Philipp Harms: International Macroeconomics, 2nd edition

Solutions to Exercises

Chapter 2

Exercise 2.1: Balance of payments transactions

a) The transaction is recorded as a *debit* entry in the *services* section of the German *goods and services account*, which is part of the *current account*. The German consumer's bank transfers the amount to the Spanish hotel's bank. This *increases* German *liabilities* vis-à-vis the rest of the world and is recorded in the *other investment* section of the German *financial account*. Note that, ceteris paribus, the transaction reduces the German current account balance and the German financial account balance by the same amount.

b) The transaction is recorded as a *credit* entry in the *goods* section of the German *goods and services account*, which is part of the *current account*. It also *increases* German *assets* vis-à-vis the rest of the world and is recorded in the *other investment* section of the German *financial account*. Note that, ceteris paribus, the transaction raises the German current account balance and the German financial account balance by the same amount. When the payment takes place six months later, this reduces one type of assets – here: trade credit – and increases another one (e.g. currency and deposits), without affecting the size of the financial account.

c) The transaction is recorded as a *credit* entry in the *goods* section of the German *goods and services account*, which is part of the *current account*, since Germany *exports* food and equipment. It also represents a *transfer* extended to residents of another country and is thus recorded as a *debit* entry in the *secondary income account,* which also is a part of the German *current account*. Note that, ceteris paribus, the transaction does neither affect the German current account balance nor the German financial account balance.

d) The transaction is recorded as an *increase of assets* in the *other investment* section of the German *financial account*. It is also recorded as an *increase in liabilities* in the *portfolio investment* section of the German *financial account*.

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): *International Macroeconomics, 2nd edition*, Tübingen (Mohr Siebeck).
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

Note that, ceteris paribus, the transaction neither affects the German financial account balance nor the German current account balance.

e) The transaction is recorded as a *debit* entry in the German *primary income account*, which is part of the *current account*. It is also recorded as an *increase in liabilities* in the *direct investment* section of the German *financial account*. Note that, ceteris paribus, the transaction reduces the German current account balance and the German financial account balance by the same amount.

Exercise 2.2: The Euro area's balance of payments

a) There are (at least) three explanations for the low figures in the Euro area balance of payments:

- The balance of payments reports *net figures*, i.e. the difference between exports and imports of goods and services, the increase in assets minus the increase in liabilities, etc. The size of the *gross* transactions behind these net figures is likely to be much bigger.

- Large net positions at the country level may net out if the Euro area is considered as a whole. Hence, if member country A runs a large current account *surplus* vis-à-vis the rest of the world, while member country B runs a large current account *deficit*, the combined current account balance of the two countries may be rather small.

- A large set of external transactions of Euro area countries actually involve *other* Euro area countries. Hence, if member country A is running a current account surplus vis-à-vis member country B, this shows up in the individual countries' balance of payments, but it cancels out in the Euro area's balance of payments.

b) The increasingly positive current account balance is clearly driven by growing net exports of goods. By comparison, the increase of net exports of services, the decline of (positive) net primary income, and the decline in (negative) net secondary income is less important.

c) The term is appropriate if "net lending (borrowing)" is interpreted as a general increase (decline) of the difference between assets and liabilities towards the rest of the world. However, if one associates "lending/borrowing" with *debt* contracts sensu stricto, the term is misleading since it masks the importance of equity-type asset sales and purchases (direct investment, portfolio equity) in a country's financial account.

d) Adding up the numbers for the years 2012 – 2014, we can state that …

- Euro area assets increased by slightly more than liabilities for direct investment,
- Euro area liabilities increased by slightly more than assets for portfolio investment,
- Euro area assets increased by more than liabilities for financial derivatives and employee stock options,
- Euro area assets increased by substantially more than liabilities for other investment,
- Euro area assets increased slightly for central bank reserves.

