Fiscal Multipliers and Policy in a Model of Goods, Labor and Credit Market Frictions

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May 2014, Mainz
Introduction

- We develop our model of credit, goods and labor market frictions to study fiscal stimulus and *fiscal multipliers*
  - Fiscal multiplier: GDP impact of an additional unit of public spendings (The New Palgrave, Chinn 2013)
  - May be 1, 0, static, dynamic (lagged impact), long term (ratio of cumulated GDP impact over cumulated spendings)
- Renewed interest for these questions since the financial crisis
  - State (as well as central bank interventions) have been massive in most Western economies.
Further, fiscal multipliers may vary:

- **within cycles**
  - may be asymmetric (stimulus vs. consolidation impact)
  - according to the position with respect to the “output gap” or to the distance to some “steady-state”

- **between cycles**
  - according to the nature of the recession; e.g. financial recessions

- according to its financing, origin of creditors
Introduction

- Empirical literature: Auerbach and Gorodnichenko (2012a and b) have found that fiscal multipliers in the US may be as large as 2.5 in a recession and close to zero in an expansion; and in the same range in a panel of OECD countries including France, Germany, Italy and Canada.

- Convergent with several recent IMF reports or papers (World Economic Outlook 2012, Batini et alii. 2012; Baum et alii. 2012, Blanchard and Leigh 2013)

- What are the existing theoretical ingredients?
Introduction

- Two main mechanisms for multipliers greater than 1:
  - Articulation with monetary policy: as the economy approaches the zero lower bound on nominal interest rates, central banks cannot prevent deflationary effects of fiscal consolidation and therefore a fiscal stimulus avoids the deflation trap
  - Price and wage rigidities
On the first effect: Christiano, Eichenbaum, and Rebelo (2011) found very large fiscal multipliers (more than 3) in a DSGE setting, while Hall (2009) argue that multiplier may be around 1.7 in a similar situation.

Think of a competitive economy first:

- output $Y_t$ as a function $f$ of hours worked $H_t$
- consumed in equilibrium by households ($C_t$) or government ($G_t$)
- log linearizing $f(H_t) = Y_t = C_t + G_t$ under the optimal choice $v'(H_t) = u'(C_t)$ where $v(H_t)$ is disutility for hours worked and $u(C_t)$ is utility from consumption
- implies a fiscal multiplier $dY_t/dG_t = \eta_u/\left(\eta_u + \eta_v\right)$
  - where $\eta_u = -C/u'' / u' \geq 0$
  - and $\eta_v$ is the elasticity of the disutility from supplying one additional unit of output

This multiplier is smaller than 1: additional supply requires additional effort (raises the implicit price of consuming)
Introduction

- The existence of a constant price markup $m \geq 1$ does not change the picture.
- Only changes the equality of marginal utilities for leisure and consumption: $mv'(H_t) = u'(C_t)$
  - hence without effect on the multiplier.
- To get round this smaller than 1 multiplier, varying markups $m_t$
  - as put by Woodford, "the key to obtaining a larger multiplier is an endogenous decline in the labor efficiency wedge"
Neo-keynesian setup

- But not enough. Think of a simple monetary policy (simple Taylor rule, nominal interest rate follows inflation and a measure of GDP gap), leading to variations in nominal interest rate
  - If there is a monetary policy can set the real interest rate $r_t$ to a constant
    - a) away from a zero interest rate trap
    - b) away from financial imperfections
  - Optimal intertemporal behaviour of households leads to
    \[
    \frac{u'(C_t)}{\beta u'(C_{t+1})} = e^{r_t}
    \]
- Under a constant real interest rate, constant consumption with curved utility
  - Hence, multipliers must be one: any additional public spending is smoothed away in future consumption
Paleo-Keynesian instead?

- Keynes: general equilibrium interaction between markets. Disequilibrium in a market (labor) lead to low demand and thus higher unemployment

- Required a disequilibrium model with rationning and agents sending signals to consume (e.g. Bénassy 1993)
  - Same idea in matching models (Wasmer 2011): aggregate function of signals of agents in Bénassy is similar to Pissarides’s rationalization (1990, chapter 1) of an aggregate matching function: a technology that makes consistent the desired demand and desired supply side of markets

- Here, we give a chance to the model to produce larger multipliers, with frictions and matching in the goods market, but also in the labor and credit markets (source of amplification)
Three symmetric frictions (C, L, G) and a timing

1. New project first requires liquidity: first match

2. Block creditor+project then requires labor: vacancies and unemployment must match

3. The three agents form a new block which must finds customers: matching between consumers and firms selling

- Agnostic: each matching block has a perfect market as a limit case
  - We have already quantified each of these blocks and investigated the respective role of each frictions on dynamics

- All this is part of a broader research agenda of multiple markets frictions
Goods markets

- Why search frictions in the goods market?
- Firm’s side:
  - Advertising, 2% of GDP and procyclical (Hall 2012)
  - Capacity utilization less than 100% (80%) so “vacant” firms
  - In Bai et alii. words: “output is not equal to the combination of inputs”
    - Eg: restaurant food is sold if there are customers (does not only depend on the number of tables)
- Consumers’ side:
  - Time use surveys: shopping time is positive and procyclical
    - Hence, consumption is also a function of transaction effort (think of consumptions of services, or of the service of durables goods)
    - Evidence of product entry and exit (Broda and Weinstein AER 10), turnover in product tastes
- Joint surplus and endogenous markups
Earlier paper (NPN-EW 2014): goods market frictions fundamentally change the dynamics

