# On the incidence of a financial transactions tax in a model with fire sales

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**Abstract**

This paper studies the impact of a financial transactions tax on a financial market where financial institutions trade with each other. Assets are marked to the market and financial institutions with negative equity are forced out of business. There are two main results: First, if all banks have enough liquidity so that they can honor their short-term obligations, a financial transactions tax is entirely neutral. Second, in a model with correlated investment risk and short-term financing of banks, a financial transactions tax contributes to financial distress and undoes other policy measures that are used to stabilize financial markets.

*Keywords:* Financial transactions tax, financial stability, financial markets, cash-in-the-market-pricing, marking-to-market.

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# 1 Introduction

The 2008/2009 financial crisis has generated a demand for new taxes to be raised in the financial sector with the objective to compensate tax payers for the costs of the financial crisis.[1] Among the conceivable tax instruments (which include bank levies, that is, insurance premia for bailout subsidies, and a financial activities tax, a tax on income generated in the financial sector), one has since received particular interest of policy makers and the public at large, namely a financial transactions tax.[2]

This paper studies the incidence of a financial transactions tax using a particular model of a financial market. In this model, the purpose of financial markets is to facilitate trade between banks with excess liquidity and banks with deficient liquidity. The freeze of interbank markets that was experienced after the collapse of Lehman brothers in September 2008, and the repercussions that this generated for the real economy, suggest that this approach is appropriate if one is interested in the implications of a financial transactions tax for financial stability. The paper also analyzes the interplay of a financial transactions tax with other policy measures that have been taken with the aim to stabilize financial markets and which, at the time of this writing, still seem to be needed. These measures include liquidity support by central banks (e.g. the opening of the FEDs discount window for investment banks in the US), or government purchases of assets from troubled financial institutions (either in the form of the TARP program in the US, or via the installation of government-backed bad banks in many European countries). The main result of the paper is that, from the viewpoint of financial stability, a financial transactions tax has undesirable consequences: It contributes to financial distress and it undermines the effectiveness of the other policy measures that have been taken in order to stabilize financial markets.

The formal analysis is based on a model that borrows essential features from the work of Allen and Gale (1994, 2004a,b). There are three periods, $T \in \{0, 1, 2\}$, and a large number of banks. In the initial period, each bank obtains funds from risk-averse debt-holders and risk-neutral providers of equity. A bank seeks to maximize its expected return on equity and has to respect a participation constraint for debt-holders. In the initial period

[1] For instance, *"At their September 2009 Pittsburgh Summit, G20 Leaders requested the International Monetary Fund to prepare a report on how the financial sector could make a fair and substantial contribution to meeting the costs associated with government interventions to repair it (IMF, 2010)"*.

[2] In September 2011, the European Commission proposed a EU-wide financial transactions tax (European Commission, 2011). Several European governments are also considering its introduction at the national level, should an EU-wide tax prove to be politically infeasible. In the US, various members of the democratic party, including its leader in the House of Representatives, have expressed their sympathy for a financial transactions tax, whereas government officials hold reservations against it. In the mid-eighties, the introduction of a financial transactions tax was seriously considered in the US, see Stiglitz (1989) and Summers and Summers (1989).

it chooses a debt structure, that is, promises to debt-holders that are due in $T = 1$ and $T = 2$, respectively, and it decides how to invest its funds. It can either invest in projects that mature in $T = 1$ or in projects that mature in $T = 2$. Both investments are risky. It becomes known in $T = 1$ whether investments are successful or fail. There is both aggregate and idiosyncratic uncertainty: The performance of the investments of an individual bank, as well as the cross-section distribution of performances are ex ante unknown.

A long-term investment cannot be liquidated, but a claim on the cash flow that it generates, henceforth simply an *asset*, can be sold in $T = 1$ on a financial (spot) market. The motive for trade is that banks differ in their liquidity needs. After the state of the world has been revealed in $T = 1$, banks with a large number of successful short-run investments have more liquidity/cash than they need in order to honor their current obligations, and are therefore willing to buy other banks' assets at an appropriate price. Banks with only a small number of successful short-run investments, henceforth referred to as fire-sellers, do not have enough liquidity and therefore need to sell assets. Moreover, there is cash-in-the-market-pricing, so that the price of an asset may fall below its "fundamental value", the price that would result if there were no limits to arbitrage. This situation arises if the demand on the financial market is restricted because only few banks have more cash than they currently need.

We model financial distress via a bankruptcy regime. A bank is bankrupt, if, at the given financial market price, the value of its liabilities exceeds the value of its assets.[3] In case of bankruptcy a bank is liquidated and its assets are sold on the financial market. In our basic model, we do not consider bank-bailouts, as in Acharya and Yorulmazer (2008). Instead we assume that, in case of bankruptcy, a bank's debt-holders are bailed out by the government, and that equity is wiped out. As an extension, we also consider the case in which the government purchases the assets of distressed financial institutions.

As any indirect tax, a financial transactions tax drives a wedge between the price that buyers pay and the price that sellers receive. A key step in the analysis is to understand how these prices change if a financial transactions tax is introduced. This makes it possible to analyze, as a second step, how a transactions tax affects the number of distressed financial institutions, government revenue, or the extent of maturity mismatch in the financial system.

An understanding of how financial market prices respond to taxation is complicated by the fact that there is generally not a unique equilibrium.[4] Still, we can show that an increase of the tax rate typically lowers the price that buyers get and therefore increases

[3] Whether a bank is bankrupt or not is determined by marking its asset to the market, as in Allen and Carletti (2008) and Cifuentes et al. (2005).

[4] The observation that there may be multiple equilibria in a model that gives rise to cash-in-the-market-pricing has previously been made by Allen and Gale (2004a) and Cifuentes et al. (2005).

the financial distress among banks that have to engage in fire sales. However, there is no robust finding on how the tax affects banks that experience losses on their long-run investments and therefore need to find cheap assets if they want to be able to honor their obligations to debt-holders in $T = 2$. We refer to such banks as *fire-buyers*. We can show that distress among fire-buyers may go down if a tax is introduced. An overall assessment of what the tax means for financial stability, then has to weight a reduced number of bankrupt fire-buyers against an increased number of bankrupt fire-sellers.

To be able to offer more definite conclusions on the impact of a financial transactions tax, we then turn to interesting special cases of the general setup. As a first benchmark, we study an economy that has no fire-selling banks, i.e. we assume that – possibly as a result of regulation – in $T = 1$ all banks have enough liquidity to honor their current obligations, but some experience problems with their long term investments. For such an economy we can show that a financial transactions tax has no impact at all on the financial market equilibrium. The tax only reduces the liquidation values for the failed banks' debt-holders, but since they are assumed to be bailed-out by the government this makes no difference for them. It also makes no difference for the government. The increase in tax revenue is exactly matched by the increase of the bail-out payments for debtors.

As our main application of interest, we then consider an economy with the following features: There is idiosyncratic liquidity risk, so that some banks have more liquidity than they need in $T = 1$ and others do not have enough of it. In addition, there is aggregate risk with respect to the performance of long-term investments. This aggregate risk affects all banks in the same way, which is meant to be descriptive of the 2008/ 2009 financial crisis, in which many banks were exposed in similar ways to the performance of subprime mortgages. Finally, we assume that banks have only little long-term debt. Under these assumptions, we obtain clear-cut results on the impact of a financial transactions tax: the price that sellers get falls, the price that buyers pay does not go up, and the number of bankruptcies rises. Moreover, these changes need not be small. If there are multiple equilibria, then the introduction of the tax may eliminate the "good" one that has only few bankruptcies and induce a jump into the "bad" one with a drastic fall in prices and a drastic increase in the number of bankruptcies. We also study the impact of various policies (liquidity support, government purchases of assets, solvency regulation) in this model and show that they would indeed have a stabilizing impact. We then show that the stabilizing impact of these policy measures may be neutralized by the introduction of a financial transactions tax.

All these observations lend themselves to the following main conclusion: As long as many financial institutions are on the brink of bankruptcy, the introduction of a financial transactions tax is likely to make things worse. If, by contrast, the financial system is by and large healthy, a financial transactions tax does not cause major problems. The formal analysis deliberately abstracts from issues that are emphasized by the proponents of a financial transactions tax, namely that it would discourage socially wasteful activities such as speculation or high-frequency trading. What the paper shows is that these potential benefits of a financial transactions tax have to be traded off against the costs in terms of financial stability – unless regulatory measures ensure that financial stability is no longer endangered.

**Related Literature.** The theoretical literature that looks at how financial institutions and financial markets are affected by taxation is surprisingly thin. Lockwood (2010) studies the optimal taxation of financial services in a dynamic Ramsey model. Philippon (2010) characterizes the optimal size of the financial sector relative to the "real" economy in an endogenous growth model. In his model, taxes can be used to improve upon the relative sizes of these sectors that would result under laissez-faire. There is also research on the treatment of financial services under a value added tax system, see Huizinga (2002), Auerbach and Gordon (2002), or Genser and Winker (1997). Some recent papers study Pigouvian taxation as a response to systemic risk-taking (Acharya e.a., 2010), maturity mismatch (Perotti and Suarez, 2011), or excessive leverage (Keen, 2011) . What all these articles have in common, however, is that they model the relation between a financial intermediary and agents who demand financial services, or they model the behavior of a representative bank. They do not model financial markets, i.e. markets where financial institutions trade financial products with each other.

This paper draws on the literature on fire sales on financial markets, and in particular on the models of Allen and Gale (1994, 2004a,b). As survey of the literature on fire sales is provided by Shleifer and Vishny (2011). Caballero and Simsek (2010) look at a setup similar to this paper. Their contribution is to model how uncertainty about the interconnectedness of financial institutions affects financial stability.

Overviews of the current practice in the taxation of the financial sector, and of the empirical and theoretical findings on financial transactions taxes are provided by Matheson (2011) and by Hemmelgarn and Nicodeme (2012). The view that financial transactions taxes are desirable as a way of discouraging speculation is, for instance, articulated in Keynes (1936),Tobin (1978), Stiglitz (1989) or Summers and Summers (1989). Recently, Brunnermeier et al. (2012) have provided a welfare analysis of speculative trade.

The remainder is organized as follows: Section 2 lays out a general model. In Section 3, we discuss equilibrium existence and comparative statics for this general framework. Section 4 contains the analysis of an economy without fire sales. Section 5 deals with the analysis of an economy with idiosyncratic liquidity risk and aggregate long-term investment risk. The last section concludes. All proofs are in an Appendix.

## 2 The Model

There is a continuum of banks of measure 1. The set of banks is denoted by $I$, with typical element $i$. There are three dates, $T \in \{0, 1, 2\}$. Bank $i$ is endowed with equity $e_i$ and debt $d_i$. It promises its debtors payments of $x_{i1}$ in period 1 and of $x_{i2}$ in period 2. We can think of debtors either as depositors as in a model of liquidity insurance, or as lenders in the commercial paper or repo market.[5] Holders of equity receive the residual value of the bank's assets (i.e. what is left after the payments promised to depositors and other debtors are made) in case there is no failure. Hence, bank $i$ has funds of $a_i := d_i + e_i$ to invest in $T = 0$. Bank $i$ lends out or invests an amount $y_{i1}^0$ with a short term perspective. These investments are risky. They fail with a certain probability, in which case they return 0 in $T = 1$. If they do not fail, they return $r_1$ in $T = 1$. For ease of notation, we let $r_1 = 1$ in the following. In addition, bank $i$ lends out an amount of $y_{i2}^0 = a_i - y_{i1}^0$ with a long run perspective. We think of these investments as risky loans to entrepreneurs, home-owners etc. A risky loan fails with a certain probability, in which case it returns 0. If it does not fail, it returns $r_2$ in $T = 2$. Whether or not a loan performs becomes known in $T = 1$. For now, we treat $e_i, d_i, x_{i1}$, $x_{i2}$, $y_{i1}^0$ and $y_{i2}^0$ as exogenous.

