^A(\beta)}{R^A(\beta)} \left( \frac{\beta \nu}{\nu - 1} \right) \frac{(\alpha_L x_L \beta + \alpha_H x_H)^{\frac{1 - \nu}{\nu}}}{\zeta} \quad (66)$$

$$u_{H}^A(\beta) = \left( \frac{\alpha_{H}}{x_L \alpha_L \beta + x_H \alpha_H} \right)^{\frac{1}{\nu}} \frac{D_{H}^A(\beta)}{R^A(\beta)} \left( \frac{\beta \nu}{\nu - 1} \right) \frac{(\alpha_L x_L \beta + \alpha_H x_H)^{\frac{1 - \nu}{\nu}}}{\zeta} \quad (67)$$

where $\zeta = \left( \frac{\varphi_L^{\frac{1}{\nu - 1}}}{\varphi_H^{\frac{1}{\nu - 1}}} \right)^{x_L \alpha_L} \left( \varphi_H^{\frac{1}{\nu - 1}} \right)^{x_H \alpha_H}.$

$R^A(\beta)$ is decreasing in $\beta$ due to the distortive effect of redistributive policy, while $D_{L}^A(\beta)$ and $D_{H}^A(\beta)$ are respectively increasing and decreasing in $\beta$ as low demand sectors are net benefitors and high demand sectors are net
contributors to the redistributive policy. $u_H^A$ is maximized for $\beta = 1$ and is thus single peaked on $\beta \in [1, \alpha_H/\alpha_L]$, while the first order condition for $u_L^A$ is given by (34). Differentiating $u_L^A(\beta)$ twice with respect to $\beta$ and evaluating it at $\beta_L^A$ (i.e. the $\beta$ for which $\partial u_L^A(\beta)/\partial \beta = 0$) shows that if the first derivative is equal to zero, the second derivative is negative. This guarantees that the function is single-peaked and that $\beta_L^A$ is the unique value of $\beta$ maximizing $u_L^A(\beta)$.

6.7 Proof of Proposition 2

The proof consists in two steps. The first step shows that there is a unique $\chi_A$ for which (35) holds with equality and the second that $0 < \chi_A < 1/2$. For simplicity, define $\chi \equiv \alpha_L x_L$ and:

$$G_A^A(\chi) = \frac{\chi^{1/\nu} \left(\beta_L^A\right)^{1+\chi \frac{\nu - 1}{\nu}}}{\chi \beta_L^A + 1 - \chi} - (1 - \chi)^{1/\nu}$$

(68)

where $\beta_L^A$ is defined by (34) and is itself a function of $\chi$. $G_A^A(\chi)$ is equal to $x_L^A u_L^A(\beta_L^A) - x_H^A u_H^A(1)$ divided by $\zeta$ and is such that if $G_A^A(\chi) \geq 0$, policy $\beta_L^A$ wins the majority of votes, while policy $\beta = 1$ wins if $G_A^A(\chi) < 0$. Since $\beta_L^A \in (1, \alpha_H/\alpha_L]$, it is immediate that $G_A^A(0) = -1$ and that $G_A^A(1) = 1$.

- Step 1: There exists a unique cutoff $\chi_A$ such that policy $\beta = 1$ wins if $\chi < \chi_A$ and policy $\beta_L^A$ wins if $\chi \geq \chi_A$

Differentiating $G_A^A(\chi)$ gives:

$$\frac{\partial G_A^A(\chi)}{\partial \chi} = \frac{1}{\nu} \frac{\chi^{1/\nu} (\beta_L^A)^{1/\nu}}{\chi (\beta_L^A)^{1+\chi \frac{\nu - 1}{\nu}} (1-\chi)} \left[1 + \chi (\beta_L^A - 1) (1-\nu)\right] + \frac{1}{\nu} (1-\chi)^{1-\nu}$$

$$+ \frac{\chi^{1/\nu} \beta_L^A (\beta_L^A)^{1-1/\nu}}{\chi (\beta_L^A)^{1+\chi \frac{\nu - 1}{\nu}} (1-\chi)} \ln(\beta_L^A) \frac{\nu - 1}{\nu} (\beta_L^A)^{\chi \frac{\nu - 1}{\nu}} + \frac{\partial G_A^A(\chi)}{\partial \beta_L^A} \frac{\partial \beta_L^A}{\partial \chi}.$$  