1. Credit and goods market frictions are *substitutable* in generating amplification of technology shocks

2. Goods market frictions are *unique* in generating persistence and hump-shaped responses to shocks

3. Bridge the gap with data, both in terms of volatility (sd of logs) and persistence (autocorr. in growth rates)

**Table:** Labor Market Second Moments

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th>Autocorr. $\Delta \theta_t$</th>
<th>$Corr(U, V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U$</td>
<td>$V$</td>
<td>$\theta$</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>0.13</td>
<td>0.14</td>
<td><strong>0.27</strong></td>
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<tr>
<td>Baseline (CLG)</td>
<td>0.09</td>
<td>0.16</td>
<td><strong>0.24</strong></td>
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<tr>
<td>Labor (L)</td>
<td>0.02</td>
<td>0.03</td>
<td><strong>0.04</strong></td>
</tr>
<tr>
<td>Credit-Labor (CL)</td>
<td>0.06</td>
<td>0.10</td>
<td><strong>0.16</strong></td>
</tr>
<tr>
<td>Goods-Labor (GL)</td>
<td>0.07</td>
<td>0.13</td>
<td><strong>0.20</strong></td>
</tr>
</tbody>
</table>
Role of goods market frictions vs. other frictions

Figure: Comparing Frictions Impulse Responses to a Positive Technology Shock.
Mechanisms in the goods market

Explanation: mechanisms related to goods market frictions:

1. Expected surplus from production *procyclical*; same for expected surplus from exchange (consumption)
   1.1 It follows that consumers’ search effort for goods: *procyclical*; eases matching in goods market, positive feedback effect on firms
   1.2 Bargained prices are *countercyclical* but total revenue of the firm still *procyclical*

2. Goods market tightness (ratio consumers to sellers): *countercyclical*
Overview of the model

- 3 types of agents: Banks, Investment projects (entrepreneurs), Workers; all three required to produce
  - Only banks have access to storage and liquidity. Conversion technology between numeraire and quasi-numeraire
  - Lifecycle of a project: search for credit, then labor, then consumers with endogenous transition rates

- Novelty in this paper:
  - Concave utility, hence non-search goods are no longer a numeraire but quasi-numeraire. 4 types of goods: search good (1), a quasi-numeraire (0), a numeraire and leisure
  - Matching frictions: Credit, Labor, Goods with two intensive search margins

<table>
<thead>
<tr>
<th>Market</th>
<th>Matching</th>
<th>Tightness</th>
<th>Price</th>
<th>Search effort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit</td>
<td>$\Omega_C(N_C, B_C)$</td>
<td>$\phi$</td>
<td>$\psi$</td>
<td>-</td>
</tr>
<tr>
<td>Labor</td>
<td>$\Omega_L(V, U)$</td>
<td>$\theta$</td>
<td>$w$</td>
<td>-</td>
</tr>
<tr>
<td>Goods</td>
<td>$\Omega_G(\bar{e}_g, t, \Xi_U, t, \bar{e}_A, tN_G, t)$</td>
<td>$\xi$</td>
<td>$\mathcal{P}$</td>
<td>$e_g$</td>
</tr>
</tbody>
</table>
Credit, labor and goods market frictions

Four stages for the firm, subscript $j$

- $j = c$: search for credit, successful with probab $p(\phi)$
  - as in Wasmer and Weil (2004) and NPN-EW (AEJ Macro 2012), ignored in the presentation, present in the calibration
  - Simply introduces an entry cost $K(\phi)$

- $j = v$: vacancy searches for labor, successful with probab $q(\theta)$

- $j = g$: search for consumer of the good, successful with probability $\lambda(\xi)$

- $j = \pi$: profits, it ends with probability $s^C, s^G, s^L$
Search in the labor market

In subsequent stages (post credit) $j = v, g, \pi$, bank and project continue to operate jointly as a firm of joint value

$$J_{j,t} = E_{j,t} + B_{j,t}$$

A number $\mathcal{V}_t$ of firms in stage $v$ (vacancies) search for unemployed workers $\mathcal{U}$

- measure of market tightness : $\theta_t = \frac{\mathcal{V}_t}{\mathcal{U}_t}$
- labor market matching : $\frac{\Omega_L(\mathcal{V}_t, \mathcal{U}_t)}{\mathcal{V}_t} = q(\theta_t)$ with $q'(\theta_t) < 0$
Matching on goods market

- Mass $\mathcal{N}_G$ of firms ready to produce makes advertising/selling effort $e_A$ to meets with a stock $\Xi_U$ of unmatched consumers who search for a product with intensity $e_g$

- Concept for tightness in the goods market: $\xi_t = \frac{\Xi_{U,t}}{\mathcal{N}_{G,t}}$

- Matching on goods market:

  Firms: $\frac{M_G(e_{g,t}\Xi_{U,t}, e_{A,t}\mathcal{N}_{G,t})}{\mathcal{N}_{G,t}} = \lambda(\xi_t, e_{g,t}, e_{A,t})$ \quad $\partial\lambda(\xi_t)/\partial\xi_t > 0$

  Consumers: $\frac{M_G(e_{g,t}\Xi_{U,t}, e_{A,t}\mathcal{N}_{G,t})}{\Xi_{U,t}} = \check{\lambda}(\xi_t, e_{g,t}, e_{A,t})$ \quad $\partial\check{\lambda}(\xi_t)/\partial\xi_t < 0$

  and $\check{\lambda}(\xi_t) = \lambda_t(\xi_t)/\xi_t$. 
Consumption and income

- Two consumption goods: numeraire $c_0$ and manufactured $c_1$, with utility $v(c_{1,t}, c_{0,t})$
  - Second good $c_1$ yields higher marginal utility but must be found (frictions): manufacturing, some services
  - First good: food exp. + utility
  - Presumption: complementarity in consumption

