Characterizing the Financial Cycle:
Evidence from a Frequency Domain Analysis

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Abstract

This paper introduces parametric spectrum estimation to the analysis of financial cycles. Our contribution is to formally test properties of financial cycles and to characterize their international interaction in the frequency domain. Existing work argues that the financial cycle is considerably longer in duration and larger in amplitude than the business cycle and that its distinguishing features became more pronounced over time. Also, a global cycle, being driven by US monetary policy, is said to be behind national financial cycles. We provide strong statistical evidence for the US and slightly weaker evidence for the UK validating the hypothesized features of the national financial cycle. In Germany, however, the financial cycle is much less visible. Similarly, a US-driven global financial cycle significantly affects national cycles in the UK but not in Germany.

Keywords: Financial Cycle, Business Cycle, Indirect Spectrum Estimation, Bootstrapping Inference.
JEL classification: C22, E32, E44.

1 Introduction

Fluctuations in financial markets play a key role in the macroeconomic dynamics of modern economies, often leading to either significant economic booms or severe economic crises, see e.g. Schularick and Taylor (2012) and Jordà et al. (2017). The last twenty years provide several examples, from the asset market crash in the early 1990s to the 2007/2008 global financial crisis which led the world economy to the brink of a new Great Depression. Against this background, a growing body of literature argues that the cyclical behavior of financial aggregates may be seen not only as a pure reflection of the real side of the economy, but also as the result of underlying changes in the general perception and attitudes towards financial and macroeconomic risk (see e.g. Borio, 2014 for a recent survey). For central banks and macroprudential regulation, the empirical and theoretical characterization of financial cycles has become one of the most central topics, see ECB (2014) and ESRB (2014).

The largest part of existing studies focuses on two aspects. The first aspect is the description of the empirical properties of financial cycles. According to Claessens et al. (2011), the most distinctive features of financial cycles seem to be that their duration is considerably longer and their amplitudes are larger than those of the classical business cycle. These results were obtained through a turning point analysis of financial aggregates using the dating algorithms of Bry and Boschan (1971) and Harding and Pagan (2002). Related studies apply frequency-based filters (Drehmann et al., 2012 and Aikman et al., 2015), model-based filters (Koopman and Lucas, 2005, Rünstler and Vlekke, 2016 and Galati et al., 2016) or wavelet analysis (Verona, 2016 and Ardila and Sornette, 2016).[1] The extended length of the financial cycle is often considered to reflect the build-up of macro-financial instability which contributed most recently to the 2007/2008 financial crisis. Note, however, that the empirical properties of the financial cycle have not yet been formally tested.

The second aspect is the analysis of the synchronization of financial cycles across countries and the possible existence of a global financial cycle. Rey (2015) argues that the global financial cycle is highly correlated with the VIX and mainly driven by US monetary policy through the risk-taking channel. Using a large recursive VAR, she finds that the global financial cycle drives capital flows, asset prices and credit growth around the world. The nexus of of the financial cycle, monetary policy and the business cycle in the US is analyzed in Juselius et al. (2016). Passari and Rey (2015) document a transmission mechanism of US monetary policy shocks on the financial conditions in the United Kingdom. Further results at the global level from a Bayesian VAR are given by Miranda-Agrippino and Rey (2015). Empirical evidence for the risk-taking channel (Borio and Zhu, 2012) of US monetary policy at the domestic level is provided by Bekaert et al. (2013) and Bruno and Shin (2015). Common to most studies of the global financial cycle is that their results are based on a time domain analysis.

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[1]For a Bayesian unobserved components approach see also Barbone Gonzalez et al. (2015).

The contribution of this paper is twofold. First, as an alternative method to describe the properties of financial cycles in the frequency domain, we use the indirect spectrum estimation approach. A distinguishing feature of this method is its estimation efficiency. The estimated parameters and their standard errors allow for formal statistical testing of the cyclical properties of financial variables. We provide a thorough univariate frequency domain analysis of credit, the credit to GDP ratio, house prices and equity prices, all of which are considered key financial cycle indicators by existing work, see e.g. Claessens et al. (2011), Drehmann et al. (2012) and Borio (2014). Including data for the US, the UK and Germany, our study is structured around tests of the following hypotheses which we identify from the literature. a) Financial cycles are longer than business cycles. b) Financial cycles have a larger amplitude than business cycles. c) Financial cycles increased in length over time. d) Financial cycles are primarily a medium-term phenomenon.

Second, we extend the literature by a multivariate frequency domain analysis of the interaction of financial cycles within countries and across countries. In contrast to an analysis in the time domain, our approach gives a more detailed picture and allows not only to analyze whether it exists a relation between financial variables at all, but also whether there is a relation at specific frequencies. For instance, if there was a relation only for a small range of frequencies it may not be discovered in the time domain. Similarly, if a significant relation is found in the time domain, it remains unclear whether the interaction is taking place at high frequencies or - as one may suspect in an analysis of financial cycles - at low frequencies. Using a multivariate indirect spectrum estimation, we can formally test frequency-specific Granger causality. Therefore, our multivariate frequency domain analysis complements and extends most recent time domain studies on the global financial cycle of, for instance, Rey (2015) and Miranda-Agrippino and Rey (2015). In particular, we investigate effects of US monetary policy through the risk-taking channel on financial and real activity in the UK and in Germany.

Our main findings are as follows. We find that the typical duration of the financial cycle, when measured by credit and house prices, has significantly increased in recent times in the United States and the United Kingdom, currently being about 15 years. Also, compared to the business cycle in these countries, we find strong statistical evidence in support of a longer duration and a larger amplitude of the financial cycle. In line with the results of Rünstler and Vlekke (2016), in the case of Germany, distinct characteristics of the financial cycle are, if at all, much less visible. A further clear result holds across all three countries: Equity has cyclical properties very similar to those of GDP and is hence mostly driven by cycles between 2 and 8 years. The multivariate analysis confirms the low frequency nature of credit and house prices. It also shows that these variables mainly interact at very low frequencies and that this interaction is statistically significant. As concerns the effect of US

monetary policy through the risk-taking channel, we find significant Granger causality at very low frequencies from the US VIX on credit and GDP in the UK. For Germany, however, the evidence is again much weaker. The effect of the US is not significant, but a tendency to a figure similar to that for the UK is visible. These findings add to our understanding of the cross-border impact of the risk-taking channel of US monetary policy. Since this channel operates mainly at medium-term frequencies, as our results indicate, and financial cycles have long-lasting effects, as argued also in Juselius et al. (2016), US monetary policy affects on the macroeconomic dynamics of other countries may last beyond the period of a business cycle.

The remainder of the paper is organized as follows. We discuss the methodology in Section 2. In Section 3, we present the data and the empirical results of our univariate analysis as well as the formal hypotheses tests. The multivariate analysis is shown in Section 4. Finally, we draw some conclusions from our study in Section 5.

2 Methodology

2.1 Univariate Spectral Analysis

The methodological starting point is the well-known fact that any covariance-stationary process has a time domain and a frequency domain representation which are fully equivalent.[2] The frequency domain representation of a time series is more suited to analyze its cyclical features. The spectrum of a process can, for instance, be used to derive the contribution of certain cycles to the overall variation of the process.[3] However, a (parsimonious) parametric time domain representation is better suited for estimation. Standard procedures can be used to consistently and efficiently estimate the parameters of a process. In order to combine the advantages of both representations, we apply the indirect spectrum estimation approach. In a nutshell, we do the estimation in the time domain and transform the estimated processes into the frequency domain to analyze their cyclical properties.

Our approach assumes that the data generating process (DGP) can be well approximated in the time domain by an autoregressive moving average (ARMA) model of the form

$$A(L)y_t = \delta + B(L)\varepsilon_t , \quad \varepsilon_t \sim \text{WN}(0, \sigma_\varepsilon^2) , \tag{1}$$

where $A(L)$ and $B(L)$ denote polynomials in the lag operator L of order p and q, respectively. For stable processes, the MA(∞) representation has the form

$$y_t - \mu = \frac{B(L)}{A(L)}\varepsilon_t , \quad \mu = \frac{\delta}{A(1)} . \tag{2}$$

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[2]This was first discussed in Wiener (1930) and Khintchine (1934).
[3]This is because the spectrum represents an orthogonal decomposition of the variance.

Expanding the series $B(L)/A(L)$ into a lag polynomial of order infinity shows that an ARMA representation can be interpreted as a filter of infinite length (which, however, depends only on a finite number of parameters). This specific filter transforms the white noise process ε_t into the observed time series y_t. As further detailed in Appendix B.1, the spectrum of y_t can be derived from the parameters of the process and is given by

$$f_y(\lambda) = \frac{|B(e^{-i\lambda})|^2}{|A(e^{-i\lambda})|^2} f_\varepsilon(\lambda) \ , \tag{3}$$

where $\lambda \in [-\pi, \ \pi]$ is the frequency, $|B(e^{-i\lambda})|^2/|A(e^{-i\lambda})|^2$ represents the transfer function, $f_\varepsilon(\lambda) = \frac{\sigma_\varepsilon^2}{2\pi}$ is the constant spectrum of the white noise process and $i^2 = -1$. Equation (3) represents the main idea of the indirect spectrum estimation and allows to derive the complete spectrum of y_t in a consistent and efficient way.[4] Normalizing the spectrum by the process variance yields the spectral density.[5] Our inference is based on bootstrapped standard errors; see Appendix B.4.

From $f_y(\lambda)$ we can identify which frequency range explains most of the variance of y_t. The spectrum will exhibit a peak at a given frequency if the cyclical variation around that frequency is particularly important for the overall variation of the process. Also, if the spectral mass is more concentrated in a given range around the peak, the corresponding cycle will show a larger amplitude causing more regular swings in the time domain.

Note that a large part of the existing insights into the characteristics of the financial cycle is based either on the analysis of turning points (e.g. Claessens et al., 2011, 2012), frequency-based filter methods (e.g. Drehmann et al., 2012, Aikman et al., 2015) or on direct spectrum estimation (e.g. Schüler et al., 2015). Compared to our approach, the turning point approach requires a pre-specified rule which is applied to an observed time series in order to find local maxima and minima. Frequency-based filter methods require a pre-specified frequency range at which the financial cycle is assumed to operate. Directly estimating spectra requires very long time series to get precise estimates. For a more detailed comparison of our approach to filters and to non-parametric spectrum estimation, see Appendices B.1 and B.3.

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[4]As described in Hamilton (1994, p. 167), "If the model is correctly specified, the estimates [of the ARMA(p,q) model] will get closer and closer to the true values as the sample size grows; hence, the resulting estimate of the population spectrum should have this same property". The approach has also been applied by A'Hearn and Woitek (2001) in an analysis of the historical properties of international business cycles.

[5]The process variance is calculated by numerically integrating the spectrum.

2.2 Multivariate Spectral Analysis

In order to analyze a *common* financial cycle, we use the indirect spectrum estimation approach in a multivariate setup. This allows us to investigate at which frequencies a relationship between certain variables is particularly strong or weak. In order to keep the notation simple, consider the concrete case of a two-dimensional VAR model of order p with variable vector $\mathbf{z}_t = (y_t, \ x_t)'$,

$$\mathbf{z}_t = A_1 \mathbf{z}_{t-1} + \cdots + A_p \mathbf{z}_{t-p} + \mathbf{u}_t \ . \tag{4}$$

The (2×2) coefficient matrices A_j, $j = 1, 2, \ldots, p$ can be estimated consistently and efficiently by ordinary least squares.[6] The two-dimensional error vector $\mathbf{u}_t = (u_y, u_x)'$ is assumed to be white noise with $\mathrm{E}(\mathbf{u}_t) = 0$ and positive definite (2×2) variance-covariance matrix $\mathrm{E}(\mathbf{u}_t \mathbf{u}_t') = \Sigma_\mathbf{u}$.

From the MA(∞) representation $\mathbf{z}_t = (I - A_1 L - \cdots - A_p L^p)^{-1} \mathbf{u}_t$ we can derive the (2×2) spectral matrix:

$$F_\mathbf{z}(\lambda) = (I - A_1 e^{-i\lambda} - \cdots - A_p e^{-ip\lambda})^{-1} \frac{\Sigma_\mathbf{u}}{2\pi} (I - A_1 e^{-i\lambda} - \cdots - A_p e^{-ip\lambda})^{-1'} \tag{5}$$

with $0 \leq \lambda \leq \pi$ and $(I - A_1 e^{-i\lambda} - \cdots - A_p e^{-ip\lambda})^{-1'}$ denoting the complex conjugate. $F_\mathbf{z}(\lambda)$ includes the real valued spectra $f_{xx}(\lambda)$ and $f_{yy}(\lambda)$ and the complex valued cross-spectra $f_{yx}(\lambda)$ and $f_{xy}(\lambda) = f_{yx}(\lambda)$

$$F_\mathbf{z}(\lambda) = \begin{pmatrix} f_{yy}(\lambda) & f_{yx}(\lambda) \\ f_{xy}(\lambda) & f_{xx}(\lambda) \end{pmatrix} \ . \tag{6}$$

The spectra $f_{yy}(\lambda)$ and $f_{xx}(\lambda)$ represent, just as in the univariate case, orthogonal decompositions of the variances of y and x in cyclical components. The spectral densities include information about the variance contributions of cycles at different frequencies.

The cross-spectra $f_{yx}(\lambda)$ and $f_{xy}(\lambda) = \overline{f_{yx}}(\lambda)$ can be used for measuring the strength of the relation between y and x. The squared coherency is given by

$$K_{yx}^2(\lambda) = \frac{|f_{yx}(\lambda)|^2}{f_{yy}(\lambda) f_{xx}(\lambda)} = K_{xy}^2(\lambda) \tag{7}$$

with $0 \leq K_{yx}^2(\lambda) \leq 1$. This measure is analogous to the coefficient of determination R^2 for a linear relation between the cycles of y and x at frequency λ. Note that, in contrast to the R^2, the coherency is invariant to different linear transformations applied to y and x. In particular, this implies that is does not change when going from the relation in levels to the

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[6]In a cointegrated VAR, ordinary least square still yields consistent estimates.

relation in first differences. Further, the coherency does not change when regressing y on x or x on y. Defining the (2×2) matrix

$$B(L) = (I - A_1 L - \cdots - A_p L^p)^{-1} P \tag{8}$$

with $PP' = \Sigma_{\mathbf{u}}$ and P as lower triangular regular matrix, the individual spectral densities of y and x can be written as

$$f_{yy}(\lambda) = \frac{1}{2\pi}(|B_{yy}(e^{-i\lambda})|^2 + |B_{yx}(e^{-i\lambda})|^2) \ . \tag{9}$$

Based on this measure, Geweke (1982, 1984) and Breitung and Candelon (2006) proposed to test for Granger causality at different frequencies using the statistic[7]

$$M_{x \to y}(\lambda) = \ln \left(1 + \frac{|B_{yx}(e^{-i\lambda})|^2}{|B_{yy}(e^{-i\lambda})|^2} \right) \ . \tag{10}$$

It holds that $M_{x \to y}(\lambda) = 0$ if $|B_{yx}(e^{-i\lambda})|^2 = 0$, i.e., when x_t does not Granger cause y_t at frequency λ. The higher the value of $M_{x \to y}(\lambda)$ the stronger the impact of x on y.