Due to the dominant role of other investments, future net primary income received on the Euro area's net international investment position will be tilted towards debt-related – i.e. non state-contingent – payments.

e) The reserve assets position in the Euro area financial account has been consistently positive between 2012 and 2014, documenting the fact that, in these years, the Euro area accumulated additional foreign reserves. However, this does not guarantee that the *value* of the Euro area's reserve assets actually increased during this period, since *valuation changes* may have pulled the value of reserve assets downward despite the positive numbers in the balance of payments. Note also that the Euro area balance of payments reports the reserves of the whole *Euro system*, i.e. the reserve assets held by the ECB *and* those held by the national central banks of Euro area member states.

f) Using the information given in the question, we can easily show that the value of the Euro area's NIIP increased by -1005.7 – (-1449.7) = 444 billion Euros between the end of 2013 and the end of 2014. However, the financial account surplus amounted to a mere 370.3 billion Euros in the year 2014. The difference between the increase of the NIIP and the ("transactions-based") net increase of assets as reported in the financial account is 444 – 370.3 = 73.7 billion Euros and represents the (positive) contribution of valuation changes to the growth of the Euro area's NIIP.

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): ***International Macroeconomics, 2nd edition*****, Tübingen (Mohr Siebeck)**.
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

Chapter 3

Exercise 3.1: The current account in a small open economy

a) A higher first-period capital stock K_1 generates a higher output Y_1 which, in turn, raises first-period savings and thus the current account CA_1. A higher total factor productivity (TFP) in the second period A_2 results in a higher output Y_2. The anticipation of the higher output (and income) in the second period lowers first-period savings and thus the first-period current account. The second channel through which A_2 influences CA_1 is investment: a higher TFP in period 2 increases the marginal productivity of capital in the second period (MPK_2), so that investment in the first period gets more attractive. As a consequence, I_1 rises and CA_1 falls.

b) Due to the assumption that $B_1 = 0$, the first-period current account balance can be written as $CA_1 = B_2 - B_1 = B_2$.

We can write

$$B_2 = Y_1 - C_1 - I_1 \Leftrightarrow C_1 = A_1(K_1)^\alpha - K_2 - B_2$$
$$C_2 = (1+r)B_2 + Y_2 - I_2 = (1+r)B_2 + A_2(K_2)^\alpha$$

For the CIES utility function used, instantaneous utility is given by

$$u(C_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma},$$

and the intertemporal Euler equation

$$u'(C_1) = \beta(1+r)u'(C_2)$$

therefore reads

$$(C_1)^{-\sigma} = \beta(1+r)(C_2)^{-\sigma}$$

Combining this with the above expressions yields

$$[A_1(K_1)^\alpha - K_2 - B_2]^{-\sigma} = \beta(1+r)[(1+r)B_2 + A_2(K_2)^\alpha]^{-\sigma}$$

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): *International Macroeconomics, 2nd edition*, Tübingen (Mohr Siebeck).
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

$$\Leftrightarrow A_1(K_1)^{\alpha} - K_2 - B_2 = [\beta(1+r)]^{-\frac{1}{\sigma}}[(1+r)B_2 + A_2(K_2)^{\alpha}]$$

$$\Leftrightarrow A_1(K_1)^{\alpha} - K_2 - [\beta(1+r)]^{-\frac{1}{\sigma}} A_2(K_2)^{\alpha} = \{[\beta(1+r)]^{-\frac{1}{\sigma}}(1+r)+1\}B_2$$

$$\Leftrightarrow CA_1 = B_2 = \frac{A_1(K_1)^{\alpha} - K_2 - [\beta(1+r)]^{-\frac{1}{\sigma}} A_2(K_2)^{\alpha}}{[\beta(1+r)]^{-\frac{1}{\sigma}}(1+r)+1}.$$

For first-period investment (which coincides with the second-period capital stock), we use

$$\text{r} = \frac{\partial Y_t}{\partial K_t} - \delta \;\Rightarrow 1+r = A_2\,\alpha\,(K_2)^{\alpha-1} \;\Leftrightarrow K_2 = \left(\frac{\alpha A_2}{1+r}\right)^{\frac{1}{1-\alpha}} = I_1$$

Substituting this into the above expression for the current account yields

$$\Rightarrow CA_1 = B_2 = \frac{A_1(K_1)^{\alpha} - \left(\frac{\alpha A_2}{1+r}\right)^{\frac{1}{1-\alpha}} - [\beta(1+r)]^{-\frac{1}{\sigma}}\left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}}(A_2)^{\frac{1}{1-\alpha}}}{[\beta(1+r)]^{-\frac{1}{\sigma}}(1+r)+1}.$$

c) Taking the derivative, we can show that $\frac{\partial CA_1}{\partial K_1} = \frac{A_1\alpha(K_1)^{\alpha-1}}{[\beta(1+r)]^{-\frac{1}{\sigma}}(1+r)+1} > 0$.