- Search cost $\sigma(e_g)$, with $\sigma'(e_g) > 0$ and $\sigma''(e_g) \geq 0$

- A short term budget constraint $\mathcal{P}_t c_{1,t} + c_{0,t} = I_t^d$ with resources pooled across categories of workers (Merz (1995) and Andofaltto (1996)):

$$I_t^d = \frac{\left(\Pi_t + N_t w_t + G_t - T_t\right)}{\bar{\Xi}} + \bar{y}^0$$

$$= \frac{\left[(\mathcal{P}_t x_t - \Omega)N_{\pi,t} - \gamma \mathcal{V}_t - \sigma_A + G_t - T_t\right]}{\bar{\Xi}} + \bar{y}^0$$

where $\bar{y}^0$ is self-production of a numeraire; $G_t$ and $T_t$ are fiscal transfers and taxes
Overview of model with goods market frictions

After hiring worker, firm must find a customer first to sell production

Additional costs (worker must be paid even if there are no sales) and dynamics where transitions rates $\lambda(\xi)$
Overview of model - Single job creation equation

Value of firm in recruiting stage equalizes cost and benefits and determines $\theta_t$:

$$\frac{\gamma}{q(\theta_t)} = \frac{1}{1 + r} \mathbb{E}_t J_{g,t+1}$$

Cost of labor frictions Expected profits
Bellman equations

\[ J_{v,t} = -\gamma_t + \frac{1}{1+r} \mathbb{E}_t [q_t J_{g,t+1} + (1-q_t) J_{v,t+1}] \]

\[ J_{g,t} = -w_t - \sigma_{A,t} + \mathbb{E}_t \left[ \frac{1-s^L}{1+r} \left( \frac{e_{A,t} \lambda_t J_{\pi,t+1}}{\bar{e}_{A,t}} + (1 - \frac{e_{A,t} \lambda_t}{\bar{e}_{A,t}}) J_{g,t+1} \right) + \frac{s^L J_{v,t+1}}{1+r} \right] \]

\[ J_{\pi,t} = \mathcal{P}_t x_t - w_t - C_Q + \mathbb{E}_t \left[ \frac{1-s^L}{1+r} \left( (1-s^G) J_{\pi,t+1} + s^G J_{g,t+1} \right) + \frac{s^L J_{v,t+1}}{1+r} \right] \]

- Small production cost $C_Q$ to ensure no production undertaken in stage $g$
Determining price $\mathcal{P}$

Consumer have utility $v(c_{1,t}, c_{0,t}) - \sigma_g(e_{g,t})$

$$D_{U,t} = v(0, c_{0,t}) - \sigma(e_{g,t}) + \frac{1}{1 + r} \mathbb{E}_t \left[ \frac{e_{g,t} \hat{\lambda}_t}{\bar{e}_{g,t}} D_{M,t+1} + \left( 1 - \frac{e_{g,t} \hat{\lambda}_t}{\bar{e}_{g,t}} \right) D_{U,t+1} \right]$$

$$D_{M,t} = v(c_{1,t}, c_{0,t}) + \frac{1 - s^L}{1 + r} \mathbb{E}_t \left[ s^G D_{U,t+1} + (1 - s^G) D_{M,t+1} \right] + \frac{s^L}{1 + r} \mathbb{E}_t D_{U,t+1}$$

▶ If we assume separability and with the short-run budget constraint:

$$v(c_{1,t}, c_{0,t}) = b_1(x_t) + b_0(I_t^d - c_{1,t} \mathcal{P}_t)$$

▶ Denote by $\Delta v$ the difference operator across states (matched or unmatched). Under linear utility for the numeraire, we have that

$$\Delta v = b_1(x_t)$$

hence independent of both prices and disposable income: fiscal multiplier here is 1: all in consumption of more numeraire

▶ We need concave utility to obtain a non-trivial multiplier
Determining price $\mathcal{P}$

- Price bargaining is a natural solution: the total surplus to the consumption relationship is $S_t^G = (J_{\pi,t} - J_{g,t}) + (D_{M,t} - D_{U,t})$

- The good’s price is determined as

$$
\mathcal{P}_t = \arg\max (J_{\pi,t} - J_{g,t})^{1-\alpha_G} (D_{M,t} - D_{U,t})^{\alpha_G}
$$

, where $\alpha_G \in (0, 1)$ is the share of the goods surplus going to the consumer.

- Sharing rule is

$$(1 - \alpha_G) (D_{M,t} - D_{U,t}) = \alpha_G v'_{0,t} \ (J_{\pi,t} - J_{g,t})$$

- Hence time varying share of consumers: depends on their marginal utility for quasi-numeraire $v'_{0,t}$:

$$
\tilde{\delta}_t = \frac{\alpha_G v'_{0,t}}{1 - \alpha_G + \alpha_G v'_{0,t}}
$$

- equal to $\alpha_G$ with linear separable utility ($v'_{0,t} = 1$);
- above $\alpha_G$ if $v'_{0,t} > 1$, converges to 1 if $v'_{0,t} \to \infty$ (starvation)
- below $\alpha_G$ if $v'_{0,t} < 1$, converges to 0 if $v'_{0,t} = 0$ (satiation of 0)
Determining price $\mathcal{P}$

- Price setting and choice of utility are key (of course): assume

$$v(c_{1,t}, c_{0,t}) = b_1(c_{1,t}) + b_0(I^d_t - \mathcal{P}_t c_{1t})$$

$$(+b_{01}v_0(c_{0,t})v_1(c_{1,t}))(\text{not needed})$$

- Under linear separable utility: no impact of $I^d_t$, because price is a weighted average of cost of producing and utility of consuming:

$$\mathcal{P}_t x_t = \alpha_G [C_Q - \sigma_A(1 - \eta_{\sigma_A})] + (1 - \alpha_G) [b_1(x_t) + \sigma_g(1 - \eta_{\sigma_g})]$$

- Under non linear utility: there is an (noying) intertemporal term in prices: same expression + future surpluses

$$\Lambda \mathbb{E}_t \left[(1 - \tilde{\delta}_t)S^G_{t+1}\right] - \Lambda \mathbb{E}_t \left\{(1 - \tilde{\delta}_{t+1})S^G_{t+1}\right\} \quad (1)$$
Determining price $\mathcal{P}$

- A Kalai solution gets it simpler computationally: it reduces by one x number of approx. terms in Chebyshev (here 3) the dimensionality of the space state.