2.3 Multivariate Spectral Analysis with Cointegration

If the variables y_t and x_t in (4) are cointegrated, the system can be represented as a vector error correction model (VECM) where the relation between the first differences is separated from the long-run relation between the levels. However, cointegration also implies that the individual time series are non-stationary and thus their spectral densities have a pole at frequency $\lambda = 0$. Needless to say, the information that all variation of a trending variable is explained by a cycle of length infinity is only of limited use.

In contrast, the spectral densities of the first differences are more interesting. In practice, the spectral matrix of $\Delta \mathbf{z}_t$ can be derived in two steps. The first step is to simply estimate the VAR in (4) under the parameter restriction that the variables are cointegrated[8]

$$\mathbf{z}_t = A_1^r \mathbf{z}_{t-1} + \cdots + A_p^r \mathbf{z}_{t-p} + \mathbf{u}_t \ . \tag{11}$$

In the second step the multivariate first difference filter $\Phi(e^{i\lambda})$ is applied to the spectral matrix of the levels. The frequency domain representation of the first difference vector time series is given by

$$F_{\Delta \mathbf{z}}(\lambda) = \Phi(e^{i\lambda}) F_{\mathbf{z}}(\lambda) \Phi(e^{-i\lambda})' \ . \tag{12}$$

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[7]For further details see e.g. Breitung and Schreiber (2017) where also a version of the test is discussed that allows for more detailed null hypotheses.

[8]Since a VECM requires to estimate less parameters than an unrestricted VAR, imposing cointegration on the VAR parameters in (4) increases estimation efficiency.

3 Univariate Empirical Analysis

3.1 Data Description

Studies as English et al. (2005), Ng (2011), and Hatzius et al. (2010) and Menden and Proaño (2017) have used principal components and factor analysis to identify the common factors of a number of financial price and quantity variables for the characterization of the financial cycle. According to Claessens et al. (2011), the core of financial intermediation should be well captured by the three distinct market segments of credit, housing and equity. Further financial variables considered in the literature are, for instance, the debt service ratio, see Juselius and Drehmann (2016). Drehmann et al. (2012) and Borio (2014) argue that the financial cycle can be most parsimoniously described in terms of credit and property prices, a result also found by Schüler et al. (2015) across European countries. Following these studies, in the univariate analysis of this section we use the most common proxy variables for the financial cycle, namely quarterly, seasonally adjusted aggregate data on credit volume, the credit to GDP ratio, house prices and equity prices. To study business cycles, we focus on real GDP. We analyze three industrialized countries: the United States (US), the United Kingdom (UK), and Germany. The US is not only the largest economy in the world, but it also has the worldwide most important financial sector. As representative countries for Europe, we consider the UK due to its leading financial sector and Germany as the largest EMU country.

We employ data transformations similar to those in Drehmann et al. (2012) to allow for a meaningful comparison to existing studies. All series are measured in logs, deflated with the consumer price index and normalized by their respective value in 1985Q1 to ensure comparability of units. Yearly growth rates are obtained by taking annual differences of each time series.[9] The only exception is the credit to GDP ratio which is expressed in percentage points and measured in deviations from a linear trend.[10] We use the longest possible sample for each individual time series which is mostly 1960Q1 until 2013Q4 for the US and the UK. Due to data availability the German time series start only in 1970Q1. The data are shown in Figures 8 to 10 in Appendix A, where they are also described in more detail.

We split the data into two subsamples with break dates closely in line with the literature to

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[9]Our results from the univariate analysis therefore refer to cycles in yearly growth rates. The multivariate analysis in Section 4.1 shows, however, that the results are robust to using quarterly growth rates. Appendix B.2 provides a theoretical discussion of the frequency domain properties of the quarterly and yearly growth rate filters.

[10]Unit root tests of the level series indicate that, with the exception of the credit to GDP ratios which are found to be trend-stationary, all other time series can be considered as integrated of order one. Therefore, working with annual growth rates for GDP, credit, housing and equity, i.e., annually differencing the data, is in line with the unit root test results. In the case of the credit to GDP ratio, we eliminated a deterministic linear trend. Results are available upon request.

analyze possible changes in the characteristics of the financial cycles over time. According to Claessens et al. (2011, 2012) and Drehmann et al. (2012), the break point is specified at 1985Q1 for the US and UK.[11] In the case of Germany, we choose the break point close to the German reunification, 1990Q2.[12] To statistically support the specified break dates which are adopted from the existing literature, we also estimate ARMA models for the full sample period and apply Chow break point tests as well impulse indicator saturation (IIS), an endogenous break point search; see Hendry (2011), Ericsson (2013) and the references therein. Chow tests provide statistical support for a break at 1985Q1 for the US and the UK. For Germany, the evidence for a break at 1990Q2 is weaker, but for GDP and credit we clearly reject the null hypothesis of no break; see Appendix C.1. Nonetheless, the German reunification is a natural break date from an economic point of view. Also, the results from the IIS break point search indicate a break at mid to late 1980s for US and UK and around 1990 for Germany; see Appendix C.2. Finally, as it will be seen in the next subsection, the computed spectra are in most cases of substantially different shape across the subsamples.[13]

In the final part of the paper we use a number of additional variables, also sampled at the quarterly frequency. We use data on the VIX index, the Fed Funds rate as well as the \$/£ and \$/€ exchange rates.

3.2 Univariate Frequency Domain Characterization of Financial Cycle Variables

We begin the discussion of our results by focusing on the spectral representation of the variables resulting from the estimated ARMA models. Further details on a concrete example of estimation and transformation are provided in Appendix B.5. As can be seen in Appendix C.1, all parameters in the final specifications are statistically significant at standard confidence levels and the estimated residuals are free from autocorrelation according to the Lagrange multiplier (LM) test.[14]

The estimated spectral densities are shown in Figures 1 to 3 in the range $[0, \pi/4]$, i.e., for periods of ∞ to 2 years.[15] We do not show frequencies in $[\pi/4, \pi]$ since almost no spectral

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[11]This is often seen as the starting point of the financial liberalization phase in mature economies. Moreover, during this period, monetary policy regimes being more successful in controlling inflation are established, see Borio (2014).

[12]From 1990Q2 on, official data for the unified Germany are available.

[13]In in Figures 8 to 10 in Appendix A, the vertical gray line highlights the sample split. A first visual inspection of the data shows that the credit and house price growth rates exhibit more pronounced swings than GDP growth, particularly in the US and the UK. In Germany, this is only true for housing. In view of equity, the figures illustrate that these series not only feature a high volatility, but their dynamics seem to be very different from the other proxy variables for the financial cycle, credit and housing.

[14]The specification of these ARMA models follows the principle of parsimony. Starting with the specification suggested by standard information criteria, we delete insignificant terms and include additional ones if necessary to guarantee the most important feature of a well-specified model: white noise residuals.

[15]When using quarterly data, all cycles of length infinity to half a year are described by the spectrum in the range from 0 to π because $f_y(\lambda)$ is an even symmetric continuous function. We approximate the continuous

mass is located in this range. The traditional business cycle range of 8 to 2 years corresponds to the frequency interval $[\pi/16\,,\pi/4]$. The financial cycle range $[\pi/64\,,\pi/16]$, as proposed by Drehmann et al. (2012), includes cycles between 32 and 8 years and is highlighted by the gray area. An initial visual inspection of Figures 1 to 3 delivers at least two noteworthy results.

Figure 1: Spectral Densities in the US. Note: Spectral densities in the gray area from frequencies $\pi/64$ to $\pi/16$ correspond to the financial cycle range between 32 and 8 years, as proposed by Drehmann et al. (2012). Frequencies $\pi/16$ to $\pi/4$ correspond to cycles in the traditional business cycle range between 8 and 2 years.

First, especially for the US and the UK, the spectral densities of credit, credit to GDP and house prices are substantially shifted to the left in the later period compared to the first one. This indicates, at least superficially, that longer cycles could have became more present. Moreover, the peaks of the spectral densities of these variables are more pronounced, suggesting that longer cycles may have become more important for the variation of the process, too. For Germany, as illustrated in Figure 3, this is only true for house prices. We obtain no clear results for German credit and credit to GDP. Therefore, it seems that German data do not provide much evidence in favor of the postulated financial cycle properties.

Second, Figures 1, 2 and 3 clearly show that in all three countries the spectral densities of GDP and equity changed very little from the period before the sample split to the one thereafter. The interesting exception is the spectrum of UK GDP growth which may indeed have experienced a change. The general impression, however, is that in almost all cases GDP

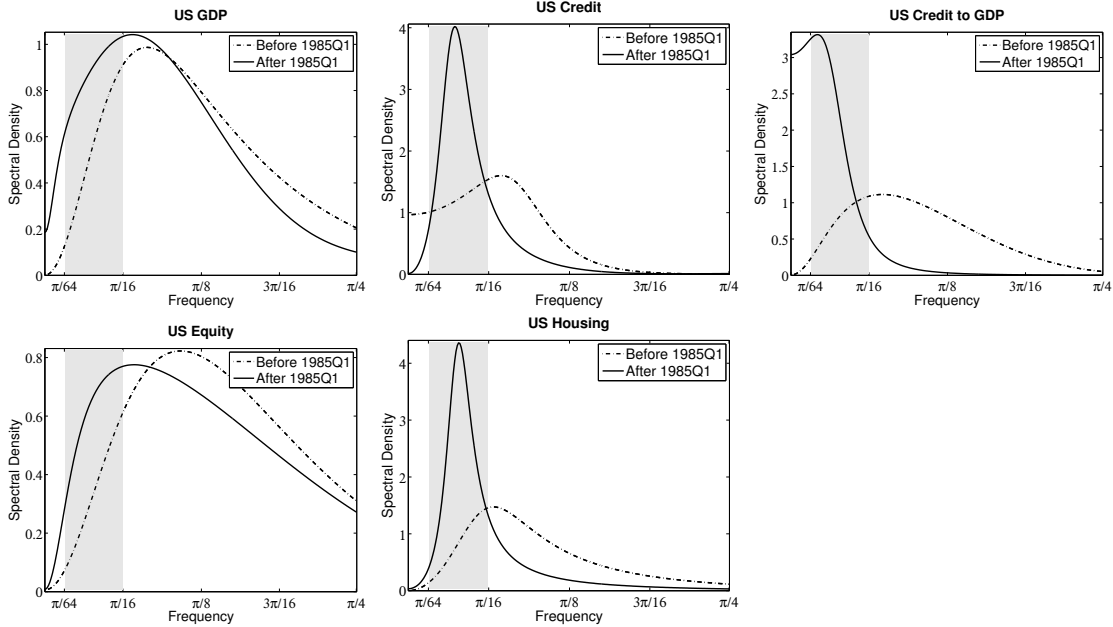spectrum by 1000 equally spaced frequency bands from 0 to π.

Figure 2: Spectral Densities in the UK. Note: Spectral densities in the gray area from frequencies $\pi/64$ to $\pi/16$ correspond to the financial cycle range between 32 and 8 years, as proposed by Drehmann et al. (2012). Frequencies $\pi/16$ to $\pi/4$ correspond to cycles in the traditional business cycle range between 8 and 2 years.

and equity show very similar spectral shapes with less pronounced peaks compared to the other variables.

It remains unclear from the graphical analysis, however, whether any of these properties changed significantly or whether certain variables have different properties. In preparation for the formal hypotheses testing in the next section, we propose four different statistics derived from the spectral densities: cycle length, amplitude, variance contribution of shorter cycles and variance contribution of longer cycles. We use these statistics to describe the main features of business and financial cycles.

Table 1 summarizes the measures for all countries under consideration and the two estimation subsamples. The first two columns include the *main cycle length* in years measured at the peak of the spectrum. It is given by $\frac{2\pi}{\lambda_{\max}}$ with $\lambda_{\max}$ as the frequency where the spectral density has its unique maximum. The remaining columns include the information about the *amplitude* and the variance contributions of certain frequency ranges. These statistics are related to the spectral mass, i.e. the area below the spectral density function, normalized to 1. For example, if the spectral mass between two frequencies, say $[\lambda_{\text{low}}, \lambda_{\text{high}}]$, would equal 0.5, we would conclude that 50% of the underlying time series' variance is explained by cycles in that frequency range. To approximate the variance contribution of the main cycle (the 'amplitude'), we report the spectral mass, measured in percentage points, which is located
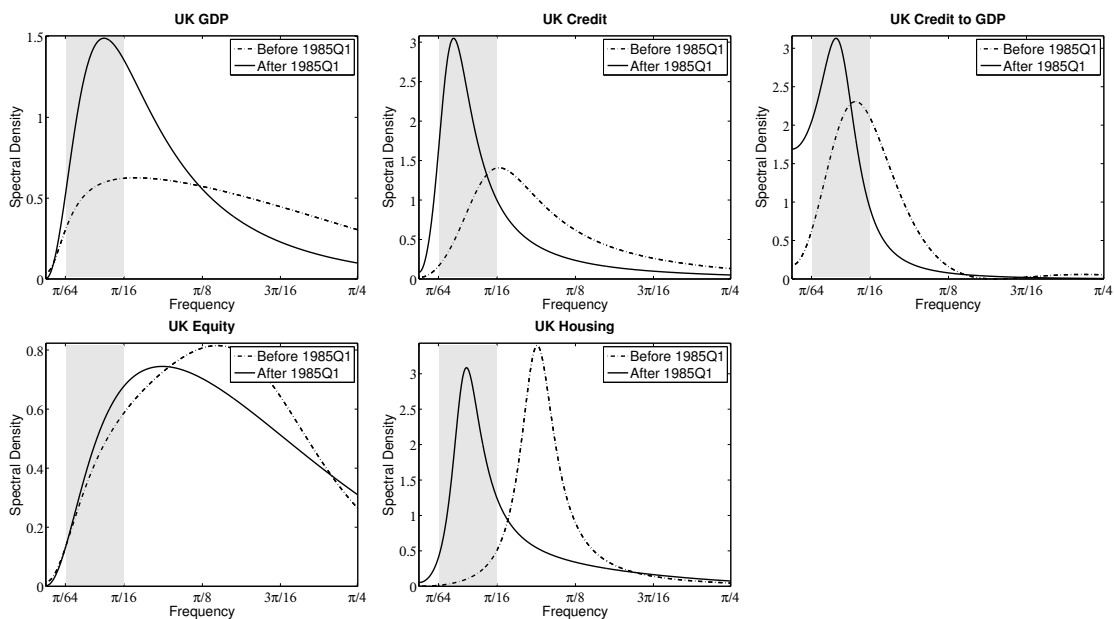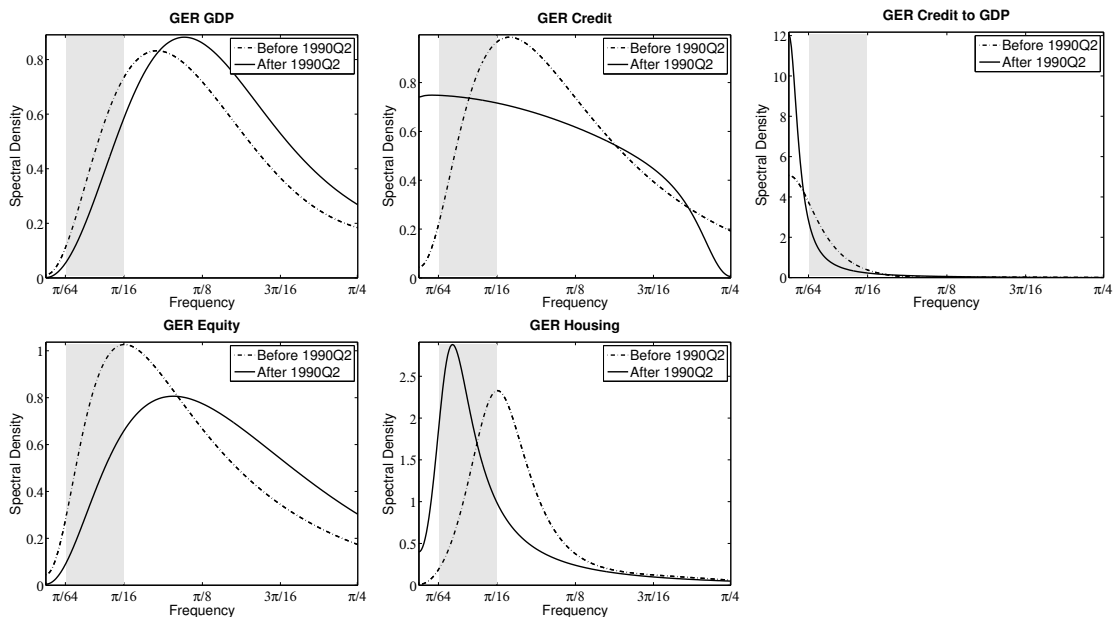
Figure 3: Spectral Densities in Germany. Note: Spectral densities in the gray area from frequencies $\pi/64$ to $\pi/16$ correspond to the financial cycle range between 32 and 8 years, as proposed by Drehmann et al. (2012). Frequencies $\pi/16$ to $\pi/4$ correspond to cycles in the traditional business cycle range between 8 and 2 years.

