Trade, firm selection, and innovation: the competition channel*

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Abstract

The availability of rich firm-level data has led researchers to uncover new evidence on the effects of trade liberalization. First, trade openness forces the least productive firms to exit the market; secondly, it induces surviving firms to increase their innovation efforts; thirdly, it increases the degree of product market competition. In this paper, we propose a tractable framework aimed at providing a coherent interpretation of these findings. We introduce firm heterogeneity into an innovation-driven growth model, where incumbent firms operating in oligopolistic industries perform cost-reducing innovation. In this environment, trade liberalization leads to lower markups, tougher firm selection, and more innovation. We break down the effects of trade on growth and welfare into those attributable to the decisions of heterogeneous firms to exit and export, the *indirect effect*, and those obtainable keeping these decisions fixed. Calibrated to match US aggregate and firm-level statistics, the model predicts that moving from a 13% variable trade costs to free trade increases the stationary annual rate of productivity growth from 1.19 to 1.29% and increases welfare by about 8% of steady-state consumption. Firm-level exit and export responses account for about one forth of the overall growth and one half of the welfare gains from trade.

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1 Introduction

An interesting set of empirical regularities about international trade has recently emerged from a large numbers of studies using firm-level data. Firstly, strong evidence suggests that trade liberalization induces the least productive firms to exit the market, reallocating both demand and resources to surviving, more productive firms; this is the so-called *selection effect* of trade resulting in an increase in aggregate productivity (e.g. Pavcnik, 2002, Topalova, 2004, and Tybout, 2003 for a survey). A second line of research has highlighted the joint selection and *innovation* effect of trade,[1] showing that trade liberalization cleans the market of inefficient firms and forces the surviving firms to innovate more (e.g. Bustos, 2010, Bloom, Draca, and Van Reenen, 2009, Aw, Roberts, and Xu, 2010). A third piece of evidence shows that trade liberalization has *pro-competitive* effects (reduces prices and markups) potentially leading to more selection and more innovation (e.g., Chen, Imbs and Scott, 2008, Corcos, Del Gatto, Ottaviano and Mion, 2010, Bugamelli, Fabiani, and Sette, 2008, Griffith, Harrison, and Simpson, 2008).[2]

Our paper has two main goals: first, to present a tractable model providing a coherent interpretation of these empirical regularities. Secondly, to perform a quantitative evaluation of the productivity and welfare gains from trade, as well as to measure the share of these gains attributable to heterogeneous firms' decisions to exit and export, and their implications for innovation.

We set up a model in which trade liberalization has pro-competitive effects through reduced markups leading to firm selection and increased innovation. A dynamic industry model with heterogeneous firms is added to a growth model with innovation by incumbents. There are two goods, a homogeneous good produced under constant returns, and a differentiated good produced with a continuum of varieties, each of them facing both variable and fixed production costs. As in Hopenhayn (1992) and Melitz (2003), productivity differs across varieties. Firms

[1] Focusing on innovation instead of directly looking at productivity has the advantage of identifying one specific channel through which improvements in productivity take place. Other studies have instead estimated productivity as a residual in the production function, facing the problem that together with technological differences, residuals captures also other differences such as market power, factor market distortion, and change in the product mix. (see i.e. Foster, Haltowanger, and Syverson, 2008, Hsieh and Klenow, 2009, and Bernard, Redding, and Schott, 2008).

[2] Further evidence from firm-level studies on the pro-competitive effect of trade can be found in Levinson (1993) for Turkey, Harrison (1994) for the Ivory Coast, Roberts (1996) for Colombia, Krishna and Mitra (1998) for India, Konings et al. (2001) for Belgium, Boulhol, Dobbelaere, and Maioli (2011) for the UK, Kee and Hoekman (2007) on UNIDO data. Boulhol (2010) surveys the literature.

in the differentiated good sector allocate labor to the production of a specific variety and to innovation activities aimed at reducing their production costs. Each variety is produced by a small number of identical firms operating in an oligopolistic market; thus quantities produced and innovation activities result from the strategic interaction among firms. The oligopolistic market structure and the cost-reducing innovation features are borrowed from static trade models of trade under oligopoly (e.g. Neary, 2009 and 2010) and from growth models with homogeneous oligopolistic firms (e.g. Peretto, 2003, Aghion et al., 2005, Licandro and Navas, 2011, and Akcigit, Ates, and Impullitti, 2012).

The open economy features two symmetric countries engaging in costly trade (iceberg type). In order to simplify the analysis, the baseline version of the model assumes no sunk entry costs into either the domestic or the export market, implying that all operating firms export, and the number of oligopolistic producers is given. Trade liberalization increases product market competition by reducing markups in the differentiated good, thus increasing market efficiency and ultimately leading to an expansion of the quantities produced by firms. Since innovation is cost-reducing, the trade-induced increase in firms' size raises their incentive to innovate. This *direct effect* can be obtained in existing models of trade and innovation with a representative firm.

The decline in the markup forces the least productive firms out of the market, reallocating resources toward surviving firms, increasing their average size and their incentive to invest in cost-reducing innovation. Hence, trade-induced firm selection increases not only the 'level' of aggregate productivity (as in Melitz, 2003) but also firms' innovation, ultimately affecting the 'growth rate' of productivity. This is the *indirect effect* of competition which strictly depends on the presence of heterogeneous firms. Since, trade increases product market competition, triggers firm selection, and promotes innovation, the model is consistent with the patterns of firm exit, innovation, and pricing decisions shown in recent empirical research.

The welfare effects of trade can also be decomposed into a direct and an indirect channel. Moreover, since innovation leads to endogenous productivity dynamics, each channel yields static and dynamic gains. The direct channel of welfare gains from trade has a static component, coming from the improved market efficiency generated by lower markups, the standard pro-competitive effect, and a dynamic component, related to the increase in firm size which stimulates innovation and productivity growth. The indirect channel, triggered by trade-induced selection, affects welfare positively through a an increase in the average *level* of productivity produced by reallocations of market shares across heterogeneous firms (static component), and through an increase in the *growth rate* of productivity triggered by faster innovation (dynamic component). Moreover, firms' exit due to selection has also a negative effect on welfare by reducing the variety of goods produced in the economy. This negative effect of selection could potentially offset the positive effects discussed above. We explore this possibility in the quantitative analysis.

In an extended version of the model, we endogenize the number of oligopolistic firms competing in each product line introducing a sunk entry cost, and adding a fixed cost of exporting leading to a trade equilibrium in which not all firms export. We calibrate this version of the model to match salient firm-level and aggregate statistics of the US economy, and solve it numerically. We find that trade liberalization increases the number of firms per product, which provides an additional source of markups reduction and an additional selection margin not featured in the baseline model: the reduction in variable trade costs and the increase in the number of firms per product line lower markups for exporters and non-exporters, thus forcing both the marginal domestic producer and the marginal exporter to be more productive. These changes lead to higher long-run growth and welfare: going from the benchmark value for variable cost of 13 percent to free trade increases the annual growth rate from 1.19 to 1.19%. About $1/4^{th}$ of this change is attributable to the indirect effect triggered by selection into the domestic (exit) and foreign market (export), and the associated reallocation of market shares across heterogeneous firms. Similarly, this reduction in the trade cost leads to a long-run welfare gain corresponding to 8% of lifetime consumption, about one half of which is accounted for by firm exit and export decisions and their interaction with innovation. Hence, the presence of heterogeneous firms accounts for a non-negligible share of the growth and welfare effects of trade liberalization.

This paper is related to the emerging literature on the joint effect of trade liberalization on selection and innovation. A first line of research introduces a one-step technological upgrading choice into an heterogeneous firm framework. Examples are Yeaple (2005), Costantini and Melitz (2007), Bustos (2010). Atkeson and Burnstein (2010) set up a dynamic model of process and product innovation with firm heterogeneity and show that trade has positive effects on process innovation that can be offset by negative effects on product innovation (entry).[3] While Atkeson and Burstein (2010) does not feature long-run growth, Baldwin and Robert-Nicoud (2008) and Gustaffson and Segerstrom (2008) introduce firm heterogeneity in an endogenous growth model of expanding varieties (Romer, 1990), showing that the effect of trade-induced firm selection on innovation and growth depends on the form of (international) knowledge spillovers characterizing the innovation technology. Although differing in the innovation type or in the innovation technology they analyze, these papers adopt a monopolistically competitive market structure with constant markups, thus they cannot account for the pro-competitive effects of trade observed in the data.

Krugman (1979) introduced the pro-competitive effects of trade in a monopolistic competition model, Brander and Krugman (1983) provided a microfundation for those effects in a pioneering model of oligopoly and trade, followed by Venables (1985), Horstman and Markusen (1986).[4] Van Long, Raff and Stahler (2011) analyze a model with oligopoly and firm heterogeneity where firms face a one-shot R&D decision affecting their productivity before entering the market. Their model is static, thus not allowing for the analysis of productivity dynamics and its interactions with trade. Recently, Melitz and Ottaviano (2008) jointly study the pro-competitive and selection effects of trade on welfare, using a linear demand system to obtain endogenous markups under monopolistic competition. We add a third channel of welfare gains: the dynamic gains coming from innovation and growth. Each one of these channels has been previously studied in the literature. In line with Melitz and Ottaviano, our contribution is not to uncover a new channel of welfare gains, but to provide a unified framework to analyze them jointly. We also differ from their paper by providing a quantitative evaluation of these channels, and by obtaining the pro-competitive effects from strategic interaction among oligopolistic firms.

A new line of research has analyzed whether the presence of firm heterogeneity and the related selection channel in recent trade models such as Melitz (2003) leads to welfare gains from trade larger than those obtainable in standard models with a representative firm such as

[3]Benedetti (2009) finds positive effects of trade liberalization on both types of innovation. Klette and Kortum (2004) and Mortensen and Lentz (2008) introduce a dynamic industry model with heterogeneous firms into a quality ladder growth model (Grossman and Helpman, 1991). They limit the analysis to the interaction between firm heterogeneity and creative destruction in closed economy, without exploring the effects of trade. Haruyama and Zhao (2008) explore the interaction between trade liberalization, selection and creative destruction in a quality ladder model of growth.

[4]See Neary (2003) and Eckel and Neary (2009) for recent applications, and Neary (2010) for a review of the literature on oligopoly and trade.

Krugman (1980). Arkolakis, Costinot, and Rodriguez-Clare (2012) show that welfare gains in a wide class of old and new trade models depend only on the change in trade share and on the Armington elasticity of trade to changes in trade costs. Atkeson and Burstein (2010) perform a similar exercise but adding innovation-driven productivity dynamics, and showing that the overall welfare contribution of "new" margins - exit, export and process innovation decisions - must be offset by changes in product innovation (entry). A key difference between the two experiments is that while Atkeson and Burstein study the welfare effects of a given change in trade costs across models, Arkolakis et al. analyze the welfare gains of a change in trade costs, provided that this change produces the same change in trade shares across models. We follow Atkeson and Burstein in focusing on the effects of changes in trade costs, and complement their analysis by introducing pro-competitive effects of trade and endogenous growth. Restricting our analysis to the steady state, our quantitative results show that the new firm-level margins can have non-negligible aggregate effects on long-run welfare and growth. Similarly, Alessandria and Choi (2007) find sizable welfare gains attributable to the response of heterogeneous firms to trade liberalization in a business cycle model with constant markups and exogenous productivity dynamics. Edmond, Midrigan and Xu (2012) find substantial welfare gains due to reallocation in a quantitative trade model with heterogeneous firm and pro-competitive effects.

Section 2 describes the baseline model with an exogenous number of competitors and studies its autarkic equilibrium. Section 3 analyses the equilibrium with two symmetric countries incurring in an iceberg trade costs but without fixed export costs. The baseline model is extended in Section 4, allowing for an endogenous number of oligopolistic firms in each product line and for selection into the export market driven by fixed export costs. Section 5 presents the calibration of the generalized model and a numerical simulation of the effects of trade liberalization on innovation and welfare. Section 6 concludes.

2 Baseline Model

This section presents a simple version of the model economy designed to point out the main properties of the suggested theory. The general version of the model used to approach the data is developed in section 4.