Long-run investments can not be liquidated; that is, it is not possible to withdraw in $T = 1$ the funds invested in $T = 0$. However, it is possible to trade claims on the returns of these investments on a financial market in $T = 1$. We henceforth refer to such claims simply as assets. The financial market equilibrium is described in more detail below. We also assume that, in $T = 1$, banks have access to a riskless storage technology in order to transfer cash available in $T = 1$ into cash available in $T = 2$.

We assume that a bank's objective is to maximize the expected return on equity, which we denote by $E[\pi_i^2]$. We assume, for simplicity, that the holders of equity require a payment only in $T = 2$, and $\pi_i^2$ is the payment per unit of equity in $T = 2$. As of $T = 0$, $\pi_i^2$ is a random quantity. It depends both on idiosyncratic risk (such as the fraction of successful investments of bank $i$) and aggregate risk (such as the price on the financial market), which is realized in $T = 1$.

**State of the economy.** In $T = 1$, each bank $i$ is hit by a shock. Formally, in $T = 1$, bank $i$ is characterized by a vector $\sigma_i = (y_{i1}, y_{i2})$, where $y_{i1} \leq y_{i1}^0$ is the volume of performing short-run investments, and $y_{i2} \leq y_{i2}^0$ is the volume of performing long-run investments, i.e., of investments that will yield a return in $T = 2$. From the perspective of the initial period, $\sigma_i$ is a random quantity; i.e., as of $T = 0$, bank $i$ does not yet know how many of its long- and short-run investments will perform. We assume, for simplicity,

[5] Consider a model with depositors who demand liquidity insurance and suppose that depositors are promised $c_{i1}$ if they withdraw early and $c_{i2}$ if they withdraw late. Then $x_{i1} = s_i c_{i1}$ and $x_{i2} = (1 - s_i)c_{i2}$, where $s_i$ is the fraction of early consumers among the depositors of bank $i$.

that the banks' liabilities $x_{i1}$ and $x_{i2}$ are not random.[6]

In the following we refer to the collection $\sigma = (\sigma_i)_{i \in I}$ as the state of the economy. There is a financial market where funds available in $T = 1$ can be traded against the long-run investments. That is, while the investments made in the initial period cannot be liquidated in $T = 1$, they can be sold on a financial market, for instance, to generate the liquidity that may be needed in order to honor promises which are due in $T = 1$. We view the price on the financial market as a function of the state of the economy. Moreover, we consider the possibility that trades on the financial market are subject to an (ad valorem) transactions tax $t$, which drives a wedge between the revenue from selling an asset, $p(\sigma)$, and the cost of buying an asset $q(\sigma)$, so that

$$q(\sigma) = (1 + t)p(\sigma) \ .$$

For most of the analysis we treat the state of the economy as fixed. To save on notation, we will suppress the dependence of the prices $p$ and $q$ on the state of the economy whenever this creates no confusion.

**Bankruptcies.** We assume that all economic agents have access to a storage technology in order to transfer resources available in $T = 1$ into resources available in $T = 2$. If instead of storing the resources available in $T = 1$, a bank buys assets on the financial market, it receives $r_2/q$ per unit invested. Hence, there will be demand on the financial market only if $r_2 \geq q$.

We assume that a bank goes bankrupt if, at the given price on the financial market, the value of its liabilities is larger than the value of its assets. To clarify the conditions under which this is the case, the following terminology is useful: In $T = 1$, bank $i$ has an endowment consisting of cash $y_{i1}$ and assets $y_{i2}$. Since each asset generates $r_2$ units of resources available in $T = 2$, we can as well think of bank $i$ as having an endowment of $y_{i1}$ resources available in $T = 1$ and of $r_2 y_{i2}$ resources available in $T = 2$. It will prove convenient to also define the net endowments $\theta_{i1} = y_{i1} - x_{i1}$ and $\theta_{i2} = r_2 y_{i2} - x_{i2}$, respectively. If there was no financial transactions tax so that $q = p$, it would be straightforward to determine whether or not a bank is bankrupt: Since $p$ is the price of a performing loan, and a performing loan generates $r_2$ units of consumption in $T = 2$, it has to be the case that, as of $T = 1$, the price of a unit of consumption available in $T = 2$ equals $p/r_2$. Hence, bank $i$ goes bankrupt if

$$\theta_{i1} + \frac{p}{r_2}\theta_{i2} < 0 \ ,$$

and stays in business otherwise. With a financial transactions tax, however, there are two prices, one which is relevant if assets are sold on the financial market, and one which is

[6] In a model of liquidity insurance, this is the case if $s_i$ is a known parameter, so that it is known a priori how many early consumers will show up in $T = 1$.

relevant if assets are bought. Which of these prices is relevant for the survival of bank $i$ depends on its net endowment.

*Safe and failed banks.* If $\theta_{i1} \geq 0$ and $\theta_{i2} \geq 0$, bank $i$ is safe in that its ability to honor its promises does not depend on market conditions. If $\theta_{i1} \leq 0$ and $\theta_{i2} \leq 0$, with a least one inequality being strict, bank $i$ fails irrespective of market conditions. In the following we denote the set of safe and failed banks by $I^*$ and $I^\dagger$, respectively.

*Fire-buyers.* Suppose that $\theta_{i1} > 0$ and $\theta_{i2} < 0$, so that bank $i$ has more resources than it needs in $T = 1$, but that it does not have enough resources in $T = 2$. Therefore, it will have to use its excess liquidity in $T = 1$ to buy assets on the financial market and the relevant price is $q$. The bank will therefore survive provided that

$$\theta_{i1} + \frac{q}{r_2}\theta_{i2} \geq 0 \; .$$

Note, in particular, that, for a bank that needs to buy assets, lower prices on the financial market make it easier to survive.[7] In the following, we denote by

$$I^*_{buy}(q) = \left\{ i \in I \mid \theta_{i1} > 0, \theta_{i2} < 0, \frac{q}{r_2} \leq -\frac{\theta_{i1}}{\theta_{i2}} \right\}$$

the set of banks that have to buy assets on the financial market and manage to survive at a price of $q$. Analogously, we denote by $I^\dagger_{buy}(q)$ the complementary set of banks that fail.

*Fire-sellers.* Now suppose instead that $\theta_{i1} < 0$ and $\theta_{i2} > 0$. This bank has a liquidity problem in $T = 1$ and therefore needs to sell assets on the financial market. The relevant price is now $p$, and the bank survives provided that

$$\theta_{i1} + \frac{p}{r_2}\theta_{i2} \geq 0 \; .$$

Note that, for a bank that needs to sell assets, lower prices on the financial market make it more difficult to survive. We denote by

$$I^*_{sell}(p) = \left\{ i \in I \mid \theta_{i1} < 0, \theta_{i2} > 0, \frac{p}{r_2} \geq -\frac{\theta_{i1}}{\theta_{i2}} \right\}$$

the set of banks that have to sell assets on the financial market and manage to survive at a price of $p$. We denote by $I^\dagger_{sell}(p)$ the set of fire-selling banks that fail.

A bank that is bankrupt gets liquidated. That is, its assets are sold on the financial market. The bank's debtors then receive the proceeds from these sales plus the cash that the bank has available in $T = 1$. In case of bankruptcy, equity holders do not receive anything.

[7] The bank could also use the storage technology, in which case it would be able to honor its promises provided that $\theta_{i1} + \theta_{i2} \geq 0$. However, since $\frac{q}{r_2} \leq 1$, it is more difficult to survive using the storage technology.

**Behavior of non-bankrupt banks.** Banks that do not go bankrupt can engage in trade on the financial market. Denote by $z_i = (z_{i1}, z_{i2})$ a portfolio consisting of cash available in $T = 1$ and holdings of the asset for bank $i$. Bank $i$ solves the following problem: Choose $z_i$ in order to maximize the return to equity

$$\pi_i^2 = \frac{r_2 z_{i2} - x_{i2} + z_{i1} - x_{i1}}{e_i}$$

subject to a budget constraint which requires that the value of the portfolio does not exceed the value of the bank's endowment,

$$z_{i2} - y_{i2} \leq \begin{cases} \frac{y_{i1} - z_{i1}}{q}, & \text{if} \quad z_{i2} \geq y_{i2} \ , \\ \frac{y_{i1} - z_{i1}}{p}, & \text{if} \quad z_{i2} \leq y_{i2} \ , \end{cases}$$

and the requirement that the bank is able to honor its promises, $z_{i1} - x_{i1} \geq 0$ and $r_2 z_{i2} - x_{i2} \geq 0$. It is straightforward to verify that safe banks and fire-buyers buy as many assets as they can if $q < r_2$, and are indifferent between holding cash and assets if $q = r_2$. Their net demand on the financial market is therefore given by

$$z_{i2} - y_{i2} = \begin{cases} \frac{\theta_{i1}}{q}, & \text{if} \quad r_2 > q \ , \\ \in [-y_{i2}, \frac{\theta_{i1}}{q}], & \text{if} \quad r_2 = q \ . \end{cases} \tag{1}$$

A fire-selling bank sells as few assets as possible if $p < r_2$ and is indifferent between holding cash and assets if $p = r_2$. This yields a (negative) net demand of

$$z_{i2} - y_{i2} = \begin{cases} \frac{\theta_{i1}}{p}, & \text{if} \quad r_2 > p \ , \\ \in [-y_{i2}, \frac{\theta_{i1}}{p}], & \text{if} \quad r_2 = p \ . \end{cases} \tag{2}$$

**Financial market equilibrium.** *Demand.* The net demand for assets on the financial market stems from the safe banks and the banks that have to buy assets and manage to survive. Recall that all these banks have more liquidity than they need in $T = 1$, so that $\theta_{i1} > 0$. Given a price of $q \in (0, r_2)$, the demand on the financial market is given by

$$D(q) := \int_{\hat{I}^*(q)} (z_{i2} - y_{i2})\, di = \int_{\hat{I}^*(q)} \frac{\theta_{i1}}{q}\, di$$

where $\hat{I}^*(q) := I^* \cup I^*_{buy}(q)$ is the set of banks that demand assets on the financial market. For $q = r_2$ the demand is bounded from above by $D(r_2)$ and from below by

$$-\int_{\hat{I}^*(q)} y_{i2}\, di \ .$$

Observe that the demand function $D$ is downward-sloping for $q \in (0, r_2)$. If $q$ goes up, this reduces the demand of each bank in $\hat{I}^*(q)$. In addition, the set $I^*_{buy}(q)$ and hence the set $\hat{I}^*(q)$ shrinks, since, for banks that have to buy, a higher price makes it more difficult to survive.

*Supply.* The net supply on the financial market has two sources: First, the assets of all banks that fail are sold. Second, some banks fire-sell to generate liquidity and thereby manage to survive. For these banks we have that $\theta_{i1} < 0$. Therefore, for $p \in (0, r_2)$ the supply on the financial market equals

$$\bar{S}(p) := \int_{\hat{I}^\dagger(p)} y_{i2}\, di - \int_{I^*_{sell}(p)} \frac{\theta_{i1}}{p}\, di ,$$

where $\hat{I}^\dagger(p) := I^\dagger \cup I^\dagger_{buy}(p(1+t)) \cup I^\dagger_{sell}(p)$. In this formula, we view the supply on the financial market as a function of the revenue $p$ that a seller can realize. Since $q = p(1+t)$ we could as well interpret market supply as a function of $q$. It will prove useful to also define

$$S(q) := \bar{S}\left(\frac{q}{1+t}\right) ,$$

a supply function which depends on the price $q$ that buyers have to pay.