(69)

If $\beta_L^A$ is interior, $\partial G_A^A(\chi)/\partial \beta_L^A = 0$ and the last product is equal to zero. If $\beta_L^A$ is constrained by the upper bound $\alpha_H/\alpha_L$, $\partial \beta_L^A/\partial \chi = 0$ and the last
product is also equal to zero. The second line above is therefore positive. By definition of $\beta^A_L$, we know that:

$$
\chi(\nu - 1)(1 - \beta^A_L) + 1 \geq 0 \quad (70)
$$

where the inequality is strict if $\beta^A_L = \alpha_H / \alpha_L$, which ensures that $\partial G^A(\chi)/\partial \chi > 0$. $G^A(\chi)$ is thus monotonically increasing on $\chi \in [0, 1]$ with $G^A(0) < 0$ and $G^A(1) > 0$, which completes the proof of step 1.

- Step 2: Proof that $\chi_A < 1/2$

To show that $\chi_A < 1/2$, it is enough to show that $G^A(1/2) > 0$ since $G^A(\chi)$ is increasing in $\chi$. It is immediate that:

$$
G^A(1/2) = 2^{\frac{\nu - 1}{2}} \left[ \left( \frac{\beta^A_L}{\beta^A_L + 1} \right)^{\frac{\nu - 1}{2\nu}} - \frac{1}{2} \right]. \quad (71)
$$

If $\beta^A_L$ were equal to one, the above expression would be zero. Since $\beta^A_L > 1$ maximizes $u^A_L$ and therefore $G^A$, it must be the case that $G^A(1/2) > 0$, which proves step 2.

6.8 The small open economy with two types of sectors

In a small open economy, $u^T_L(\beta)$ and $u^T_H(\beta)$ are given by:

$$
u L(\beta) = \frac{\alpha^L_T \beta \zeta}{(x_L\alpha_L\beta + x_H\alpha_H)^{\frac{1}{2}}} \left[ \frac{\beta^A_L}{\beta^A_L + 1} \right]$$

$$
u H(\beta) = \frac{\alpha^H_T \beta \zeta}{(x_L\alpha_L\beta + x_H\alpha_H)^{\frac{1}{2}}} \left[ \frac{\beta^A_L}{\beta^A_L + 1} \right]$$

$45$
In a similar manner to the autarkic case, $R^T(\beta)$ is decreasing in $\beta$ due to the distortive effect of redistributive policy, while $D^T_L(\beta)$ and $D^T_H(\beta)$ are respectively increasing and decreasing in $\beta$ as low demand sectors are net benefactors and high demand sectors are net contributors to the redistributive policy. $u^T_H$ is maximized for $\beta = 1$ and is thus single peaked on $\beta \in [1, \alpha_H/\alpha_L]$, while the first order condition for $u^T_L$ is given by (37). The single peakedness of $u^T_L(\beta)$ is shown in the same manner as the single peakedness of $u^A_L(\beta)$ in the appendix 6.6.

6.9 Proof of Proposition 3

The proof of Proposition 3 requires to show that there exists a unique $\chi_T$ such that policy $\beta = 1$ wins if $\chi < \chi_T$ and policy $\beta^T_L$ wins if $\chi \geq \chi_T$.