2.1 Economic Environment

The economy is populated by a continuum of identical consumers of measure one. Time is continuous and denoted by t, with initial time $t = 0$. Preferences of the representative consumer are

$$U = \int_0^\infty (\ln X_t + \beta \ln Y_t)\ \mathrm{e}^{-\rho t}\ \mathrm{d}t, \tag{1}$$

with discount factor $\rho > 0$. There are two types of goods: a homogeneous good, taken as the numeraire, and a differentiated good. Consumers are endowed with a unit flow of labor, which can be transformed one-to-one into the homogeneous good. This implies that equilibrium wages are equal to one. The amount Y of the labor endowment is allocated to homogeneous good production, which enters utility with weight $\beta > 0$.

The differentiated good is produced with a continuum of varieties of endogenous mass $M_t \in [0, 1]$, according to

$$X_t = \left(\int_0^{M_t} x_{jt}^\alpha\ \mathrm{d}j \right)^{\frac{1}{\alpha}}, \tag{2}$$

where x_{jt} represents consumption of variety j, and $1/(1-\alpha)$ is the elasticity of substitution across varieties, with $\alpha \in (0, 1)$. Each variety j in $[0, M_t]$ is produced by n identical firms, manufacturing with symmetric technologies perfectly substitutable goods. In a more general framework, the degree of substitution across these n goods may be finite but larger than the degree of substitution across varieties. However, introducing another degree of imperfect substitutability across goods would complicate notation without adding any key insight.

Firms use labor to cover a fixed production cost $\lambda > 0$ and variable costs. Productivity $\tilde{z}_t$ differs across varieties, but firms producing the same variety are assumed to be equally productive. From now, index j will be omitted and varieties will be identified with their productivity. A firm producing a variety with productivity $\tilde{z}_t$ has the following production technology

$$l_t = \tilde{z}_t^{-\eta} q_t + \lambda \tag{3}$$

where l represent labor and q production . Variable costs are assumed to be decreasing in the firm's state of technology, with $\eta > 0$. Notice that this technology structure can capture relevant, although not all, features of firm heterogeneity observed in the data in the following way: first, we imagine to divide the set of firms in groups of n units, those producing the closest possible goods in terms of their substitutability -we assume perfect substitutability, for simplicity- and we call the goods they produce a 'variety'. Second, we assume that the degree of substitutability across varieties is constant; and finally, to keep the model tractable, we assume homogeneity in productivity within varieties, but heterogeneity across varieties. Since we have a continuum of products, this simplified technology structure does not capture only heterogeneity across the 'few' observable sectors in the data, but also the productivity difference across the 'many' firms producing different and imperfectly substitutable goods.

Innovation activities are undertaken by incumbents according to the following technology

$$\dot{\tilde{z}}_t = Ak_t h_t, \tag{4}$$

where h_t represents labor allocated to innovation, $A > 0$ is an efficiency parameter, and k_t is a firm-specific externality which will be defined later. We assume, for simplicity, that all firms producing the same good have the same initial productivity $\tilde{z}_0 > 0$.

Irrespective of their productivity, varieties exit the market at rate $\delta > 0$. Exiting varieties are replaced by new varieties in order for the mass of operative varieties to remain constant at the steady-state equilibrium. Below we derive the equilibrium, restricting the analysis to the steady state.

2.2 Households

The representative household maximizes utility subject to its instantaneous budget constraint. The corresponding first order conditions are

$$Y = \beta E, \tag{5}$$

$$\frac{\dot{E}}{E} = r - \rho, \tag{6}$$

$$p_{jt} = \frac{E}{X_t^{\alpha}}\, x_{jt}^{\alpha-1}, \tag{7}$$

where r is the interest rate and p_{jt} is the price of good j. Total household expenditure on the composite good X is

$$E = \int_0^M p_{jt} x_{jt}\, \mathrm{d}j.$$

Because of log preferences, total spending in the homogeneous good is β times total spending in the differentiated good. Equation (6) is the standard Euler equation implying $r = \rho$ at the stationary equilibrium, and (7) is the inverse demand function for variety $j \in [0, M]$. Variables Y, E, M are also constant in steady state (index t is then omitted to simplify notation).

2.3 Production and Innovation

Firms producing the same variety behave non-cooperatively and maximize the expected present value of their net cash flow:

$$V_{ijs} = \int_s^\infty \pi_{ijt}\; e^{-(\rho+\delta)(t-s)}\; \mathrm{d}t,$$

where $\pi_{ijt} = (p_{ijt} - \tilde{z}_{ijt}^{-\eta})q_{ijt} - h_{ijt} - \lambda$ are profits of firm i producing variety j. We solve this differential game focusing on Nash Equilibrium in open loop strategies. Let $a_{ijt} = (q_{ijt}, h_{ijt})$, $t \geq s$, be a strategy for firm i producing j at time t. Let us denote by a_{ij} firm i strategy path for quantities and innovation. At time s a vector of strategy path $(a_{1j}, ..., a_{ij}, ..., a_{nj})$ is an equilibrium in market j if

$$V_{ijs}(a_{1j}, ..., a_{ij}, ..., a_{nj}) \geq V_{ijs}(a_{1j}, ..., a'_{ij}, ..., a_{nj}) \geq 0,$$

for all firms $\{1, 2, ..., n\}$, where in $(a_{1j}, ..., a'_{ij}, ..., a_{nj})$ only firm i deviates from the equilibrium path. The first inequality states that firm i maximizes its value taking the strategy paths of the others as givens, and the second requires firm i's value to be positive.[5]

The characterization of the open loop Nash equilibrium proceeds as follows: a time s firm producing a particular variety solves (let us suppress indexes i and j to simplify notation)

$$V_s = \max_{[q_t, h_t]_{t=s}^\infty} \int_s^\infty \left[(p_t - \tilde{z}_t^{-\eta})q_t - h_t - \lambda\right] \mathrm{e}^{-(\rho+\delta)(t-s)}\; \mathrm{d}t, \qquad st. \tag{8}$$

[5] We choose the open loop equilibrium because it is easier to derive in closed form solution. The drawback of focusing on the open loop equilibriun is that it does not generally have the property of subgame perfection, as firms choose their optimal time-paths strategies at the initial time and stick to them forever. In closed loop and in feedback strategies, instead, firms do not pre-commit to any path and their strategies at any time depend on the whole past history. The Nash equilibrium in this case is strongly time-consistent and therefore sub-game perfect. Unfortunately, closed loop or feedback equilibria generally do not allow a closed form solution and often they do not allow a solution at all. The literature on differential games has uncovered classes of games in which the open loop equilibrium degenerates into a closed loop and therefore is subgame perfect (e.g. Reingaum, 1982, Fershtman, 1987, and Cellini and Lambertini, 2005). A sufficient condition for the open loop Nash equilibrium to be subgame perfect is that in the first order conditions for a firm the state variable of other firms do not appear. In our model, this condition holds when $\eta = 1$ since in this case the externality k in the FOC (10) does not depend on the productivity of direct competitors.

$$
\begin{aligned}
p_t &= \frac{E}{X_t^{\alpha}}\ x_t^{\alpha-1} \\
x_t &= \hat{x}_t + q_t \\
\dot{\tilde{z}}_t &= A k_t h_t \\
\tilde{z}_s &> 0.
\end{aligned}
$$

In a Cournot game a firm takes as given the path of its competitors' production $\hat{x}_t$, the path of the externality k_t, as well as the path of the aggregates E and X_t, and the exogenous exit rate δ. The first order conditions for the problem above are,

$$\tilde{z}_t^{-\eta} = \theta \underbrace{\frac{E}{X_t^{\alpha}} x_t^{\alpha-1}}_{p_t}, \tag{9}$$

$$1 = \upsilon_t A k_t, \tag{10}$$

$$\frac{\eta \tilde{z}^{-\eta-1}}{\upsilon_t} q_t = \frac{-\dot{\upsilon}_t}{\upsilon_t} + \rho + \delta, \tag{11}$$

where υ_t is the costate variable. From (9), firms charge a markup over marginal costs, with $\theta \equiv (n-1+\alpha)/n$, being the inverse of the markup. This is the well known result in Cournot-type equilibria that the markup depends on the perceived demand elasticity, which is a function of both the demand elasticity and the number of competitors.

Firms producing the same variety are assumed to face the same initial conditions, resulting in a symmetric equilibrium with $x_t = n q_t$. As shown in the appendix, substituting (9) into (2) we obtain the demand for variable inputs

$$\tilde{z}_t^{-\eta} q_t = \theta e\, \frac{z}{\bar{z}} \tag{12}$$

where $e \equiv E/nM$ is expenditure per firm, z is a measure of firm detrended productivity, $z \equiv \tilde{z}_t^{\hat{\eta}}\, e^{-\hat{\eta} g t}$, with $\hat{\eta} \equiv \eta\alpha/(1-\alpha)$, and g is the growth rate of productivity that will be defined below, and

$$\bar{z} \equiv \frac{1}{M} \int_0^M z_j \; \mathrm{d}j$$

is the average of detrended productivity z. Notice that the amount of resources allocated to a firm in (12) is the product of average expenditures per firm, the inverse of the markup and the relative productivity of the variety the firm produces. When the environment becomes more competitive, θ increases, prices lower, produced quantities increase and firms demand more inputs. Moreover, (12) shows that more productive firms produce more.

Let us now define the externality k_t,

$$k_t = \frac{\bar{z}}{z}\tilde{z}_t, \tag{13}$$

The externality has two components. First, there is a spillover effect coming from the productivity of direct competitors, as represented by $\tilde{z}$. Similar assumptions are standard in innovation-driven growth models to obtain positive growth in the long run (e.g. Aghion and Howitt, 1992, Grossman and Helpman 1991, and Romer, 1990). Second, the term $\bar{z}/z$ introduces increasing difficulty in innovation. Innovation is more difficult for firms with a productivity above $\bar{z}$, the average productivity in the composite sector. Decreasing returns to innovation allow us to obtain symmetric growth rates across firms, yielding an equilibrium stationary distribution of productivity. This assumption of increasing innovation difficulty is commonly used in R&D-driven growth models to eliminate counterfactual scale effects, and stationarize models with growing populations. Jones (1995), Kortum (1997) and Segerstrom (1998) among others, provide robust empirical evidence supporting the increasing difficulty assumption.

Under this specification of the externality k, the growth rate of productivity

$$g \equiv \frac{\dot{\tilde{z}}}{\tilde{z}} = \eta A\theta e - \rho - \delta, \tag{14}$$

is the same for all $\tilde{z}$. As shown in the appendix, this is obtained using (10), (11), (12) and the definition of k. Equilibrium innovation for firm z can be derived using (4), (14) and the definition of the externality,

$$h = \left[\eta\theta e - \left(\frac{\rho+\delta}{A}\right)\right]\frac{z}{\bar{z}}, \tag{15}$$

where innovation resources h are positively related to the firm's relative productivity $z/\bar{z}$.

The specific form of the externality k allows for the growth rate to be equal across varieties, offsetting the positive effect that the relative productivity has on innovation and growth. Equation (15) shows that more productive firms invest more in innovation. Intuitively, more productive firms are larger, as shown in (12), and since innovation is cost-reducing, a higher production implies higher incentives to innovate. This is consistent with the empirical evidence showing that more productive firms spend more in innovation (e.g. Lentz and Mortensen, 2008, and Aw, Roberts, and Xu, 2010, Akcigit and Kerr, 2011). The assumption of increasing innovation difficulty implies that, although more productive firms innovate more, all firm grow at the same rate in the steady-state equilibrium. Moreover, since there is no innovation in the homogeneous good sector, (1) and (3) imply that the growth rate of output in this economy is

$$g_{out} = (1 - \beta)\eta g.$$

In a stationary equilibrium, all firms grow at the same rate and, as a consequence, their productivities grow at the same rate as the average productivity, implying that their demand for variable inputs, as described by (12), is constant along the balance growth path. More importantly, the result that in a stationary equilibrium productivity grows at the same rate for all firms implies that firms stay in their initial position in the productivity distribution, and the model remains highly tractable.