The first-period current account increases in K_1, as a higher capital stock raises output and thus savings in the first period.

Taking the derivative, we can show that

$$\frac{\partial CA_1}{\partial A_2} = \frac{-\left(\frac{\alpha}{(1+r)(1-\alpha)}\left(\frac{\alpha A_2}{1+r}\right)^{\frac{\alpha}{1-\alpha}} + [\beta(1+r)]^{-\frac{1}{\sigma}}\left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{1-\alpha}\right)(A_2)^{\frac{\alpha}{1-\alpha}}\right)}{[\beta(1+r)]^{-\frac{1}{\sigma}}(1+r)+1} < 0$$

As noted above, there are two channels of influence of A_2 on CA_1: First, $\frac{\partial Y_2}{\partial A_2} > 0$ and $\frac{\partial B_2}{\partial Y_2} < 0$, so that $A_2 \uparrow \Rightarrow B_2 \downarrow \Rightarrow CA_1 \downarrow$. Second, $\frac{\partial F_K(K_2)}{\partial A_2} > 0$ $\Rightarrow \frac{\partial I_1}{\partial A_2} > 0$, so that $A_2 \uparrow \Rightarrow I_1 \uparrow \Rightarrow CA_1 \downarrow$. This implies that a ceteris paribus increase of second-period TFP unambiguously lowers the first-period current account.

Exercise 3.2: The German current account

The German reunification of 1990 affected both German savings and investment (and thus the current account): German savings decreased as both public and private consumption grew quicker than income. At the same time, Germany witnessed a boom of both private and public investment, driven by the need to overhaul the public infrastructure and the private capital stock in the former German Democratic Republic.

Exercise 3.3: Current account balances and the interest rate in a two-country model with exogenous incomes

a) As both countries in the setting are assumed to be large, their saving and investment decisions affect the world interest rate, which emerges as an equilibrium price on the international capital market.

b) Educated guess: $\frac{Y_2^F}{Y_2^H} = \phi \;\Rightarrow\; \frac{\partial\left(\frac{Y_2^F}{Y_2^H}\right)}{\partial\phi} > 0 \quad \Rightarrow \frac{\partial CA_1^F}{\partial\phi} < 0 \;, \frac{\partial CA_1^H}{\partial\phi} > 0$. The higher the parameter ϕ, the higher country F's relative income in period 2, so that country F has a stronger incentive to borrow in period 1, while country H has a stronger incentive to lend.

As for the real world interest rate, a higher second-period income gap raises country F's borrowing, i.e. its supply of assets. For savers from country H to buy these assets, the interest rate has to rise, i.e. $\frac{\partial r^*}{\partial\phi} > 0$.

c) $B_2^c = Y_1^c - C_1^c$; $C_2^c = (1+r)B_2^c + Y_2^c$.

For country H: $B_2^H = Y_1^H - C_1^H$; $C_2^H = (1+r)B_2^H + Y_2^H \Leftrightarrow C_1^H = Y_1^H - B_2^H$

Intertemporal Euler equation: $u'(C_1^H) = \beta(1+r)\,u'(C_2^H)$

$$\Leftrightarrow \frac{1}{Y_1^H - B_2^H} = \beta(1+r)\frac{1}{(1+r)B_2^H + Y_2^H} \Leftrightarrow Y_1^H - B_2^H = \frac{1}{\beta(1+r)}[(1+r)B_2^H + Y_2^H]$$

$$\Leftrightarrow Y_1^H - \frac{1}{\beta(1+r)}Y_2^H = \left(1+\frac{1}{\beta}\right)B_2^H \Leftrightarrow CA_1^H = B_2^H = \frac{\beta}{\beta+1}Y_1^H - \frac{1}{(\beta+1)(1+r)}Y_2^H$$

$$\Leftrightarrow CA_1^H = \frac{1}{\beta+1}\left(\beta Y_1^H - \frac{Y_2^H}{1+r}\right)$$