- In the general case with decreasing marginal utility in quasi-numeraire, prices react (positively) to $I^d_t$:

$$\tilde{\delta}_t [\mathcal{P}_t x_t - C_Q + \sigma_A (1 - \eta \sigma_A)] = (1 - \tilde{\delta}_t) \left\{ b_1(x_t) + b_0 (I^d_t - \mathcal{P}_t x_t) - b_0 (I^d_t) + \sigma_g \right\}$$

- Higher income leads to more consumption of goods 0 in both matched and unmatched states, thus to reduces the quasi-numeraire utility gap from spending income into the search good.

- Same as complementarity between $c_{0,t}$ and $c_{1,t}$.
Optimal search efforts

\[
\bar{e}_{c,t} \sigma'_c(e^*_{c,i,t}) = \frac{\lambda_t}{1 + r} \mathbb{E}_t \tilde{\delta}_{t+1} S^G_{t+1}. \tag{2}
\]

\[
\bar{e}_{A,t} \sigma'_A(e_{A,t}) = \frac{\lambda_t}{1 + r} (1 - s^L) (1 - s^C) \mathbb{E}_t (1 - \tilde{\delta}_{t+1}) S^G_{t+1}. \tag{3}
\]

- Since the marginal expected surplus is procyclical, so are the search efforts of both sides of the goods market.
  - Empirically confirmed by Hall (2013) for advertising efforts
  - Further, combining the optimality conditions for the intensive search margins with bargaining, we have:

\[
\frac{\bar{e}_{A,t} \sigma'_A(e_{A,t})}{\bar{e}_{c,t} \sigma'_c(e_{c,t})} = \xi_t \times \frac{\mathbb{E}_t (1 - \tilde{\delta}_{t+1}) S^G_{t+1}}{\mathbb{E}_t \tilde{\delta}_{t+1} S^G_{t+1}} (1 - s^L) (1 - s^C) > 0
\]

- Strategic complementarity arising from bilateral search effort
Indeed! time use survey suggest procyclical search for goods

<table>
<thead>
<tr>
<th>Year</th>
<th>Goods and Services</th>
<th>of which Consumer Goods</th>
<th>of which Grocery shopping</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>0.81</td>
<td>0.40</td>
<td>0.11</td>
</tr>
<tr>
<td>2004</td>
<td>0.83</td>
<td>0.41</td>
<td>0.10</td>
</tr>
<tr>
<td>2005</td>
<td>0.80</td>
<td>0.41</td>
<td>0.11</td>
</tr>
<tr>
<td>2006</td>
<td>0.81</td>
<td>0.40</td>
<td>0.10</td>
</tr>
<tr>
<td>2007</td>
<td>0.78</td>
<td>0.39</td>
<td>0.10</td>
</tr>
<tr>
<td>2008</td>
<td>0.77</td>
<td>0.38</td>
<td>0.10</td>
</tr>
<tr>
<td>2009</td>
<td>0.76</td>
<td>0.38</td>
<td>0.11</td>
</tr>
<tr>
<td>2010</td>
<td>0.75</td>
<td>0.37</td>
<td>0.10</td>
</tr>
<tr>
<td>2011</td>
<td>0.72</td>
<td>0.37</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Source: [http://www.bls.gov/tus/](http://www.bls.gov/tus/), Civilian Population, Table 1 and Table A1,
Procyclical search for goods

Time spent purchasing goods and service – ATUS 2003–2012

- Aggregate
- Employed
- Unemployed
- Not in Labor Force
For simplicity, we assume a wage rule that takes the functional form

\[ w_t = \chi_w (P_t x_t)^{\eta_w}, \]  

(4)

where \( \eta_w \) is the elasticity of wages to the marginal production of labor in terms of the quasi-numeraire \( P_t x_t \).

Adapted from Blanchard-Gali (2010), allows us to focus on the role played by the elasticity of wages to prices for fiscal multipliers.

Petrosky-Nadeau and Wasmer (2014) and Appendix explore wage variants such as Nash-bargaining, one for which the firm is in stage \( g \), the other when the firm is in stage \( \pi \).
Laws of motions

Consumers:

\[
\Xi_{U,t+1} = (1 - \tilde{\lambda}_t)\Xi_{U,t} + \left[ s^C + \left( 1 - s^C \right) \left( s^L + \left( 1 - s^L \right) s^G \right) \right] \Xi_{M,t}
\]  

(5)

\[
\Xi_{M,t+1} = \left( 1 - s^C \right) \left( 1 - s^L \right) \left( 1 - s^G \right) \Xi_{M,t} + \tilde{\lambda}_t \Xi_{U,t}
\]  

(6)

Firm:

\[
\mathcal{N}_{G,t+1} = \left( 1 - s^C \right) \left( 1 - s^L \right) \left[ (1 - \lambda_t)\mathcal{N}_{G,t} + s^G\mathcal{N}_{\pi,t} \right] + q(\theta_t)\mathcal{V}_t
\]  