2.4 Exit and Entry

From the previous section, it can be easily shown that the profit is a linear function of the relative productivity $z/\bar{z}$

$$\pi(z/\bar{z}) = (1-\theta)\, ez/\bar{z} - \underbrace{\left(\eta\theta e - \frac{\rho+\delta}{A}\right) z/\bar{z}}_{h} - \lambda. \tag{16}$$

Produced quantities and innovation effort depend both on the distance from average productivity $z/\bar{z}$. In the following, we assume η to be small enough such that $1 - (1+\eta)\theta > 0$, a sufficient condition for profits to depend positively on z. Let us denote by z^* the stationary cutoff productivity below which varieties exit the market. At a stationary state, the cutoff productivity makes firm's profits and firm's value equal to zero, implying

$$e = \frac{\frac{\lambda}{z^*/\bar{z}} - \frac{\rho+\delta}{A}}{1-(1+\eta)\theta}. \tag{EC}$$

We refer to it as the exit condition, a negative relation between e and z^*.[6]

Next, we assume that there is a mass of unit measure of potential varieties of which $M \in [0,1]$ are operative. We also assume that at each period non operative varieties draw a productivity z from an initial productivity distribution $\Gamma(z)$, which is assumed to be continuous in $(z_{\min}, \infty)$, with $0 \leq z_{\min} < \infty$. Let us denote by $\mu(z)$ the stationary equilibrium density distribution defined on the z domain. The endogenous exit process related to the cutoff point

[6] Notice that problem (8) does not explicitly include positive cash flow as a restriction. By doing so and then imposing the exit condition (EC), we implicitly forbid firms with $z < z^*$ to innovate and potentially grow at some growth rate smaller than g. If they were allowed to do so, they will optimally invest in innovation up to the point in which the cash flow would be zero. In such a case, firms with initial productivity smaller than the cutoff value will be growing at a rate smaller than g, moving to the left of the distribution and eventually exiting. Such an extension would make the problem unnecessarily cumbersome without affecting the main results.

z^* implies $\mu(z) = 0$ for all $z < z^*$. Since the equilibrium productivity growth rates are the same irrespective of z, in a stationary environment, surviving firms remain always at their initial position in the distribution Γ. Consequently, the stationary equilibrium distribution is $\mu(z) = f(z)/(1 - \Gamma(z^*))$, for $z \geq z^*$, where f is the density associated to the entry distribution Γ.

We can now write $\bar{z}$ as a function of z^*

$$\bar{z}(z^*) = \frac{1}{1 - \Gamma(z^*)} \int_{z^*}^{\infty} z f(z) \, \mathrm{d}z. \tag{17}$$

Since varieties exit at the rate δ, stationarity requires

$$(1 - M)(1 - \Gamma(z^*)) = \delta M. \tag{18}$$

This condition states that the exit flow, δM, equals the entry flow defined by the number of entrants, $1 - M$, times the probability of surviving, $1 - \Gamma(z^*)$. Consequently, the mass of operative varieties is a function of the productivity cutoff z^*,

$$M(z^*) = \frac{1 - \Gamma(z^*)}{1 + \delta - \Gamma(z^*)}. \tag{OV}$$

It is easy to see that M is decreasing in z^*, going from $1/(1 + \delta)$ to zero.[7]

2.5 Stationary Equilibrium

The labor market clearing condition can be written as

$$n \int_0^M (l_j + h_j) \, \mathrm{d}j + Y = n \int_0^M \left(\tilde{z}_j^{-\eta} q_j + h_j + \lambda \right) \, \mathrm{d}j + \beta E = 1.$$

The labor endowment is allocated to production and R&D activities in the composite sector, as well as to production in the homogeneous sector. The first equality is obtained substituting l from (3) and Y from (5). Let us change the integration domain from varieties $j \in [0, M]$ to

[7] Note that the entry distribution Γ is assumed to depend on detrended productivity z. This assumption is crucial for the economy to be growing at a stationary equilibrium. Incumbent firms are involved in innovation activities making their productivity grow at the endogenous rate g. This makes the distribution of incumbent firms move permanently to the right. By defining the entry distribution as a function of detrended productivity z, we allow the productivity of entrants to grow on average at the same rate as that of incumbent firms. This is a form of technological spillover or learning-by-doing from incumbents to new entrants, sustaining a stationary equilibrium is a growing economy with exit and entry. A similar assumption has been previously used to support a stationary equilibrium in models of random (exogenous) growth with heterogeneous productivity such as Luttmer (2007), Poschke (2009) and Gabler and Licandro (2007).

productivities $z \in [z^*, \infty]$ and use (4), (12) and (14) to rewrite the market clearing condition as

$$\int_{z^*}^{\infty} \left((1+\eta)\ \theta e\ z/\bar{z} - \frac{\rho+\delta}{A} z/\bar{z} + \lambda \right)\ \mu(z)\ \mathrm{d}z + \beta e = \frac{1}{nM}.$$

Since $\int_{z^*}^{\infty} \mu(z)\, \mathrm{d}z = \int_{z^*}^{\infty} z/\bar{z}\ \mu(z)\, dz = 1$, after integrating over all varieties we obtain

$$e = \frac{\frac{1}{nM(z^*)} + \frac{\rho+\delta}{A} - \lambda}{\beta + (1+\eta)\theta}, \qquad \text{(MC)}$$

a positive relation between e and z^*.

Assumption 1. The entry distribution is such that

$$z^*/\bar{z}(z^*) \text{ is increasing in } z^*, \qquad \text{(a)}$$

and the following parameter restrictions hold:

$$\frac{\bar{z}_e}{z_{\min}} > \frac{\rho+\delta}{A} \qquad \text{(b)}$$

$$1+\eta < \frac{\Psi}{\theta} \qquad \text{(c)}$$

where

$$\Psi = \frac{\frac{(1+\delta)}{n} + \frac{\rho+\delta}{A}(1+\beta) - \lambda\left(1 + \beta\frac{\bar{z}_e}{z_{\min}}\right)}{\frac{(1+\delta)}{n} + \lambda\left(\frac{\bar{z}_e}{z_{\min}} - 1\right)}$$

and $\bar{z}_e$ is the average productivity at entry.

Assumption (a) makes the (EC) curve decreasing in z^*. As discussed in Melitz (2003), many common distributions satisfies condition (a).[8] Assumptions (b) and (c) guarantee that the (EC) curves cuts the (MC) curve from above. Notice that, as in Melitz, if the productivity distribution is Pareto, the (EC) is horizontal and conditions (b) and (c) are sufficient to guarantee the existence of equilibrium.[9]

Proposition 1 *Under Assumption 1, there exits a unique interior solution* (e, z^*) *of (MC) and (EC), with* M *determined by (OV).*

[8] More precisely, condition (a) in assumption 1 is satisfied by the lognormal, exponential, gamma, Weibul, or truncation on $(0, +\infty)$ of the normal, logistic, extreme value, or Laplace distributions. See Melitz (2003).

[9] Consistently with evidence on US firm size distribution (e.g. Axtell, 2001, and Luttmer, 2007), in the quantitative analysis we will assume that firms' size/productivity is distributed Pareto.

Figure 1 provides a graphical representation of the equilibrium.

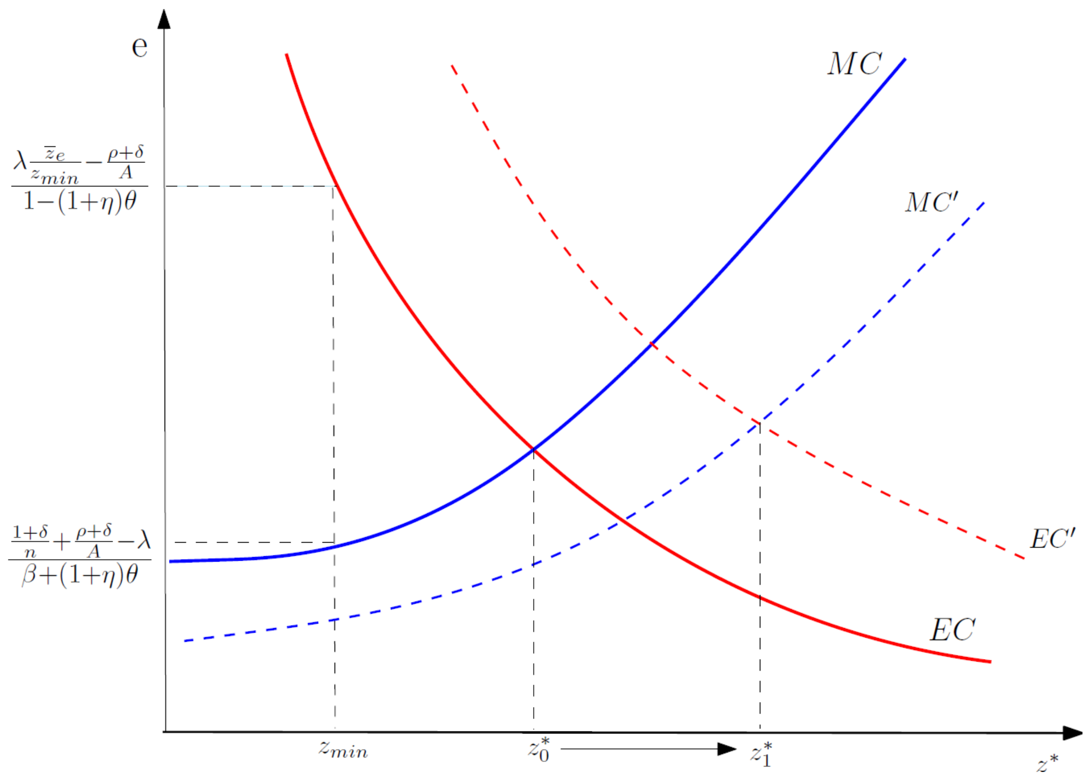


Figure 1. Steady-state equilibrium in closed economy

Next, we provide a first glance at the effects of trade openness by analyzing the effects of an exogenous increase in product market competition, a reduction in the markup rate $1/\theta$ which, as $\theta \equiv (n-1+\alpha)/n$, can potentially be produced by either an increase in the substitutability parameter α, or by an increase in the number of firms n.

Proposition 2 *An increase in θ raises the productivity cutoff z^*, reduces the number of operative varieties $M(z^*)$, has an ambiguous effect on the labor resources allocated to the homogeneous sector e and increases the growth rate g.*

Proof. Figure 1 shows the effect of an increase in the degree of competition (reduction in the markup $1/\theta$) on the equilibrium values of z^* and e. An increase in θ shifts both the (EC) and the (MC) curves to the right, thereby increasing the equilibrium productivity cutoff z^*. Depending on the relative strengths of the shift of the two curves e can increase or decrease, but the average growth rate g always increases. From (14), the effect on g of a change on θ is determined by its effect on θe. Multiplying the market clearing condition (MC) by θ, we obtain θe as a function of θ and $M(z^*)$, and since in equilibrium $M(z^*)$ is decreasing in θ, we can conclude that θe is increasing in θ. ■

Two mechanisms contribute to increasing growth, a direct effect and an indirect effect. In a Cournot equilibrium, an increase in competition reduces markups and allows for an increase in produced quantities; this can be easily seen from (12) which shows that the quantity produced is positively related to θ. The increase in quantities is feasible since the homogeneous good becomes relatively more expensive (i.e. the relative efficiency of the differentiated sector increases), and consumers' demand moves away from it towards the differentiated sector. Since the benefits of cost-reducing innovation are increasing in the quantity produced, the higher static efficiency associated with lower markups affects positively innovation and growth. This mechanism does not depend on firm heterogeneity: it is easy to check that assuming away the dependence of M on z^*, setting $M = 1$, the equilibrium growth rate derived from (MC) and (EC) becomes independent of the cutoff z^*, but is still increasing in θ. This *direct effect* of competition on growth can in fact be found in representative firm models of growth with endogenous market structure (see e.g. Peretto, 1996 and 2003, and Licandro and Navas, 2011).

The *indirect effect* is instead specifically related to the heterogeneous firms structure of the model. A reduction in the markup raises the productivity threshold above which firms can profitably produce, the cutoff z^*, thus forcing the least productive firms to exit the market. As a consequence, market shares are reallocated from exiting to more surviving firms, thereby increasing their market size and their incentive to innovate. Therefore this selection mechanism leads to higher aggregate productivity level and higher innovation and productivity growth.

3 Open Economy

Consider a world economy populated by two symmetric countries with the same technologies, preferences, and endowments as described in the previous section. We assume that trade costs are of the iceberg type: $\tau > 1$ units of goods must be shipped abroad for each unit sold at destination. Costs τ can represent transportation costs or trade barriers created by policy. For simplicity in the baseline model we do not assume entry costs in the export market, thus all surviving firms sell both to the domestic and foreign markets.