around $\lambda_{\max}$. We choose a symmetric frequency band with length of about $\frac{\pi}{20}$.[16] We also calculate the spectral mass for pre-defined ranges which are often used in the literature, i.e., 8 to 32 years for the financial cycle and 2 to 8 years for the business cycle. The values in brackets below the point estimates are the 95% bootstrap confidence intervals.

According to the point estimates in Table 1, the duration of the financial may have increased from the first to the second subsample. Averaging over the four financial variables, we find cycles of 6.5, 6.6 and 7.6 years during the first period, and 14.4, 12.7 and 11.7 years during the second period in the US, UK, and Germany, respectively.[17] However, our estimates do not show much of a change in the length of the business cycle. This would be in line with the standard notion that business cycles have a duration between 2 and 8 years, see Hodrick and Prescott (1997).

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[16]Using a band is necessary because the variance contribution of single frequency is zero by definition. The band is defined as $\lambda_{\max} \pm \frac{\pi}{1000} \cdot 25$ and its size therefore equals 5% of all frequencies. In that sense it is referred to as "small". During computation we discretize the frequencies into 1000 steps.

[17]For Germany, cases where we do not obtain finite standard errors are not taken into account.

Table 1: Frequency Domain: Length and Variance Contribution of Cycles

| | | length of main cycle *in years* | | spectral mass *in percent* located... | | | | | |
| | | | | ...at the amplitude of the main cycle | | ...at longer-term cycles (8 to 32 years) | | ...at shorter-term cycles (2 to 8 years) | |
		pre	post	pre	post	pre	post	pre	post
US	GDP	6.2 [4.6, 7.2]	7.1 [4.9, 17.2]	29.9 [22.3, 35.8]	31.9 [27.2, 42.3]	17.2 [9.9, 23.1]	26.8 [10.9, 39.4]	71.4 [66.6, 74.9]	63.3 [45.1, 77.7]
	credit	6.9 [5.7, 12.2]	13.9 [12.8, 14.7]	46.1 [39.1, 59.7]	75.4 [71.9, 78.6]	37.8 [16.9, 48.6]	76.2 [71.2, 79.2]	51.4 [34.9, 80.6]	19.3 [15.4, 23.3]
	credit to GDP	6.8 [5.1, 9.4]	23.8 [14.7, 45.4]	33.7 [27.8, 36.7]	86.4 [75.2, 91.2]	23.8 [13.6, 31.2]	61.8 [49.0, 71.6]	66.0 [54.7, 71.0]	6.8 [4.1, 12.8]
	housing	7.6 [4.0, 9.6]	12.8 [10.2, 14.7]	41.4 [20.6, 72.3]	73.0 [61.4, 81.0]	26.0 [3.3, 37.7]	72.6 [55.7, 80.6]	66.2 [53.5, 89.9]	24.9 [17.5, 41.3]
	equity	4.6 [3.3, 5.6]	7.0 [5.4, 8.5]	25.3 [21.7, 30.9]	23.9 [21.0, 27.1]	10.6 [3.9, 15.0]	19.0 [14.1, 22.3]	74.2 [67.5, 82.0]	65.4 [61.6, 69.3]
UK	GDP	6.9 [6.2, 8.1]	10.9 [8.2, 12.5]	19.4 [19.3, 19.5]	41.8 [32.0, 47.0]	16.3 [14.9, 17.4]	38.3 [25.8, 45.5]	58.6 [57.3, 59.6]	55.8 [48.3, 65.2]
	credit	7.8 [4.6, 9.4]	17.8 [15.1, 20.0]	39.5 [21.4, 54.3]	60.6 [53.7, 64.9]	26.2 [8.1, 38.5]	62.4 [55.1, 65.5]	63.5 [53.5, 77.3]	27.6 [23.0, 35.3]
	credit to GDP	9.8 [5.7, 13.9]	14.3 [8.8, 31.2]	62.1 [46.5, 79.4]	73.2 [52.0, 87.1]	54.5 [18.2, 63.2]	68.3 [42.6, 83.4]	39.7 [29.5, 76.6]	13.1 [6.3, 41.3]
	housing	5.3 [3.7, 6.4]	13.1 [11.1, 14.3]	69.2 [35.0, 96.8]	58.1 [33.5, 72.2]	5.1 [0.3, 16.6]	58.2 [30.0, 72.7]	93.0 [76.8, 99.4]	37.8 [24.9, 63.0]
	equity	3.6 [2.4, 6.4]	5.4 [3.3, 6.8]	25.2 [20.7, 47.9]	23.0 [20.6, 27.1]	12.1 [2.2, 19.5]	14.0 [4.3, 18.1]	75.3 [61.5, 89.5]	68.4 [62.8, 77.2]
GER	GDP	5.6 [5.0, 7.8]	4.5 [2.9, 5.7]	25.4 [20.7, 32.1]	27.0 [21.7, 33.1]	13.8 [11.2, 23.2]	9.4 [3.0, 15.4]	62.2 [58.0, 62.8]	74.2 [66.0, 81.6]
	credit	6.9 [6.1, 8.5]	45.4 [−, −]	29.8 [25.2, 36.9]	23.3 [−, −]	20.7 [16.5, 28.6]	22.2 [−, −]	67.9 [63.6, 69.3]	56.6 [−, −]
	credit to GDP	∞ [−, −]	∞ [−, −]	86.9 [69.7, 94.5]	91.4 [45.2, 96.5]	45.6 [27.7, 51.8]	23.5 [10.1, 54.4]	4.1 [1.7, 10.4]	5.7 [2.2, 41.2]
	housing	7.9 [4.9, 12.8]	18.5 [11.6, 20.8]	59.8 [33.2, 72.8]	60.6 [35.8, 70.3]	39.3 [11.2, 55.8]	60.4 [33.2, 68.7]	59.4 [38.7, 84.4]	27.7 [19.6, 55.0]
	equity	7.9 [4.8, 10.2]	4.9 [3.0, 6.4]	30.7 [18.6, 40.7]	24.9 [21.5, 31.4]	23.78 [10.4, 32.9]	12.0 [2.9, 17.4]	63.0 [56.0, 71.1]	72.5 [65.3, 82.3]

Notes: The terms *pre* and *post* refer to the sample periods 1960Q1 until 1984Q4 and 1985Q1 until 2013Q4, respectively, for the US and the UK. In the case of Germany the samples are 1970Q1 until 1990Q1 and 1990Q2 until 2013Q4. The length of the main cycle is defined as $\frac{2\pi}{\lambda_{max}}$, where λ_{max} is the frequency where the spectral density has its unique maximum. The amplitude of the main cycle is defined as the spectral mass in a "small" frequency band symmetric around λ_{max} with a length of about $\frac{\pi}{20}$. 95% bootstrap confidence intervals are given in brackets. By "−" it is indicated that there is no distinct solution (maximum and percentiles) in the frequency range $0 < \lambda \leq \pi$. Accordingly, a main cycle length of "∞" is sometimes obtained because all spectral mass is located at frequency zero, pointing to a unit root.

3.3 Hypotheses Tests

Let us now see which of the above impressions survive statistical testing. We investigate four distinct hypotheses. a) The financial cycle is longer than the business cycle. b) The financial cycle increased in length over time. c) The financial cycle has a larger amplitude than the business cycle. d) The importance of the financial cycle increase over time.

3.3.1 Hypothesis a): The financial cycle is longer than the business cycle.

As a starting point, we test whether the financial cycle tends to be a medium-term phenomenon with a significantly longer cycle length than that of the business cycle as suggested, by Claessens et al. (2011), Drehmann et al. (2012) and Borio (2014), among others. Table 2

Table 2: Is the Financial Cycle Longer Than the Business Cycle?

		pre break		post break	
		$\widehat{t}$-stat	p-value	$\widehat{t}$-stat	p-value
	credit	0.26	0.399	2.45	0.007
US	credit to GDP	0.47	0.319	0.64	0.260
	housing	0.76	0.224	1.95	0.025
	equity	−1.57	0.942	−0.04	0.514
	credit	0.64	0.259	4.28	0.000
UK	credit to GDP	1.19	0.118	0.26	0.399
	housing	−1.86	0.968	1.72	0.042
	equity	−1.82	0.966	−3.63	0.999
	credit	1.37	0.085	−	−
GER	credit to GDP	−	−	−	−
	housing	0.90	0.185	5.39	0.000
	equity	1.40	0.080	0.34	0.366

H_0: The financial cycle and the business cycle are of equal length.
H_1: The financial cycle is longer than the business cycle.

Notes: $\widehat{t}$-stat represents the estimated value of the t-statistic of a one-sided two-sample t-test. By "−" it is indicated that we could not obtain finite bootstrap standard deviations which implies that it is not possible to conduct a t-test.

reports the test results of the null hypothesis that the financial cycle and the business cycle are of equal length against the alternative hypothesis that the financial cycle is longer than the business cycle. According to the one-sided two-sample t-tests, the null hypothesis cannot be rejected in the first subsample for any of the countries; at least at the 5%-level. This holds also for the second subsample for the credit to GDP ratio and equity. The cycle lengths of credit and housing, however, are significantly longer than the one of GDP. With respect to the information given in Table 1 this is not surprising. In the second subsample the average financial cycle is twice as long as the average business cycle.

3.3.2 Hypothesis b): The financial cycle has a larger amplitude than the business cycle.

Next, we test whether the financial cycle and the business cycle feature the same amplitude in the two subperiods. Table 3 shows a clear result for equity, where the amplitude is never

Table 3: Does the Financial Cycle Have a Larger Amplitude Than the Business Cycle?

		pre break		post break	
		$\widehat{t}$-stat	p-value	$\widehat{t}$-stat	p-value
US	credit	2.64	0.004	6.48	0.000
	credit to GDP	0.92	0.178	7.13	0.000
	housing	0.82	0.207	5.05	0.000
	equity	-1.09	0.861	-1.19	0.884
UK	credit	2.27	0.012	3.97	0.000
	credit to GDP	5.01	0.000	3.19	0.000
	housing	3.01	0.001	1.66	0.049
	equity	0.77	0.219	-4.36	0.999
GER	credit	0.96	0.169	$-$	$-$
	credit to GDP	8.29	0.000	4.80	0.000
	housing	3.17	0.001	3.54	0.000
	equity	0.81	0.209	-0.52	0.698

H_0: The financial cycle and the business cycle have the same amplitude.
H_1: The financial cycle has a larger amplitude than the business cycle.

Notes: The approximate amplitude is defined as the spectral mass in the symmetric frequency band with a length of about $\frac{\pi}{20}$ around $\lambda_{\max}$, where $\lambda_{\max}$ is the frequency where the spectral density has its unique maximum. $\widehat{t}$-stat represents the estimated value of the t-statistic of a one-sided two-sample t-test. By "$-$" it is indicated that we could not obtain finite bootstrap standard deviations which implies that it is not possible to conduct a t-test.

found to be different from the one of GDP. For the other variables we get somewhat mixed evidence in the first sample period, as the null is rejected only in a few cases. In the second period, however, the results are again very clear. Apart from equity, we strongly reject for all variables in all countries. This implies that, particularly in recent times and when taking credit, the credit to GDP ratio and housing as indicators, the financial cycle is characterized by a significantly larger amplitude than the business cycle.

3.3.3 Hypothesis c): The financial cycle increased in length over time.

We now analyze whether the medium-term nature of the financial cycle is a recent phenomenon, i.e., whether its length increased over time. Statistical evidence is shown in Table 4. The results clearly support the hypothesis for credit and housing, both having longer cycles during the second sample period. These results thus provide statistical support for the

Table 4: Did the Financial Cycle Increase in Length Over Time?

H_0: The length of the financial cycle has not changed over time.
H_1: The length of the financial cycle has increased over time.

		$\widehat{t}$-stat	p-value
US	credit	2.32	0.010
	credit to GDP	0.66	0.256
	housing	2.59	0.005
	equity	2.35	0.009
UK	credit	5.69	0.000
	credit to GDP	0.33	0.370
	housing	7.26	0.000
	equity	0.84	0.200
GER	credit	—	—
	credit to GDP	—	—
	housing	3.01	0.001
	equity	−1.74	0.959

Notes: $\widehat{t}$-stat represents the estimated value of the t-statistic of a one-sided two-sample t-test that compares the cycle lengths in the pre and post sample period. By "−" it is indicated that we could not obtain finite bootstrap standard deviations which implies that it is not possible to conduct a t-test.

findings of Claessens et al. (2011), Drehmann et al. (2012) and Borio (2014). However, the tests also show that it should be looked closely at each variable. For the credit to GDP ratio and equity we find almost no evidence for an increase in the cycle length.