For $p = r_2$, $\bar{S}(r_2)$ is only a lower bound on the supply on the financial market, since, at this price, the surviving fire-sellers would be willing to sell all their assets. The corresponding upper bound is

$$\int_{\hat{I}^\dagger(r_2) \cup I^*_{sell}(r_2)} y_{i2}\, di .$$

Without further assumptions, the slope of the supply function cannot be determined. If $p$ goes up, this implies that the set $I^\dagger_{sell}(p)$ shrinks, which tends to reduce the supply, but the set $I^*_{sell}(p)$ expands, which tends to increase it. Moreover, for each bank in $I^*_{sell}(p)$, the number of assets that are put on the market goes down. Finally, $I^\dagger_{buy}(p(1+t))$ increases because it gets more difficult to survive as a buyer. The interplay of all these effects is such that that the slope of the supply function can not be signed a priori.

*Equilibrium.* A price $q \in (0, r_2]$ is an equilibrium price if

$$D(q) = S(q) , \quad \text{or if} \quad D(q) > S(q) \quad \text{and} \quad q = r_2 .$$

Why is $q = r_2$ an equilibrium price even if $D(r_2) > S(r_2)$? At this price, banks on the demand side are willing to hold any number of assets smaller than $D(r_2)$. If $D(r_2) > S(r_2)$ it is therefore possible to arrange trade between buyers and sellers so that, at the given price of $q = r_2$, every non-bankrupt bank gets a profit-maximizing portfolio.

**Proposition 1** *Suppose that $D$ and $S$ are continuous functions on $[0, r_2]$, and that $\hat{I}^*(0) \neq \emptyset$. Then, an equilibrium price exists.*

The Proposition asserts that a financial market equilibrium exists under very weak conditions. All that is needed, apart from continuity, is that demand exceeds supply at the minimal price of $q = 0$. Recall that the net demand of a bank that has excess liquidity in

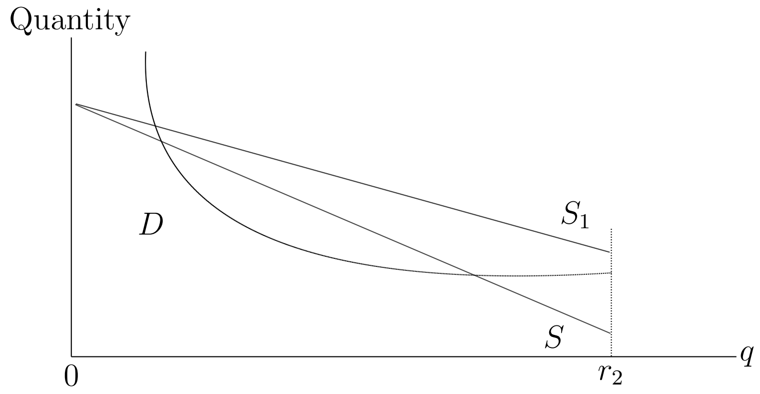


Figure 1: *Multiple equilibria.*

$T = 1$ equals $\frac{\theta_{i1}}{q}$. This grows without limits as $q$ converges to 0. Hence, what is needed for existence is that there is a bank that survives and has more cash than it needs in $T = 1$.

Obviously, an equilibrium exists if the demand and the supply curve intersect over the range of admissible financial market prices $q \in (0, r_2]$. An equilibrium exists even if they do not intersect. If there is no intersection, this means that, for all admissible $q$, demand exceeds supply in the sense that $D(q) > S(q)$. But this implies, in particular that $D(r_2) > S(r_2)$ so that $r_2$ is an equilibrium price.

There may exist multiple equilibria as illustrated in Figure 1. The reason is that a model with fire-sales may give rise to a downward-sloping supply curve: the fire-sellers have to sell a lot if the price is low, and they can afford to sell little if the price is high.[8] The different equilibria cannot be Pareto-ranked. An equilibrium with a low price is preferred by those on the demand side. In fact, $q < r_2$ means that they can buy assets below their net present value and make a speculative profit. By contrast, for banks that have to fire-sell equilibria with a high price are preferred. Higher prices make it more likely that they survive and that their shareholders receive a profit. If the government provides protection to the banks' debtors (via deposit insurance or bail-out guarantees), it will prefer the equilibrium with the lowest number of bankruptcies. However, it is not clear a priori that a higher financial market price is desirable from the government's perspective. This holds only if all banks at the brink of survival are fire-sellers. Fire-buyers need low asset prices in order to survive.

# 3 Impact of a financial transactions tax

We can illustrate the impact of an increased tax $t$ graphically. Consider the supply function $S : q \mapsto S(q)$ with $S(q) = \bar{S}\left(\frac{q}{1+t}\right)$, which gives the supply on the financial

[8] In Section 5 we will look at a more specific version of this model, which shows that the situation in Figure 1 arises under a set of plausible assumptions.

market as a function of the price $q$ that a buyer has to pay on the financial market. Now, if we change the tax from a level of $t$ to a level of $t_1 > t$ this yields a new supply function $S_1(q) = \bar{S}\left(\frac{q}{1+t_1}\right)$. Graphically, we can depict this as a shift of the supply curve to the right: A given supply is now associated with a higher price $q$ for the buyers in order to compensate for the fact that the sellers receive a smaller fraction of the buyer's total spending. For the case of a downward-sloping supply curve, this is illustrated in Figure 1.[9]

Figures 1 and 2 show how an increase of the tax rate $t$ can affect the set of equilibria. In Figure 1 there are initially three equilibria, namely the two intersections of the supply and the demand curve and the equilibrium at $q = r_2$. After a rightward shift of the supply curve, there is only one equilibrium left. Hence, the increase of $t$ implies that we move from a situation with three equilibria to a situation with a unique equilibrium. In Figure 2, by contrast, the structure of the set of equilibria remains unaffected. Before and after the change of the tax rate, there are three equilibria.

Even if the structure of the set of equilibria remains unchanged as in Figure 2, it is not straightforward to determine whether the price on the financial market goes up or down in response to an increase of the financial transactions tax. If we focus on the "high-price equilibrium" or the "equilibrium in the middle", we would conclude that the increase of $t$ drives the financial market price for buyers, $q$, up. The "low-price equilibrium", by contrast, has a lower value of $q$ after an increase of the tax rate. A typical approach is to view the "equilibrium in the middle" as irregular or unstable, and the 'low-price equilibrium" and the "high-price equilibrium" as stable.[10] As the following Proposition shows, the focus on stable equilibria indeed makes it possible to offer one general conclusion:

**Proposition 2** *Suppose that $D$ and $S$ are continuously differentiable and denote by $t_0$ an initial tax rate. Consider an equilibrium that is not irregular. There exists $\epsilon > 0$ so that moving to a higher tax rate $t_1 \in (t_0, t_0 + \epsilon)$ gives rise to a new equilibrium, which is in the neighborhood of the old equilibrium and such that:*

*i) The price that sellers receive goes down: $p_1 < p_0$, where $p_1$ and $p_0$ are the equilibrium prices for sellers before and after the tax change, respectively.*

*ii) For fire-selling banks it is more difficult to survive in the new equilibrium: $p_1 < p_0$ implies that $I^*_{sell}(p_1) \subset I^*_{sell}(p_0)$.*

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[9] Since we have modeled the financial transactions tax as an ad valorem tax, the shift to the right is larger the larger is $q$.

[10] See Mas-Collel e.a. (1995). The reasoning is that excess demand drives prices up, and that excess supply drives prices down. Hence, starting from any non-equilibrium price, we will not approach the "equilibrium in the middle", but either the "low-price equilibrium" or the "high-price equilibrium".

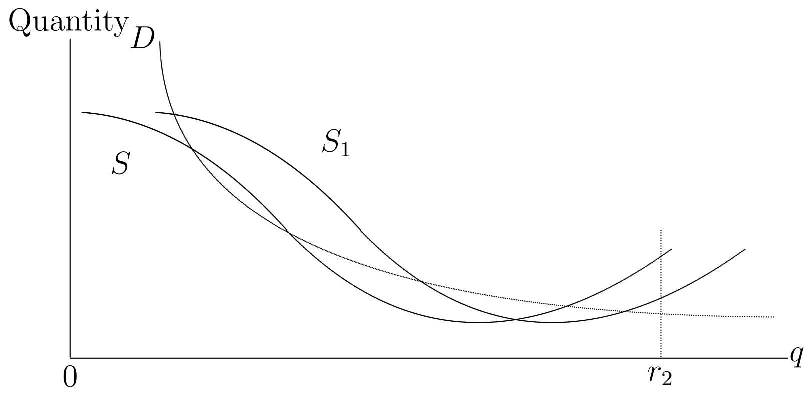


Figure 2: *Comparative static effects of an increase of the financial transactions tax: Regular versus irregular equilibria.*

The Proposition shows that there is a rather general comparative statics result: If we are in a regular equilibrium, then, an increase of the tax rate will depress the revenue $p$ that fire-selling banks realize per asset that they put on the financial market. As a consequence, it gets harder for them to generate the liquidity that is needed in order to survive, and (weakly) more fire-sellers will go bankrupt.

However, even if we are prepared to focus on stable equilibria, it is difficult to come up with more general statements on what happens on financial markets if a financial transactions tax is introduced. This can be illustrated with the help of Figure 2. If we focus on the "low-price equilibrium" in Figure 2, then the implications of a small increase of the tax rate are the following: The equilibrium price $q$ that buyers have to pay goes down. Hence, for fire-buying banks it is easier to survive in the new equilibrium since $q_1 < q_0$ implies that $I^{\dagger}_{buy}(q_0) \subset I^{\dagger}_{buy}(q_1)$. By contrast, if we focus on the "high-price-equilibrium" in Figure 2, we would get comparative statics results which are reminiscent of those from a conventional partial equilibrium analysis: Buyers have to pay more; i.e., $q$ goes up. Sellers get less; i.e., $p$ goes down. The equilibrium quantity goes down. Finally, for all banks – for fire-selling as well as for fire-buying banks – it gets more difficult to survive.

Given this inconclusiveness of the general framework, we will analyze various interesting special cases in the following. In Section 5, we will look at a specific model with correlated long-run investment risk and idiosyncratic liquidity risk, which may be descriptive of the events following the collapse of Lehman brothers in September 2008. Before we turn to this case, we will, as a benchmark, investigate an economy in which fire sales do not arise.

# 4 No fire sales

**Assumption 1** *For all $\sigma$ and all $i$, $\theta_{i1} \geq 0$.*

Under Assumption 1, all banks have enough cash in $T = 1$ to honor their current obligations. To make this assumption appear "natural" consider the following environment: The short-run investment is virtually risk-free, i.e. short-run investments fail with a very small probability. Consequently, Assumption 1 is satisfied if all banks choose to hold enough cash.[11] Alternatively, regulatory measures such as reserve requirements may force banks to hold sufficient liquidity. In this case, we may think of Assumption 1 not as resulting from a choice, but as a constraint on the behavior of banks.

**Proposition 3** *Suppose that Assumption 1 holds and fix some arbitrary state $\sigma$ of the economy. Suppose that $q_0$ and $p_0$ are equilibrium prices for buyers and sellers, respectively, at a tax rate of $t = 0$ and denote by $\hat{I}_0^\dagger$ the set of bankrupt banks in this equilibrium. Then, for any tax rate $t \neq 0$, there is an equilibrium so that:*

*i) The equilibrium price for buyers is given by $q_0$.*

*ii) The equilibrium price for sellers is $\frac{q_0}{1+t}$.*

*iii) The set of bankrupt banks equals $\hat{I}_0^\dagger$.*

The Proposition establishes a neutrality result. An increase of the tax rate does neither affect the price that buyers have to pay on the financial market nor the number of banks that go bankrupt. It only affects the revenue of a seller. Since the price that buyers pay is not affected by taxation, the price that sellers receive must go down if the tax rate goes up. The intuition is as follows: Assumption 1 eliminates fire-sales. Therefore banks that face a risk of bankruptcy are those with $\theta_{i2} < 0$. These banks have to buy assets on the financial market. Otherwise they will not be able to honor their promises to debtors in $T = 2$. Hence, whether these banks survive depends only on the price that buyers face. As a consequence, the equilibrium condition for the buyers' price, $S(q) = D(q)$, becomes independent of the tax rate.