3.1 Stationary Equilibrium

Since the two countries are perfectly symmetric, we can focus on one of them. Let q_t and $\tau\check{q}_t$ be the quantities produced by a firm for the domestic and the foreign markets, respectively, and let $q_{x,t}$ be total firm's output . Firms solve a problem similar to that in closed economy (see appendix). The first order conditions are:

$$\tilde{z}_t^{-\eta} = \left((\alpha-1)\frac{q_t}{x_t}+1\right)p_t$$

$$\tau\tilde{z}_t^{-\eta} = \left((\alpha-1)\frac{\check{q}_t}{x_t}+1\right)p_t$$

$$1 = v_t A k_t(\tilde{z}),$$

$$\frac{\eta\tilde{z}_t^{-\eta-1}}{v_t}\underbrace{(q_t+\tau\check{q}_t)}_{q_{x,t}} = \frac{-\dot{v}_t}{v_t}+\rho+\delta.$$

Variable x represents here the total output offered in the domestic market by both local and foreign firms, $x_t = nq_t + n\check{q}_t$. By symmetry it is equal to the total supply in the foreign market. Because of the trade costs, firms face different marginal costs and set different markups for the domestic and foreign markets. Under Cournot competition countries export and import goods that are perfectly substitutable even in the presence of positive variable trade costs.[10]

In the appendix, we show that the first two conditions above yield the following

$$\tilde{z}_t^{-\eta} q_{x,t} = \theta_x e\ z/\bar{z} \tag{19}$$

where $\tilde{z}$ and $\bar{z}$ are defined as in autarky and

$$\theta_x = \frac{2n-1+\alpha}{n(1+\tau)^2(1-\alpha)}\left[\tau^2(1-n-\alpha)+n(2\tau-1)+(1-\alpha)\right] \tag{20}$$

is the inverse of the average markup faced by a firm in both the domestic and foreign market. Notice that θ_x is decreasing in variable trade costs τ, with θ_x reaching its maximum value $\theta_{\max} \equiv (2n-1+\alpha)/2n$ when $\tau = 1$, the polar case of no iceberg trade costs; the autarky value $\theta = (n-1+\alpha)/n$ is reached when $\tau = \bar{\tau} \equiv n/(n+\alpha-1)$, the alternative polar case of prohibitive trade costs implying that both economies do not have any incentive to trade.

[10] This is a standard result in the literature of trade under oligopoly since the pioneering contribution of Brander and Krugman (1983). Intuitively, in imperfectly competitive markets frms equal marginal revenues (not prices) to marginal costs. In the presence of variable export costs, marginal costs of exporting are higher than those of selling domestically. Hence, setting marginal revenues equal marginal costs leads exporters to sell a lower quantity in the foreing market, compared to domestic sales. This leads to intra-industry trade, or "cross-hauling", of highly similar goods. See Neary (2010) for a recent overview of models of oligopoly and trade.

Using the last two first order conditions above and proceeding as in the closed economy, we find that the growth rate of productivity

$$g \equiv \frac{\dot{\tilde{z}}}{\tilde{z}} = \eta A \theta_x e - \rho - \delta \tag{21}$$

takes the same functional form as in the closed economy. Consequently, opening to trade only affects the equilibrium growth rate through changes in the markup θ_x. As in the closed economy case, we focus on the characterization of the steady-state equilibrium. The productivity cutoff is determined by the exit condition

$$\pi(z^*/\bar{z}) = (1-\theta_x)\, e\, z^*/\bar{z} - \underbrace{\left(\eta\theta_x e - \frac{\rho+\delta}{A}\right) z^*/\bar{z}}_{h} - \lambda = 0.$$

which yields

$$e = \frac{\frac{\lambda}{z^*/\bar{z}(z^*)} - \frac{\rho+\delta}{A}}{1-(1+\eta)\,\theta_x}. \tag{EC^T}$$

Since firms compensate their losses in local market shares with their shares in the foreign market, profits are only affected by the change in the markup. Consequently, the exit condition has the same functional form as in (EC) except for θ_x.

The market clearing condition, proceeding as in the closed economy, becomes

$$e = \frac{\frac{1}{nM(z^*)} + \frac{\rho+\delta}{A} - \lambda}{\beta + (1+\eta)\,\theta_x}. \tag{MC^T}$$

which is equal in all aspects to (MC) except for the markup, with θ_x instead of θ. Equations (EC^T) and (MC^T) yield the equilibrium (e, z^*) in the open economy, with $M(z^*)$ determined by (OV). The equilibrium growth is defined by (21).

Proposition 3 *Under Assumption 1 and for $\tau \in [1, \bar{\tau}]$, there exists a unique interior solution $(e, \tilde{z}^*)$ of (MC^T)-(EC^T).*

Proof. At $\bar{\tau} = n/(n+\alpha-1)$ the markups under trade and autarky are equal, $\theta_x = \theta$, and the prohibitive level of trade costs is reached. Thus, for $\tau \geq \bar{\tau}$ firms do not have incentives to export, and trade does not take place. For $\tau < \bar{\tau}$ the proof of existence and unicity is similar to that in the closed economy, and we omit it for brevity. ■

3.2 Trade Liberalization

When countries are symmetric, trade openness does not affect firms' market shares because the reduction in local market sales due to foreign competition is offset by increased participation in the foreign market. For this reason, (MC^T) and (EC^T) are formally equivalent to (MC) and (EC) except for θ. We can then apply proposition 2 to study the effects of trade liberalization. The economy with costly trade is characterized by a level of product market competition higher than in autarky, with $\theta_x > \theta$, due to the participation of foreign firms in the domestic market. A larger number of firms in the domestic market raises product market competition, thus lowering the markup rate. From the definition of θ and the equilibrium value of θ_x we obtain

$$\theta_x - \theta = \frac{\tau (1-\alpha)^2 - n(\tau - 1)^2 (n + \alpha - 1)}{n (1+\tau)^2 (1-\alpha)}, \tag{22}$$

which is positive for any non-prohibitive level of trade costs ($\tau < \bar{\tau}$). Differentiating the expression above it is easy to see that the distance between θ_x and θ is decreasing in τ, which implies that θ_x is decreasing in τ (see appendix). Hence we have two results, first, when a country goes from autarky to costly trade, it experiences an increase in product market competition. Secondly, incremental trade liberalization increases product market competition as well. When trade is completely free, $\tau = 1$, product market competition reaches its maximum level, $\theta_{\max} \equiv (2n - 1 + \alpha)/2n$. Notice that $\theta_{\max}$ has the same functional form as the inverse of the markup in autarky but with the number of firms doubled. Once established that trade reduces markups, it is easy to see that trade liberalization has the same effects on selection and innovation as those produced by an exogenous change in the markup in closed economy of proposition 2 shown in figure 1. These results can be summarized in the following proposition.

Proposition 4 *Trade liberalization, both in the form of moving from autarchy to costly trade and reducing variable trade costs, makes markets more competitive by lowering markups, increases the productivity cutoff z^* and the productivity growth rate g.*

The effect of trade on innovation and growth can be decomposed into two components: a direct effect induced by changes in the markup, which can be obtained also in a representative firm economy, and an indirect effect trigger by firm selection. The *direct effect* following trade liberalization is produced by the reduction of the oligopolistic inefficiency in the differentiated goods sector, which raises the quantity produced by each firm.[11] As innovation is cost re-

[11] Recall that, since countries are symmetric, firms fully compensate the shares lost in the local market by an increase in their exports.

ducing, the marginal benefit from a reduction in costs is increasing in the quantity produced, therefore lower markups trigger higher innovation. The *indirect effect* works through exiting of less productive firms triggered by the reduction in the markup: the market shares of exiting firms are reallocated towards surviving firms, thus increasing their quantity produced and their incentive to innovate. Thus, the selection effect of trade liberalization not only raises the level of productivity as in Melitz (2003) but also its growth rate.

Proposition 4 contains the main predictions of the model matching the three pieces of evidence established by the empirical literature discussed in the introduction, and providing a coherent interpretation. i) Trade liberalization increases product market competition thereby reducing prices and markups. This is the pro-competitive effect of trade found in an extensive body of empirical analysis using firm-level data. ii) The trade-induced reduction in markups forces the less productive firms out of the market, reallocating market shares toward more productive surviving firms, thereby increasing the level of productivity; this is the consensus selection effect found in the data. iii) Reallocating market shares toward surviving firms, trade increases their incentive to innovate, as recently found in several empirical studies discussed in the introduction.

Notice that trade liberalization has an anti-variety effect, it reduces the number of produced and consumed varieties M. This is a consequence of the assumption that there is a perfect overlap between the varieties produced by the two economies. The standard pro-variety effect of trade (e.g. Krugman 1980) could be generated by introducing asymmetry in the set of goods produced by the two countries. However, a model with asymmetric countries would complicate the algebra substantially, without adding much to the main mechanism we want to highlight (the effect of trade-induced selection on innovation and growth). Although such an extension would be relevant in the quantitative analysis of the gains from trade, as we will see later.

Proposition 5 *The growth effect of moving from autarky to costly or free trade is decreasing in n. While the growth effect of incremental trade liberalization is increasing in n.*

As it can be easily seen from (20), the distance between open and closed economy markups is decreasing in n. This implies that opening up to trade is more beneficial, in terms of productivity gains, for less competitive countries. On the other hand, differentiating the absolute value of (20) with respect to n we obtain

$$\frac{\partial\left(|\partial\theta_x/\partial\tau|\right)}{\partial n}=\frac{2(\tau-1)\left(2n-1+\alpha\right)}{n^2\left(1+\tau\right)^3}>0.$$

Hence, once a country has opened to trade, further reductions in trade costs produce larger productivity gains the lower the oligopolistic inefficiency in the domestic market. Summarizing, less competitive closed economies benefit more from opening up to trade, and more competitive open economies experience a higher growth effect of further trade liberalization.

Finally, we analyze the role of firm heterogeneity in shaping the effects of trade on growth and welfare. Similarly to Atkeson and Burstein (2010), changes in variable trade costs have two effects on growth and welfare. The first is the direct effect of a reduction in trade cost attainable keeping the exit decision fixed. The second effect is indirect and comes from the decision of firms to exit and its interaction with the innovation process.[12] From the steady-state equilibrium growth rate (14) and using (MC^T) to substitute for e we obtain

$$\frac{\partial g}{\partial\left(1/\tau\right)}=\underbrace{g_1\frac{\partial\theta_x}{\partial\left(1/\tau\right)}}_{\texttt{Direct Effect}}+\underbrace{g_2\frac{\partial z^*}{\partial\theta_x}\frac{\partial\theta_x}{\partial\left(1/\tau\right)}}_{\texttt{Indirect Effect}}\tag{23}$$

where g_1 and g_2 are obtained differentiating g with respect to θ_x and z^*respectively. As described in propositions 2 and 4, the increase in product market competition θ_x produced by a reduction in τ increases aggregate growth directly through the reallocation of resources from the homogeneous to the differentiated good, and indirectly reallocating resources between firms producing different varieties of the differentiated good. Keeping the decision to exit fixed, z^* and the mass of firms M are constant, the indirect effect disappears and trade does not trigger firm selection. As shown in the appendix, aggregate steady-state welfare can be written as

$$U=\frac{1}{\rho}\left[\ln(\bar{z}^{\frac{1-\alpha}{\alpha}}M^{\frac{1}{\alpha}}\theta ne)+\ln(\beta nMe)\right]+\frac{1}{\rho^2}\frac{1-\alpha}{\alpha}\hat{\eta}g.\tag{24}$$

Using (MC^T) to substitute for e we can break down the welfare effects of a reduction in trade costs into the direct and indirect component,

$$\frac{\partial U}{\partial\left(1/\tau\right)}=\underbrace{u_1\frac{\partial\theta_x}{\partial\left(1/\tau\right)}+u_4g_1\frac{\partial\theta_x}{\partial\left(1/\tau\right)}}_{\texttt{Direct Effect}}+\underbrace{u_2\frac{\partial z^*}{\partial\theta_x}\frac{\partial\theta_x}{\partial\left(1/\tau\right)}+u_3\frac{\partial\bar{z}}{\partial\theta_x}\frac{\partial\theta_x}{\partial\left(1/\tau\right)}+u_4g_2\frac{\partial z^*}{\partial\theta_x}\frac{\partial\theta_x}{\partial\left(1/\tau\right)}}_{\texttt{Indirect Effect}}\tag{25}$$

[12] Notice that Atkeson and Bursten (2010) consider the innovation decision only in the indirect effect, while our model features innovation both in the direct and indirect effect. Moreover, in their indirect effect they include also the firm decision to export. We introduce this decision in the extended model in the following section.

where u_1, u_2, u_3 and u_4 are the derivatives of U with respect to θ_x, z^*, $\bar{z}$ and g respectively. The direct effect, which keeps exit constant, has two parts: the first is produced by a reduction in the markup that reduces prices in the differentiated sector, thus increasing consumers' welfare. This is the standard pro-competitive effect of trade introduced in the literature by Krugman (1979) in reduced form and microfunded by Brander and Krugman (1983). The second is related to the direct effect of a trade-induced increase in competition on innovation and growth. As discussed above, this effect can be found in models of trade and growth with endogenous market structure and homogeneous firms, such as Peretto (2003) and Licandro and Navas (2011). The indirect effect has now two components that can potentially offset each other: on the one hand, as in Meltiz (2003), the reduction in the active mass of goods brought about by selection reduces welfare - first term of the indirect effect in (25). On the other hand, the increase in productivity due to reallocations of market shares toward more productive firms affects welfare positively. The second term of the indirect effect in (25) shows the static effect of trade due to increases in the *level* of productivity, as in Melitz. Finally, the third component features the dynamic gains from trade triggered by the increase in the *growth rate* of productivity, which is specific to our economy with firm heterogeneity and endogenous technical change.