For simplicity, define:

$$G^T(\chi) = \chi \frac{1}{\nu} \left( \frac{\beta^T_L(\beta)}{\beta^T_L(\beta)} \right)^\nu + (1 - \chi) \beta^T_L(\beta) \left( \frac{\beta^T_L(\beta)}{\beta^T_L(\beta)} \right)^\nu + (1 - \chi) \frac{1}{\nu} \left( \frac{\beta^T_L(\beta)}{\beta^T_L(\beta)} \right)^\nu + 1 - \chi$$

(74)

where $\beta^T_L$ is defined by (37) and is itself a function of $\chi$. $G^T(\chi)$ is equal to $x^T_L u^T_L(\beta^T_L) - x^T_H u^T_H(1)$ divided by $\zeta$ and is such that if $G^T(\chi) \geq 0$, policy $\beta^T_L$ wins the majority of votes, while policy $\beta = 1$ wins if $G^T(\chi) < 0$. Since $\beta^T_L \in (1, \alpha_H/\alpha_L]$, it is immediate that $G^T(0) = -1$ and that $G^T(1) = 1$.

Differentiating $G^T(\chi)$ gives:

$$\frac{\partial G^T(\chi)}{\partial \chi} = \frac{1}{\nu} \chi \left( \frac{\beta^T_L(\beta)}{\beta^T_L(\beta)} \right)^\nu + (1 - \chi) \frac{\beta^T_L(\beta)}{\beta^T_L(\beta)} \left( \frac{\beta^T_L(\beta)}{\beta^T_L(\beta)} \right)^\nu + (1 - \chi) \frac{1}{\nu} \left( \frac{\beta^T_L(\beta)}{\beta^T_L(\beta)} \right)^\nu + 1 - \chi \frac{1}{\beta^T_L(\beta)} \frac{\partial G^T(\chi)}{\partial \beta^T_L(\beta)} \frac{\partial \beta^T_L(\beta)}{\partial \chi}.$$  

If $\beta^T_L$ is interior, $\partial G^T(\chi)/\partial \beta^T_L = 0$ and the last term drops out. If $\beta^T_L$ is constrained by its upper bound $(\alpha_H/\alpha_L)$, on the other hand, $\partial \beta^T_L(\beta)/\partial \chi = 0$. In both cases, the last term above drops out.
By definition of $\beta^T_L$, we know that:

$$\chi (\beta^T_L)^\nu + \nu \chi \left( (\beta^T_L)^{\nu-1} - (\beta^T_L)^\nu \right) + 1 - \chi \geq 0 \Leftrightarrow \frac{(\beta^T_L)^\nu - (\beta^T_L)^{\nu+1}}{\chi (\beta^T_L)^\nu + 1 - \chi} \geq -\frac{\beta^T_L}{\nu \chi}.$$  \hfill (76)

Plugging the above inequality in (75) and rearranging, we obtain:

$$\frac{\partial G^T(\chi)}{\partial \chi} \geq \frac{1}{\nu} \chi \frac{1}{\chi (\beta_L)^\nu + 1 - \chi} + \frac{1}{\nu} (1 - \chi) \frac{1}{\nu - 1} > 0.$$  \hfill (77)

$G^T(\chi)$ is a monotonically increasing function from $G^T(0) < 0$ to $G^T(1) > 0$. There exists therefore a unique cutoff $\chi_T$ such that policy $\beta = 1$ wins if $\chi < \chi_T$ while policy $\beta^T_L$ wins if $\chi \geq \chi_T$.

### 6.10 Proof of Proposition 4

The proof of Proposition 4 consists of two parts. In a first step, we prove that $\chi_T < \chi_A$. In a second step, we show that $(\beta^A_L)^{\frac{1}{\nu}} < (\beta^T_L) \leq (\beta^A_L)$.

- **Part 1:** $\chi_T < \chi_A$

  Since by definition $\beta^T_L$ maximizes $u^T_L(\beta)$, we have that:

  $$x_L^{\frac{1}{\nu}} u^T_L(\beta^T_L) \geq x_L^{\frac{1}{\nu}} u^T_L \left( (\beta^A_L)^{\frac{1}{\nu}} \right).$$  \hfill (78)

  We now show that for any $\beta > 1$:

  $$x_L^{\frac{1}{\nu}} u^T_L \left( (\beta)^{\frac{1}{\nu}} \right) \geq x_L^{\frac{1}{\nu}} u^A_L(\beta).$$  \hfill (79)