**Profits and Liquidation Value.** In $T = 0$, bank $i$ chooses its investments $y_{i1}^0$, and $y_{i2}^0$ and the promises to debtors. Let $h_{0i} = (y_{i1}^0, y_{i2}^0, x_{i1}, x_{i2})$ be the vector of variables which are determined in the initial period. Denote by $q(\sigma, t)$ and $p(\sigma, t)$ the equilibrium prices

[11] Such a decision may be motivated by the hope to reduce the risk of bankruptcy or by the hope to have cash reserves that make it possible to reap a profit if the financial market price $q$ falls below its "fair value" of $r_2$.

for buyers and seller's respectively, which depend both on the state of the economy $\sigma$ and the tax rate $t$. We can now write bank $i$'s expected profit as

$$\Pi_i^2(h_{0i}, t) = E_\sigma[E[\pi_i^2 \mid \sigma, p(\sigma, t), q(\sigma, t), h_{0i}]] ,$$

where the inner expectation is taken conditional on the state $\sigma$ of the economy, a given tax rate of $t$, and financial market prices of $p(\sigma, t)$ and $q(\sigma, t)$, and the bank's choices in $T = 0$, respectively, and the outer expectation is taken with respect to the random variable $\sigma$.

**Proposition 4** *Under Assumption 1, a bank's expected profit is unaffected by the tax rate $t$. Formally, for any $h_{01}$, and for any pair of tax rates $t$ and $t'$ with $t \neq t'$ we have that*

$$\Pi_i^2(h_{0i}, t) = \Pi_i^2(h_{0i}, t') .$$

The Proposition is a consequence of limited liability and of Proposition 3. In case of bankruptcy, limited liability implies that the holders of equity get a payoff of zero. Otherwise profits depend on the price $q$ which is unaffected by the tax rate. An increase of the tax rate therefore affects only the payoff for the debtors who, in case of bankruptcy, receive the liquidation value $y_{i1} + p(\sigma, t)y_{i2}$ of the bank's asset. By Proposition 3, $p(\sigma, t)$ goes down if the tax rate goes up, which implies that the debtors get less.

**Investment decisions, Maturity structure of debt.** We now turn to the bank's initial choices. The bank chooses $h_{0i}$ in order to maximize $\Pi_i^2(h_{0i}, t)$ subject to a participation constraint for its debtors.

We assume that the bank's debtors are risk averse and, for simplicity, that they are indifferent regarding the timing of the bank's payment. Hence, if bank $i$ does not go bankrupt, the debtors' utility equals $u(x_{i1} + x_{i2})$, where $u$ is a strictly concave utility function. If bank $i$ does go bankrupt, then the debtors receive the liquidation value of the bank's asset, and possibly, some government support $b_i$, a bailout or a payment from an insurance system for financial institutions. In this case their utility equals $u(y_{i1}+py_{i2}+b_i)$. We view the bailout as a function of the bank's initial choices $h_{0i}$, and the state of the economy $\sigma$, and write $b_i = b_i(h_{0i}, \sigma)$. We focus on the case of a complete bailout policy.

**Assumption 2** *For all $h_{0i}$ and for all states $\sigma$ in which bank $i$ goes bankrupt,*

$$b_i(h_{0i}, \sigma) = x_{i1} + x_{i2} - y_{i1} - p(\sigma)y_{i2} .$$

Under Assumption 2, a debtor's expected utility can be written as

$$U_i(h_{0i}) \quad = \quad u(x_{i1} + x_{i2}) .$$

The full bailout assumption implies that a debt-holder's expected utility does neither depend on the state of the economy, nor on the fortune of the financial institution. Assumption 2 can be justified in a couple of ways. For commercial banks the assumption is descriptive because of deposit insurance. For investment banks, the recent experience suggests that a full bailout assumption seems like a reasonable approximation.[12] Finally, we have assumed that debtors' are risk-averse. If we think of the government as a potential provider of insurance to debtors and assume, moreover, that the government is not as risk-averse as debtors, then full bailouts seem to be an optimal arrangement.

Bank $i$'s problem now is to choose $h_{0i} = (y^0_{i1}, y^0_{i2}, x_{i1}, x_{i2})$ in order to maximize $\Pi^2_i(h_{0i}, t)$ subject to the participation constraint for debtors, $u(x_{i1}+x_{i2}) \geq \bar{u}_i$, where $\bar{u}_i$ is a minimal utility level that debtors request in exchange for their willingness to lend to the bank, and the constraint that a bank's total investments is bounded by the funds that are made available by its depositors and its holders of equity $y^0_{i1} + y^0_{i2} = a_i$. We denote by $H^*_{0i}(t)$ the set of solutions to this optimization problem.

**Proposition 5** *Under Assumptions 1 and 2, $H^*_{0i}(t) = H^*_{0i}(t')$, for any pair of tax rates $t$ and $t'$.*

The Proposition follows immediately from two observations: (i) As shown in Proposition 4, the expected profit that is generated by any initial decision $h_{0i}$ does not depend on the tax rate, and (ii) under a full bailout assumption, also the bank's constraints do not depend on the tax rate. Consequently, the set of optimal choices for bank $i$ does not depend on the tax rate.

Propositions 3 and 5 identify conditions under which a financial transactions tax is inconsequential for the behavior of banks or for financial stability. There is als no effect on the government budget: The tax reduces the liquidation value for debtors in case a bank fails and generates some tax revenue for the government. If there is always a full bailout by the government, then the tax induced reduction of the liquidation value has to be compensated for by an increase of the payments to debtors. In the next section we will eliminate Assumptions 1 so that there are indeed banks with liquidity problems. We will see that this changes the analysis in a drastic way.

[12] The failure of Lehman brothers is the prominent exception where debtors were not made whole. But given the drastic consequences that have been attributed to this political decision, it seems unlikely that this will happen again any time soon.

# 5 Fire sales

Assumption 3 - 6 below shift the focus to a financial system with the following features: Banks have primarily short-term debt and, moreover, their long-term performance is correlated. In addition, there is idiosyncratic liquidity risk.

Assumption 3 restricts attention to a symmetric environment. While banks may be heterogeneous ex post – that is, after the shock has hit in $T = 1$ –, they are identical ex ante.

**Assumption 3** *In $T = 0$, all banks are identical. This implies, in particular, that they all have the same structure of liabilities, i.e. there exist numbers $X_1$ and $X_2$ so that $x_{i1} = X_1$ and $x_{i2} = X_2$, for all $i$. Also, they choose the same short-run and long-run investments in $T = 0$. We denote the common investment levels by $Y_1^0$ and $Y_2^0$, respectively.*

**Assumption 4** *There is aggregate risk with respect to the performance of long-run investments. All banks are equally affected by this aggregate risk. Formally, we assume that if bank $i$ invests $y_{i2}^0$ into long-term projects, the volume of successful investments equals $y_{i2} = \sigma y_{i2}^0$, where $\sigma \in [0, 1]$ is an aggregate shock that affects all banks equally.*

All banks engage in the same long-run investment in $T = 2$, and they will all experience the same outcome, which can be good (high values $\sigma$) or bad (low values of $\sigma$). As an example, suppose that all banks have lent to a pool of home-owners. Within this pool, there is a certain rate of defaults. Since all banks have lent to the same pool, the default rate is the same for all banks. Put differently, their long-term lending risks are perfectly correlated. In a symmetric equilibrium where all banks choose the same investments – so that there is a number $Y_2^0$ such that $y_{i2}^0 = Y_2^0$, for all $i$ – this implies, in particular, that they will experience the same performance of their long-run investments in $T = 1$. Hence, there exists a number $Y_2 = \sigma Y_2^0$ so that $y_{i2} = Y_2$, for all $i$.

**Assumption 5** *We assume that $r_2 Y_2 > X_2$, or, equivalently, that $\theta_2 := r_2 Y_2 - X_2 \geq 0$, with probability 1. We also assume that $X_1 > \theta_2$ with probability 1.*

The assumption that $\theta_2 := Y_2 - X_2 > 0$ with probability 1 shifts the focus on fire-sales. (In states with $\theta_2 < 0$ there would be no fire-sellers. Instead there would be fire-buyers.) If banks have a lot of short-term debt and only little long term debt, this means that $X_1$ will be large relative to $X_2$. This situation is typical for investment banks who obtain a lot of their financing on the repo or commercial paper market. A stylized model of this could be to set $X_2 = 0$. In this case, the assumption that $\theta_2 \geq 0$, with probability 1, would follow naturally from the fact that $Y_2$ cannot become negative.

The assumption that $X_1 > \theta_2$ with probability 1 is made for ease of exposition. Economically, this assumption says that short-run liabilities are large relative to the net endowment $\theta_2$ in period 2. As will become clear below, this assumption implies that even at a price of $p = r_2$, i.e. with the most favorable conditions for sellers on the financial market, a bank that experiences a complete failure of its short-run investments, so that $y_{i1} = 0$, will be unable to avoid bankruptcy. Put differently, the assumption ensures that for all $p \in [0, r_2]$, $I^{\dagger}_{sell}(p) \neq \emptyset$.[13]

**Assumption 6** *There is idiosyncratic liquidity risk: Bank i's performing short-run investments are given by $y_{i1} = \beta_i Y_1^0$, where $\beta_i \in [0, 1]$. From the perspective of $T = 0$, $\beta_i$ is a random variable that has a uniform distribution, and is stochastically independent from $Y_2$. Also, with an appeal to the law of large numbers, we assume that the cross-section distribution of $\beta_i$ is a uniform distribution with probability 1, so that there is no aggregate liquidity risk. Finally, we assume that $Y_1^0 > X_1$.*

According to this assumption, the short-run investments gives a random return of $\beta_i$ per unit invested, where the random variable $\beta_i$ captures an idiosyncratic investment risk. The expected return of bank $i$ is $E[\beta_i] = \frac{1}{2}$. Due to the law of large numbers this is also the average return among all banks. The idiosyncratic liquidity risk is what generates a motive for trade on the financial market. Some banks lose a lot on their short-run investments and therefore have to sell some of their assets in order to honor their promises and to avoid bankruptcy. Other banks lose very little on their short-run investments and therefore have money left that they can invest on the financial market. The assumption that $Y_1^0 > X_1$ ensures that there are some banks with more money than they need in $T = 1$. Without this assumption there would be no demand on the financial market.

**Proposition 6** *Under Assumptions 3 - 6 the demand and the supply functions are, respectively, given by*

$$D(q) = \frac{1}{q}\frac{(Y_1^0 - X_1)^2}{2Y_1^0} \quad \text{and} \quad S(q) = \frac{Y_2}{Y_1^0}X_1 - \frac{\theta_2}{2r_2Y_1^0}\left(Y_2 + \frac{X_2}{r_2}\right)\frac{q}{1+t} .$$

Since $\theta_2 > 0$ the supply curve is linear and downward sloping, as in Figure 1. The demand function $D$ is downward sloping and convex, again as in Figure 1. This implies that the excess demand function $Z(q) := D(q) - S(q)$ is also globally convex so that there are

[13]Without this assumption, we would have to distinguish explicitly between low prices such that $I^{\dagger}_{sell}(p) \neq \emptyset$ and high prices with $I^{\dagger}_{sell}(p) = \emptyset$. This would be only a minor complication. The supply function derived formally in Proposition 6 would look somewhat different, but it would still be a downward-sloping function.

at most two solutions to the equation $Z(q) = 0$. Generically, the set of equilibria will therefore have one of the following structures:

(a) There is only one equilibrium with $q = r_2$. This is the case if there is no $q \in [0, r_2]$ with $Z(q) = 0$.