(7)

\[
\mathcal{N}_{\pi,t+1} = \left( 1 - s^C \right) \left( 1 - s^L \right) \left[ (1 - s^G)\mathcal{N}_{\pi,t} + \lambda_t\mathcal{N}_{G,t} \right].
\]  

(8)

Workers

\[
\mathcal{U}_{t+1} = \left[ s^C + \left( 1 - s^C \right) s^L \right] (1 - \mathcal{U}_t) + (1 - f(\theta_t))\mathcal{U}_t
\]  

(9)

\[
1 - \mathcal{U}_t = \mathcal{N}_{G,t} + \mathcal{N}_{\pi,t} = \mathcal{N}_{t+1}.
\]  

(10)

Vacancies with imperfect markets: prior to vacancy creation, projects  \( \mathcal{E}_{C,t} \) and creditors (banks)  \( \mathcal{B}_{C,t} \) must meet:

\[
\mathcal{V}_{t+1} = p(\phi)\mathcal{E}_{C,t} + [1 - q(\theta_t)]\mathcal{V}_t + s^L\mathcal{N}_{t+1}
\]

\[
= \phi p(\phi)\mathcal{B}_{C,t} + [1 - q(\theta_t)]\mathcal{V}_t + s^L\mathcal{N}_{t+1}.
\]  

(11)
Extension to credit markets

- The transition rates for investment projects and creditors are given by:

\[
\frac{\Omega_C(N_C, B_C)}{N_C} = p(\phi) \quad \text{with} \quad p'(\phi) < 0 \quad (12)
\]

\[
\frac{\Omega_C(N_C, B_C)}{B_C} = \tilde{p}(\phi) = \phi p(\phi) \quad \text{with} \quad \tilde{p}'(\phi) > 0 \quad (13)
\]

- Double free-entry of creditors and projects + Nash-bargaining (share \(\alpha_C\) to the bank) leads to:

\[
J_{v,t} = E_{v,t} + B_{c,t} \equiv \frac{\kappa_B}{\phi^* p(\phi^*)} + \frac{\kappa_I}{p(\phi^*)} = K
\]

\[
\phi^* = \frac{\kappa_B}{\kappa_I} \frac{1 - \alpha_C}{\alpha_C} \forall t
\]
Overview of model - Single job creation equation

Value of firm in recruiting stage equalizes cost and benefits and determines $\theta_t$:

$$
K(\phi^*)(1 + o_t) + \frac{\gamma}{q(\theta_t)} = \frac{1 - s^c}{1 + r} \mathbb{E}_{t}J_{g,t+1}
$$

Cost of credit frictions  Cost of labor frictions  Expected profits
Illustration with log linearization of the job creation condition

Dynamics of job creation:

\[
\hat{\theta}_t = \frac{1}{\eta_L} \times \frac{S_g}{S_g - K} \times E_t \hat{S}_{g,t+1}
\]

- Elasticity
- Labor frictions
- Credit frictions
- Goods frictions

- Financial multiplier from small firm surplus \( S_g - K(\phi^*) \)
Equilibrium

The equilibrium is a set of policy and value functions for the consumers \( \{ D_{M,t}, D_{U,t}, \bar{e}_{g,t} \} \) and firms \( \{ J_{c,t}, J_{v,t}, J_{g,t}, J_{\pi,t}, \bar{e}_{A,t} \} \); a set of prices rules in goods, labor and credit; and stocks, measures of tightness in the markets for goods, labor and credit \( \{ B_{C,t}, N_{C,t}, V_{t}, N_{G,t}, N_{\pi,t}, \Xi_{U,t}, \Xi_{M,t}, U_{t} \} \) and \( \{ \xi_{t}, \theta_{t}, \phi \} \), overall 22 variables, such that 22 equations are satisfied:

1. Consumers’ value follows functions, with the search-effort optimality conditions for consumers
2. Firm’s value functions in the vacancy stage, in the consumer stage and in the profit stage, together with the optimal advertisement condition
3. Three definitions of credit, labor and goods market tightness as a ratio of the searching stocks
4. A credit block with free-entry in the credit market for both projects and banks (2 equations)
5. Prices in the goods, labor and credit markets are determined by Nash bargaining given by conditions
6. Stock in the goods and labor markets follow seven law of motions, a fixed total number of consumers, finally two identities between the number of firms and the number of employed workers on the one hand, and between the number of matched firms and the number of matched consumers on the other hand

- Risk free rate $r = 0.04$ yearly

- Productivity: $\log x_t = \rho_x \log x_{t-1} + \varepsilon_t^x$, 
  $\rho_x = 0.983$, $\sigma_x = 0.0072$

- Wage rule: bargaining or alternatively some recent macro shortcut 
  $w_t = \chi_w(P_t x_t)^{\eta_w}$

- Den Haan et alii. (2012):
  
  $M_j(X_1, X_2) = \frac{X_1 X_2}{(X_1^{\nu_C} + X_2^{\nu_C})^{1/\nu_C}}$, $j = C, L, G$
Calibration: labor and credit markets

- **Labor targets:**
  - 5.8% unemployment rate, job separation $s^c + (1 - s^c)s^L = 0.043$ (Davis et alii. 2006), implies $f$
  - Elasticity of wages to productivity $\approx 0.75$,
  - $W/P = 0.75$

- **Credit market target:**
  - credit market’s share of GDP $\Sigma = \frac{B_{\pi} - B_g w - B_l \gamma - B_c \kappa}{Y} = 2.5\%$
Calibration: goods markets