  From (66) and (72), this is equivalent to showing that:

  $$\chi \beta^{\frac{\nu-1}{\nu}} + 1 - \chi > \beta^{\frac{\nu-1}{\nu}}.$$  \hfill (80)

  For $\beta = 1$, the above expression would hold with equality. Differentiating both sides with respect to $\beta$ shows that the left hand side grows quicker than
the right hand side, and therefore that for any $\beta > 1$, the above inequality holds. The inequality (79) reflects the fact that with policy $\beta^{\frac{1}{\nu}}$ in a small open economy, the misallocation and the redistribution are equal to those in autarky with policy $\beta$, but the inefficiency of the misallocation is stronger in autarky, making the utility of all workers lower.

Combining (78) and (79), we can show that for all $\chi \in [0,1]$: 

$$G^T(\chi) > G^A(\chi)$$

(81)

and in particular: $G^T(\chi_A) > G^A(\chi_A) = 0$. Since $G^T$ is increasing in $\chi$, it implies that $\chi_T > \chi_A$.

- Part 2: $(\beta^A)^{1/\nu} < \beta_L^T \leq \beta^A_L$

Equation (37) can be rewritten as:

$$\left[ \frac{1}{x_L\alpha_L(\nu - 1)} + 1 - (\beta_L^T)^{\nu} \right] + \frac{\nu}{\nu - 1} \left( (\beta_L^T)^{\nu - 1} - 1 \right) = 0.$$  

(82)

By definition of a redistributive policy, $\beta_L^T \geq 1$, which implies that the left hand side above is weakly decreasing in $\beta_L^T$. At $\beta_L^T = 1$, the left hand side is positive. It must therefore be the case that $\beta_L^T > 1$ for the above equation to hold. If the square bracket were equal to zero, we would have that $\beta_L^T = (\beta^A)^{1/\nu}$. In this case, the redistribution and the misallocation would be the same in the open and in the closed economy case. The additional term to the square bracket on the left hand side is positive for $\beta_L^T > 1$ and reflects the fact that the marginal inefficiency of a stronger misallocation is less strong in autarky than in an open economy. It must therefore be the case for the above to hold that $^{19} \beta_L^T > (\beta^A)^{1/\nu}$.

^{19}Note that if $\beta^A_L = \alpha_H/\alpha_L$, the inequality also holds as $\beta_L^T$ is larger than the shadow value of $\beta^A_L$ to the power $1/\nu$, and is therefore larger than $(\alpha_H/\alpha_L)^{1/\nu}$. 
To show that $\beta_T^L \leq \beta_A^L$, evaluate (37) at the value $\beta_T^L = \beta_A^L$ for $\beta_A^L$ interior. We obtain:

$$x_H \alpha_H \left[ 1 - \left( 1 + \frac{1}{x_L \alpha_L (\nu - 1)} \right)^{\nu - 1} \right] < 0.$$  \hspace{1cm} (83)

Since the left hand side of (37) is decreasing in $\beta_T^L$, it must be the case that $\beta_T^L < \beta_A^L$ if $\beta_A^L$ is interior. $\beta_T^L$ and $\beta_A^L$ are only equal if both correspond to maximum redistribution: $\beta_T^L = \beta_A^L = \alpha_H / \alpha_L$. 

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Figure 1: Utility in autarky of a worker with $Z=1.3$. The dashed curves represent $u^A_H(\beta)$ and $1.3u^A_L(\beta)$. The plain sections of the curve represent the actual utility as a function of $\beta$. The parameters are: $x_L = 1$, $x_H = 0.5$, $\alpha_L = 0.5$, $\alpha_H = 1$, $\nu = 2$, $\varphi_L = \varphi_H = 1$, $z^M_H = 1$.

Figure 2: Equilibrium policy as a function of $x_L$. The dashed line characterizes equilibria in a small open economy while the plain line shows equilibria in autarky. The parameters are: $\alpha_L = 1$, $\alpha_H = 2$, $\nu = 3$. 

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