(b) There is one regular equilibrium with $q < r_2$. This is the case if there is exactly one $q \in [0, r_2]$ with $Z(q) = 0$. This situation arises in Figure 2 with the supply function $S_1$.

(c) There is a both a regular and an irregular equilibrium with $q < r_2$. In addition, there is an equilibrium with $q = r_2$. This is the case if there are $q, q' \in [0, r_2]$ with $Z(q) = 0$ and $Z(q') = 0$, see Figure 1.

In case there are multiple equilibria as in $(c)$ we may again dismiss the irregular equilibrium as implausible, in which case we are left with a "low-price equilibrium" with $q < r_2$ and an equilibrium with $q = r_2$. Moreover, under Assumptions 3 - 6, there is an unambiguous relation between the financial market price and the number of bankruptcies. The measure of bankrupt banks is given by

$$\int_{i \in I^{\dagger}_{sell}\left(\frac{q}{1+t}\right)} di = \int_0^{X_1 - \frac{q}{(1+t)r_2}\theta_2} \frac{1}{Y_1^0}\, ds = \frac{1}{Y_1^0}\left(X_1 - \frac{q}{(1+t)r_2}\theta_2\right) ,$$

which is a strictly decreasing function of $q$. Hence, from the viewpoint of financial stability – or from the perspective of a government that has to bail out debtors in case of bankruptcy – the "fair-price equilibrium" is also the "good equilibrium" and the "low-price equilibrium" is the "bad equilibrium."[14]

Assumptions 3 - 6 make it possible to obtain additional comparative statics results on the effects of a small increase of the tax $t$. Proposition 2 offered a definite answer only with respect to the change of the price $p$ that sellers receive, but not with respect to the price $q$ that buyers have to pay after a tax increase. Now, if we focus on a "low-price equilibrium" and consider a small tax increase (graphically, a small rightward shift of the supply curve) this will lead to a decrease of the equilibrium value of $q$. If, by contrast, we focus on a "fair-price equilibrium", then we have $q = r_2$ before and after the tax change. These observations are summarized in the following Proposition, which is a strengthened version of Proposition 2 and which we state without proof.

**Proposition 7** *Suppose that Assumptions 3 - 6 hold and that there is an initial tax rate $t_0$. Consider an equilibrium that is not irregula. There exists $\epsilon > 0$ so that moving to a higher tax rate $t_1 \in (t_0, t_0 + \epsilon)$ gives rise to a new equilibrium which is in the neighborhood of the old equilibrium and has the following properties:*

[14] In the more general framework of Section 3 this conclusion is not available, beacuse lower prices are good for fire-buyers. Here, fire buyers are assumed away, see Assumption 5.

*i) The new equilibrium price $p$ that sellers receive is lower.*

*ii) The new equilibrium price $q$ that buyers pay is not higher.*

*iii) The new equilibrium has more bankrupt banks.*

Proposition 7 deals with the consequences of a tax increase that leads to small price adjustments and which leaves the structure of the set of equilibria unchanged. An increase of the tax rate may however change the set of equilibria. Suppose we are initially in a situation with three equilibria, the bad "low-price equilibrium", the irrelevant "irregular equilibrium" and the good "fair-price equilibrium". An increase of the tax rate leaves the intercept of the supply curve unaffected but makes it flatter. Hence, if the tax rate is chosen sufficiently high, only the "low-price equilibrium" will be left, see the left part of Figure 3. Alternatively, the initial situation may be one in which the "fair-price equilibrium" is the only equilibrium because the supply curve and the demand curve do not intersect. Again, a big enough increase of the tax rate leads to a situation where the "low-price-equilibrium" is the only equilibrium, see the right part of Figure 3.

**Proposition 8** *Suppose that Assumptions 3 - 6 hold and that*

$$r_2 > \frac{(Y_1^0 - X_1)^2}{Y_2 X_1} . \tag{3}$$

*Suppose that there is an initial tax rate $t_0$ and that, at this tax rate, there exists a "fair-price equilibrium". There exists $\epsilon > 0$ so that moving to a higher tax rate $t_1 \geq t_0 + \epsilon$ gives rise to a new equilibrium set that contains only a "low-price equilibrium".*

Proposition 8 shows that the introduction of a financial transactions tax can have drastic consequences for financial stability. Suppose that we are in an initial situation in which assets are fairly priced and the number of bankruptcies is modest. Then, an increase of the financial transactions tax may kill the "fair-price equilibrium" and so that only the "low-price equilibrium" is left. Consequently, there would be a large increase in the number of bankruptcies.

## 5.1 Interaction with other policy measures

The experience of financial instability after the collapse of Lehman brothers in September 2008 has led to a number of policy measures which aimed at restoring confidence on financial markets. These policy measures included liquidity support by central banks (e.g. providing access to the Fed's discount window to investment banks in the US) and measures (such as the installation of bad banks in Europe or the toxic asset relief program in the US) with the intention to stop the vicious spiral of asset price falls, which trigger

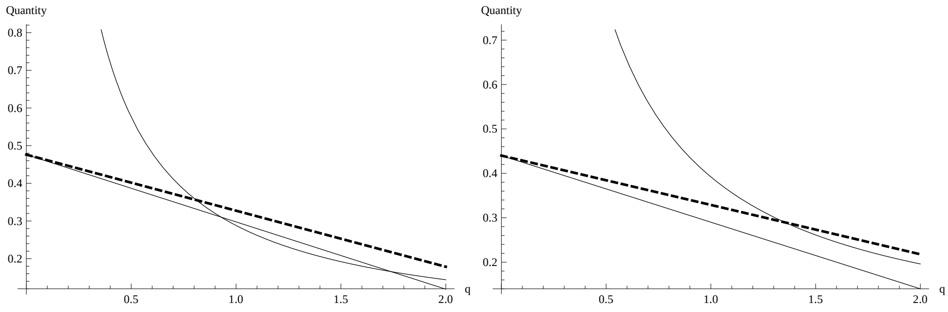


Figure 3: *Numerical examples: The parameters choices are $\theta_2 = X_1 = X_2 = Y_2 = 1$, $r_2 = 2$ and $Y_1^0 = 2.1$, in the figure on the left, and $Y_1^0 = 2.5$, in the figure on the right. The convex curves are the demand functions. The solid linear curves are the supply functions for a tax rate of $t = 0$. The dashed linear curves are the supply functions for a tax rate of $t = 0.2$ in the left figure and for tax rate of $t = 0.35$ in the right figure.*

fire-sales, which trigger further reductions in asset prices etc. In addition, the excessive reliance of banks on short term funding via the commercial paper or the repo market, has been identified as a major source of financial fragility. This problem could possibly be addressed via cash or solvency requirements that force banks to make sure that they have enough liquidity to honor their short-term promises to debtors. In the following we will discuss these policy measures in the context of our model and study their interaction with a financial transactions tax. Our main finding is that these measure are all effective in the sense that they (i) make it more likely that there is a "fair-price equilibrium" and (ii) reduce the number of bankruptcies in a "low-price equilibrium". Hence, these measure have good implications from the perspective of financial stability. We will then show that a financial transactions tax has exactly the opposite effect: It makes the "fair-price equilibrium" less likely and increases the the number of bankruptcies in a "low-price equilibrium", i.e., it undoes the beneficial effects of other policy measures that serve to stabilize financial markets.

**Liquidity Support.** We formalize liquidity support by a central bank as a perfectly elastic supply of funds to non-bankrupt banks at an interest rate of 0. The restriction of lending to non-bankrupt banks ensures that the latter will indeed be able to pay back in $T = 2$ what they have borrowed in $T = 1$.

**Assumption 7** *Suppose that all non-bankrupt banks can borrow in $T = 1$ as much as they want at an interest rate of* 0.

**Proposition 9** *Under Assumptions 3-7, the "fair-price equilibrium" is the only equilibrium. An increase of the financial transactions tax*

*i) does not affect the equilibrium price $q$ that buyers have to pay.*

*ii) depresses the equilibrium price $p$ that sellers receive and therefore increases the number of bankrupt banks.*

The fact that banks can borrow as much as they want kills the "low-price equilibrium", which is the bad equilibrium from a financial stability viewpoint. The reason is that, at a price of $q < r_2$, buyers on the financial market get the assets on the financial market below their fair price and make a profit. This will trigger a huge demand for liquidity, which is then used to buy assets on the financial market. Consequently, at any price $q < r_2$, the demand on the financial market exceeds the supply, and this pushes prices up to the equilibrium level of $q = r_2$. Now, the beneficial effect of liquidity support is counteracted by the financial transactions tax. If the price for buyers is fixed at $q = r_2$, then $p = \frac{r_2}{1+t}$, so that the price for sellers goes down if the tax rate $t$ goes up which implies that more fire-selling banks go bankrupt.

**TARP/ Bad banks.** An alternative way of restoring financial stability is to limit the amount of assets that are sold on the financial market. In 2008 the US government bought assets from distressed financial institutions via the the toxic asset relief program (*TARP*). An alternative measure is the installation of *bad banks*, i.e. of banks that hold rather than sell assets, and which are backed by a government guarantee. The main rationale for these policy measures was to limit the extent of adverse selection problems, by taking the worst, or most-difficult-to-value assets from the financial market. In the given model, we have abstracted from issues of adverse selection since we assumed that (i) it becomes commonly known in $T = 1$ which long-term investments fail and which ones perform, and (ii) that all performing long-term investments yield the same return in $T = 2$. We show in the following that, even in the given framework with no adverse selection problem, such programs can play a useful role simply because they limit the volume of fire-sales in a period of financial instability. We model the implications of a *TARP* or *Bad bank*-program via the following assumption:

**Assumption 8** *The assets of bankrupt banks are not sold on the financial market.*

The financial market price still determines how many banks go bankrupt. The measure of bankrupt banks is given by the formula

$$\int_{i \in I^{\dagger}_{sell}\left(\frac{q}{1+t}\right)} di = \int_0^{X_1 - \frac{q}{(1+t)r_2}\theta_2} \frac{1}{Y_1^0}\, ds = \frac{1}{Y_1^0}\left(X_1 - \frac{q}{(1+t)r_2}\theta_2\right) .$$

However, the price that clears the financial market equates the demand for assets with the supply of assets by fire-selling banks who manage to survive. Without the government intervention, the supply would be larger since also the assets of bankrupt banks would be sold on the financial market. The equilibrium price $q$ is therefore higher and the number of bankruptcies smaller than otherwise.

**Proposition 10** *Under Assumptions 3-6 and 8, a financial market equilibrium has the following properties:*

*i) The supply curve is increasing, implying that the equilibrium is unique.*

*ii) If the unique equilibrium is a "low-price equilibrium", i.e. an equilibrium with $q < r_2$, then the equilibrium price is higher and the number of bankruptcies is smaller than in a "low price equilibrium" that exists if bankrupt banks sell their assets on the financial market.*

*iii) An increase of the tax rate leads to a lower price $p$ for sellers and an increased number of bankruptcies. The price $q$ for buyers either goes up or remains constant.*

Under Assumption 8 the supply function is increasing, rather than decreasing. The dominant force is that a higher price increases the number of fire-selling banks that can avoid bankruptcy which in turn increases the supply on the market. An implication of this is that there is a unique financial market equilibrium. To the extent that equilibrium multiplicity may be considered a problematic source of uncertainty for market participants (e.g. because it implies that it is difficult to determine the "true" value of an asset) it is a beneficial side effect of a *TARP* or *Bad-Bank* program that this multiplicity is eliminated.