In the next section we provide a quantitative evaluation of the growth and welfare gains from trade and of their transmission channels. Before performing our quantitative analysis we generalize the model along two relevant dimensions: first, we allow vertical entry, that is we assume that in order to enter the market firms pay a fixed cost $\phi > 0$. This implies that the number of firms per product n will be endogenously determined. Second, we introduce a sunk cost of exporting leading to a market structure in which not all firms export, thus introducing the equilibrium decision of firms to serve the export market.

4 Generalization

Following Melitz (2003), we assume that exporting firms face not only a variable trade cost but also a fixed export cost λ_x.[13] Under symmetric countries, this assumption implies that some produced varieties are exported and others not, splitting the set of operative varieties

[13] As in Melitz, this is equivalent to a sunk cost for entering the export market: since productivity is known when firms decide whether to export or not, firms are indifferent on whether to pay a sunk export cost f_x or its annualized value $\lambda_x \equiv f_x/(\rho+\delta)$. Sunk export costs can be costs of setting distribution channels abroad, learning about foreign regulatory system, advertising etc.

into traded and non-traded goods. In equilibrium, markets for non-exporters behave as in the benchmark autarky model, but markets for exporters behave as under the costly trade economy discussed in the previous section. The only difference between these markets is in the markup, $1/\theta$ for non-exporters and $1/\theta_x$ for exporters, with θ and θ_x as defined above. With this difference in mind, we can proceed to solve for firms' quantities and innovation as in the open economy model of section 3.1.

Non-exporter and exporter demands for variable inputs, instead of (12) and (19), become

$$\tilde{z}_t^{-\eta} q_t = \theta e \left(\frac{\bar{p}}{p(z)} \right)^{\frac{\alpha}{1-\alpha}} \tag{26}$$

$$\tilde{z}_t^{-\eta} q_{x,t} = \theta_x e \left(\frac{\bar{p}}{p_x(z)} \right)^{\frac{\alpha}{1-\alpha}}, \tag{27}$$

respectively, where $q_{x,t}$ is now the production of exporting firms including domestic and foreign sales. Detrended prizes derive from (9) and the definition of stationary productivity $z \equiv \tilde{z}_t^{\hat{\eta}} e^{-\hat{\eta} g t}$, yielding $p(z)^{\frac{\alpha}{\alpha-1}} = \theta^{\frac{\alpha}{1-\alpha}} z$ and $p_x(z)^{\frac{\alpha}{\alpha-1}} = \theta_x^{\frac{\alpha}{1-\alpha}} z$. The average detrended price is

$$\bar{p}^{\frac{\alpha}{\alpha-1}} = \left(\theta^{\frac{\alpha}{1-\alpha}} \int_{z^*}^{z_x^*} z\mu(z)\mathrm{d}z + \theta_x^{\frac{\alpha}{1-\alpha}} \int_{z_x^*}^{\infty} z\mu(z)\mathrm{d}z \right).$$

In the particular case where $\theta = \theta_x$, the ratios $(\bar{p}/p_x(z))^{\frac{\alpha}{1-\alpha}}$ and $(\bar{p}/p(z))^{\frac{\alpha}{1-\alpha}}$ become both equal to $z/\bar{z}$ as in (12) and (19) in the baseline formulation of the previous sections.

In order to keep the model stationary, we assume that the externality in the innovation technology follows

$$k_t = \frac{\theta_x e}{\tilde{z}_t^{-\eta} q_t} \tilde{z}_t,$$

instead of (13). After substituting (12) in (13), it is easy to see that this assumption is equivalent to (13) with θ_x instead of θ. Similarly to the baseline model, there is a spillover coming from the productivity of direct competitors, represented here by $\tilde{z}$, and a catching-up component coming from the distance between resources allocated to the consumption of exported goods, $\theta_x e$, and the resources allocated to the production of the $\tilde{z}$ good, $\tilde{z}_t^{-\eta} q_t$. In the case of an exported good, the externality is identical to (13), but in the case of a non-exported good it adds a multiplicative term $\theta_x/\theta > 1$, meaning that spillovers go from exporters facing international competition to non-exporters. In the special case of $\theta_x = \theta$, the externality is equal to that in the baseline model (13).

The first order conditions of firms' dynamic problem are similar to (10)-(11), and using the new definition of k, it can be shown that the growth rate of productivity is the same for exporters and non-exporters and reads

$$g \equiv \frac{\dot{\tilde{z}}}{\tilde{z}} = \eta A \theta_x e - \rho - \delta. \tag{28}$$

The steady-state innovation employment is

$$h_x\left(z\right) = \left(\frac{\bar{p}}{p_x(z)}\right)^{\frac{\alpha}{1-\alpha}} \left(\eta\theta_x e - \frac{\rho+\delta}{A}\right) \tag{29}$$

for exporters and

$$h\left(z\right) = \left(\frac{\bar{p}}{p(z)}\right)^{\frac{\alpha}{1-\alpha}} \left(\eta\theta_x e - \frac{\rho+\delta}{A}\right) \frac{\theta}{\theta_x} \tag{30}$$

for non-exporters. Since $p_x(z) < p(z)$ and $\theta/\theta_x < 1$ we can conclude that exporting firms invest in innovation more than non-exporters. There are two productivity cutoffs, one for exporters, z_x^*, defined by

$$\pi_x(z_x^*) = \left\{\left[1-(1+\eta)\theta_x\right]e + \frac{\rho+\delta}{A}\right\} \left(\frac{\bar{p}}{p_x(z_x^*)}\right)^{\frac{\alpha}{1-\alpha}} - \lambda - \lambda_x = 0,$$

and another for non-exporters

$$\pi(z^*) = \left\{\left[1-(1+\eta)\theta\right]e + \frac{\rho+\delta}{A}\frac{\theta}{\theta_x}\right\} \left(\frac{\bar{p}}{p(z^*)}\right)^{\frac{\alpha}{1-\alpha}} - \lambda = 0.$$

They lead to the following equilibrium cutoffs conditions, respectively,

$$e = \frac{\left(\lambda+\lambda_x\right)\left(\frac{p_x(z_x^*)}{\bar{p}}\right)^{\frac{\alpha}{1-\alpha}} - \frac{\rho+\delta}{A}}{1-\left(1+\eta\right)\theta_x} \tag{XC}$$

and

$$e = \frac{\lambda\left(\frac{p(z^*)}{\bar{p}}\right)^{\frac{\alpha}{1-\alpha}} - \frac{\rho+\delta}{A}\frac{\theta}{\theta_x}}{1-\left(1+\eta\right)\theta}. \tag{EC'}$$

Firms entering the economy pay a fixed entry cost $\phi > 0$ before they observe the productivity z of the good they will produce. Free entry implies that the expected value of the firm must be equal to the entry cost

$$\left(1-\Gamma(z^*)\right)\frac{\bar{\pi}}{(\rho+\delta)} = \phi,$$

where average profits are given by

$$\bar{\pi} = \int_{z^*}^{z_x^*} \left[(p(\tilde{z}) - \tilde{z}^{-\eta})q - h\left(z\right) - \lambda\right]\mu(z)dz + \int_{z_x^*}^{\infty} \left[(p(\tilde{z}) - \tilde{z}^{-\eta})q_x - h_x\left(z\right) - \lambda - \lambda_x\right]\mu(z)dz$$

which yields the following expression for the free entry condition

$$\left(1-\left(1+\eta\right)\bar{\theta}\right)e + \frac{\rho+\delta}{A}\frac{\bar{\theta}}{\theta_x} - \lambda - \frac{1-F(z_x^*)}{1-F(z^*)}\lambda_x = \frac{\rho+\delta}{1-F(z^*)}\phi, \tag{FE}$$

where

$$\bar{\theta} = \theta \int_{z^*}^{z_x^*} \left(\frac{\bar{p}}{p(z)} \right)^{\frac{\alpha}{1-\alpha}} \mu(z) \mathrm{d}z + \theta_x \int_{z_x^*}^{\infty} \left(\frac{\bar{p}}{p_x(z)} \right)^{\frac{\alpha}{1-\alpha}} \mu(z) \mathrm{d}z.$$

Finally, in the market clearing condition we have to take into account that not all firms export, which leads to

$$\int_{z^*}^{z_x^*} \left(\tilde{z}^{-\eta} q + h(z) + \lambda \right) \mu(z) \mathrm{d}z + \int_{z_x^*}^{\infty} \left(\tilde{z}^{-\eta} q_x + h_x(z) + \lambda + \lambda_x \right) \mu(z) \mathrm{d}z + \beta e + \frac{1 - M(z^*)}{M(z^*)} \phi = \frac{1}{nM(z^*)}$$

where $(1 - M(z^*))\phi$ is the amount of labor devoted to entry. From (18) and the definition of $\bar{\theta}$ above, the market clearing condition can be written as

$$\left(\beta + (1+\eta)\,\bar{\theta}\right) e + \left(\lambda + \frac{1 - F(z_x^*)}{1 - F(z^*)} \lambda_x + \frac{\delta}{1 - F(z^*)} \phi \right) - \frac{\rho + \delta}{A} \frac{\bar{\theta}}{\theta_x} = \frac{1 + \delta/(1 - F(z^*))}{n}. \quad \text{(MC')}$$

A stationary equilibrium for this economy is a vector $\{z^*, z_x^*, e, n\}$ solving the system (EC')-(XC)-(FE)-(MC'), with $M(z^*)$ determined by (OV). As shown in the appendix the present value of steady-state welfare can be written as

$$U_\tau = \frac{1}{\rho} \left[\ln \left(\frac{M^{\frac{1}{\alpha}} ne}{\bar{p}} \right) + \beta \ln(\beta n M e) \right] + \frac{1}{\rho^2} \eta g, \tag{31}$$

which is similar to that in the baseline model with the inverse of the average price $1/\bar{p}$ replacing $\theta \bar{z}^{\frac{1-\alpha}{\alpha}}$.

5 Quantitative Analysis

The purpose of this section is twofold. First, we explore numerically the equilibrium properties of the generalized model with endogenous n and fixed export costs, showing that the core results in Proposition 2 hold and that a richer sets of results can be obtained with this extended version. Secondly, we measure the quantitative effects on productivity growth of a reduction in variable trade costs τ, and we asses the relevance of our mechanisms by breaking down the contribution of the direct and the indirect effect to trade-induced productivity and welfare gains.

5.1 Calibration

Similarly to most calibrated models of firm dynamics, we target the US economy, for which micro data are widely available (see i.e. Bernard et al., 2003, Luttmer, 2007, Alessandria-Choi, 2007). Consistent with the available evidence on firm size distribution, we assume that the entry distribution is Pareto with shape parameter κ, and scale parameter $z_{\min}$ (see e.g. Axtell, 2001, and Luttmer, 2007). We have to calibrate 12 parameters $(\alpha, \tau, \delta, \rho, \beta, \eta, A, \lambda, \lambda_x, \phi, \kappa, z_{\min})$. The discount factor ρ is equal to the interest rate in steady state, thus we set it to 0.05 following the business cycle literature. Anderson and Wincoop (2004) summarize the tariff and non-tariff barriers to trade using TRAINS (UNCTAD) data: for industrialized countries tariffs are on average 5% and non tariff barriers are on average 8%. We take the sum of these two costs and set $\tau = 1.13$. Using Census 2004 data, we set $\delta = 0.09$ to match the average enterprise annual death rate in manufacturing observed in period 1998-2004.[14] Rauch (1999) classifies goods into homogeneous and differentiated, and finds that differentiated goods represents between 64.6 and 67.1 percent of total US manufactures, depending on the chosen aggregation scheme. We set $\beta = 0.5$ to get the share of differentiated goods $1/(1+\beta)$ equal to 2/3. We normalize the minimum value of the productivity distribution $z_{\min}$ to 1, without loss of generality.