3.3.4 Hypothesis d): The importance of the financial cycle increased over time.

Finally, we investigate whether the financial cycle has become more important over time. We address this question by testing whether the main cycle's amplitude has increased, i.e. whether a larger part of the overall process variance is explained by the financial cycle. A higher amplitude would be reflected by a higher peak of the main cycle, implying an increase in its variance contribution. As can be seen from Table 5, the t-tests strongly support the alternative hypothesis of a significant increase for US credit, credit to GDP and housing. In the UK, the t-test results deliver a significant result only for the credit series. Germany seems again to be characterized by different dynamics, as the main cycles' amplitudes of the credit to GDP ratio as well as the growth rates of house prices and equity do not appear to have changed between the two time periods.

One explanation of the different findings for Germany is given by Rünstler and Vlekke, 2016. The private home ownership rate is considerably lower in Germany than in the US and the UK. While we find long cycles in German house prices, they have not become more important

over time. Also, the cycles in credit and credit to GDP and not as pronounced as in the US and the UK.

Table 5: Did the Importance of the Financial Cycle Increase Over Time?

		$\widehat{t}$-stat	p-value
		H₀: The main cycle's amplitude has not changed over time.	
		H₁: The main cycle's amplitude has increased over time.	

H₀: The main cycle's amplitude has not changed over time.
H₁: The main cycle's amplitude has increased over time.

		$\widehat{t}$-stat	p-value
US	credit	5.38	0.000
	credit to GDP	11.06	0.000
	housing	2.18	0.015
	equity	-0.49	0.688
UK	credit	2.28	0.011
	credit to GDP	0.89	0.186
	housing	-0.59	0.722
	equity	-0.28	0.611
GER	credit	$-$	$-$
	credit to GDP	0.31	0.379
	housing	0.06	0.478
	equity	-0.93	0.825

Notes: The approximate amplitude is defined as the spectral mass in the symmetric frequency band with a length of about $\frac{\pi}{20}$ around $\lambda_{\max}$, where $\lambda_{\max}$ is the frequency where the spectral density has its unique maximum. $\widehat{t}$-stat represents the estimated value of the t-statistic of a one-sided two-sample t-test. By "$-$" it is indicated that we could not obtain finite bootstrap standard deviations which implies that it is not possible to conduct a t-test.

Altogether, the results show a statistically significant change of the overall shape of the spectral density over time, particularly for credit and housing. In more recent times, the largest share of spectral mass of the credit and housing series in the US and UK is clearly located at cycles longer than the business cycle. Therefore, our results also support the conjecture of Drehmann et al. (2012) and Borio (2014) that particularly credit and house prices capture the most important features of financial cycles. Credit represents a direct financing constraint to the economy and real estate is one of the assets that is mainly financed by credit. Besides, house prices can be related to the perception of value and risk in the economy. For a detailed discussion of the role of subjective and possibly biased perceptions of value and risk in the housing market, see Geanakoplos et al. (2012).

Our results also highlight that credit and house prices should be at the center of interest when monitoring the financial cycle. To some extend, this is already the case. With the intention of curbing the financial cycle, the European Systemic Risk Board, for instance, recommends the credit to GDP ratio as financial cycle indicator to guide the setting of countercyclical buffer rates, see ESRB (2014). However, our results indicate that the credit to GDP ratio shows only a few of the hypothesized financial cycle properties. This implies that there is considerable

statistical uncertainty associated with the estimation of the frequency domain representation of that time series. It seems that when credit and GDP are considered individually, they are quite informative, but when forming the ratio, the combined measure provides only a mixed picture.

A formal interpretation of that finding is suggested by the representation of credit and GDP in terms of their p reciprocal roots $\{z_1, ..., z_p\}$ of their characteristic equations. Assuming for simplicity that both time series are pure AR(p) processes, they have representations $y_t^{\text{credit}} = (1 - z_1^{\text{credit}})^{-1} \cdot (1 - z_2^{\text{credit}})^{-1} \cdot ... \cdot (1 - z_p^{\text{credit}})^{-1} \varepsilon_t^{\text{credit}}$ and $y_t^{\text{GDP}} = (1 - z_1^{\text{GDP}})^{-1} \cdot (1 - z_2^{\text{GDP}})^{-1} \cdot ... \cdot (1 - z_p^{\text{GDP}})^{-1} \varepsilon_t^{\text{credit}}$. When forming the credit to GDP ratio $y_t^{\text{credit}}/y_t^{\text{GDP}}$, common (or similar) roots in numerator and denominator will (almost) cancel out and it is unclear what kind of dynamics the resulting polynomial actually shows. Therefore, the cyclical properties of the credit to GDP ratio are very difficult to interpret.

A clearer result of our univariate analysis is that for none of the countries considered equity shows any of the financial cycle characteristics. This finding is consistent with (near) market efficiency. If equity prices are a good approximation of all the information available to market participants, the strength of serial correlation should be limited.

4 Multivariate Empirical Analysis: Characterizing a Common Financial Cycle in the Frequency Domain

4.1 Credit and House Prices Interaction

A widespread conjecture is that there exists *one* financial cycle that is common to several financial variables. If the common financial cycle was important, variables which are affected by it should *first* exhibit similar dynamics and, *second*, should be related to each other at low frequencies. As to the first aspect, the findings from our previous univariate analysis indicate that similar cyclical properties are only found for credit and house prices, which both show long cycles with large amplitudes. Therefore, we start our multivariate analysis by looking at these two variables within each country.[18]

As to the second aspect, it remains the question where to suspect a low frequency relation; between the levels of credit and house prices, the growth rates or both? Since the levels of credit and house prices are non-stationary, a cointegration relation would represent the interesting case of a common long-run trend, i.e. a relation at the lowest possible frequency $\lambda = 0$. However, a probably more popular view in the literature is to think of the common cycle as a low frequency common component of very persistent but still stationary financial

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[18]Note that a multivariate analysis is compatible with a univariate study. Even if the true data generating process was a VAR, there still exists a univariate ARMA representation of each time series in the system, see e.g. Maravall and Mathis (1994).

variables, like the growth rates of credit and house prices. Evidently, focusing only on the growth rates could lead to the conclusion that no relation exists while in fact there is a relation in levels. The same holds when focusing only on levels.

Therefore, we estimate vector error correction models (VECM) in the time domain to have a representation that separates the relations in levels from the one in first differences and allows us to test for cointegration. After transforming the VECMs into the frequency domain, we consider the coherency measure. A convenient feature of the coherency is that it is invariant to linear transformations and therefore remains unchanged when moving from the levels to first differences. Therefore, we can also use the coherency for the first differences to check at which frequencies we have the strongest correlation between house price and credit growth.

Note that the first differences in the VECMs represent *quarterly* growth rates. This is another interesting aspect, because it allows us to check whether our results from the univariate analysis of yearly growth rates are robust to using quarterly growth rates.

A formal test of the hypothesis of a common cycle, for example in the series of US credit and house prices, is to check whether there is significant interaction between the levels and the first differences of these variables. However, this would only be a necessary condition. The sufficient condition would be that the interaction takes place at low frequencies. We start with an analysis of the levels. Table 6 shows cointegration test results. In all countries

Table 6: Cointegration Tests

			pre				post	
	H_0 (rank)	lags	trace test	$\lambda_{\max}$-test	H_0 (rank)	lags	trace test	$\lambda_{\max}$-test
US	$r = 0$	4	35.4 (0.002)	29.1 (0.001)	$r = 0$	5	28.0 (0.027)	21.1 (0.028)
	$r \leq 1$	4	6.3 (0.419)	6.3 (0.419)	$r \leq 1$	5	6.9 (0.355)	6.9 (0.355)
UK	$r = 0$	3	24.1 (0.083)	17.0 (0.106)	$r = 0$	2	23.3 (0.102)	18.0 (0.078)
	$r \leq 1$	3	7.0 (0.343)	7.0 (0.343)	$r \leq 1$	2	5.2 (0.562)	5.2 (0.562)
GER	$r = 0$	4	36. (0.002)	27.2 (0.003)	$r = 0$	4	26.0 (0.048)	20.2 (0.038)
	$r \leq 1$	4	9.0 (0.179)	9.0 (0.179)	$r \leq 1$	4	5.8 (0.483)	5.8 (0.483)

Notes: This table shows the Johansen (1995) cointegration test results. We use two test statistics, the $\lambda_{\max}$ and the trace statistic. The lag length of each VECM is specified by using standard information criteria and residual autocorrelation tests to confirm that the lag order is sufficient to guaranty white noise residuals. Since all series are trending, the deterministic terms in the VECMs allow for different drifts of the time series.

and for both sample periods, we find one significant cointegration relation between credit and house prices. We also obtain clear-cut evidence against a second cointegration relation, which formally implies that the data are in fact non-stationary and contain one common stochastic trend. The long-run equilibrium relation of these two variables is also clearly visible in the frequency domain when regarding the coherencies in Figure 4. As implied by cointegration, at frequency $\lambda = 0$ the coherency is equal to 1.

Figure 4 is also very informative about the interaction at cycles of finite length. It can be seen that for the US and UK the center of correlation has shifted from business cycle frequencies before the sample split to much lower frequencies thereafter. For the case of Germany, the coherency remains more stable.
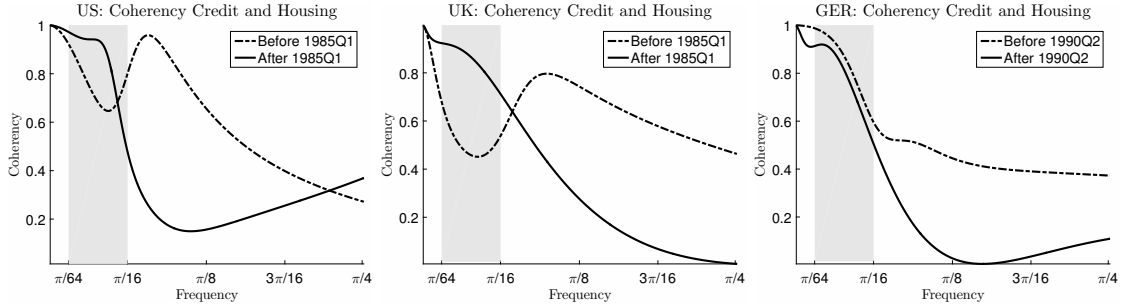

Figure 4: Coherency Between Credit and House Prices. Note: This Figure shows the coherency between credit growth and house prices. Since the coherency is invariant to linear transformations, it applies to the levels and to the first differences of the time series.

Results for the dynamics of each quarterly growth rate are provided in Figure 5. We derived the individual spectral densities using the estimated VECM parameters as outlined in Section 2.3. While the shapes of the spectral densities differ to some extend, the overall picture is fairly in line with the one obtained from the univariate analysis in the previous section.

As explained in detail in Appendix B.2, this can be interpreted as follows. If most of the spectral mass of the true data generating process is located at frequencies lower than, say, $\pi/16$, it makes only little difference between using the $(1-L)$-filter or the $(1-L^4)$-filter. Acting similar at low frequencies, both filters will provide an accurate picture of the cyclical properties of the stationary component of the times series under investigation.

The above results from the multivariate analysis of credit and house prices within countries have shown correlation at very low frequencies. However, this can only be a first indication of a *common cycle*. In order to go a little further towards a more detailed analysis of the global cycle, we estimate four bivariate cross-country VARs consisting of the US plus either the UK or Germany. We now look at frequency-wise Granger causality for credit and house prices. The Granger causality across frequencies is calculated according to (10) and measures the causality from the US to the second country in the system. Figure 6 shows the results. While the exact magnitude of the measures is not informative, it can still be observed that the impact of the US on the UK and on Germany is concentrated around specific frequencies and has moved over time towards lower ones.
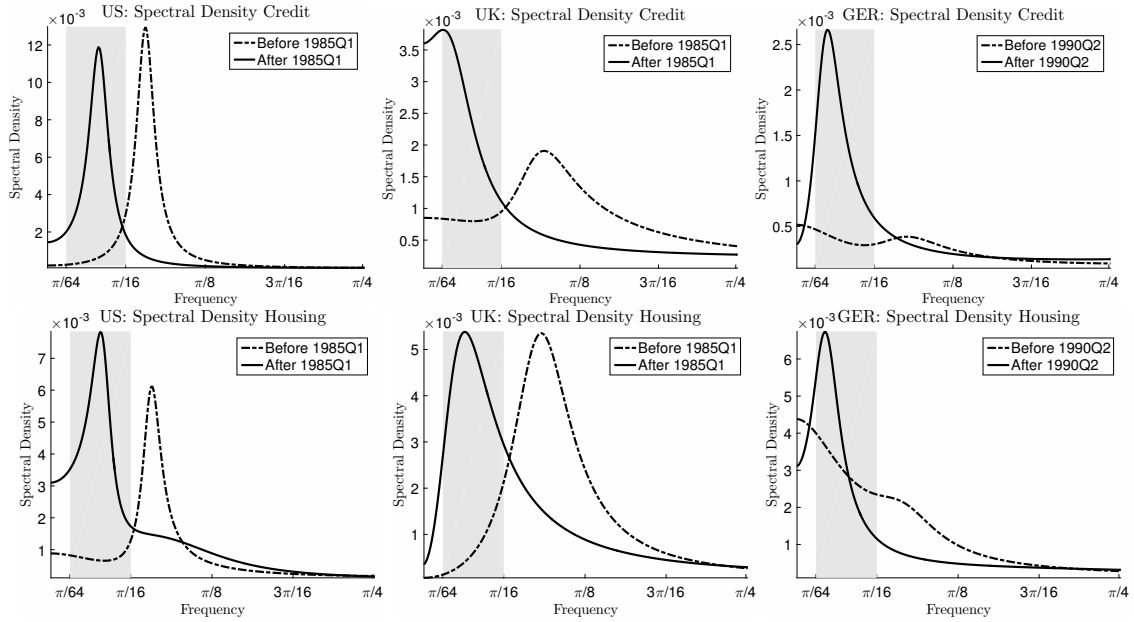
Figure 5: Spectral Densities of First Differences. Note: Individual spectral densities for quarterly growth rates of credit and house prices derived from country-specific VECM representations.
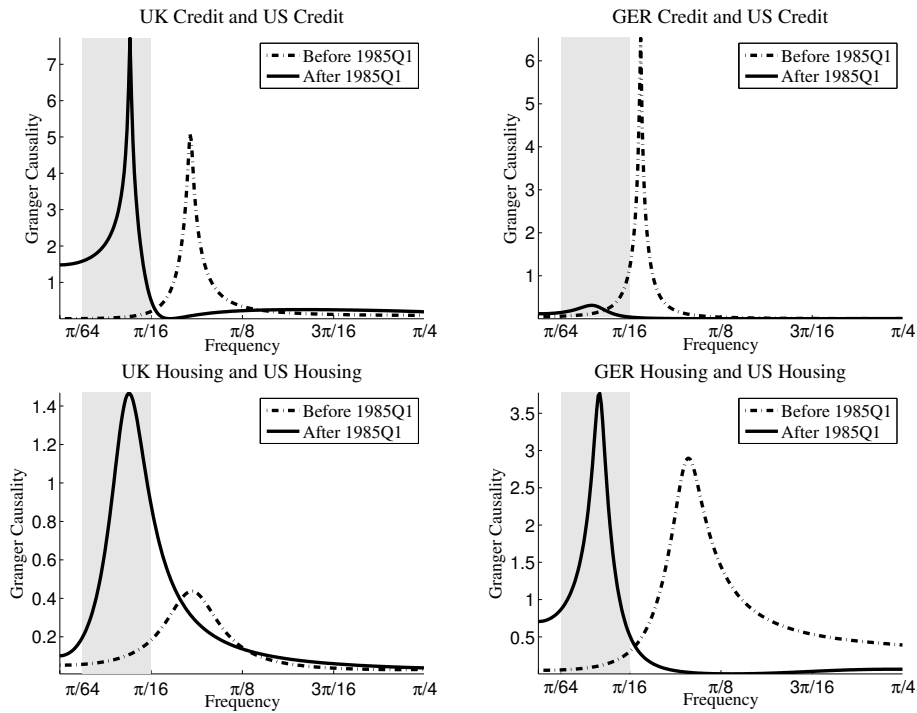

Figure 6: Cross-Country Granger Causality Note: This Figure shows the Granger causality measure of Breitung and Candelon (2006). The measures are derived from bivariate cross-country VARs and show the causality from the US to the second country in the system.