With an increasing supply and a decreasing demand function, we get the "usual" comparative static effects of a tax increase on prices: the price $p$ for buyers falls and the price $q$ for seller rises, or remains constant. The fall of $p$, means, once more, that there is more financial distress. Figure 4 illustrates the implications of a *TARP* or *Bad-Bank* program graphically, and also what happens if under such a program the tax rate is increased. In the left part, there is a unique "low-price equilibrium" with and without the program. The program gives rise to higher values of $q$ and $p$, and hence fewer bankruptcies in case there is no taxation $t = 0$. If a tax is introduced the supply curve shifts to the right. This implies that $q$ goes up, $p$ goes down and the number of bankruptcies rises even further. In the right part, there are multiple equilibria without the program. With the program, only the "fair-price equilibrium" remains. Again, if a tax is introduced, the supply curve shifts to the right, implying that $q$ stays put at a level of $r_2$, $p$ goes down, and the number of bankruptcies goes up.

**Equity requirements.** The typical rationalization of equity requirements goes as follows: They provide a buffer against losses; that is, if a significant fraction of a bank's investments fail, then, if there is sufficient equity, holders of equity will carry the losses,

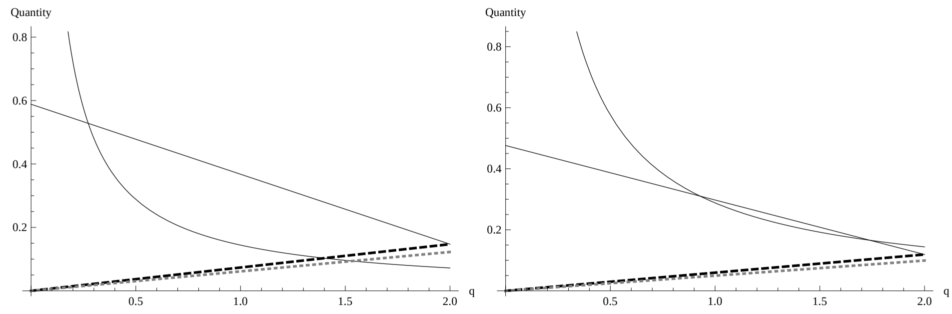


Figure 4: *Numerical examples: The parameters choices are* $\theta_2 = X_1 = X_2 = Y_2 = 1$*,* $r_2 = 2$ *and* $Y_1^0 = 1.7$*, in the figure on the left, and* $Y_1^0 = 2.1$ *in the figure right. The convex curves are the demand functions. The solid linear curves are the supply functions for a tax rate of* $t = 0$ *if there is no TARP-program. The dashed black curves are the supply functions with a TARP-program and a tax rate of* 0*. The dotted gray curves are the supply curves with a TARP-program and a tax rate of* $t = 0.2$*.*

while the bank will be able to honor the promises to its depositors and therefore avoid bankruptcy. In the given model, we can think of an equity requirement as follows: We take the amount of debt $d_i$ and the accompanying claims $(x_{i1}, x_{i2})$ of debtors as given. We then study the consequences of an increase of $e_i$ for the financial market equilibrium. Since $a_i = e_i + d_i$, this means that, in $T = 0$, a bank has additional funds that can be invested either with a short-term perspective or a long-term perspective.

The following Proposition shoes that increased equity per se does not improve financial stability. Its impact depends on how the additional funds are invested. If they are invested with a long-term perspective then financial stability may go down (financial market prices fall and bankruptcies rise), and it improves if they are invested with a short-term perspective. For ease of exposition, we focus on a situation where banks have only short-term debt so that $X_2 = 0$.[15]

**Assumption 9** *Banks have only short-term debt, so that* $X_1 > 0$ *and* $X_2 = 0$*.*

**Proposition 11** *Suppose Assumptions 3-6 and Assumption 9 hold. Suppose there is a "low-price-equilibrium" with an equilibrium price* $q^*$*.*

*i) The equilibrium price* $q^*$ *is an increasing function of* $Y_1^0$*.*

*ii) The equilibrium price* $q^*$ *is a decreasing function of* $Y_2 = \sigma Y_2^0$*.*

Since $X_2 = 0$, all banks trivially have sufficient assets to honor their obligations in $T = 2$. Hence, the only source of financial distress is that banks may have insufficient liquidity in $T = 1$. This problem is not alleviated if the amount invested with a long-run perspective

[15]The Proposition trivially extends to situations where $X_2$ is not too large.

goes up. The problem is even aggravated since those banks that fail throw more assets on the financial market and thereby generate downward pressure on the market price. By contrast, additional investments with a short-run perspective have a beneficial effect. At an aggregate level, they increase the liquidity that is available in $T = 1$. This increase of aggregate liquidity implies that the demand on the financial market is higher, and this leads to a upward pressure on the financial market price.

The important insight is that an equity requirement does not by itself contribute to financial stability. To have a beneficial effect, additional equity must be invested in such a way that the proceeds from the investment are available in the short-run. Again, as an implication of Proposition 7, such a beneficial use of equity will be neutralized by the introduction of a financial transactions tax, which drives the price $p$ that seller's receive down.

## 5.2 Investment decisions, Maturity structure of debt

We now turn to a bank's choice of $h_{0i} = (y^0_{i1}, y^0_{i2}, x_{i1}, x_{i2})$ in the initial period. We again impose Assumption 2 so that debtors are always made whole by the government in case bank $i$ goes bankrupt. Bank $i$'s problem therefore is to choose $h_{0i}$ in order to maximize $\Pi^2_i(h_{0i}, t)$ subject to the constraints that $u(x_{i1} + x_{i2}) = \bar{u}_i$ and $y^0_{i1} + y^0_{i2} = a_i$. Assume, for ease of notation, that non-negativity constraints can be neglected. Then, a solution

$$h^*_{01}(t) = ((y^{0*}_{i1}(t), y^{0*}_{i2}(t), x^*_{i1}(t), x^*_{i2}(t)))$$

satisfies the following first order conditions

$$\frac{\partial \Pi^2_i(h^*_{0i}(t), t)}{\partial y^0_{i1}} = \frac{\partial \Pi^2_i(h^*_{0i}(t), t)}{\partial y^0_{i2}} \,, \tag{4}$$

and

$$\frac{\partial \Pi^2_i(h^*_{0i}(t), t)}{\partial x_{i1}} = \frac{\partial \Pi^2_i(h^*_{0i}(t), t)}{\partial x_{i2}} \,. \tag{5}$$

which require, respectively, that the marginal returns of long- and short-term investments are equalized and that the marginal costs (in terms of forgone profit) of long and short-term debt are equalized. The following Proposition shows how the solution to this system of equations changes if the tax rate changes.

**Proposition 12** *Suppose that, for all $\sigma$, $q(\sigma, t)$ and $p(\sigma, t)$ are regular equilibrium prices and differentiable functions of the tax rate $t$. Also suppose that, for all $i$, $y^0_{i1} > x_{i1}$ and $\theta_{i2} := r_2 \sigma y^0_{i2} - x_{i2} > 0$, with probability 1, and that bank $i$'s optimal decision in $T = 0$ is characterized by the first order conditions in (4) and (5). Then, bank $i$ responds in the following way to an increase of the tax rate:*

*i) It increases its short-run investments and decreases its long-run investments.*

*ii) It decreases its short-run debt and increases its long-run debt.*

The Proposition restricts attention to local changes of equilibrium prices in response to taxation. In addition it imposes Assumption 5 for each bank separately, i.e. it is assumed that, for all $i$, $y^0_{i1} > x_{i1}$ and $\theta_{i2} := r_2\sigma y^0_{i2} - x_{i2} > 0$, with probability 1. Under these assumptions, there is a beneficial effect of an increase of the financial transaction tax: Banks reduce their maturity mismatch, i.e., they reduce their short-term debt and increase their short-term investments.[16] The intuition is straightforward: The tax-induced increase of financial distress makes it more attractive to be a safe bank rather than a fire-seller. The chance of being safe bank is increased if maturity mismatch is reduced.

For reasons of tractability, the Proposition limits attention to local changes whose impact can be analyzed by looking at derivatives. We have seen previously, however, that tax increases can change the equilibrium structure so that, for some states $\sigma$, there is a jump from a "fair-price equilibrium" to a "low-price equilibrium". Allowing for such drastic shifts would reinforce the conclusions of Proposition 12. What is essential for the Proposition is the price response to a tax increase, namely that the price $p$ falls in every state and that the price $q$ either remains constant or falls. Now, the jump from a "fair-price equilibrium" to a "low-price equilibrium" has exactly this structure: both prices fall. Hence, Proposition 12 would extend if this possibility was acknowledged.

To sum up, we have seen that a financial transactions tax has two different, and opposing, implications for financial stability. In the short-run, for given investment decisions and debt structures, such a tax threatens financial stability. It depresses the prices on the financial markets and thereby makes life more difficult for a distressed financial institution. In the long run, however, banks have an incentive to adjust their investments and their debt structure so that maturity mismatch is reduced and distress becomes less likely. At first glance, this raises the question of how to strike a balance between these two effects. However, the beneficial corrective effects of the transactions tax could be generated in a different way, namely by a tax that addresses maturity mismatch directly without having the destabilizing impact of a tax on financial transactions.

# 6 Concluding Remarks

This paper has looked at a stylized model of a financial market in order to trace out the implications of a financial transactions tax for financial stability. Key features of this model are that banks with liquidity needs may have to sell assets and that banks'

[16]Recall from Proposition 11 that this leads to an increase of financial market prices and hence to fewer bankruptcies.

assets are marked to the market. If many banks have to sell assets at the same time, this depresses the price, and, consequently, a large number of banks will be driven out of business. In such an environment, a financial transactions tax reduces the liquidity that is generated by the fire-sale of an asset because part of the revenue has to be shared with the government. Hence, distressed financial institutions have to sell more assets to meet their liquidity needs, and therefore also the number of distressed financial institutions rises. A financial transactions tax therefore has a destabilizing short-run effect.

The analysis has also shown that there may be a beneficial long-run effect. If banks understand the implications of a financial transactions tax, they have an incentive to reduce their maturity mismatch, i.e., to reduce the risk of ending up in the position of a fire-seller. It would be inappropriate, however, to advocate a financial transactions tax on this basis. A tax instrument, or some other regulatory tool, that addresses the discrepancy between banks' liquid assets and their short-term debt directly would have the same beneficial long-run consequences, without being destabilizing in periods of financial crisis.

The model in the paper was meant to be descriptive of the state of financial markets in the recent financial crises. It therefore has deliberately abstracted from normative considerations. For instance, it was simply assumed that those who lend to a distressed bank will be bailed out by the government, without discussing the desirability of such bailouts. Also, it was noted that maturity mismatch lies at the heart of the crisis, without asking what an optimal degree of maturity mismatch would look like. These questions are of obvious importance for the design of optimal taxes and an optimal regulatory framework for the financial sector. They are, however, beyond the scope of this paper.