For country F (analogous derivation): $CA_1^F = B_2^F = \frac{1}{\beta+1}\left(\beta Y_1^F - \frac{Y_2^F}{1+r}\right)$.

d) Equilibrium on the international capital market requires: $B_2^H(r) + B_2^F(r) = 0$, i.e. one country's supply of assets has to meet the other country's demand.

$$\Rightarrow \frac{1}{\beta+1}\left(\beta Y_1^H - \frac{Y_2^H}{1+r}\right) + \frac{1}{\beta+1}\left(\beta Y_1^F - \frac{Y_2^F}{1+r}\right) = 0$$

$$\Leftrightarrow \frac{1}{\beta+1}\left[\beta Y_1^H - \frac{Y_2^H}{1+r} + \beta Y_1^H - \frac{\phi Y_2^H}{1+r}\right] = 0 \Leftrightarrow 2\beta Y_1^H - \left(\frac{1}{1+r} + \frac{\phi}{1+r}\right)Y_2^H = 0$$

$$\Leftrightarrow 2\beta Y_1^H = \frac{1+\phi}{1+r}Y_2^H \Leftrightarrow 1+r^* = \frac{(1+\phi)Y_2^H}{2\beta Y_1^H} \Leftrightarrow r^* = \frac{(1+\phi)Y_2^H}{2\beta Y_1^H} - 1$$

Apparently, r^* is increasing in ϕ, which confirms the guess from part b).

e) We can derive country H's first-period current account by substituting the equilibrium gross interest rate $(1+r^*)$ into the expression for CA_1^H that we have derived in part c). This yields

$$CA_1^H = \frac{1}{\beta+1}\left(\beta Y_1^H - \frac{2\beta}{(1+\phi)}\frac{Y_1^H}{Y_2^H}Y_2^H\right) = \frac{\beta}{\beta+1}\frac{(1+\phi-2)}{1+\phi}Y_1^H = \left(\frac{\beta}{\beta+1}\right)\left(\frac{\phi-1}{1+\phi}\right)Y_1^H$$

$\frac{\partial CA_1^H}{\partial \phi} > 0$, which confirms the guess from part b).

Exercise 3.4: Current account balances and the interest rate in a two-country model with investment

a) As in exercise 3.1, we can write $B_2^c = Y_1^c - C_1^c - I_1^c$

$\Leftrightarrow C_1^c = A_1^c\left(K_1^c\right)^\alpha - K_2^c - B_2^c$; $C_2^c = (1+r)B_2^c + A_2^c\left(K_2^c\right)^\alpha$.

Using the intertemporal Euler equation $u'(C_1^c) = \beta(1+r)u'(C_2^c)$ and the assumption on the utility function, we get $(C_1^c)^{-1} = \beta(1+r)(C_2^c)^{-1}$

$$\Leftrightarrow \left[A_1^c(K_1^c)^\alpha - K_2^c - B_2^c\right]^{-1} = \beta(1+r)\left[(1+r)B_2^c + A_2^c(K_2^c)^\alpha\right]^{-1}$$

$$\Leftrightarrow A_1^c(K_1^c)^\alpha - K_2^c - B_2^c = \left[\beta(1+r)\right]^{-1}\left[(1+r)B_2^c + A_2^c(K_2^c)^\alpha\right]$$

$$\Leftrightarrow A_1^c(K_1^c)^\alpha - K_2^c - \frac{A_2^c(K_2^c)^\alpha}{\beta(1+r)} = \frac{1+\beta}{\beta}B_2^c$$

$$\Leftrightarrow B_2^c = \frac{\beta}{1+\beta}\left[A_1^c\left(K_1^c\right)^\alpha - K_2^c\right] - \frac{A_2^c(K_2^c)^\alpha}{(1+\beta)(1+r)}.$$

Using the fact that $1+r = A_2^c\alpha\left(K_2^c\right)^{\alpha-1}$, we can derive $K_2^c = \left(\frac{\alpha A_2^c}{1+r}\right)^{\frac{1}{1-\alpha}}$.