Parameters (α, η, A, λ, λ_x, κ, ϕ) are jointly calibrated in order to match seven steady-state moments predicted by the model to the corresponding firm-level and aggregate statistics. The annual growth rate of productivity is set equal to 1.19 percent, following evidence in Corrado, Hulten and Sichel (2009).[15] The R&D to GDP ratio is set equal to 2.5 percent, the US average in the post-War period (National Science Foundation, 2011). These targets are relevant in calibrating the technology parameters A and η. Bernard, Jensen, Eaton, and Kortum (2003) using 1992 Census data for US manufacturing firms report the following statistics: first, exporters are about 33 percent more productive than non-exporters on average; second, the standard deviation of firm productivity is 0.75. Using the World Bank World Development Indicators (2011) we compute an export share of output of 7.9 percent in 2009. We target these statistics since they are relevant in determining the fixed costs λ and λ_x, and the shape parameter of the productivity distribution κ. The average markup is set to 22 percent, an intermediate value in the range of estimates reported in Basu (1996), which is useful in calibrating the elasticity parameter α. Finally, Djankov, La Porta, Lopez-de-Silanes, and Shleifer (2002) find that the total regulatory entry cost for the U.S. in 1999 were 1.6 percent of GDP per capita, which we target in order to calibrate the entry cost ϕ. This leads to the following calibrated parameters: $\alpha = 0.6$, $\eta = 0.0655$, $A = 20.1$, $\lambda_x = 0.023$, $\lambda = 0.038$, $\kappa = 3.33$, $\phi = 0.06$. Table 2 shows that

[14] For each year the death rates are computed as follows: taking year 2000 as an example, the death rate is the ratio of firms dead between March 2000 and March 2001 to the total number of firms in March 2000. Data can be downloaded at http://www.sba.gov/advo/research/data.html#ne, file data_uspdf.xls.

[15] Since the model does not include tangible capital, investment in tangible capital has to be subtracted from total income in the data to compute labor productivity. After this adjustment, Corrado, Hulten and Sichel (2009) report an average growth of labor productivity of 1.19% a year in the period 1973-2003.

the calibrated model provides a sufficiently good fit of the targeted statistics.

Table 2. Model fit

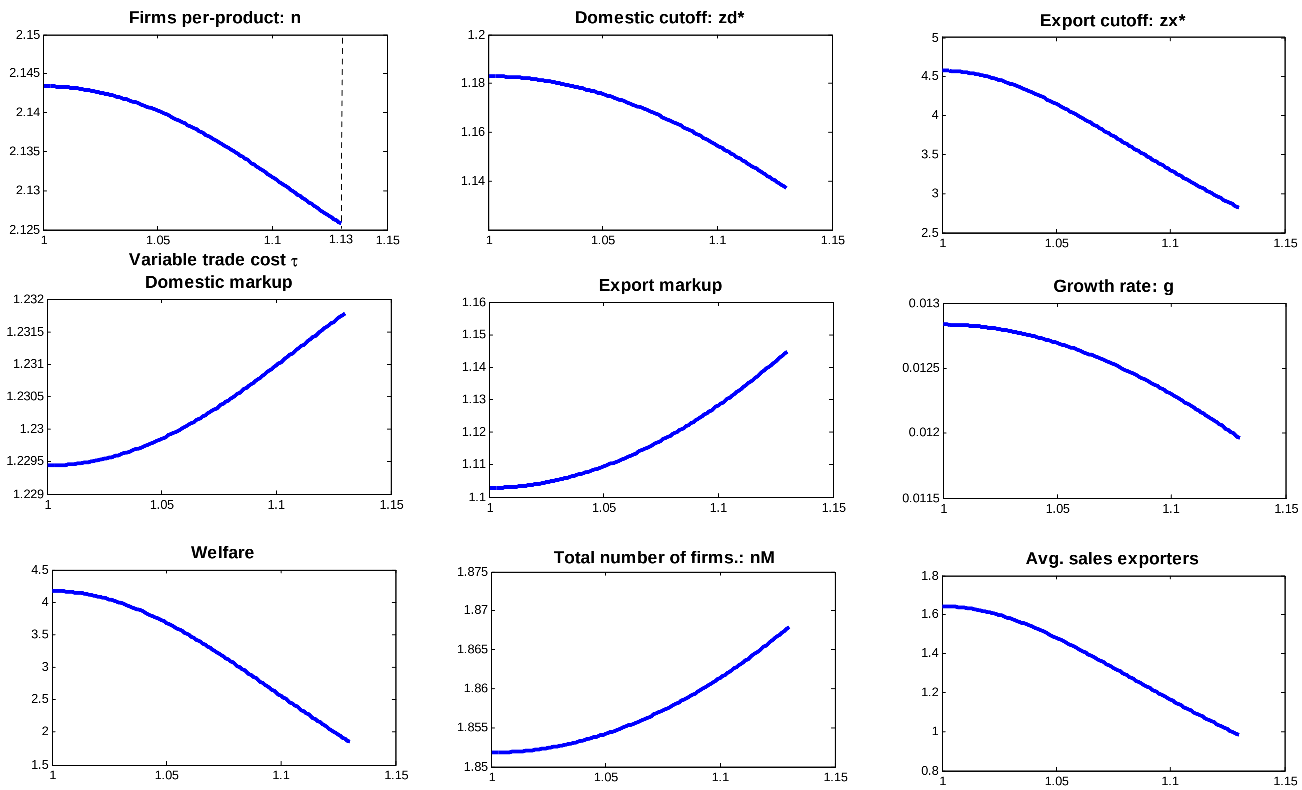
Target	Data	Source	Model
Growth rate (%)	1.19	Corrado et al. (2009)	1.19
Innovation/GDP (%)	2.5	NSF (2011)	2.4
Productivity std.dev.	0.75	Bernard et al. (2003)	0.77
Export share	0.079	WDI (2011)	0.079
Exporters premium	0.33	Bernard et al. (2003)	0.35
Entry costs/GDP (%)	1.6	Djankov et al.(2002)	1.6
Markup rate (%)	22	Basu (1996)	22

The benchmark numerical solution also matches the following constraints. Firstly, the equilibrium number of firms $n = 2.06$ is small enough for the equilibrium prohibitive tariff, $n/(n + \alpha - 1) = 1.24$, to be higher than the calibrated variable trade cost $\tau = 1.13$. Secondly, we check that conditions (b) and (c) in Assumption 1 hold, which as in the simple model make the profit function increasing in z. Under a Pareto distribution, condition (b) becomes $\kappa/(\kappa - 1) > (\rho + \delta)/A$. As expected, the calibrated model yields $z^* > z_{\min} = 1$, implying that not all entrants get the chance to produce profitably, and $z_x^* > z^*$ stating that only the most productive firms export.

5.2 Trade Liberalization

Here, we use the calibrated economy to simulate the steady-state equilibrium response to a reduction in trade costs τ. More precisely, we analyze the response of product market competition, selection and innovation, when the iceberg trade cost goes from its benchmark value to free trade, $\tau \to 1$. Figure 1 below shows the results.

Figure 2: Trade liberalization



The three main results are: firstly, trade liberalization has a positive effect on the number of firms per product n. Second, both domestic and foreign markups are reduced, thus trade increases competition for both exporters and non-exporters. A consequence of tougher competition is that the domestic and the export markets become more selective, which leads to an increase in z^* and z_x^*. Finally, trade liberalization increases steady-state growth and welfare. Since we are doing steady-state welfare comparison, we interpret this exercise as comparing the welfare of two global economies similar in all features except for the variable trade cost.

We now explore the economic mechanism behind these results. A reduction in variable trade costs makes the export markets more competitive, thereby reducing export markups and inducing the marginal exporters to exit the foreign market. As a consequence the productivity cutoff z_x^* increases. Notice that a reduction in the iceberg trade cost affects the export cutoff z_x^* in the opposite direction compared to Melitz (2003). In that paper, countries produce and trade different varieties, implying that there is no direct competition between domestic and foreign goods. Therefore, reducing trade costs implies that exporters benefit from an expansion of their market which leads to larger profits and a lower productivity threshold for exporting. In our model, this effect is more than compensated by the competition effect that reduces profits, thus making the export market more selective.

The entry decision depends on expected profits $(1-\Gamma(z^*))\,\bar{\pi}/(\rho+\delta)$, where $\bar{\pi}$ is an average of domestic and export profits. From the baseline model we know that $\theta < \theta_x$, therefore in the neighborhood of z_x^* the profits of exporters are lower than those of non exporters. A reduction in τ reduces the share of exporters in the economy, thus increasing the average profits for entering firms $\bar{\pi}$, stimulating entry and ultimately leading to a higher equilibrium number of firms per product. Moreover, trade-induced increase in competition produces a reallocation of resources from the homogeneous good to all varieties (exporters and non-exporters) in the differentiated sector. This has an additional positive effect on the average profits and induces more entry. A larger n then reduces the domestic markup $1/\theta$ and raises the domestic cutoff z^*, thereby forcing the least productive domestic firms to exit. Finally, a higher n also strengthens the reduction in the export markup produced by trade liberalization, thus further increasing the export cutoff z_x^*.

Another interesting result is that although lowering trade costs makes the export market more competitive, there are more exporters per variety, each exporting more. In Figure 2, we can see that both the total number of domestic firms producing each variety, n, and the average sales of exporters increase. These two predictions are in line with the empirical evidence on US firms.[16] Interestingly, although trade liberalization increases the level of competition and reduces markups, there is an indirect 'market-size' effect that increases average firm size, sales and profits. Similarly to Melitz and Ottaviano (2008), the endogenous market structure of our model implies that trade liberalization has a positive effect on firms' production that outweigths the direct competition effect on prices and markups and allows surviving firms to be bigger, sell more, and earn higher profits on average.

[16] Bernard, Jensen and Schott (2006) find that a reduction in trade costs increases the volume of export. Bernard, Redding, and Schott (2010) find that the number of firms per product increases with a reduction in trade costs. Although they find that the number of both exporting firms and products increases, with the former incresing more than the latter.

5.3 Decomposing the Growth and Welfare Effect of Trade

In this section we assess the contribution of the direct and indirect channels to the overall growth and welfare effect of trade liberalization shown in (23) and (25). As in the baseline model trade-induced increases in competition affect growth directly by increasing market efficiency and indirectly through firm selection. Compared to the growth decomposition in (23), in this extended model selection operates through two different margins, the production and export cutoffs, which were previously merged in a unique margin. In assessing the contribution to productivity and welfare of the firm level response to trade liberalization we follow Atkeson and Burstein (2010) and perform two exercises: first, we evaluate the contribution of selection, the indirect channel, to trade and growth by computing the effects of a reduction in variable trade costs keeping the decision to exit and export as given and compare them with the results of the calibrated benchmark model. More precisely, we compute the welfare and growth gains from a given reduction in τ in the full model simulated above, and then repeat the same experiment but keeping constant the cutoffs z^* and z_x^*, in order to pin down the direct effect; the indirect effect is then obtained as a residual. Secondly, we compare the gains in the benchmark model with those obtainable in a specification of the model with exogenous exit decision, all firm exporting, and no heterogeneity in productivity. We calibrate this version of the model in order to obtain the same export share and the same growth rate of the benchmark model. This second exercise goes beyond the break down of the direct and indirect channel of gains from trade, and allows for a quantitative comparison between a model of trade and growth with a representative firm and one featuring heterogeneous firms. We perform both these exercises for a marginal reduction in trade costs, with τ going from 1.13 to 1.17 (a 10% reduction), and for more pronounced liberalization, moving τ from its benchmark value of 1.13 to 1, a world without variable trade costs.