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# Appendix

**Proof of Proposition 1.** We first show that $D(q) > S\left(\frac{q}{1+t}\right)$ as $q$ approaches 0. Hence, at the minimal price of $q = p = 0$, demand exceeds supply on the financial market. Note that, for any $q$, $S\left(\frac{q}{1+t}\right)$ is bounded from above. This follows from the assumptions that in $T = 0$, each bank has a limited amount of funds that can be invested in the long asset, and that the number of performing assets in $T = 1$ cannot not exceed the investments made in $T = 0$. From

$$D(q) = \int_{\hat{I}^*(q)} \frac{\theta_{i1}}{q} \, di$$

and the assumption that $\hat{I}^*(0) \neq \emptyset$ it follows that

$$\lim_{q \to 0} D(q) = \infty \ .$$

Now, since $D$ and $S$ are assumed to be continuous, there are only two possibilities: Either they do not intersect over the domain $(0, r_2]$. In this case it has to be true that $D(r_2) > S\left(\frac{r_2}{1+t}\right)$ so that $r_2$ is an equilibrium price. Or they do intersect, implying the existence of a price $q^* < r_2$ so that $D(q^*) = S\left(\frac{q^*}{1+t}\right)$, in which case $q^*$ is an equilibrium price. $\square$

**Proof of Proposition 2.** For both types of equilibria, i.e. those with $D(q) = S(q)$ and $D' < S'$ and those with $D(r_2) > S(r_2)$, moving to a higher tax rate $t_1 \in (t_0, t + \epsilon)$ gives rise to a new equilibrium which is in the neighborhood of the old equilibrium. This can be illustrated graphically: The tax increase generates a "small" rightward shift of the supply curve. If the initial equilibrium is one with $D(q) = S(q)$ and $D' < S'$ then, because of continuous differentiability of $D$ and $S$, the new equilibrium will inherit both of these properties. If, by contrast, the initial equilibrium is such that $q_0 = r_2$ and $D(r_2) > S(r_2)$, then, because of the continuity of $D$ and $S$, after a small rightward shift of $S$ it is still the case that $D(r_2) > S(r_2)$ implying that $q_1 = r_2$ is still an equilibrium price. With $p_0 = \frac{q_0}{1+t_0} = \frac{r_2}{1+t_0}$ and $p_1 = \frac{q_1}{1+t_1} = \frac{r_2}{1+t_1}$, the conclusion $p_0 < p_1$ then follows immediately for the latter type of equilibrium.

Now, consider an equilibrium with $D(q) = S(q)$ and $D'(q) < S'(q)$. We denote by $q^*(t)$ the equilibrium price for buyers as a function of the tax rate $t$. This price is implicitly defined by the equation $D(q^*(t)) = \bar{S}\left(\frac{q^*(t)}{1+t}\right)$. Analogously we define $p^*(t) := \frac{q^*(t)}{1+t}$. Straightforward calculations yield

$$p^{*\prime}(t) = \frac{q^*(t)}{(1+t)^2} \frac{D'(q^*(t))(1+t)}{\bar{S}'\left(\frac{q^*(t)}{1+t}\right) - D'(q^*(t))(1+t)} \tag{6}$$

For any $q$, and any given tax rate $t$, we have that $S(q) = \bar{S}\left(\frac{q}{1+t}\right)$. This implies that

$$S'(q)(1+t) = \bar{S}'\left(\frac{q}{1+t}\right) . \tag{7}$$

Using (7), we can rewrite equation (6) as

$$p^{*\prime}(t) = \frac{q^*(t)}{(1+t)^2} \frac{D'(q^*(t))}{S'(q^*(t)) - D'(q^*(t))} . \tag{8}$$

Since $D' < 0$, $p^{*\prime}(t) < 0$ if and only if $S' > D'$, i.e., if the initial equilibrium is regular.

To complete the proof we note that the claim in ii) is an immediate consequence of the definition of the set $I^*_{sell}(p)$.

□

**Proof of Proposition 3.** *ad i)* Under Assumption 1, there are no failed banks and no fire-selling banks so that $I^\dagger = I^\dagger_{sell}(p) = I^*_{sell}(p) = \emptyset$. The set of bankrupt banks consists entirely of fire-buying banks that fail, i.e., of the set $I^\dagger_{buy}(q)$. Note that whether or not a bank fails depends only on the price $q$ for buyers, but not on the price $p$ for sellers. The supply on the financial market is therefore given by

$$S(q) = \int_{I^\dagger_{buy}(q)} y_{i2}\, di .$$

Observe that this supply function depends only on $q$. Consequently, the equilibrium condition – $q$ is an equilibrium price if (i) $D(q) = S(q)$ and $q \leq r_2$, or (ii) $D(q) > S(q)$ and $q = r_2$ – is independent of the tax rate. Therefore, if $q_0$ fulfills the equilibrium condition for a tax rate of 0 then it fulfills the equilibrium condition for any tax rate.

*ad ii)* Immediate from part i) and the condition that $q = (1+t)p$.

*ad iii)* Immediate from part i) and the observation in the proof of part i) that the set of bankrupt banks depends only on $q$. □

**Proof of Proposition 4.** Under Assumption 1, there are only three types of banks: safe ones, fire-buying banks that go bankrupt, and fire-buying banks that survive. Fix a state of the economy $\sigma$. Since banks are protected by limited liability, if $\theta_{i2} < 0$ and $\frac{q(\sigma,t)}{r_2} > -\frac{\theta_{i1}}{\theta_{i2}}$, a bank's profit is zero. Otherwise the bank makes a profit. Profit maximizing behavior of safe and fire-buying banks implies that the profit equals $\theta_{i2} + \frac{r_2}{q(\sigma,t)}\theta_{i1}$. We can therefore write

$$\begin{aligned}&E[\pi_i^2 \mid \sigma, p(\sigma,t), q(\sigma,t), h_{0i}] = \\ &E\left[\mathbf{1}\left(\theta_{i2} + \tfrac{r_2}{q(\sigma,t)}\theta_{i1} \geq 0\right)\left(\theta_{i2} + \tfrac{r_2}{q(\sigma,t)}\theta_{i1}\right) \mid \sigma, t, q(\sigma,t), h_{0i}\right] .\end{aligned}$$

Observe that the right hand side of this equation does not depend on $p(\sigma,t)$. Consequently, the ex ante expected profit

$$\Pi_i^2(h_{0i}, t) = E_\sigma\left[E\left[\mathbf{1}\left(\theta_{i2} + \frac{r_2}{q(\sigma,t)}\theta_{i1} \geq 0\right)\left(\theta_{i2} + \frac{r_2}{q(\sigma,t)}\theta_{i1}\right) \mid \sigma, q(\sigma,t), h_{0i}\right]\right] ,$$

also does not depend on $p(\sigma,t)$. It does depend on $q(\sigma,t)$. In Proposition 3, however, we have shown that for all $\sigma$, $t$ and $t'$, $q(\sigma,t) = q(\sigma,t')$. Consequently, we have for every $\sigma$, $t$ and $t'$ that

$$\begin{aligned}&E\left[\mathbf{1}\left(\theta_{i2} + \tfrac{r_2}{q(\sigma,t)}\theta_{i1} \geq 0\right)\left(\theta_{i2} + \tfrac{r_2}{q(\sigma,t)}\theta_{i1}\right) \mid \sigma, q(\sigma,t), h_{0i}\right] = \\ &E\left[\mathbf{1}\left(\theta_{i2} + \tfrac{r_2}{q(\sigma,t')}\theta_{i1} \geq 0\right)\left(\theta_{i2} + \tfrac{r_2}{q(\sigma,t')}\theta_{i1}\right) \mid \sigma, q(\sigma,t'), h_{0i}\right]\end{aligned}$$

and therefore also that

$$\Pi_i^2(h_{0i}, t) = \Pi_i^2(h_{0i}, t') .$$

$\square$

**Proof of Proposition 5.** By Proposition 4, under Assumption 1,

$$\Pi_i^2(h_{0i}, t) = \Pi_i^2(h_{0i}, t') ,$$

for any pair of tax rates $t$ and $t'$ and any vector $h_{0i}$. Hence, there exists a function $\bar{\Pi}_i^2 : h_{0i} \mapsto \bar{\Pi}_i^2(h_{0i})$ such that

$$\bar{\Pi}_i^2(h_{0i}) = \Pi_i^2(h_{0i}, t),$$

for any tax rate $t$. Under Assumption 2, bank $i$'s problem is therefore equivalent to the problem of choosing $h_{01}$ in order to maximize $\bar{\Pi}_i^2(h_{0i})$ subject to the constraints $u(x_{i1} + x_{i2}) \geq \bar{u}_i$ and $y_{i1}^0 + y_{i2}^0 = a_i$. Neither the objective function nor any of the constraints depends on the tax rate $t$. Hence, if $h_{0i}$ is an optimal choice for some tax rate, then it is an optimal choice for any tax rate. $\square$

**Proof of Proposition 6.** The assumption that $\theta_2 > 0$ implies that $I^\dagger = I^*_{buy}(q) = I^\dagger_{buy}(q) = \emptyset$, for all $q$. The demand for assets on the financial market is therefore given by

$$D(q) = \int_{i \in I^*} \frac{\theta_{i1}}{q} \, di$$

Under Assumptions 3 - 6, this can be written as

$$D(q) = \int_{X_1}^{Y_1^0} \frac{s - X_1}{q} \frac{1}{Y_1^0} \, ds = \frac{1}{q} \frac{(Y_1^0 - X_1)^2}{2Y_1^0} \, .$$

The supply on the financial market stems from banks in $I^\dagger_{sell}(p)$ who go bankrupt and whose assets are liquidated and from banks in $I^*_{sell}(p)$ that fire-sell and manage to survive. The supply of the former is given by

$$\bar{S}^\dagger(p) := \int_{i \in I^\dagger_{sell}(p)} Y_2 \, di = Y_2 \int_0^{X_1 - \frac{p}{r_2}\theta_2} \frac{1}{Y_1^0} \, ds = \frac{Y_2}{Y_1^0} \left( X_1 - \frac{p}{r_2} \theta_2 \right) \, .$$

The supply of the latter is given by

$$\bar{S}^*(p) := -\int_{i \in I^*_{sell}(p)} \frac{\theta_{i1}}{p} \, di = \int_{X_1 - \frac{p}{r_2}\theta_2}^{X_1} \frac{X_1 - s}{p} \frac{1}{Y_1^0} \, ds = \frac{1}{2} \frac{p}{Y_1^0} \left( \frac{\theta_2}{r_2} \right)^2$$

Summing these two expressions and exploiting that $\theta_2 = r_2 Y_2 - X_2$ yields

$$\bar{S}(p) = \frac{Y_2}{Y_1^0} X_1 - \frac{\theta_2}{2 r_2 Y_1^0} \left( Y_2 + \frac{X_2}{r_2} \right) p \, ,$$

and

$$S(q) = \frac{Y_2}{Y_1^0} X_1 - \frac{\theta_2}{2 r_2 Y_1^0} \left( Y_2 + \frac{X_2}{r_2} \right) \frac{q}{1+t} \, .$$

□

**Proof of Proposition 8.** The supply curve has an intercept of $\frac{Y_2}{Y_1^0} X_1$. The assumption $r_2 > \frac{(Y_1^0 - X_1)^2}{Y_2 X_1}$ ensures that a horizontal supply curve with this intercept (the limit case which arises as $t \to \infty$) intersects the demand curve at a price $q < r_2$. To see this, note that the price which equates $D(q)$ and $\frac{Y_2}{Y_1^0} X_1$ is given by

$$q = \frac{(Y_1^0 - X_1)^2}{Y_2 X_1} \, .$$

If (3) is violated, then there is no intersection of the supply and the demand curve at a price $q < r_2$, whatever the tax rate. Put differently, the "fair-price equilibrium" would be the only equilibrium, whatever the tax rate. By contrast, if (3) holds, then there is $\hat{t}$ so that for all $t \geq \hat{t}$, the "low-price equilibrium" is the only equilibrium. □