- Price mark-up over cost of 15%
- Broda and Weistein (2010):
  - average rate of product entry $\bar{\lambda} = 0.22$
  - average product exit rate implies $\tau = 0.01$
- Cost of time searching in the goods market corresponds to approximately 7% of wage income
- $\Phi$: obtained from the target on the share of expenditures on primary goods (food consumed at home plus utilities): 15%
## Calibration (monthly)

<table>
<thead>
<tr>
<th>Labor market:</th>
<th>Value</th>
<th>Sources or Target:</th>
</tr>
</thead>
<tbody>
<tr>
<td>job-separation rate</td>
<td>$s^L$</td>
<td>0.043 → Davis et al. (2006)</td>
</tr>
<tr>
<td>matching curvature</td>
<td>$\nu_L$</td>
<td>1.25 → Den Haan et al. (2000)</td>
</tr>
<tr>
<td>vacancy cost</td>
<td>$\gamma$</td>
<td>0.14 → Silva and Toledo (2007)</td>
</tr>
<tr>
<td>wage elasticity</td>
<td>$\eta_w$</td>
<td>0.65 → Wage elasticity</td>
</tr>
<tr>
<td>wage level parameter</td>
<td>$\chi_w$</td>
<td>0.80 → Wage to productivity ratio</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Goods market:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>goods exit rate</td>
<td>$\tau$</td>
<td>0.01 → Broda and Weinstein (2010)</td>
</tr>
<tr>
<td>matching curvature</td>
<td>$\nu_G$</td>
<td>1.40 → Goods market transition rate</td>
</tr>
<tr>
<td>cost function level parameter</td>
<td>$\chi_\sigma$</td>
<td>0.5 → American Time Use Survey</td>
</tr>
<tr>
<td>cost function elasticity</td>
<td>$\eta_\sigma$</td>
<td>2 → Quadratic cost</td>
</tr>
<tr>
<td>consumer bargaining weight</td>
<td>$\delta$</td>
<td>0.30 → Share of expenditure on essential good</td>
</tr>
<tr>
<td>marginal utility of $c_1$</td>
<td>$\Phi$</td>
<td>1.15 → Price Markup</td>
</tr>
<tr>
<td>production cost</td>
<td>$\Omega$</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Credit market:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>separation rate</td>
<td>$s^C$</td>
<td>0.01/3 → Bernanke et al (1999)</td>
</tr>
<tr>
<td>bank bargaining weight</td>
<td>$\beta$</td>
<td>0.62 → Petrosky-Nadeau and Wasmer (2013)</td>
</tr>
<tr>
<td>search costs</td>
<td>$\kappa_B = \kappa_I$</td>
<td>0.1 → Petrosky-Nadeau and Wasmer (2013)</td>
</tr>
<tr>
<td>risk-free rate</td>
<td>$r$</td>
<td>0.01/3 → 3 month U.S. T-bill</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Technology</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>persistence parameter</td>
<td>$\rho_x$</td>
<td>0.95^{1/3} → B.L.S labor productivity</td>
</tr>
<tr>
<td>standard deviation</td>
<td>$\sigma_x$</td>
<td>0.007 → B.L.S. labor productivity</td>
</tr>
</tbody>
</table>
Government and debt

- Fiscal policies with some persistence known to economic agents and denote by $\rho_G$ the first order auto-correlation of the policy
- For a given impulse policy $G_{t_0}$ at time $t_0$, we denote by $G_{t_0}^{cum}(T)$ the cumulated spending at horizon $t$, and

\[
G_{t_0}^{cum}(t) = \sum_{k=0}^{t} G_{t_0+k} = \sum_{k=0}^{t} \rho_G^k G_{t_0}
\]

\[
G_{t_0}^{cum}(\infty) = \sum_{k=0}^{\infty} \rho_G^k G_{t_0} = \frac{G_{t_0}}{1 - \rho_G}
\]

- Debt repaid at a fixed interest rate $r_G$, and is fully repaid in the long-run - quite at a slow pace though
- Short-run ($T = 0$) and the longer-run multipliers:

\[
M_G(T) = \frac{\sum_{k=0}^{T} \Delta^{cum} Y_{t_0+k}}{\sum_{k=0}^{T} G_{t_0+k}}
\]
Exercise: fiscal stimulus (blue) or consolidation (red)
Stimulus: debt increases then decreases slowly
GDP reacts quite fast
Fiscal shock affects forward value of goods surplus
Firms advertise more (or less in consolidation)
Same for consumer search effort
Some procyclicality of wages
Employment reacts more slowly - persistence
Fiscal multiplier quite high: 1.4
Summary: compare a productivity shock and a fiscal stimulus

- **Sequence:**

  1. Positive fiscal stimulus $\Rightarrow$ consumers consume more the quasi-numeraire, whether matched or unmatched

  2. At a constant price, more total surplus from consumption of the search good because of decreasing marginal utility (sacrifice of buying search goods is lower after the fiscal expansion)

  3. Hence higher price, **more firms advertising, more effort to enter the market**

  4. **Consumers meet search goods more frequently**

    4.1 Hence **increase in consumer search effort**

  5. **Amplification, hump-shape and the shock is more persistent.**
Technological shock: the same except for prices

Figure: Propagation of a Positive Technology Shock
Summary: compare a productivity shock and a fiscal stimulus

- Sequence for a positive fiscal stimulus

1. Positive surplus shock ⇒ more firms enter
2. More competition to sell goods (both goods market tightness and price *decline*)
3. Overall effect still positive for firms, BUT in addition...
4. .. consumers get higher income (only after the two matching lags) and also meet search goods more frequently
   4.1 Hence strong increase in consumer search effort
5. Amplification, hump-shape and the shock is more persistent.
Sensitivity: specification of goods market frictions
We explore three additional variants.