Substituting this into the above expression yields:

$$B_2^c = \frac{\beta}{1+\beta}\left[A_1^c\left(K_1^c\right)^\alpha - \left(\frac{\alpha A_2^c}{1+r}\right)^{\frac{1}{1-\alpha}}\right] - \frac{A_2^c}{(1+\beta)(1+r)}\left(\frac{\alpha A_2^c}{1+r}\right)^{\frac{\alpha}{1-\alpha}}$$

$$= \frac{\beta}{1+\beta}\left[A_1^c\left(K_1^c\right)^\alpha - \left(\frac{\alpha A_2^c}{1+r}\right)^{\frac{1}{1-\alpha}}\right] - \frac{1}{(1+\beta)}\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{A_2^c}{1+r}\right)^{\frac{1}{1-\alpha}}$$

$$= \frac{\beta}{1+\beta}A_1^c\left(K_1^c\right)^\alpha - \frac{1}{1+\beta}\left(\beta\alpha^{\frac{1}{1-\alpha}} + \alpha^{\frac{\alpha}{1-\alpha}}\right)\left(\frac{A_2^c}{1+r}\right)^{\frac{1}{1-\alpha}}$$

$$= \frac{\beta}{1+\beta}A_1^c\left(K_1^c\right)^\alpha - \frac{1}{1+\beta}\alpha^{\frac{\alpha}{1-\alpha}}(\alpha\beta+1)\left(\frac{A_2^c}{1+r}\right)^{\frac{1}{1-\alpha}}$$

Note that this is a special case of the equation we have derived for B_2 in problem 3.1 b). You can easily check this by setting $\sigma = 1$ in that expression. (Recall from the Appendix to Chapter III that $\lim_{\sigma\to 1}\frac{C_t^{1-\sigma}-1}{1-\sigma} = \ln C_t$)

b) As in problem 3.3, equilibrium on the international capital market requires that $B_2^F + B_2^H = 0$. This implies

$$\frac{\beta}{1+\beta}\left[A_1^H\left(K_1^H\right)^\alpha + A_1^F\left(K_1^F\right)^\alpha\right] = \frac{1}{1+\beta}\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha\beta)\left[\left(\frac{A_2^H}{1+r}\right)^{\frac{1}{1-\alpha}} + \left(\frac{A_2^F}{1+r}\right)^{\frac{1}{1-\alpha}}\right]$$

$$\Leftrightarrow (1+r)^{\frac{1}{1-\alpha}} = \frac{\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha\beta)\left[\left(A_2^H\right)^{\frac{1}{1-\alpha}} + \left(A_2^F\right)^{\frac{1}{1-\alpha}}\right]}{\beta\left[A_1^H\left(K_1^H\right)^{\alpha} + A_1^F\left(K_1^F\right)^{\alpha}\right]}$$

$$\Leftrightarrow (1+r^*) = \left(\frac{\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha\beta)\left[\left(A_2^H\right)^{\frac{1}{1-\alpha}} + \left(A_2^F\right)^{\frac{1}{1-\alpha}}\right]}{\beta\left[A_1^H\left(K_1^H\right)^{\alpha} + A_1^F\left(K_1^F\right)^{\alpha}\right]}\right)^{1-\alpha}$$

Note that the world interest rate is the higher the higher the countries‘ second-period TFP, since this reduces savings and thus raises asset supply. Conversely, a high first-period income enhances savings and thus reduces the interest rate.