**Proof of Proposition 9.** Suppose there is an equilibrium with $q < r_2$. Then, if a bank lends $q$ units of money from the central bank it can buy one asset and this asset will return $r_2$ in $T = 2$. In $t_2$ the bank has to repay $q$ and therefore makes a profit of $r_2 - q > 0$. Hence, all banks will lend unlimited amounts from the central bank, and there will be an arbitrarily large demand on the financial market. Since the supply of assets on the financial market is bounded, this implies that $q < r_2$, and $D(q) > S(q)$. This contradicts the assumption that there is an equilibrium with $q < r_2$. There can also be no equilibrium with $q > r_2$, because in this case all non-bankrupt banks strictly prefer to hold cash over the purchase of assets on the financial market. Consequently, in any equilibrium, it has to be the case that $q = r_2$. This implies that, in equilibrium, $p = \frac{q}{1+t} = \frac{r_2}{1+t}$, so that an increase of $t$ reduces $p$. The measure of bankrupt banks is given by

$$\int_{i \in I^{\dagger}_{sell}\left(\frac{q}{1+t}\right)} di = \int_0^{X_1 - \frac{q}{(1+t)r_2}\theta_2} \frac{1}{Y_1^0}\, ds = \frac{1}{Y_1^0}\left(X_1 - \frac{q}{(1+t)r_2}\theta_2\right) .$$

With $q = r_2$ this simplifies to $\frac{1}{Y_1^0}\left(X_1 - \frac{1}{(1+t)}\theta_2\right)$, an expression that is strictly increasing in $t$. □

**Proof of Proposition 10.** *ad i)* It follows from the argument in the proof of Proposition 6 that, under, under Assumption 10, supply is given by a function $S^*$, with

$$S^*(q) = \frac{1}{2}\frac{q}{(1+t)Y_1^0}\left(\frac{\theta_2}{r_2}\right)^2 .$$

Obviously, this curve is increasing and hence has a unique intersection $q'$ with the demand curve. If $q' \leq r_2$, then $q'$ is the unique equilibrium price. Otherwise $r_2$ is the unique equilibrium price.

*ad ii)* Under Assumption 10 supply is given by $S^*$. Absent the government intervention supply is given by $S(q) = S^*(q) + S^{\dagger}(q)$, where $S^{\dagger}$ is defined formally in the proof of Proposition 6. Under Assumption 5 we have that $S^{\dagger} > 0$, for all $q < r_2$. Hence, for all $q < r_2$, we have that $S(q) > S^*(q)$. Consequently, if $q'$ is such that $S(q') = D(q')$, then $D(q') > S^*(q')$ and $q'' > q'$ for any $q''$ satisfying $D(q'') = S(q'')$.

*ad iii)* Consider a "low-price equilibrium." A small tax increase shifts the supply curve to the right and yields a new "low-price equilibrium". Since $S' > 0$ and $D' < 0$, the new equilibrium has a higher value of $q$ and, by Proposition 2, a strictly lower value of $p$, and therefore a strictly larger number of bankrupt banks. A larger tax increase may eventually turn this "low-price equilibrium" into a "fair-price equilibrium". However, by the preceding argument, $p$ falls and bankruptcies rise all along the way to the "fair-price equilibrium". Once we are in a "fair-price equilibrium", further tax increases leave the equilibrium value of $q$ unaffected but depress the equilibrium value of $p$ further, which leads to more bankruptcies. □

**Proof of Proposition 11.** Under Assumptions 3-6 and Assumption 9 demand and supply are given by

$$D(q) = \tfrac{1}{q}\tfrac{(Y_1^0 - X_1)^2}{2Y_1^0} \quad \text{and} \quad S(q) = \tfrac{Y_2}{Y_1^0} X_1 - \tfrac{1}{2Y_1^0}(Y_2)^2 \tfrac{q}{1+t} \; .$$

The equation $D(q) = S(q)$ has two solutions. We focus on a "low-price-equilibrium" that is on the smallest $q$ such that $D(q) = S(q)$, henceforth referred to as $q^*$. Straightforward computations yield

$$q^* = \frac{(1+t)X_1 - \sqrt{(1+t)^2(X_1)^2 - (1+t)(Y_1^0 - X_1)^2}}{Y_2} \; ,$$

an expression which increases in $Y_1^0$ and decreases in $Y_2$. □

**Proof of Proposition 12.** *Step 1.* We first show that, under Assumptions 3-6,

$$\begin{aligned} \Pi_i^2(h_{0i}, t) \quad = \quad & \frac{1}{2\, e_i\, y_{i1}^0} E_\sigma \left[ \frac{r_2}{q(\sigma, t)} (y_{i1}^0 - x_{i1})^2 + 2(y_{i1}^0 - x_{i1})(r_2\, \sigma\, y_{i2}^0 - x_{i2}) \right. \\ & \left. + \frac{p(\sigma, t)}{r_2} (r_2\, \sigma\, y_{i2}^0 - x_{i2})^2 \right] \; , \end{aligned} \tag{9}$$

To see this, note that, for any given pair of financial market prices $p$ and $q$, we can write bank $i$'s expected return on equity as

$$E[\pi_i^2 \mid \sigma, p, q, h_{0i}] = \frac{1}{e_i} \left( V_i^{buy}(\sigma, q, h_{0i}) + V_i^{sell}(\sigma, p, h_{0i}) \right) \; , \tag{10}$$

where

$$V_i^{buy}(\sigma, q, h_{0i}) := E\left[ \mathbf{1}\left( \theta_{i1} \geq 0, \theta_{i1} + \frac{q}{r_2}\theta_{i2} \geq 0 \right) \left( \theta_{i2} + \frac{r_2}{q}\theta_{i1} \right) \mid \sigma, q, h_{0i} \right]$$

is the expected return on equity conditional on bank $i$ having excess liquidity in $T = 1$ and therefore being a buyer on the financial market in $T = 1$, and

$$V_i^{sell}(\sigma, p, h_{0i}) := E\left[ \mathbf{1}\left( \theta_{i1} \leq 0, \theta_{i1} + \frac{p}{r_2}\theta_{i2} \geq 0 \right) \left( \theta_{i2} + \frac{r_2}{p}\theta_{i1} \right) \mid \sigma, p, h_{0i} \right]$$

is the expected return on equity conditional on bank $i$ having to fire-sell on the financial market in $T = 1$. In these expressions $\mathbf{1}$ is the indicator function and expectations are taken with respect to the random variables $\theta_{i1}$ and $\theta_{i2}$, respectively. Assumptions 5 and 6 imply that

$$\begin{aligned} V_i^{buy}(\sigma, q, h_{0i}) \quad &= \quad \textstyle\int_{x_{i1}}^{y_{i1}^0} \left( \theta_{i2} + \frac{r_2}{q}(s - x_{i1}) \right) \frac{1}{y_{i1}^0} ds \\ &= \quad \left( 1 - \frac{x_{i1}}{y_{i1}^0} \right) \left( r_2\, \sigma\, y_{i2}^0 - x_{i2} + \frac{1}{2}\frac{r_2}{q(\sigma)}(y_{i1}^0 - x_{i1}) \right) \; . \end{aligned}$$

Assumptions 5 and 6 also imply that

$$\begin{aligned} V_i^{sell}(\sigma, q, h_{0i}) \quad &= \quad \textstyle\int_{x_{i1} - \frac{p}{r_2}\theta_{i2}}^{x_{i1}} \left( \theta_{i2} + \frac{r_2}{p}(s - x_{i1}) \right) \frac{1}{y_{i1}^0} ds \\ &= \quad \frac{1}{2}\frac{p(\sigma)}{r_2}\frac{(r_2\, \sigma\, y_{i2}^0 - x_{i2})^2}{y_{i1}^0} \; . \end{aligned}$$

Evaluating these expressions for $V_i^{sell}(\sigma, q, h_{0i})$ and $V_i^{sell}(\sigma, p, h_{0i})$ at the equilibrium prices $q(\sigma, t)$ and $p(\sigma, t)$, respectively, and substituting them into (10) yields

$$\begin{aligned} &E[\pi_i^2 \mid \sigma, p(\sigma,t), q(\sigma,t), h_{0i}] = \\ &\tfrac{1}{e_i}\left(\left(1-\tfrac{x_{i1}}{y_{i1}^0}\right)\left(r_2\ \sigma\ y_{i2}^0 - x_{i2} + \tfrac{1}{2}\tfrac{r_2}{q(\sigma,t)}(y_{i1}^0 - x_{i1})\right) + \tfrac{1}{2}\tfrac{p(\sigma,t)}{r_2}\tfrac{(r_2\ \sigma\ y_{i2}^0 - x_{i2})^2}{y_{i1}^0}\right)\ . \end{aligned} \tag{11}$$

Our claim now follows from the law of iterated expectations which implies that

$$\Pi_i^2(h_{0i}, t) = E_\sigma\left[E[\pi_i^2 \mid \sigma, p(\sigma,t), q(\sigma,t), h_{0i}]\right]$$

and some straightforward algebraic manipulations.

*Step 2.* Fix some arbitrary vector $h_{01}$. Using equation (9) we can compute the following expressions

$$\begin{aligned} \frac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial y_{i1}^0} &= \frac{-1}{2\ e_i\ (y_{i1}^0)^2} E_\sigma\left[\frac{r_2}{q(\sigma,t)^2}\frac{\partial q(\sigma,t)}{\partial t}((y_{i1}^0)^2 - (x_{i1})^2)\right. \\ &\quad \left. + \frac{\partial p(\sigma,t)}{\partial t}(r_2 \sigma y_{i2}^0 - x_{i2})^2\right]\ , \end{aligned}$$

$$\frac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial x_{i1}} = \frac{1}{e_i\ y_{i1}^0} E_\sigma\left[\frac{r_2}{q(\sigma,t)^2}\frac{\partial q(\sigma,t)}{\partial t}(y_{i1}^0 - x_{i1})\right]\ ,$$

$$\frac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial y_{i2}^0} = \frac{1}{e_i\ y_{i1}^0} E_\sigma\left[\frac{1}{r_2}\frac{\partial p(\sigma,t)}{\partial t} r_2 \sigma (r_2 \sigma y_{i2}^0 - x_{i2})\right]\ ,$$

and

$$\frac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial x_{i2}} = \frac{-1}{e_i\ y_{i1}^0} E_\sigma\left[\frac{1}{r_2}\frac{\partial p(\sigma,t)}{\partial t}(r_2 \sigma y_{i2}^0 - x_{i2})\right]\ .$$

It follows from Proposition 7 that, for all $\sigma$,

$$\frac{\partial q(\sigma,t)}{\partial t} \leq 0 \quad \text{and} \quad \frac{\partial p(\sigma,t)}{\partial t} < 0.$$

Using these results and the assumption that, $y_{i1}^0 > x_{i1}$ and $\theta_{i2} := r_2 \sigma y_{i2}^0 - x_{i2} > 0$, with probability 1, makes it possible to verify the following statements

$$\tfrac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial y_{i1}^0} > 0, \quad \tfrac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial x_{i1}} \leq 0, \quad \tfrac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial y_{i2}^0} < 0 \quad \text{and} \quad \tfrac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial x_{i2}} > 0\ .$$

Consequently, if we have an initial situation $h_{0i}$ in which the first order condition in (4) is satisfied, and then increase the tax rate, we have

$$\frac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial y_{i1}^0} > \frac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial y_{i2}^0}\ ,$$

i.e. it becomes optimal to increase $y_{i1}^0$ and to decrease $y_{i2}^0$. Analogously, if initially the first order condition (5) is satisfied, then after a tax increase we have

$$\frac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial x_{i2}} > \frac{\partial^2 \Pi_i^2(h_{0i},t)}{\partial t\, \partial x_{i1}}\ ,$$

so that it becomes optimal to increase $x_{i2}$ and to decrease $x_{i1}$. □