A 10% decrease in the iceberg trade cost τ raises the growth rate from its benchmark value 0.0119 to about 0.0121, which represents 1.3% increase. Selection accounts for 22% of the total increase in growth generated by trade liberalization. Therefore about $1/4^{th}$ of the effect on aggregate growth comes from the indirect channel driven by heterogeneous firms' response to liberalization. In the table we also show the robustness of the result to a 20 percent increase from the benchmark of the key parameters.[17]

[17] We have chosen a 20 percent increase from the benchmark because local sensitivity analysis allows us to

Table 2

DECOMPOSING GROWTH AND WELFARE EFFECTS OF TRADE LIBERALIZATION

(Small trade liberalization: $\tau = 1.13 \rightarrow \tau = 1.117$)

	Bench	$\kappa = 4$	$\phi = 0.072$	$\beta = 0.6$	$\lambda = 0.0456$	$\lambda_x = 0.0276$
Growth effect: $-\partial \bar{g}/\partial \tau \Delta \tau$ (%)	1.3	1.32	1.25	1.27	1.15	1.28
Indirect (%)	22	18	19	21	26	21
Welfare effect: ω (%)	1	0.15	0.52	0.06	2.9	1.02
Indirect (%)	31	28	32	30	29	29

With the same procedure used in (25), we break down the overall welfare effect of trade into its direct and indirect component. In order to make welfare comparisons between the stationary solutions of the benchmark economy and the counterfactual economy, as described in the previous section, we use a consumption equivalent measure. Variable ω in table 2 measures the percentage gains in terms of lifetime total consumption of a 10 percent reduction in the iceberg trade cost.[18] Table 2 shows that in the benchmark economy, a 10 percent reduction in τ increases lifetime consumption by 1 percent, about $1/3^{rd}$ of which is attributable to the indirect channel. Hence, the negative impact of exiting varieties on welfare does not completely offset the positive welfare effects generated by improvements in the level and growth rate of productivity.

In table 3 we repeat the exercise for a more substantial reduction in trade costs, going from the benchmark value for variable cost to free trade. Now trade liberalization yields substantial gains: the growth rate increases by about 7 percentage points (from 1.19 to 1.29%) and welfare rises by 8% of equivalent consumption. Resources reallocation across heterogeneous firm still plays an important role, driving about $1/4^{th}$ of the overall growth effect and about one half of the welfare gains.[19]

obtain accurate results. We have also checked the sensitivity of the results when the benchmark parameters are doubled and halved and we obtain similar qualitative results, which are available upon request.

[18] The compensating variation is computed as follows: let us denote by Ω the steady-state equilibrium allocation $\{X, Y\}$, of the benchmark calibration and by $U(\Omega)$ the corresponding present-value welfare function, which results from substituting the equilibrium path in the utility function (1). Let us do the same for the counterfactual economy and denote by $U_c = U(\Omega_c)$ the level of welfare attained at equilibrium, where Ω_c represents the solution path of the counterfactual economy. Finally, let us define our consumption equivalent measure as the real number ω that makes $U(\Omega_\omega) = U_c$, where Ω_ω is the equilibrum allocation $\{\omega X, \omega Y\}$, that results from increasing consumption in the stationary state of the benchmark economy at the rate $\omega - 1$. It measures the percentage gains in terms of lifetime total consumption of comparing the benchmark with the counterfactual economy. From utility (1) it follows

$$\frac{1+\beta}{\rho} \log(\omega) = U(\Omega_c) - U(\Omega).$$

[19] Alessandria and Choi (2007) set up a version of Melitz (2003) with exogenous productivity dynamics and

Table 3

DECOMPOSING GROWTH AND WELFARE EFFECTS OF TRADE LIBERALIZATION

(Large trade liberalization: $\tau = 1.13 \rightarrow \tau = 1$)

	Bench	$\kappa = 4$	$\phi = 0.072$	$\beta = 0.6$	$\lambda = 0.0456$	$\lambda_x = 0.0276$
Growth effect: $-\partial\bar{g}/\partial\tau\Delta\tau$ (%)	7.2	7.2	7	7.1	6.5	7.2
Indirect (%)	25	28	22	24	29	22
Welfare effect: ω (%)	8	0.8	4	0.5	25	8
Indirect (%)	49	30	50	47	49	46

In our third experiment we explore a different way to put the welfare gains from reallocation of resources across heterogeneous firms in perspective. We compare the gains in the benchmark model with those obtainable in a version with exogenous exit decision, all firm exporting, and no heterogeneity in productivity, calibrated to match the same initial export share and growth rate of the benchmark economy. The results reported in table 4 show that in this version of the model a move from the benchmark τ to free trade leads to negligible growth effects and small welfare gains.[20] Precisely, trade liberalization increases the growth rate by less than a percentage point and yields a welfare gain of 1.3% of long-run consumption. Hence, the comparison of these outcomes with those in the benchmark model suggests that introducing firm heterogeneity in a model with oligopoly trade and growth increases the quantitative gains from trade substantially.

Table 4

GROWTH AND WELFARE GAINS ACROSS MODELS

(Large trade liberalization: $\tau = 1.13 \rightarrow \tau = 1$)

	Benchmark	Representative Firm
Growth effect: $-\partial\bar{g}/\partial\tau\Delta\tau$ (%)	7.2	0.09
Welfare effect: ω (%)	8	1.3

We can draw two conclusions from these experiments. First, the results seem to suggest that introducing pro-competitive effects and endogenous growth into a trade model with heterogeneous firms can potentially tame the neutrality result found in Atkeson and Burstein (2010), showing that firm-level response to trade (i.e. selection into domestic and export market) can have non-negligible effects on aggregate innovation and welfare.[21] Second, we show that introducing firm heterogeneity in a model of oligopoly trade and growth (e.g. Peretto, 2003,

calibrate it to the US economy. In line with our results, they find that reducing the variable trade cost from 8% to free trade leads to a welfare gain of 1.03% of lifetime consumption, about 30% of which attributable to firm heterogeneity.

[20] We only report the gains from the large liberalization for brevity.

[21] Atkeson and Burstein (2010) shows that slow transitional dynamics can reduce the welfare effects attributable to the indirect channel even in those specification of their model in which trade has large effects on

and Licandro and Navas, 2011) yields additional gains from trade spurring from the static and dynamic effects of firm selection.

6 Conclusion

In this paper, we built a rich and tractable model of trade with heterogeneous firms and innovation-driven productivity dynamics, in order to account for a set of empirical regularities on the effects of trade liberalization. In our framework, the competition channel is at the roots of the selection and innovation effects of trade. Consistent with the empirical evidence, trade liberalization increases product market competition by reducing markups, drives inefficient firms out the market, selects the most productive firms into the export market, and forces surviving firms to innovate more. Endogenous markups are derive directly from oligopolistic competition among firms. Because of the innovation-driven dynamics of productivity, trade-induced firm selection affects both the level and the growth rate of productivity, leading to static and dynamic welfare gains.

Calibrating the model to match US firm-level and aggregate statistics we show that up to about $1/4^{th}$ of the growth and one half of the welfare effect of a non-marginal reduction in variable trade costs can be attributed to firm selection margins (domestic and export). Thus suggesting that firm-level decisions can play a role in shaping the aggregate response of an economy to trade liberalization.

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steady-state productivity and output. Since we focus on the steady state we cannot check whether the speed of transition affects our quantitative results. Nevertheless, comparing our steady-state results to theirs we find noticeble size differences and less indications of potential neutrality of firm-level responses to trade. Computing the transitional dynamics of the model is certainly an important task for future research.

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A Derivation of equation (12), the stationary growth rate, and the welfare function

Equation (12). Rearranging (9), we obtain $x_t = \tilde{z}_t^{\eta \frac{1}{1-\alpha}} (\theta E / X_t^{\alpha})^{\frac{1}{1-\alpha}}$. Substituting it into (2) yields

$$X_t^{\alpha} = \left(\int_0^M \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j \right)^{1-\alpha} (\theta E)^{\alpha},$$

where $\hat{\eta} \equiv \eta\alpha/(1-\alpha)$. Using this into the expression for x above, we find

$$x_t^{\alpha} = (\theta E)^{\alpha} \, \tilde{z}_t^{\hat{\eta}} \left(\int_0^M \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j \right)^{-\alpha}.$$

Substituting these expressions for x and X into (9), considering that in a symmetric equilibrium $x = nq$, and using the definition of stationary productivity $z \equiv \tilde{z}_t^{\hat{\eta}} \, e^{-\hat{\eta} g t}$ we obtain (12).

Welfare equation. Using the expression for X obtained above and the definition of stationary productivity $z \equiv \tilde{z}_t^{\hat{\eta}} \, e^{-\hat{\eta} g t}$ we get

$$X_t^{\alpha} = \left(M \bar{z} e^{\hat{\eta} g t} \right)^{1-\alpha} (\theta E)^{\alpha},$$

which substituted into the discounted utility (1) and using (5) yields the steady-state welfare function

$$\begin{aligned} U(0) &= \int_0^{\infty} (\ln X_t + \beta \ln Y_t) \, \mathrm{e}^{-\rho t} \, \mathrm{d}t \\ &= \frac{1}{\rho} \left[\ln(\bar{z}^{\frac{1-\alpha}{\alpha}} M^{\frac{1}{\alpha}} \theta n e) + \ln(\beta n M e) \right] + \frac{1}{\rho^2} \frac{1-\alpha}{\alpha} \hat{\eta} g. \end{aligned}$$

Steady-state growth. The stationary growth rate (14) is obtained differentiating (10) with respect to time, which yields $\dot{v}/v = \dot{k}/k = \dot{\tilde{z}}/\tilde{z}$, where the second equality is obtained using $k_t(\tilde{z}) = (\bar{z}/z)\tilde{z}_t$ in which by definition $\bar{z}$ and z are stationary. Plugging $\dot{v}/v = \dot{k}/k = \dot{\tilde{z}}/\tilde{z}$, (12), and $1/v = Ak$ from (10) into (11) we obtain (14).

B Equilibrium existence

Proof of proposition 1. Since M is decreasing in z^*, the (MC) locus is increasing starting at

$$\frac{\frac{(1+\delta)}{n} + \frac{\rho+\delta}{A} - \lambda}{\beta + (1+\eta)\theta},$$

when $z^* = z_{min}$, and going to infinity when z^* goes to infinity. Under Assumption 1(a), the (EC) locus is decreasing, starting at

$$\frac{\lambda \frac{\bar{z}_e}{z_{\min}} - \frac{\rho+\delta}{A}}{1-(1+\eta)\theta},$$

for $z^* = z_{min}$, and going to $[\lambda - (\rho+\delta)/A] \,/\, [1-(1+\eta)\theta]$ when z^* goes to ∞. Assumption 1.b implies $\Psi < 1$ and substituting this into 1.c leads to $1+\eta < 1/\theta$, which guarantees that profits (16) are increasing in productivity z. Since $\Psi < 1$ it is easy to show that 1.c is a sufficient condition for the intercept of the (EC) curve be larger than the (MC) curve at $z^* = z_{min}$, which implies single-crossing of the two equilibrium conditions.