4.2 US Monetary Policy Transmission and the Global Financial Cycle

Rey (2015), Passari and Rey (2015) and Miranda-Agrippino and Rey (2015) suggest that the correlation of national financial cycles is driven by a global financial cycle which is shown to be highly correlated with the VIX volatility index and is therefore related to agents' risk appetite.[19] The VIX, in turn, is influenced by US monetary policy, see e.g. Bekaert et al. (2013). Thus, the existence of a global cycle represents an additional international transmission channel of US monetary policy which may weaken the effectiveness of national monetary policies.

Our analysis in the previous section provided some indications which seem to be in line with the idea of a global financial cycle. The frequency domain perspective allowed us to show that the effects of the US financial cycle on the financial cycles in the UK and in Germany have shifted from higher to lower frequencies in the last decades. However, the analysis was based on bivariate systems of credit and house prices only.

As a natural next step, we extend our VARs by a number of variables to investigate in more detail Rey's (2015) global financial cycle hypothesis. The models are estimated for the sample 1990Q1-2013Q4.[20] We then study Granger causality in the time and in the frequency domain. The spectral matrix of the VAR models is derived from the estimated time domain parameters. On this basis, we calculate p-values of the Breitung and Candelon (2006) test with the null hypothesis of no Granger causality at a given frequency.

For the US, we estimate a VAR model consisting of the variables credit, real GDP, house prices (all three in logs), the real short-term interest rate (calculated as in Bruno and Shin, 2015 as the target Fed Funds rate minus the CPI inflation rate) and the log VIX, in this order. The models for the UK and Germany include the real exchange rate as an additional endogenous variable. This reflects that the US may be considered a large and rather closed economy while the UK and Germany are comparatively small and open economies. Besides, the exchange rate will control for the traditional transmission mechanism of US monetary policy on other countries.

Table 7 shows p-values for the time domain Granger tests with the null hypothesis of no causality, see Toda and Yamamoto (1995). Figure 7 illustrates the p-values of the Breitung and Candelon (2006) test for Granger causality in the frequency domain for credit, housing prices and GDP.[21]

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[19]The notion of the existence of such a common financial cycle is not new: Seminal works by Fisher (1933), Keynes (1936), von Mises (1952), Hayek (1933) and Minsky (1982) stressed the inherently procyclical behavior of the financial system and the role of bounded rational behavior by financial market participants.

[20]The shorter length for this multivariate analysis is due to the data availability of the VIX series, which starts only at 1990Q1. As in the univariate analysis, the lag order of the VAR models is determined through standard information criteria and, if necessary, adjusted until all autocorrelation is removed from the residuals.

[21]The p-value graphs for the other variables are available upon request.

Table 7: Time domain Granger non-causality tests (*p*-values), Estimation sample: 1990:1-2013Q4.

. . . is not Granger-caused by . . .	Credit	GDP	RER	Housing Prices	Real Interest Rate	VIX
			United States			
Credit	–	0.231	–	0.001	0.640	0.363
GDP	0.211	–	–	0.154	0.257	0.370
RER	–	–	–	–	–	–
House Prices	0.009	0.093	–	–	0.132	0.175
Real Interest Rate	0.771	0.984	–	0.948	–	0.638
VIX	0.853	0.172	–	0.122	0.520	–
			United Kingdom			
Credit	–	0.205	0.814	0.445	0.553	0.094
GDP	0.359	–	0.509	0.120	0.979	0.001
RER	0.906	0.364	–	0.004	0.124	0.265
House Prices	0.059	0.853	0.106	–	0.999	0.304
Real Interest Rate	0.046	0.439	0.029	0.517	–	0.274
VIX	0.714	0.104	0.545	0.641	0.181	–
			Germany			
Credit	–	0.001	0.318	0.003	0.698	0.509
GDP	0.171	–	0.118	0.458	0.338	0.376
RER	0.545	0.191	–	0.421	0.879	0.872
House Prices	0.778	0.495	0.622	–	0.242	0.232
Real Interest Rate	0.521	0.184	0.596	0.947	–	0.639
VIX	0.474	0.100	0.709	0.257	0.319	–

For the US, we find significant time domain Granger causality from credit to housing and the other way around. From the frequency domain we obtain the information that this relation is particularly significant at lower frequencies. This corroborates the results from the coherency analysis of the bivariate VARs in the previous section (see Figure 4). While Granger causality remains a reduced form concept, our findings are in line with the widespread notion that house prices were a key factor of the pronounced credit expansion in the US during the 1990s and 2000s which led to 2007/2008 crisis and the following collapse in economic growth. In the time domain, where relations at specific frequencies cannot be identified, US GDP appears not to be significantly influenced by housing or credit, at least not at the 10%-level. However, looking in the frequency domain, we can see that house prices do significantly affect GDP, but they do so only at lower frequencies. Similarly, Figure 7 also shows a significant low frequency Granger-causal relationship from the VIX to house prices.

Does US monetary policy transmit through the risk taking channel and affect national financial cycles in other countries? If so, there should be significant Granger causality *at low frequencies* between the US VIX and foreign credit (or house prices). In the UK, we obtain time domain evidence for a significant relation between the VIX and credit. In the frequency domain, we see that the significance of this relation is driven by low frequencies. The same result holds for the effect of the US VIX on macroeconomic activity in the UK.
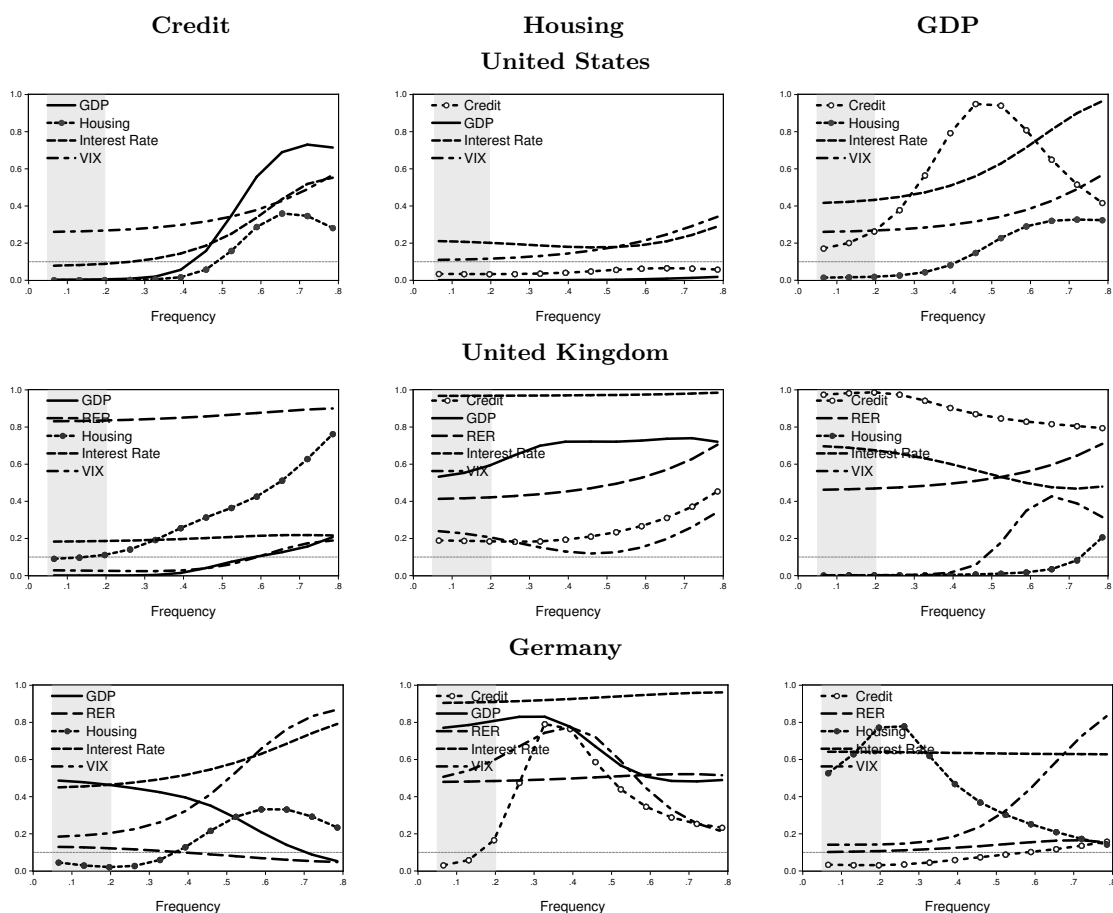
Figure 7: Breitung and Candelon (2006) *p*-values for frequency domain Granger non-causality (low *p*-values indicate a rejection of the null hypothesis of non-causality).

For Germany, the results look different. The time domain tests indicate that the US VIX is neither Granger causal to credit or house prices nor to GDP. However, the frequency domain reveals that the picture may not be too different from that of the UK. The VIX does not affect credit and GDP significantly (at the 10%-level), but the significance clearly increases to levels of less than 20% towards lower frequencies. Therefore, it could well be the case that the higher dimensional VAR for Germany would need just a bit more data to be able to deliver the same results as for the UK.

5 Concluding Remarks

What are the main characteristics of the financial cycle? Is it a medium-term phenomenon, meaning it is longer than the business cycle, as suggested in the literature, or does it share similar characteristics with the business cycle? How are financial cycles related within countries and what could explain internationally related financial cycles? In this paper we intended

to shed some light on these and other related questions by estimating the data generating processes of financial and business cycle variables using standard econometric methods. We extended the literature by formally testing various cyclical properties and economic interpretations that have been suggested by existing work. Most of our analysis of the financial cycle was performed in the frequency domain, while the estimation was carried out in the time domain. Our approach allowed us to take into account all possible cycles without *a priori* assuming different ranges for financial and business cycles. The efficiency of the parametric spectrum estimation made it possible to test for time-variation of cyclical properties via subsample analysis and to test for Granger causality in a multivariate setting.

For the US and the UK, we found strong evidence that growth rates of credit and house prices feature longer cycles and larger amplitudes than the business cycle. We also found time variation of these properties and showed that they have become more visible recent times. The statistical significance of our findings contributes to establishing stylized facts of the properties of financial cycles. Further, we found statistical evidence at low frequencies supporting the relevance of Rey's (2015) global financial cycle hypothesis and thus of the risk-taking channel of US monetary policy.

Our results highlight the need for closely monitoring the evolution of financial cycles and for policy-measures to curb extreme fluctuations (as, for instance, counter-cyclical capital buffer rates) in order to mitigate the effects of the global financial cycle.

An interesting and straightforward extension of our approach that could be applied in both a univariate and a multivariate framework would be to use time-varying parameter models. These could possibly provide deeper insights into the question of *when* the financial cycle has developed its distinguishing features and *what* events have caused the parameter changes.

Appendix A   Data Sources and Definitions

All series are measured in logs and deflated using the consumer price index. All series are normalized by their respective value in 1985Q1 to ensure comparability of the units. We obtain annual growth rates by taking annual differences of the time series. The only exception is the credit to GDP ratio which is expressed in percentage points.

Table 8: Definition and Sources of the Data

	Source	Identifier	Notes
GDP	OECD.Stat	CARSA	national currency
CPI	OECD.Stat	Consumer Prices	national index
credit	Datastream	USBLCAPAA, UKBLCAPAA, BDBLCAPAA	national currency, credit to private non-financial sector from all sectors
housing	OECD.Stat	House Prices	national index
equity	IMF	USQ62...F, UK62...F, BDQ62.EPC	national index

Figure 8: Real GDP and Financial Variables in the United States. Note: All series are annual growth rates, except the credit to GDP ratio, which represents deviations from a linear trend measured in percentage points. The vertical gray line shows the sample split.