c) Substituting $(1+r^*)$ from part b) into the expression for $CA_1^H = B_2^H$ from part a) yields:

$$CA_1^H = \frac{\beta}{1+\beta}A_1^H\left(K_1^H\right)^{\alpha} - \frac{1}{1+\beta}\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha\beta)\left(A_2^H\right)^{\frac{1}{1-\alpha}}\frac{\beta\left[A_1^H\left(K_1^H\right)^{\alpha} + A_1^F\left(K_1^F\right)^{\alpha}\right]}{\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha\beta)\left[\left(A_2^H\right)^{\frac{1}{1-\alpha}} + \left(A_2^F\right)^{\frac{1}{1-\alpha}}\right]}$$

$$\Leftrightarrow CA_1^H = \frac{\beta}{1+\beta}A_1^H\left(K_1^H\right)^{\alpha} - \frac{\beta}{1+\beta}\frac{\left(A_2^H\right)^{\frac{1}{1-\alpha}}}{\left(A_2^H\right)^{\frac{1}{1-\alpha}} + \left(A_2^F\right)^{\frac{1}{1-\alpha}}}\left[A_1^H\left(K_1^H\right)^{\alpha} + A_1^F\left(K_1^F\right)^{\alpha}\right]$$

$$\Leftrightarrow CA_1^H = \frac{\beta}{1+\beta}\left[\frac{A_1^H\left(K_1^H\right)^{\alpha}}{A_1^H\left(K_1^H\right)^{\alpha} + A_1^F\left(K_1^F\right)^{\alpha}} - \frac{\left(A_2^H\right)^{\frac{1}{1-\alpha}}}{\left(A_2^H\right)^{\frac{1}{1-\alpha}} + \left(A_2^F\right)^{\frac{1}{1-\alpha}}}\right]\left[A_1^H\left(K_1^H\right)^{\alpha} + A_1^F\left(K_1^F\right)^{\alpha}\right]$$

Note: Whether country H exhibits a current account surplus in period 1 depends on its first-period income relative to the first-period income of country F, and its second-period TFP (relative to country F’s second-period TFP).

d) $\frac{\partial r^*}{\partial A_2^c} > 0$; $\frac{\partial r^*}{\partial K_1^c} < 0$

$$\frac{\partial CA_1^H}{\partial A_2^H} < 0 \;;\; \frac{\partial CA_1^H}{\partial A_2^F} > 0;\; \frac{\partial CA_1^H}{\partial K_1^H} > 0 \;;\; \frac{\partial CA_1^H}{\partial K_1^F} < 0$$

Interpretation:

$$\left.\begin{array}{l} A_2^H \uparrow \Rightarrow Y_2^H \uparrow \Rightarrow B_2^H \downarrow \\ A_2^H \uparrow \Rightarrow MPK_2^H \uparrow \Rightarrow I_1 \uparrow \end{array}\right\} \Rightarrow CA_1^H \downarrow$$

$K_1^H \uparrow \Rightarrow Y_1^H \uparrow \Rightarrow S_1^H \uparrow \Rightarrow CA_1^H \uparrow$ and $r^* \downarrow$ (If I_1 =const and $\delta = 1$)

$$\left.\begin{array}{l} A_2^F \uparrow \Rightarrow Y_2^F \uparrow \\ A_2^F \uparrow \Rightarrow I_1 \uparrow \end{array}\right\} \Rightarrow CA_1^F \downarrow \text{ and } CA_1^H \uparrow \text{ and } r^* \uparrow$$

$K_1^F \uparrow \Rightarrow Y_1^F \uparrow \Rightarrow S_1^F \uparrow \Rightarrow CA_1^F \uparrow$ and $CA_1^H \downarrow$ and $r^* \downarrow$

Exercise 3.5: International investment and factor prices

a) We start by analyzing the case of **financial autarky**, i.e. a situation in which the country has no access to the international capital market.

We start by computing the aggregate capital stock in period 2

The assumption of *identical homothetic preferences* and of a *constant returns to scale production function* for all firms allows to use the shortcut of a *representative consumer* and a *representative firm*. Note also that, due to our assumption that the labor force is assumed to coincide with the entire population, which is normalized to one in both countries, the aggregate capital stock equals the capital stock *per capita*.