1. Baseline: search effort by consumers

\[ \sigma'(e_{c,i,t}) = \frac{\tilde{\lambda}_t/\bar{e}_{c,t}}{1 + r} \mathbb{E}_t [(D_{1,t+1} - D_{0,t+1})] \]

2. Firm’s advertising effort

\[ \sigma'_A(e_{A,t}) = \frac{\lambda_t/\bar{e}_{A,t}}{1 + r} \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}] (1 - s^L) (1 - s^c) \]

3. Two sided effort: leads to a strategic complementarity

\[ \frac{\bar{e}_{A,t} \sigma'_A(e_{A,t})}{\bar{e}_{c,t} \sigma'_c(e_{c,t})} = \left( \frac{1 - \delta}{\delta} \right) (1 - s^L) (1 - s^c) \xi_t \]

\[ \text{e.g. } = \left( \frac{\bar{e}_{A,t}}{\bar{e}_{c,t}} \right)^2 \text{ if quadratic costs} \]

4. Constant consumer effort
**Sensitivity: specification of goods market frictions**

**Table:** Labor market second moments: Extensions

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th>Autocorr. $\Delta \theta_t$</th>
<th>$Corr(U, V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U$</td>
<td>$V$</td>
<td>$\theta$</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>0.13</td>
<td>0.14</td>
<td>0.27</td>
</tr>
<tr>
<td>Variant 1. Baseline</td>
<td>0.09</td>
<td>0.16</td>
<td>0.24</td>
</tr>
<tr>
<td>Variant 2. Firm effort</td>
<td>0.08</td>
<td>0.24</td>
<td>0.28</td>
</tr>
<tr>
<td>Variant 3. Two sided effort</td>
<td>0.11</td>
<td>0.22</td>
<td>0.27</td>
</tr>
<tr>
<td>Variant 4. Constant $e_c$</td>
<td>0.07</td>
<td>0.21</td>
<td>0.23</td>
</tr>
</tbody>
</table>
Sensitivity: specification of wages

Table: Business Cycle Moments - Sensitivity to the Specification of Wages

<table>
<thead>
<tr>
<th></th>
<th>Baseline wage setting</th>
<th>Nash bargained wages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \zeta_{w,p} = 0.85 ) (instead of 0.75)</td>
<td></td>
</tr>
<tr>
<td>LMG frictions</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>Vacancies</td>
<td>10.51</td>
<td>0.98</td>
</tr>
<tr>
<td>Unemployment</td>
<td>4.66</td>
<td>-0.44</td>
</tr>
<tr>
<td>Labor tightness</td>
<td>13.74</td>
<td>0.90</td>
</tr>
<tr>
<td>Wage</td>
<td>0.95</td>
<td>0.93</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Notes: H.-P. filtered (a) sd relative to GDP; (b) contemp. correlation with GDP.
Sensitivity: other parameters

- Wage Elasticity

![Graph showing Sensitivity to Wage Elasticity]

- Good market matching function, elasticity

![Graph showing Sensitivity to Good Market Matching Function]
Sensitivity: other parameters

- Consumer search cost elasticity

![Graph showing the relationship between labor market tightness (θ) and the deviation from steady state over quarters. The graph compares baseline response with increased and lower search cost elasticities.]
1. Credit and goods market imperfections amplify the response of labor market tightness to productivity
2. Only goods market imperfections provide persistence
3. Results robust to changes in goods market specification.
Conclusion

This work: we have a simple framework to deal with aggregate fiscal stimulus or consolidation

- Fiscal multipliers requires a concave utility
- Real effects you expect: increases in output and employment; but you can quantify them.
- E.g.: are fiscal multipliers countercyclical? Next talk when simulations will be ready
Bargained Wage

The wage rule in this case is

\[ w_t = \begin{cases} \argmax (S_{g,t} - S_{l,t})^{1-\alpha} (W_{g,t} - U_t)^\alpha & \text{in stage } g \\ \argmax (S_{\pi,t} - S_{l,t})^{1-\alpha} (W_{\pi,t} - U_t)^\alpha & \text{in stage } \pi, \end{cases} \]

where \( W_g \) and \( W_\pi \) are the asset values of employment to a worker, and \( U \) is the value of unemployment. The resulting wage rules are

\[ w_t = \begin{cases} \alpha \theta_t \left( \gamma + \frac{r + s^c}{1 + r} K \right) - \alpha \left( 1 + s^L \frac{1 - s^c}{1 + r} \right) K & \text{in stage } g \\ + (1 - \alpha) b \end{cases} \]

\[ w_t = \begin{cases} \alpha \left( \mathcal{P}_t x_t - \Omega - \left[ 1 + s^L \frac{1 - s^c}{1 + r} \right] K \right) & \text{in stage } \pi. \\ + \alpha \theta_t \left( \gamma + \frac{r + s^c}{1 + r} K \right) + (1 - \alpha) b \end{cases} \]
Average hours per day men and women spent in various activities

NOTE: Data include all noninstitutionalized persons age 15 and over. Data include all days of the week and are annual averages for 2008. Travel related to these activities is not included in these estimates.