Figure 9: Real GDP and Financial Variables in the United Kingdom. Note: All series are annual growth rates, except the credit to GDP ratio, which represents deviations from a linear trend measured in percentage points. The vertical gray line shows the sample split.
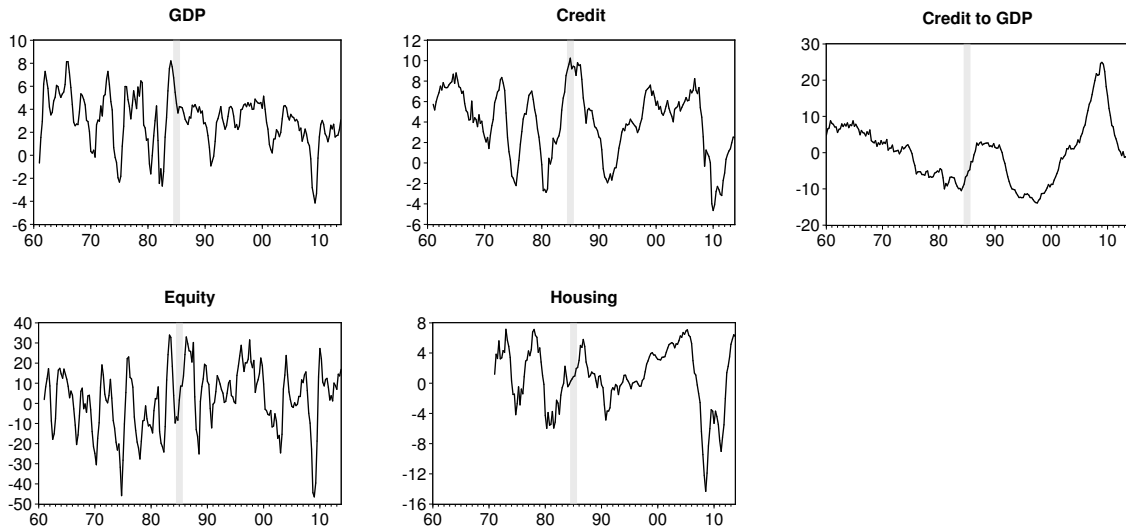

Figure 10: Real GDP and Financial Variables in Germany. Note: All series are annual growth rates, except the credit to GDP ratio, which represents deviations from a linear trend measured in percentage points. The vertical gray line shows the sample split.
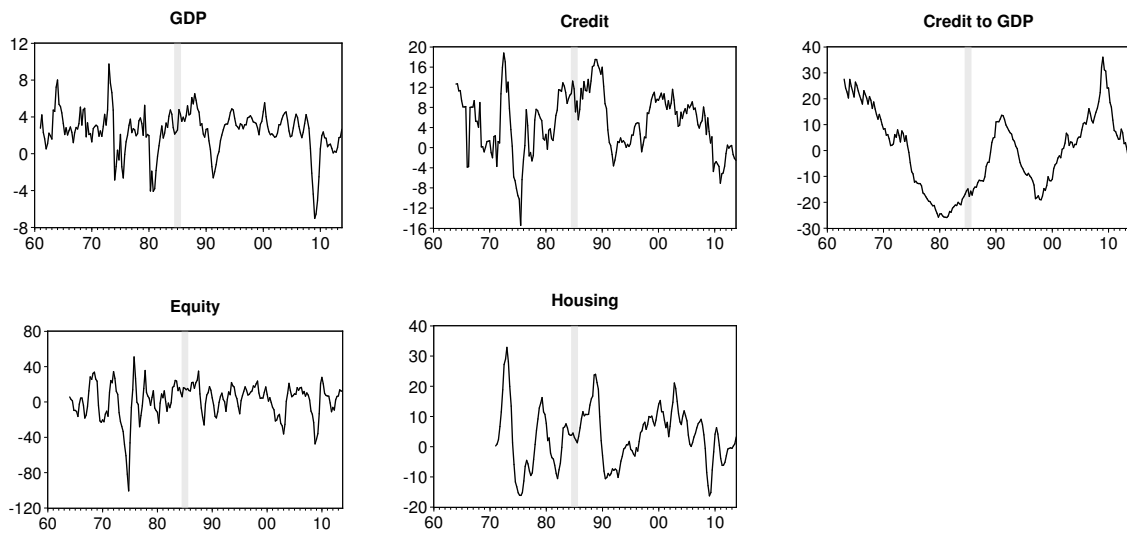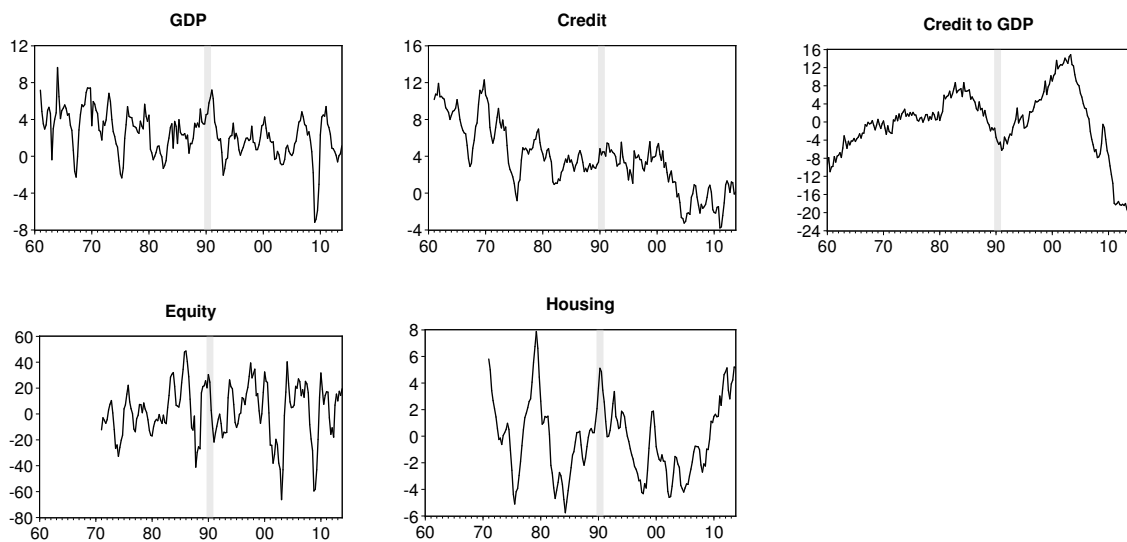
Appendix B   Methodological Discussion

B.1   Comparison of Indirect Spectrum Estimation and Filtering

A popular approach in the analysis of the financial cycle is to work with frequency-based filters (Drehmann et al., 2012, Aikman et al., 2015) which are usually based on Baxter and King (1999) or refinements thereof, see e.g. Christiano and Fitzgerald (2003). When filters are used to analyze cycles, a crucial point is that the frequency range has to be pre-specified by the researcher. This may lead to results biased into the direction of the pre-defined range. The intuition behind this difficulty is best described by briefly discussing the general functioning of filters.

In the time domain, any linear filter can be written in the form of a two-sided moving average

$$y_t = \sum_{j=-m}^{n} a_j x_{t-j} \quad . \tag{13}$$

The filtered series y_t depends simultaneously on the properties of the filter (the coefficients a_j) and the data (the observed series x_t). To convert equation (13) into the frequency domain, one uses the corresponding filter function

$$C(\lambda) = \sum_{j=-m}^{n} a_j e^{-i\lambda j} \ , \qquad i^2 = -1 \ , \tag{14}$$

and its transfer function

$$T(\lambda) = |C(\lambda)|^2 \quad , \tag{15}$$

where $\lambda \in [-\pi, \ \pi]$ denotes the frequency range, see e.g. Wolters (1980a, 1980b). The relationship between the spectra of the original and the filtered series is given by

$$f_y(\lambda) = T(\lambda) f_x(\lambda). \tag{16}$$

When compared to the time domain, a major advantage of the representation in (16) is that now the filter effect $T(\lambda)$ is separated from the data $f_x(\lambda)$.

For the spectrum of the filtered series $f_y(\lambda)$ to accurately identify cycles in the spectrum of the observed series $f_x(\lambda)$, a so-called band pass filter must be used. Such a filter has the value 1 at the frequency range of interest $[\lambda_{\text{low}}, \ \lambda_{\text{high}}]$ and 0 otherwise. In view of equation (16), this implies $f_y(\lambda) = f_x(\lambda)$ for $\lambda \in [\lambda_{\text{low}}, \ \lambda_{\text{high}}]$ and $f_y(\lambda) = 0$ for $\lambda \notin [\lambda_{\text{low}}, \ \lambda_{\text{high}}]$. Theoretically, the ideal time domain filter can be achieved by moving averages of infinite order. Yet, in practice, this is not possible since only a limited number of observations is available.

The main idea of frequency-based filters is to pre-specify a range from λ_{low} to λ_{high} and to choose finite values for m and n in equation (13) to find the weights a_j which approximate the ideal filter as good as possible. Due to the approximation, the spectrum of the filtered series is in general different from the one of the observed series in $[\lambda_{\text{low}}, \lambda_{\text{high}}]$ and reflects a mixture of filter and data properties in the form of $T(\lambda)f_x(\lambda)$. More precisely, it can be shown that any filter has the tendency to overstate the contributions of cycles in the pre-defined frequency range $[\lambda_{\text{low}}, \lambda_{\text{high}}]$ to the overall variance of the underlying time series. Because of this tendency, artificial cycles may be produced even if the true data generating process has no cycles.[22]

In this study, we emphasize that the appropriate λ_{low} and λ_{high} are unknown. Indeed, while there seems to be general consensus in the literature about the relevant frequency range for the business cycle (2 to 8 years), the relevant frequency range for the financial cycle is less clear. For instance, Drehmann et al. (2012) construct their financial cycle measure from the underlying series with *a priori* chosen cycle length between 8 and 30 years.

B.2 The Effect of Quarterly and Yearly Growth Rate Filters: Δ vs. Δ_4

Our univariate analysis of financial cycles focuses on *yearly growth rates* of a number of financial variables. This allows us to compare our results to those provided by existing studies. The fact that most of the literature focuses on yearly growth rates also highlights that the dynamics of yearly changes are of general interest to economists.

Nonetheless, when calculating yearly growth rates, a specific filter, the $(1 - L^4)$-filter, is applied to the level of the time series. This raises the concern if and how this filter changes the cyclical properties of the data. Another interesting aspect is whether it makes a difference when using the $(1 - L)$-filter to calculate *quarterly* instead of yearly growth rates.

What we do know about the level data is that they are non-stationary. Hence, the spectral density collapses at frequency $\lambda = 0$ because all the variation of the series is explained by the cycle with length infinity. Therefore, the more relevant question must be whether using the $(1 - L^4)$ or $(1 - L)$-filter gives a good approximation of the cyclical properties of the stationary component of the time series which remains when the unit root is removed. The correct answer, in turn, depends crucially on the DGP.

Consider the DGP

$$(1 - L^4)A(L)y_t = \varepsilon_t \tag{17}$$

with $A(L)$ stationary and ε_t white noise. The process in (17) is still fairly general and could

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[22]This issue has been raised already in the 1950's and 1960's. For a discussion of the general problem see e.g. König and Wolters (1972), King and Rebelo (1993), Baxter and King (1999), Christiano and Fitzgerald (2003), Harvey and Trimbur (2003) and Murray (2003).

well be the generating process behind the present data. The true spectrum of the level of y_t is given by

$$f_y(\lambda) = [2(1 - \cos(4\lambda))]^{-1} \cdot [|A(e^{-i\lambda})|^2]^{-1} \cdot f_{\varepsilon_t}(\lambda)$$

$$= [2(1 - \cos(4\lambda))]^{-1} \cdot f_{\Delta_4}(\lambda) \cdot \frac{\sigma^2_{\varepsilon_t}}{2\pi}$$

In that case, the $(1-L^4)$-filter, which has the frequency domain representation $[2(1-\cos(4\lambda))]$, should indeed be applied because it would give us exactly (up to a constant scale factor $\frac{\sigma^2_{\varepsilon_t}}{2\pi}$) the true spectrum of the cyclical component. Of course, the crucial assumption is that true spectrum of the cyclical component is $f_{\Delta_4}(\lambda) = [|A(e^{-i\lambda})|^2]^{-1}$. Naturally, the same argument could be made against the $(1 - L^4)$-filter by assuming the DGP was $(1 - L)A(L)y_t = \varepsilon_t$.

Sticking to the example that the true DGP is (17), using the $(1-L)$-filter would not produce the correct spectrum of the stationary component of y_t. This can be seen from Figure 11 which shows the transfer functions of both filters. The $(1 - L)$-filter would dampen the true spectrum at lower frequencies. Similarly, if the DGP was $(1-L)A(L)y_t = \varepsilon_t$, the $(1-L^4)$-filter would dampen the true spectrum for $\lambda < \pi/12$ and intensify frequencies for $\pi/12 < \lambda < \pi/4$.
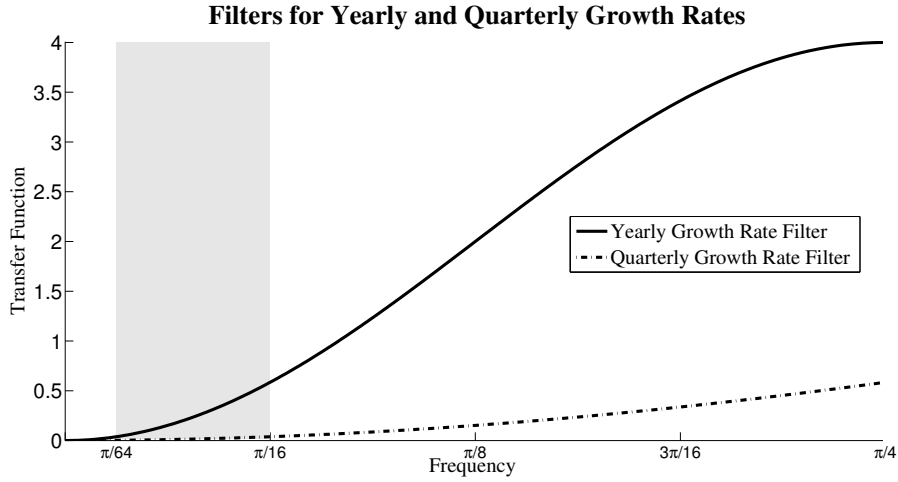
Figure 11: Comparing the Filters $(1 - L^4)$ and $(1 - L)$. Note: The Figure shows the transfer functions of the yearly and quarterly growth rate filters, i.e. $2(1 - \cos(4\lambda))$ and $2(1 - \cos(\lambda))$.

To assess the relevance of this issue in an empirical analysis of financial cycles in growth rates, it is instructive to state that the $(1 - L^4)$-filter can be decomposed as follows

$$(1 - L^4) = (1 - L)(1 + L + L^2 + L^3) \quad . \tag{18}$$

According to (18), the $(1-L^4)$-filter is equivalent to taking first differences and then forming a moving average of the remainder. Figure 12 shows the transfer functions of both components of the $(1-L^4)$-filter. The transfer function of the $(1+L+L^2+L^3)$-filter reflects what is done to the data on top of forming first differences when using the $(1-L^4)$-filter. Note that the transfer function of the $(1+L+L^2+L^3)$-filter is quite flat in the gray region which reflects the financial cycle area.