Equilibrium in autarky is characterized by

$$\frac{1}{C_1^c} = \beta(1+r^{c,a})\frac{1}{C_2^c} \Leftrightarrow C_2^c = \beta(1+r^{c,a})C_1^c$$

Note: $C_1^c = \left(K_1^c\right)^\alpha - I_1^c = \left(K_1^c\right)^\alpha - K_2^c$; $C_2^c = \left(K_2^c\right)^\alpha$

$$\alpha\left(K_2^c\right)^{\alpha-1} = 1+r \Leftrightarrow K_2^c = \left(\frac{\alpha}{1+r}\right)^{\frac{1}{1-\alpha}}$$

Substituting these expressions into the intertemporal Euler equation yields

$$\left(\frac{\alpha}{1+r^{c,a}}\right)^{\frac{\alpha}{1-\alpha}} = \beta(1+r^{c,a})\left[\left(K_1^c\right)^\alpha - \left(\frac{\alpha}{1+r^{c,a}}\right)^{\frac{1}{1-\alpha}}\right],$$

with $r^{c,a}$ as the **autarky interest rate** in country *c*.

Simplifying this expression yields

$$\left(\frac{\alpha}{1+r^{c,a}}\right)^{\frac{\alpha}{1-\alpha}} = \beta(1+r^{c,a})\left(K_1^c\right)^{\alpha} - \beta\alpha^{\frac{1-\alpha+\alpha}{1-\alpha}}(1+r^{c,a})^{-\frac{\alpha}{1-\alpha}}$$

$$\alpha^{\frac{\alpha}{1-\alpha}}\left(1+r^{c,a}\right)^{-\frac{\alpha}{1-\alpha}} + \beta\alpha^{\frac{1-\alpha}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}(1+r^{c,a})^{-\frac{\alpha}{1-\alpha}} = \beta(1+r^{c,a})\left(K_1^c\right)^{\alpha}$$

$$\alpha^{\frac{\alpha}{1-\alpha}}\left(1+r^{c,a}\right)^{-\frac{\alpha}{1-\alpha}}(1+\alpha\beta) = \beta(1+r^{c,a})\left(K_1^c\right)^{\alpha}$$

$$\frac{\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha\beta)}{\beta\left(K_1^c\right)^{\alpha}} = (1+r^{c,a})^{\frac{1}{1-\alpha}} \Leftrightarrow (1+r^{c,a}) = \left[\frac{\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha\beta)}{\beta\left(K_1^c\right)^{\alpha}}\right]^{1-\alpha}$$

Note that $(1+r^{F,a}) = \left(\frac{1}{\phi}\right)^{(1-\alpha)\alpha}(1+r^{H,a})$, i.e. the interest rate in the initially poorer country F is higher than the interest rate in the initially richer country H.

Combining the result on the interest rate with the above expression for the optimal capital stock yields the optimal second-period capital stock in country *c*:

$$K_2^{c,a} = \left(\frac{\alpha}{1+r^{c,a}}\right)^{\frac{1}{1-\alpha}} = \alpha^{\frac{1}{1-\alpha}}\beta\left(K_1^c\right)^{\alpha}\alpha^{-\frac{\alpha}{1-\alpha}}(1+\alpha\beta)^{-1} = \frac{\alpha\beta}{1+\alpha\beta}\left(K_1^c\right)^{\alpha}$$

Note:

$$\frac{K_2^{F,a}}{K_2^{H,a}} = \frac{\frac{\alpha\beta}{1+\alpha\beta}\left(\phi K_1^H\right)^{\alpha}}{\frac{\alpha\beta}{1+\alpha\beta}\left(K_1^H\right)^{\alpha}} = \phi^{\alpha} < 1$$

Interpretation: The country with the higher capital stock in period 1 exhibits the higher capital stock in period 2.

b) We now analyze the case of **financial integration**, i.e. a situation in which agents are free to trade assets on a perfect international capital market.

We can use the solution to problem 3.4 b) (setting $A_1^H = A_1^F = A_2^H = A_2^F = 1$ and using the subscript „FI" to denote variables under financial integration):

$$1+r^{*}=\left[2\frac{\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha\beta)}{\beta\left[\left(K_1^H\right)^{\alpha}+\left(K_1^F\right)^{\alpha}\right]}\right]^{1-\alpha}=\left[\frac{2}{1+\phi^{\alpha}}\frac{\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha\beta)}{\beta\left(K_1^H\right)^{\alpha}}\right]^{1-\alpha}=\left[\frac{2\phi^{\alpha}}{1+\phi^{\alpha}}\frac{\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha\beta)}{\beta\left(K_1^F\right)^{\alpha}}\right]^{1-\alpha}$$

Comparing these results to the above expression for the countries' autarky interest rates reveals that $(1+r^{*})>(1+r^{H,a})$ and that $(1+r^{*})<(1+r^{F,a})$.