C Firm problem in open economy

Each firm solves the following problem

$$V_s = \max_{\left(q^D_{D,t}, q^F_{D,t}, z_{D,t}\right)_s^\infty} \int_s^\infty \left[\left(p_{D,t} - \frac{1}{\tilde{z}^\eta_{D,t}}\right) q^D_{D,t} + \left(p_{F,t} - \frac{\tau}{\tilde{z}^\eta_{D,t}}\right) q^F_{D,t} - h_{D,t} - \lambda\right] e^{-\int_s^T (r_z+\delta)\ dz}\ dt$$

$$\begin{aligned}
&s.t.\\
&p_{D,t} = \frac{E_{D,t}}{X^\alpha_{D,t}}\, x^{\alpha-1}_{D,t} \qquad \textit{and} \quad p_{F,t} = \frac{E_{F,t}}{X^\alpha_{F,t}}\, x^{\alpha-1}_{F,t}\\
&x_{D,t} = \hat{x}^D_{D,t} + q^D_{D,t} + x^D_{F,t} \quad \textit{and} \quad x_{F,t} = \hat{x}^F_{D,t} + q^F_{D,t} + x^F_{F,t}\\
&\dot{\tilde{z}}_{D,t} = A\ \hat{z}_{D,t} h_{D,t}\\
&z_{D,s} > 0,
\end{aligned}$$

where $p_{j,t}$, $E_{j,t}$ and $X_{j,t}$ are the domestic price, expenditure and total composite good respectively for country $j = D, F$, and q^j_i is the quantity sold from source country i to destination country j. Writing down the current-value Hamiltonian and solving it yields the following first order conditions

$$\left[(\alpha-1)\,\frac{q^D_{D,t}}{x_{D,t}} + 1\right] p_{D,t} = \frac{1}{\tilde{z}^\eta_{D,t}} \tag{32}$$

$$\left[(\alpha-1)\,\frac{q^F_{D,t}}{x_{D,t}} + 1\right] p_{F,t} = \frac{\tau}{\tilde{z}^\eta_{D,t}} \tag{33}$$

$$1 = v_{D,t} A \hat{z}_{D,t}, \tag{34}$$

$$\frac{\eta \tilde{z}^{-\eta-1}_{D,t}}{v_{D,t}} \left(q^D_{D,t} + \tau q^F_{D,t}\right) = \frac{-\dot{v}_{D,t}}{v_{D,t}} + r_t + \delta, \tag{35}$$

Since the two countries are symmetric, $q^D_{D,t} = q^F_{F,t} \equiv q_t$, $q^F_{D,t} = q^D_{F,t} = \breve{q}_t$, $x_{D,t} = x_{F,t} \equiv x_t$, $E_{D,t} = E_{F,t}$, $X_{D,t} = X_{F,t}$, $p_{D,t} = p_{F,t}$. From (32) and (33) and using $q_t/x_t +\ \breve{q}_t/x_t = 1/n$ yields

$$\left[(\alpha - 1)\frac{q_t}{x_t} + 1\right] = \frac{2n - 1 + \alpha}{n(1+\tau)} \equiv \theta_D \tag{36}$$

$$\left[(\alpha - 1)\frac{\breve{q}_t}{x_t} + 1\right] = \tau\frac{2n - 1 + \alpha}{n(1+\tau)} \equiv \theta_F = \tau\theta_D \tag{37}$$

which allows us to rewrite (32) and (33) as follows

$$\theta_D \frac{E_t}{X_t^\alpha}\ x_t^{\alpha-1} = \frac{1}{\tilde{z}_t^\eta} \ \text{ and } \tau\theta_D\frac{E_t}{X_t^\alpha}\ x_t^{\alpha-1} = \frac{\tau}{\tilde{z}_t^\eta}.$$

Multiplying the above equations by q_t and $\breve{q}_t$ and summing up we obtain

$$\frac{q_t + \tau\breve{q}_t}{\tilde{z}_t^\eta} = n\left[\theta_D\frac{q_t}{x_t} + \tau\theta_D\frac{\breve{q}_t}{x_t}\right]\frac{E_t}{n}\left(\frac{x_t}{X_t}\right)^\alpha.$$

Using $x_t = \{[1/\tilde{z}_t^\eta](X_t^\alpha/\theta_D E_t)\}^{\frac{1}{\alpha-1}}$, it is easy to prove that $(x_t/X_t)^\alpha = z/\bar{z}$. From (36) and using $q_t/x_t +\ \breve{q}_t/x_t = 1/n$ we obtain

$$\frac{q_t + \tau\breve{q}_t}{\tilde{z}_t^\eta} = \theta_x e_t \frac{z}{\bar{z}} \tag{38}$$

where

$$\theta_x = \frac{2n - 1 + \alpha}{n(1+\tau)^2(1-\alpha)}\left[\tau^2(1 - n - \alpha) + n(2\tau - 1) + 1 - \alpha\right]$$

is the inverse of the markup in the open economy.

D Exit in open economy

The productivity cutoff is determined solving the following equation

$$\pi_t(\tilde{z}^*) = \left(p_t - \frac{1}{\tilde{z}_t^{*\eta}}\right)q_t + \left(p_t - \frac{\tau}{\tilde{z}_t^{*\eta}}\right)\breve{q}_t - h_t - \lambda = 0$$

Using $p_t = 1/\theta_D\tilde{z}^\eta_{D,t}$ and $h_t = \eta\theta_x e_t\tilde{z}_t - (\rho + \delta)/A$ obtained from (34) and (35) yields

$$\frac{1}{\theta_D}\frac{q_t + \breve{q}_t}{\tilde{z}_t^{*\eta}} - \left(\frac{q_t + \tau\breve{q}_t}{\tilde{z}_t^{*\eta}}\right)(1+\eta) + \frac{\rho+\delta}{A} - \lambda = 0.$$

With the same procedure used to derive (38) we obtain

$$\frac{q_t + \breve{q}_t}{\tilde{z}_t^{*\eta}} = \theta_D e_t\ z_t/\bar{z}_t$$

which, together with (38), yields

$$[1 - (1+\eta)\theta_x]\,e_t z_t^*/\bar{z}_t + \frac{\rho+\delta}{A} - \lambda = 0.$$

This expression is similar to (EC) except for the markup $1/\theta_x$ in the place of $1/\theta$.

E Pro-competitive effect

Differentiating θ_x with respect to τ

$$\frac{\partial \theta_x}{\partial \tau} = -\frac{2(\tau-1)\,(2n-1+\alpha)^2}{n\,(1+\tau)^3\,(1-\alpha)} \leq 0,$$

thus trade liberalization reduces the markup. Moreover, taking the absolute value of this derivative and differentiating it with respect to n we find

$$\frac{\partial\left(\left|\partial\theta^T/\partial\tau\right|\right)}{\partial n} = \frac{2(\tau-1)\,(2n-1+\alpha)}{n^2\,(1+\tau)^3} > 0,$$

which implies that the competition effect of incremental trade liberalization is decreasing in the number of firms n.

F Derivations for the generalized model

Variable costs. We want to derive the variable costs for non-exporters $\tilde{z}^{-\eta}q$ and exporters $\tilde{z}^{-\eta}q_x$. The first order condition for non-exporters will be again (9), simply stating that their price, given by the inverse market demand, is equal to a markup θ over marginal costs $\tilde{z}^{-\eta}$

$$\tilde{z}_t^{-\eta} = \theta\frac{E}{X_t^\alpha}x_t^{\alpha-1} \tag{39}$$

The exporter will solve the same problem as in the open economy version for the benchmark model (section 3), and face a price equal to a markup θ_x over marginal costs $\tilde{z}^{-\eta}$, that is

$$\tilde{z}_t^{-\eta} = \theta_x\frac{E}{X_t^\alpha}x_{xt}^{\alpha-1} \tag{40}$$

Rearranging we obtain $x_t^\alpha = \tilde{z}_t^{\hat{\eta}}\left(\theta E/X_t^\alpha\right)^{\frac{\alpha}{1-\alpha}}$ and $x_{xt}^\alpha = \tilde{z}_t^{\hat{\eta}}\left(\theta E/X_t^\alpha\right)^{\frac{\alpha}{1-\alpha}}$, where $\hat{\eta} \equiv \eta\alpha/(1-\alpha)$. Substituting these into (2) we obtain

$$X_t^\alpha = \left(\theta^{\frac{\alpha}{1-\alpha}}\int_0^{M_d}\tilde{z}_{jt}^{\hat{\eta}}\,\mathrm{d}j + \theta_x^{\frac{\alpha}{1-\alpha}}\int_0^{M_x}\tilde{z}_{jt}^{\hat{\eta}}\,\mathrm{d}j\right)^{1-\alpha} E^\alpha$$

where with a slight abuse of notation we temporarily define the mass of exporting firm M_x and the mass of non-exporters M_d. Substituting back into the expressions for x_t^α and x_{xt}^α yields

$$x_t^\alpha = \frac{\theta^{\frac{\alpha}{1-\alpha}}E^\alpha\tilde{z}_t^{\hat{\eta}}}{\left(\theta^{\frac{\alpha}{1-\alpha}}\int_0^{M_d}\tilde{z}_{jt}^{\hat{\eta}}\,\mathrm{d}j + \theta_x^{\frac{\alpha}{1-\alpha}}\int_0^{M_x}\tilde{z}_{jt}^{\hat{\eta}}\,\mathrm{d}j\right)^\alpha} \text{ and } x_t^\alpha = \frac{\theta_x^{\frac{\alpha}{1-\alpha}}E^\alpha\tilde{z}_t^{\hat{\eta}}}{\left(\theta^{\frac{\alpha}{1-\alpha}}\int_0^{M_d}\tilde{z}_{jt}^{\hat{\eta}}\,\mathrm{d}j + \theta_x^{\frac{\alpha}{1-\alpha}}\int_0^{M_x}\tilde{z}_{jt}^{\hat{\eta}}\,\mathrm{d}j\right)^\alpha}.$$

Plugging these into (39) and (40) and using $z \equiv \tilde{z}_t^{\hat{\eta}} \, e^{-gt}$ and the symmetric equilibrium condition $x = nq$ we obtain (26) and (27).

Welfare. Rearranging (9), we obtain $x_t = \tilde{z}_t^{\eta \frac{1}{1-\alpha}} \left(\theta E / X_t^{\alpha}\right)^{\frac{1}{1-\alpha}}$ for non exporters and $x_{xt} = \tilde{z}_t^{\eta \frac{1}{1-\alpha}} \left(\theta_x E / X_t^{\alpha}\right)^{\frac{1}{1-\alpha}}$ for exporters. Substituting it into (2) yields

$$X_t^{\alpha} = \left(\theta^{\frac{\alpha}{1-\alpha}} \int_0^{M_d} \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j + \theta_x^{\frac{\alpha}{1-\alpha}} \int_0^{M_x} \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j \right)^{1-\alpha} E^{\alpha},$$

where with a slight abuse of notation we temporarily define the mass of exporting firm M_x and the mass of non-exporters M_d. Proceeding as in the benchmark model above, we use the definition of stationary productivity $z \equiv \tilde{z}_t^{\hat{\eta}} \, e^{-g\hat{\eta}t}$ we obtain

$$X_t = \left(M e^{\hat{\eta}gt}\right)^{\frac{1-\alpha}{\alpha}} \frac{E}{\bar{p}} = \frac{enM^{1/\alpha}}{\bar{p}} e^{\frac{1-\alpha}{\alpha}\hat{\eta}gt}.$$

Substituting it into the discounted utility (1) and using (5) yields the steady-state welfare function (31).

G Calibration

Here we derive the moments used in the internal calibration of parameters α, η, A, λ, λ_x, κ, and ϕ. Since the model assumes no productivity growth in the homogeneous good sector, the overall growth rate of labor productivity that we match to the data is

$$\bar{g} = \frac{1}{(1+\beta)} \eta g_{\tau},$$

were g_{τ} is the growth rate of the extended model derived in (28) and η comes from (3). The average R&D/GDP is

$$\bar{h} = \int_{z^*}^{z_x^*} h(z)\mu(z)dz + \int_{z_x^*}^{\infty} h_x(z)\mu(z)dz = \frac{\bar{\theta}}{\theta_x} \left[\eta\theta_x e - \left(\frac{\rho+\delta}{A}\right) \right] nM,$$

where $h(z)$ and $h_x(z)$ are taken from (29) and (30), the equilibrium productivity density under Pareto distribution is $\mu(z) = \kappa z^{*\kappa} z \kappa^{-\kappa-1}$, and the national income is pinned down by the size of population normalized to one. The standard deviation of the productivity distribution is

$$std(z) = \frac{z^*}{\kappa - 1} \left(\frac{\kappa}{\kappa - 2}\right)^{1/2},$$

and the share of exporters is

$$expshare = \frac{1 - F(z_x^*)}{1 - F(z^*)} = \left(\frac{z^*}{z_x^*}\right)^{\kappa}.$$

The average markup is

$$\bar{\mu} = \int_{z^*}^{z_x^*} \frac{1}{\theta}\mu(z)dz + \int_{z_x^*}^{\infty} \frac{1}{\theta_x}\mu(z)dz = \frac{1}{\theta}\left[1 - \left(\frac{z^*}{z_x^*}\right)^{\kappa}\right] + \frac{1}{\theta_x}\left(\frac{z^*}{z_x^*}\right)^{\kappa}.$$

Finally the productivity advantage of exporters is computed as the percentage difference between the average productivity of exporters $\bar{z}_x = \frac{1-F(z_x^*)}{1-F(z^*)} \int_{z_x^*}^{\infty} z\mu(z)dz$ and the average productivity of non exporters $\bar{z} = \int_{z^*}^{z_x^*} z\mu(z)dz$, yielding

$$expremium = \left(\frac{z^*}{z_x^*}\right) - \left(\frac{z^*}{z_x^*}\right)^{\kappa}.$$