Decomposing the Yearly Grwoth Rate Filter $(1-L^4)=(1-L)(1+L+L^2+L3)$
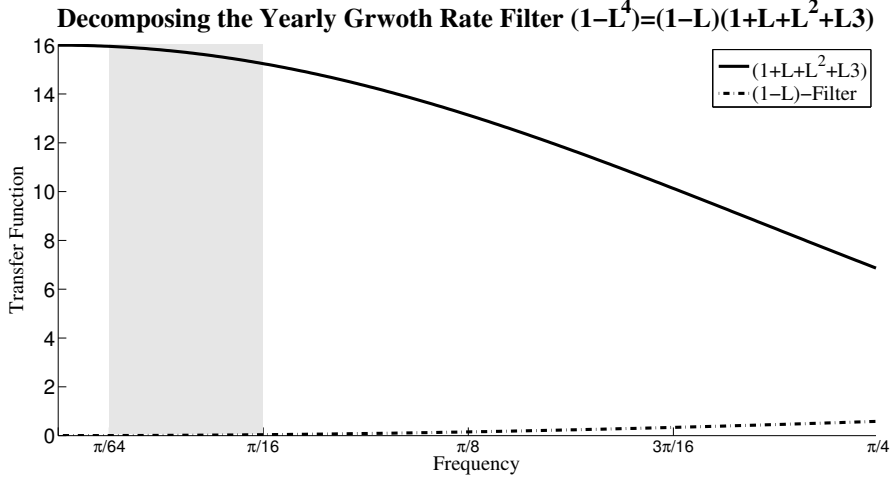
Figure 12: Decomposing the Filter for Yearly Growth Rates. Note: The figure shows the transfer functions of the two components $(1-L)$ and $(1+L+L^2+L^3)$ of the $(1-L^4)$-filter.

Let us now make the assumption that all (or most) of the true spectral mass of the cyclical component of y_t is located at longer cycles. If that was actually the case, then, compared to taking first differences, the $(1-L^4)$-filter scales up the true spectrum by a roughly constant number. However, when forming the spectral densities by dividing through the area below the spectrum, the scaling effect becomes irrelevant. Therefore, given all spectral mass is truly located at very low frequencies, the $(1-L^4)$-filter and the $(1-L)$-filter should deliver similar spectral densities. Since we have shown in Section 4.1, that for quarterly growth rates of credit and housing the spectral densities are indeed very similar to those obtained from the univariate analysis of yearly growth rates, we conclude that these long cycles are in fact present in the true data generating process.

B.3 Comparison of Indirect and Direct Spectrum Estimation

A direct non-parametric estimation of a spectrum produces generally less precise estimates than the parametric (indirect) approach.[23] In order to get consistent estimates, one has to

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[23]Two interesting applications can be found in Aikman et al. (2015) and Schüler et al. (2015). Aikman et al. (2015) directly estimate the spectral density for UK real loan growth using yearly data from 1880-2008

use a kernel estimator. Thus, the estimates depend on the specific kernel and its bandwidth parameter M, implying that a large amount of observations is required to have the necessary degrees of freedom. For instance, if one estimates the spectrum from T observations by transforming the first M estimated autocovariances, one has $C \cdot \frac{T}{M}$ degrees of freedom. The constant C is kernel-specific and usually takes on values of around 3. A small value of M decreases the variance but increases the bias of the estimator. Having in mind the data in our empirical analysis, let us consider a sample size of $T = 100$. In that case, a reasonable choice for M may be 20, leading to only about 15 degrees of freedom. In contrast, the indirect estimation approach, starting from an ARMA model with e.g. 5 parameters, leaves us with 95 degrees of freedom and hence implies a strong efficiency gain.

An important additional consequence of this reduction in estimation uncertainty is that it facilitates to study possible changes in the spectrum across different time periods. Since the indirect spectrum estimation is based on time domain estimates, it allows the use of conventional break point tests, as e.g. the Chow test, or impulse indicator saturation to endogenously determine breaks.[24]

B.4 Bootstrap Inference

To statistically assess the characteristics of the financial cycle across variables and different sample periods, the inference in the empirical part is based on bootstrap methods.[25] We apply the following bootstrap procedure, see e.g. Benkwitz et al. (2001).

1. Estimate the parameters of the ARMA model in equation (1).

2. Generate bootstrap residuals $u_1^*, \ldots, u_T^*$ by randomly drawing with replacement from the set of estimated residuals.

3. Construct a bootstrap time series recursively using the estimated parameters from step 1 and the bootstrap residuals from step 2.

4. Reestimate the parameters from the data generated in step 3.

5. Repeat step 2 to step 4 5000 times.

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to analyze both the business and the financial cycle, while Schüler et al. (2015) also directly estimate spectral measures to characterize financial cycles across Europe.

[24]When estimating spectral densities directly, comparable structural break tests are also available in the frequency domain, see Ahmed et al. (2004) and Pancrazi (2015). Their advantage is that they allow to analyze breaks of (integrated) spectral densities in any frequency range of interest. Generally, however, they are not as precise as tests in the time domain since they are based on a non-parametric approach. During the further procedure, this paper also uses frequency-wise testing to address a number of specific questions, but our testing relies on the indirectly estimated spectra and their bootstrap standard errors.

[25]To the best of our knowledge, analytical expressions for such standard errors are not available.

6. From the bootstrap distributions of the statistics of interest, e.g., cycle length, amplitude etc., we compute the standard errors and the corresponding 95% confidence intervals.

B.5 Estimation Example

In order to provide an example of how we use the estimated ARMA models to obtain the spectral densities, consider the following process of US real GDP growth during the pre-1985 period with t-values in parentheses (cf. Appendix C.1) and the notation as in equation (1):

$$y_t = \underset{(2.97)}{0.002} + \underset{(19.21)}{1.144} y_{t-1} - \underset{(-4.37)}{0.224} y_{t-3} - \underset{(-42.04)}{0.985} u_{t-4} + u_t \quad .$$

These estimates are applied to equation (3) to calculate the corresponding spectrum as

$$f_y(\lambda) = \frac{|1 - 0.985 \, e^{-i4\lambda}|^2}{|1 - 1.144 \, e^{-i\lambda} + 0.224 \, e^{-i3\lambda}|^2} \frac{\sigma^2}{2\pi} \quad ,$$

with $e^{-i\lambda} = \cos(\lambda) - i \cdot \sin(\lambda)$ by Euler's relation and $i^2 = -1$. According to this procedure, we calculated the spectra for all countries and sample periods under consideration. Dividing $f_y(\lambda)$ by the variance of y_t yields the spectral densities.

Appendix C   Detailed Time Domain Estimation Results

C.1 Estimated ARMA Models

Table 9: ARMA Models for the US

parameters	GDP total	GDP pre	GDP post	credit total	credit pre	credit post	credit to GDP total	credit to GDP pre	credit to GDP post	housing total	housing pre	housing post	equity total	equity pre	equity post
const	0.028 (18.7)	0.031 (21.3)	0.022 (2.95)	0.039 (24.2)	0.044 (5.10)	0.038 (10.5)	-1.180 (-0.50)	0.239 (1.40)	-1.795 (-0.43)	0.009 (4.15)	0.001 (0.14)	0.010 (2.48)	0.050 (2.04)	-0.049 (-5.77)	0.043 (3.25)
AR(1)	1.202 (29.9)	1.144 (19.2)	1.318 (27.1)	1.185 (79.7)	1.237 (19.6)	1.170 (47.3)	1.251 (28.8)	0.873 (18.4)	1.381 (35.8)	1.373 (18.7)	1.130 (17.2)	1.664 (18.5)	1.350 (20.8)	1.302 (13.5)	1.285 (14.3)
AR(2)										-0.332 (-2.58)		-0.827 (-4.66)	-0.365 (-5.64)	-0.410 (-4.33)	-0.333 (-3.77)
AR(3)	-0.230 (-5.76)	-0.224 (-4.37)	-0.328 (-6.64)		-0.346 (-5.69)		-0.600 (-6.49)		-0.402 (-10.3)	0.270 (2.10)		0.484 (2.68)			
AR(4)							0.939 (62.68)			-0.342 (-4.64)	-0.201 (-3.28)	-0.345 (-3.69)			
AR(5)				-0.222 (-14.9)		-0.201 (-7.92)	-0.611 (-10.2)								
MA(3)					0.403 (3.68)		0.360 (3.95)								
MA(4)	-0.978 (-92.2)	-0.985 (-42.0)	-1.144 (-18.1)	-0.904 (-29.5)	-0.374 (-3.75)	-0.929 (-31.6)	-0.429 (-5.65)	0.212 (2.76)		-0.973 (-37.2)	-0.983 (-29.5)	-0.932 (-25.9)	-0.977 (-65.2)	-0.943 (-47.2)	-0.972 (-60.5)
MA(5)					0.241 (2.21)	0.487 (5.95)		-0.516 (-8.47)							
MA(6)					0.302 (2.95)										
MA(7)							-0.185 (-2.75)								
MA(12)			0.182 (3.02)					-0.429 (-5.87)							
diagnostics															
Chow	3.37 (0.01)			1.74 (0.14)			3.66 (0.00)			3.97 (0.00)			3.38 (0.01)		
LM(4)	0.75	0.89	0.35	0.82	0.16	0.21	0.47	0.62	0.28	0.14	0.69	0.83	0.94	0.51	0.99
LM(8)	0.36	0.46	0.42	0.53	0.38	0.14	0.79	0.94	0.54	0.11	0.63	0.69	0.81	0.44	0.99
LM(12)	0.13	0.34	0.17	0.22	0.44	0.35	0.57	0.68	0.63	0.14	0.57	0.56	0.71	0.26	0.93

Notes: The terms *pre* and *post* refer to the sample periods 1960Q1 until 1984Q4 and 1985Q1 until 2013Q4, respectively. *Total* denotes the full sample period. Below the parameter estimates *t*-values are given in parentheses. Chow tests with *p*-values in parentheses refer to a break in parameters at 1985Q1. For LM(*k*) tests of no autocorrelation up to order *k* the table shows *p*-values.

Table 10: ARMA Models for the UK

parameters	GDP			credit			credit to GDP			housing			equity		
	total	pre	post	total	pre	post	total	pre	post	total	pre	post	total	pre	post
const	0.024 (23.3)	0.021 (5.95)	0.023 (10.8)	0.057 (12.7)	0.043 (2.71)	0.037 (3.71)	−4.367 (−2.87)	−0.094 (−0.07)	−1.538 (−0.37)	0.037 (7.24)	0.001 (0.16)	0.039 (4.64)	0.032 (3.03)	−0.024 (−0.83)	0.023 (3.05)
AR(1)	1.101 (32.0)	0.941 (27.2)	1.464 (17.2)	1.113 (36.4)	1.174 (39.6)	1.060 (19.6)	1.099 (37.3)	1.046 (17.8)	1.151 (36.0)	1.480 (36.9)	1.321 (33.0)	1.573 (17.1)	1.309 (18.0)	1.372 (11.8)	1.206 (13.3)
AR(2)			−0.291 (−1.98)									−0.291 (−1.65)	−0.400 (−3.37)	−0.571 (−3.05)	−0.289 (−3.32)
AR(3)			−0.198 (−2.33)				0.479 (4.43)			−0.596 (−8.03)	−0.446 (−12.1)	−0.300 (−3.17)	0.166 (1.37)	0.446 (2.32)	
AR(4)	−0.135 (−3.82)			0.267 (2.28)	−0.192 (−6.34)	−0.137 (−2.29)	−0.589 (−7.04)	−0.120 (−2.01)					−0.235 (−1.98)	−0.596 (−3.19)	
AR(5)				−0.400 (−4.00)					−0.192 (−5.99)	0.095 (2.27)			0.123 (1.70)	0.280 (2.40)	
MA(4)	−0.922 (−113.6)	−0.946 (−34.9)	−0.986 (−81.8)	−1.148 (−14.9)	−0.954 (−39.2)	−0.930 (−30.4)	−0.474 (−5.33)			−1.120 (−12.3)	−0.956 (−41.1)	−1.261 (−12.6)	−0.979 (−74.5)	−0.939 (−30.1)	−0.979 (−38.6)
MA(6)							−0.229 (−3.47)		−0.246 (−2.42)						
MA(8)							−0.237 (−3.40)	0.150 (3.02)		0.156 (1.75)		0.317 (3.33)			
MA(12)				0.174 (2.41)				−0.832 (−28.0)							
diagnostics															
Chow	1.86 (0.12)			5.54 (0.00)			1.92 (0.07)			3.24 (0.01)			1.90 (0.07)		
LM(4)	0.83	0.62	0.77	0.88	0.74	0.44	0.83	0.75	0.72	0.15	0.98	0.99	0.95	0.40	0.70
LM(8)	0.39	0.39	0.85	0.41	0.74	0.78	0.83	0.81	0.52	0.18	0.99	0.57	0.85	0.82	0.92
LM(12)	0.43	0.40	0.56	0.51	0.81	0.84	0.71	0.31	0.36	0.30	0.99	0.87	0.94	0.94	0.98

Notes: The terms *pre* and *post* refer to the sample periods 1960Q1 until 1984Q4 and 1985Q1 until 2013Q4, respectively. *Total* denotes the full sample period. Below the parameter estimates t-values are given in parentheses. Chow tests with p-values in parentheses refer to a break in parameters at 1985Q1. For LM(k) tests of no autocorrelation up to order k the table shows p-values.

Table 11: ARMA Models for Germany

parameters	GDP total	GDP pre	GDP post	credit total	credit pre	credit post	credit to GDP total	credit to GDP pre	credit to GDP post	housing total	housing pre	housing post	equity total	equity pre	equity post
const	0.017 (8.59)	0.022 (14.1)	0.013 (14.2)	−0.159 (−0.16)	0.035 (15.0)	−0.047 (−0.83)	–	−2.168 (−0.71)	−0.933 (−0.10)	−0.005 (−1.93)	−0.001 (−0.30)	−0.004 (−0.31)	0.038 (5.12)	0.065 (1.66)	0.029 (2.06)
AR(1)	1.032 (24.7)	0.820 (12.2)	1.201 (12.4)	1.091 (28.2)	1.159 (16.8)	1.018 (22.3)	0.999 (55.4)	0.586 (5.11)	1.094 (19.7)	1.545 (19.9)	1.769 (23.0)	1.440 (14.0)	1.233 (16.6)	1.076 (16.9)	1.291 (13.2)
AR(2)			−0.363 (−3.80)					0.298 (2.53)		−0.429 (−3.06)	−0.811 (−10.7)	−0.337 (−2.54)	−0.307 (−4.17)		−0.386 (3.97)
AR(3)					−0.223 (−3.75)					−0.139 (−1.77)					
AR(4)	−0.072 (−1.72)			−0.092 (−2.37)		−0.725 (−8.61)			−0.518 (−3.09)					−0.136 (−2.16)	
AR(5)						0.688 (9.74)			0.396 (2.39)			−0.116 (−2.80)			
MA(4)	−0.751 (−9.90)	−0.615 (−5.34)	−0.782 (−6.94)	−0.647 (−8.86)	−0.912 (−28.6)		0.402 (5.74)	0.520 (5.11)	0.764 (5.93)	−0.736 (−10.9)	−0.751 (−7.59)	−0.905 (−24.5)	−0.969 (−74.5)	−0.923 (−21.3)	−0.957 (−45.0)
MA(8)	−0.228 (−3.03)	−0.302 (−2.68)	−0.182 (−1.63)	−0.239 (−3.18)				0.814 (19.3)							
MA(12)						−0.907 (−38.2)		0.563 (6.28)		−0.198 (−3.09)	−0.183 (−1.87)				
diagnostics															
Chow	3.63 (0.00)			3.68 (0.00)			1.74 (0.18)			1.22 (0.30)			0.30 (0.88)		
LM(4)	0.96	0.80	0.15	0.88	0.66	0.32	0.43	0.76	0.83	0.57	0.74	0.58	0.57	0.31	0.86
LM(8)	0.28	0.90	0.58	0.20	0.86	0.19	0.43	0.40	0.92	0.59	0.95	0.75	0.89	0.73	0.87
LM(12)	0.42	0.97	0.86	0.38	0.92	0.31	0.54	0.50	0.59	0.76	0.81	0.81	0.78	0.57	0.80

Notes: The terms *pre* and *post* refer to the sample periods 1970Q1 until 1990Q1 and 1990Q2 until 2013Q4, respectively. *Total* denotes the full sample period. Below the parameter estimates *t*-values are given in parentheses. Chow tests with *p*-values in parentheses refer to a break in parameters at 1990Q2. For LM(k) tests of no autocorrelation up to order k the table shows *p*-values.