SOURCE: Bureau of Labor Statistics
Model Equations

\[ D_{0,t} = U(0, c_{0,t}) - \sigma(e_{i,t}) + \frac{1}{1 + r} \mathbb{E}_t \left[ \frac{e_{i,t} \tilde{\lambda}_t}{\tilde{e}_t} D_{1,t+1} + \left( 1 - \frac{e_{i,t} \tilde{\lambda}_t}{\tilde{e}_t} \right) D_{0,t+1} \right] \]

\[ D_{1,t} = U(c_{1,t}, c_{0,t}) + \left( \frac{1 - s^c}{1 + r} \right) \mathbb{E}_t (1 - s^L) \left[ \tau D_{0,t+1} + (1 - \tau) D_{1,t+1} \right] + \frac{s^c + (1 - s^c) s^L}{1 + r} \mathbb{E}_t D_{0,t+1}. \]

\[ S_{c,t} = 0 \Leftrightarrow \frac{\kappa_B}{\hat{p}_t} + \frac{\kappa_I}{p_t} = S_{l,t} \]

\[ S_{l,t} = -\gamma + \frac{1 - s^c}{1 + r} \mathbb{E}_t \left[ q_t S_{g,t+1} + (1 - q_t) S_{l,t+1} \right] \]

\[ S_{g,t} = -\omega_t + \frac{1 - s^c}{1 + r} \mathbb{E}_t \left[ (1 - s^L) \left[ \lambda_t S_{\pi,t+1} + (1 - \lambda_t) S_{g,t+1} \right] + s^L S_{l,t} \right] \]

\[ S_{\pi,t} = \mathcal{P}_t x_t - \omega_t - \Omega + \frac{1 - s^c}{1 + r} \mathbb{E}_t \left[ (1 - s^L) \left[ (1 - \tau) S_{\pi,t+1} + \tau S_{g,t+1} \right] \right] \]
Model Equations

\( \bar{e}_t \sigma^' (e^*_t) = \frac{\tilde{\lambda}_t}{1 + r} E_t \left[ (D_{1,t+1} - D_{0,t+1}) \right]. \) \hspace{1cm} (20)

\( (1 - \delta) (D_{1,t} - D_{0,t}) = \delta (S_{\pi,t} - S_{g,t}), \) \hspace{1cm} (21)

\( (1 - \beta) B_{l,t} = \beta J_{l,t}, \) \hspace{1cm} (22)

\( w_t = \begin{cases} 
\alpha \theta_t \left( \gamma + \frac{r+s^c}{1+r} K \right) - \alpha \left( 1 + s^L \frac{1-s^c}{1+r} \right) K + (1 - \alpha) b \\
\alpha \left( \mathcal{P}_t x_t - \Omega - \left[ 1 + s^L \frac{1-s^c}{1+r} \right] K \right) + \alpha \theta_t \left( \gamma + \frac{r+s^c}{1+r} K \right) + (1 - \alpha) b
\end{cases} \)
Model Equations

\begin{align*}
    \mathcal{C}_{0,t+1} &= (1 - \tilde{\lambda}_t) \mathcal{C}_{0,t} + \left[ s^c + (1 - s^c) \left( s^L + (1 - s^L) \tau \right) \right] \mathcal{C}_{0,t+1} \\
    \mathcal{C}_{1,t+1} &= (1 - s^c) (1 - s^L) (1 - \tau) \mathcal{C}_{1,t} + \tilde{\lambda}_t \mathcal{C}_{0,t} \\
    \mathcal{N}_{g,t+1} &= (1 - s^c) (1 - s^L) \left[ (1 - \lambda_t) \mathcal{N}_{g,t} + \tau \mathcal{N}_{\pi,t} \right] + q(\theta_t) \mathcal{N}_{g,t} \\
    \mathcal{N}_{\pi,t+1} &= (1 - s^c) (1 - s^L) \left[ (1 - \tau) \mathcal{N}_{\pi,t} + \lambda_t \mathcal{N}_{g,t} \right]. \\
    u_{t+1} &= \left[ s^c + (1 - s^c) s^L \right] (1 - u_t) + (1 - f(\theta_t)) u_t \\
    1 - u_t &= \mathcal{N}_{g,t} + \mathcal{N}_{\pi,t}.
\end{align*}
Broda and Weinstein (2010) carefully documented the magnitude of flows of entry and exit of goods in a typical household’s consumption basket.

- Unique data set of 700,000 products with bar codes purchased by 55,000 households (40% of all expenditures on goods in the CPI).
- They find large flows of entries and exits of "products" in a typical consumer’s consumption basket, actually four times more than is found in labor markets, and that a large share of product turnover happens within firms.

Over their 9 year sample period, 1994–2003, the product entry rate, defined as the number of new product codes divided by the stock, is 0.78 quarterly. The product exit rate, defined as the number of disappearing product codes over the stock, is 0.72, and 0.24 when weighted by expenditure share.

Further, net product creation is strongly procyclical and primarily driven by creation rather than destruction, which is weakly countercyclical.
Credit market frictions

Search on credit markets:

- Creditors ($B_C$) and investment projects ($N_c$) meet to form a firm
- Search costs: creditors $\kappa_B$; investment projects $\kappa_I$
- Measure of credit market tightness: $\phi_t = \frac{N_{c,t}}{B_{c,t}}$

Matching on credit markets:

$$\frac{M_C(B_{c,t}, N_{c,t})}{N_{c,t}} = p(\phi_t) \text{ with } p'(\phi_t) < 0.$$
Search on credit markets

Value of search on credit markets with free entry:

\[ E_{c,t} = -\kappa_I + p(\phi_t)E_{l,t} = 0 \] (free-entry)
\[ B_{c,t} = -\kappa_B + \phi_t p(\phi_t)B_{l,t} = 0 \] (free-entry)

With Nash bargaining, \((1 - \beta)B_{l,t} = \beta E_{l,t}\), implies

\[ \phi^* = \frac{\kappa_B}{\kappa_I} \frac{1 - \beta}{\beta} \forall t \]

Total transaction costs in stage \(c\) summarized as

\[ K(\phi^*) \equiv \frac{\kappa_I}{p(\phi^*)} + \frac{\kappa_B}{\phi^* p(\phi^*)} \]