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C.2 Testing for Breaks: Impulse Indicator Saturation

As outlined in Section 3.1 on the data, we want to analyze possible changes in the characteristics of the financial cycle. Thereby, we follow the literature, which commonly specifies the break at 1985Q1, arguing that around this time the process of financial liberalization around the world started to speed up considerably (see e.g. Claessens et al., 2011, 2012 and Drehmann et al., 2012). Chow tests provide some support for the break date 1985Q1; see test results in Appendix C.1.

To provide additional statistical support without a priori specifying a break date (as it is necessary for the Chow test), we conducted an impulse indicator saturation analysis. Following Hendry (2011) and Ericsson (2013) (see also the references therein), we use the split-half approach. That is, we include impulse dummies for all data points of the first half of the sample and estimate the model over the full sample. From the estimation results, we identify the significant dummies (see Block 1 in Figures 13 to 15). Similarly, we then repeat the estimation including dummies only for the second half of the sample (see Block 2). Thereafter, we estimate the model including all dummies which were significant in first two steps of the test procedure. Finally, we retain all dummies being significant at the 10%-level (see Block Combination).

From the test results in Figures 13 to 15, it can be seen that for most variables a considerable amount of dummies remains significant in one half of the sample while in the other half only a few dummies survive. Often the few surviving dummies are found in the second sample half and only pick up outliers during the financial crisis. While the test procedure does not provide specific break dates, it gives in most cases fairly strong indication that the model parameters are subject to structural change. In particular, the results indicate a break around mid to late 1980s for US and UK and around 1990 for Germany.

Note that the IIS procedure is quite general. It is possible to interpret many existing tests as special cases of IIS, as e.g. recursive estimation, rolling regression, the Chow (1960) test, the Andrews (1993) endogenous break point test, the Bai and Perron (1998) multiple breakpoint test and the tests of extended constancy in Ericsson et al. (1998).
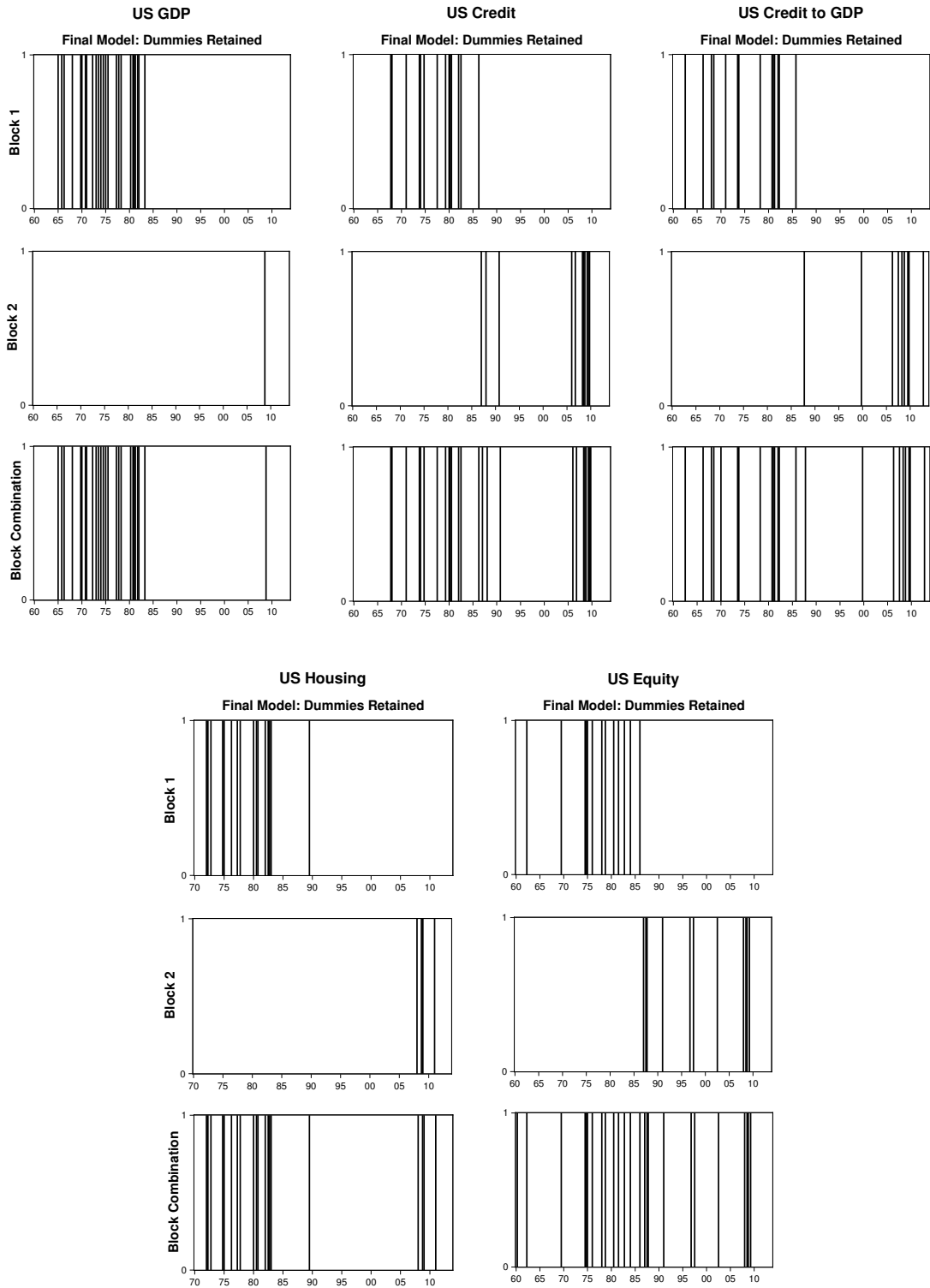
Figure 13: Endogenous Structural Break Point Search for the US. Note: The figure shows the results from the Impulse Indicator Saturation procedure using the split-half approach.
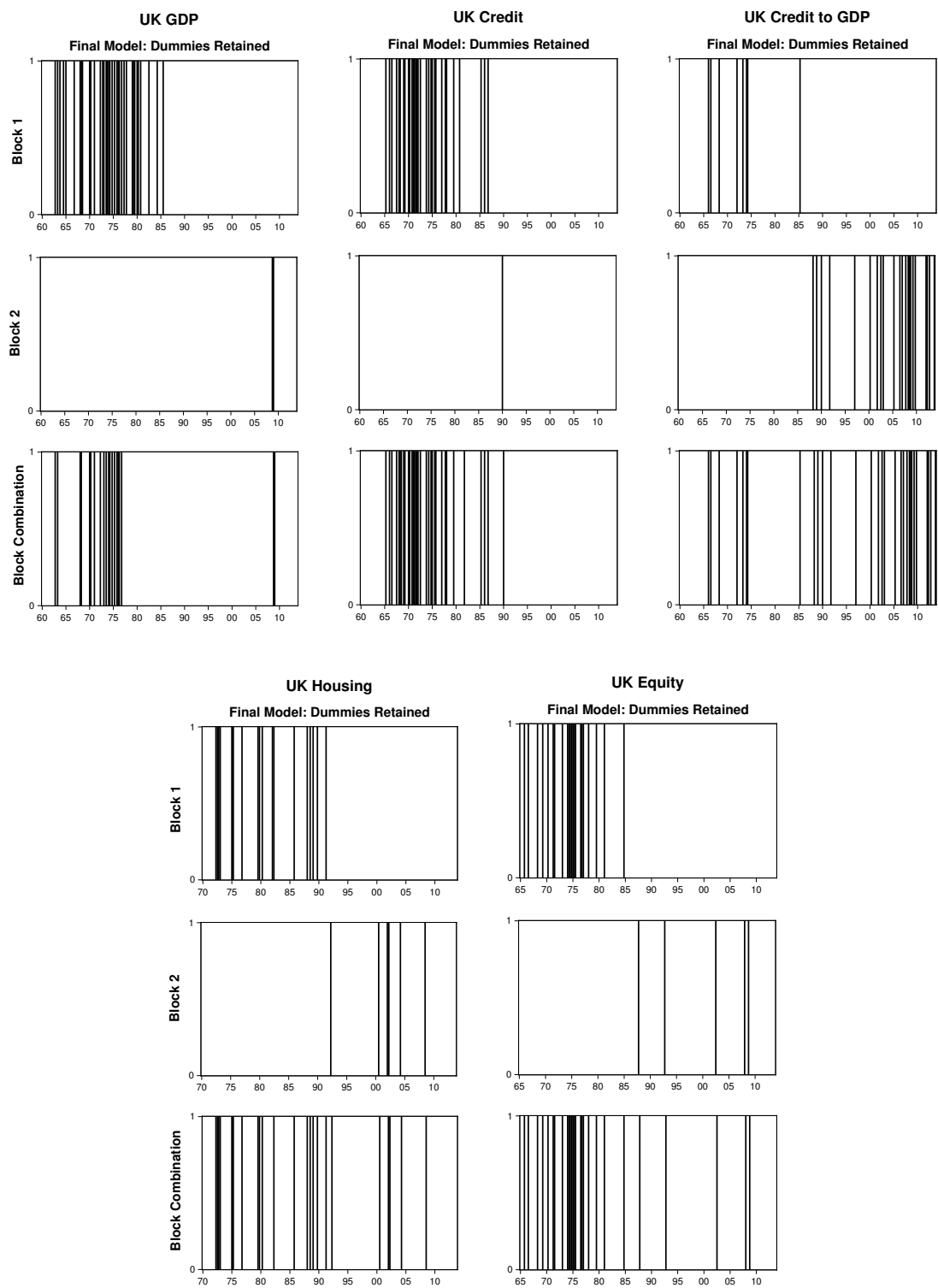
Figure 14: Endogenous Structural Break Point Search for the UK. Note: The figure shows the results from the Impulse Indicator Saturation procedure using the split-half approach.
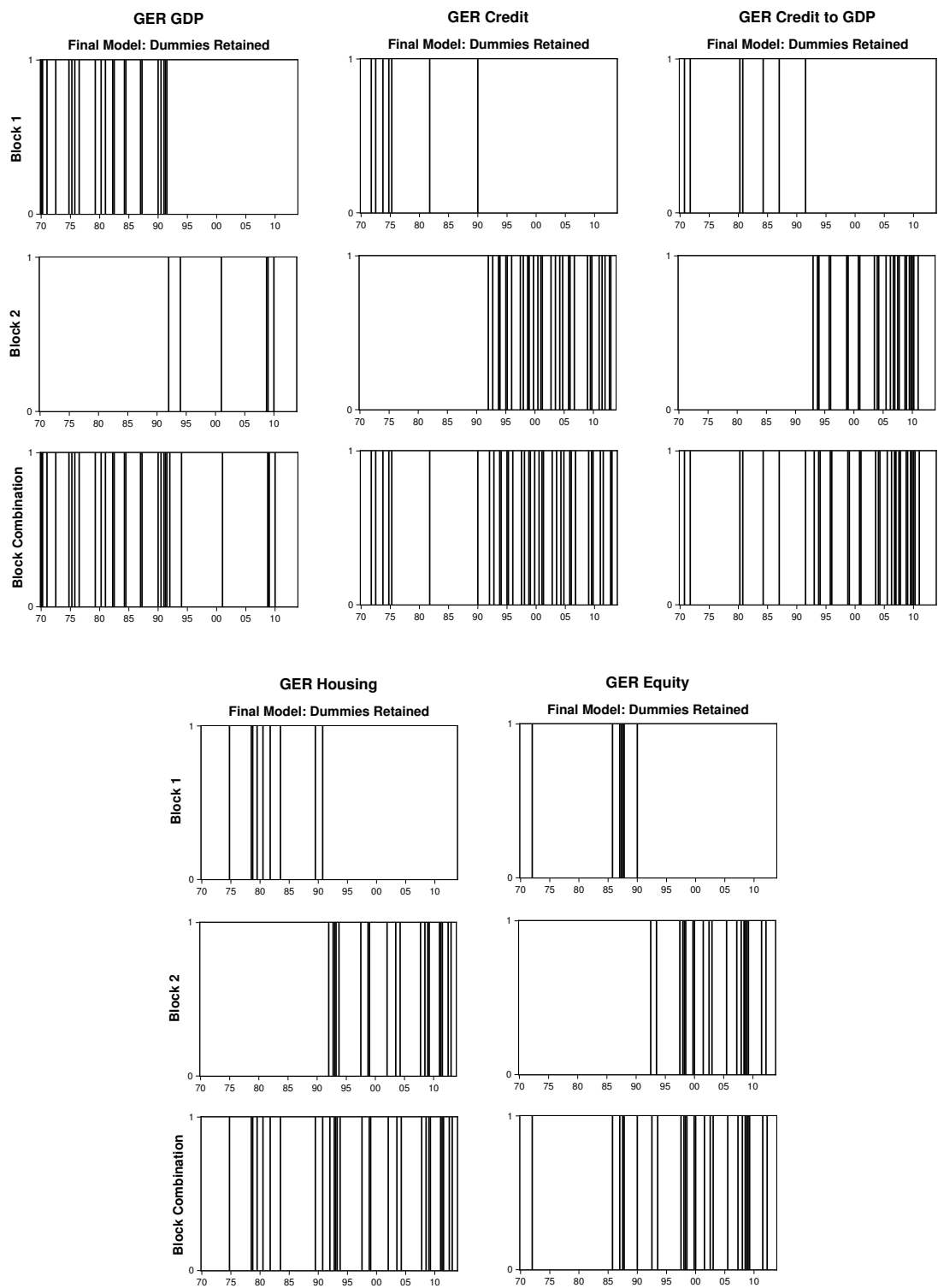
Figure 15: Endogenous Structural Break Point Search for the GER. Note: The figure shows the results from the Impulse Indicator Saturation procedure using the split-half approach.

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