Under the above assumptions, financial integration – which implies that the same (world) interest rate r* prevails in both countries, implies that the countries‘ second-period capital stocks (per capita) are equalized.

$$K_2^{H,FI}=K_2^{F,FI}=\left(\frac{\alpha}{1+r^{*}}\right)^{\frac{1}{1-\alpha}}=\alpha^{\frac{1}{1-\alpha}}\frac{\beta\left[\left(K_1^H\right)^{\alpha}+\left(K_1^F\right)^{\alpha}\right]}{\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha\beta)\cdot 2}=\frac{\alpha\beta}{1+\alpha\beta}\frac{\left[\left(K_1^H\right)^{\alpha}+\left(K_1^F\right)^{\alpha}\right]}{2}$$

$$=\frac{\alpha\beta}{1+\alpha\beta}\frac{\left(1+\phi^{\alpha}\right)}{2}\left(K_1^H\right)^{\alpha}$$

Comparing the second-period capital stock under financial autarky and financial integration yields:

$$K_2^{H,FI}<K_2^{H,a}\text{ since }\frac{1+\phi^{\alpha}}{2}<1$$

$$K_2^{F,FI}>K_2^{F,a}\text{ since }\frac{1+\phi^{\alpha}}{2}>\phi^{\alpha}$$

Hence, the initially richer country H exhibits a *lower* second-period (domestic) capital stock under financial integration than under financial autarky, while initially poorer country F exhibits a *higher* second-period (domestic) capital stock under financial integration than under financial autarky.

c) GDPs (per capita) for the two countries in period 2 under financial autarky are given by

$Y_2^{c,a}=\left(K_2^{c,a}\right)^{\alpha}$, i.e.

$$Y_2^{H,a}=\left(\frac{\alpha\beta}{1+\alpha\beta}\left(K_1^H\right)^{\alpha}\right)^{\alpha}$$

$$Y_2^{F,a}=\left(\frac{\alpha\beta}{1+\alpha\beta}\left(K_1^F\right)^{\alpha}\right)^{\alpha}=\left(\frac{\alpha\beta}{1+\alpha\beta}\left(\phi K_1^H\right)^{\alpha}\right)^{\alpha}=\left(\frac{\alpha\beta}{1+\alpha\beta}\left(K_1^H\right)^{\alpha}\right)^{\alpha}\cdot\phi^{\alpha^2}$$

Hence, under autarky, the initially rich country H has a higher GDP in period 2 than the initially poor country F.

By contrast, both countries have the same period-2 GDP under financial integration:

$$Y_2^{H,FI}=Y_2^{F,FI}=\left(\frac{\alpha\beta}{1+\alpha\beta}\frac{\left(1+\phi^{\alpha}\right)}{2}\left(K_1^H\right)^{\alpha}\right)^{\alpha}$$

Observation:

$Y_2^{H,FI}<Y_2^{H,a}$, $Y_2^{F,FI}>Y_2^{F,a}$

Hence, country H (F) would have had a higher (lower) period-2 GDP under financial autarky than under financial integration.

Note, however, that a part of capital stock located in F in period 2 is owned by residents of country H. Hence:

$Y_2^{GNI,H}>Y_2^{H,FI}$, i.e. second-period GNI is *greater* than GDP for country H

$Y_2^{GNI,F}<Y_2^{F,FI}$, i.e. second-period GNI is *smaller* than GDP for country F

More specifically:

$Y_2^{GNI,c}=Y_2^{c,FI}+r\cdot B_2^c=Y_2^{c,FI}+r\cdot CA_1^c$ (we can use result from 3.4)

We know from 3.4 c):

$$CA_1^H=\frac{\beta}{1+\beta}\left[\frac{\left(K_1^H\right)^{\alpha}}{\left(K_1^H\right)^{\alpha}+\left(K_1^F\right)^{\alpha}}-\frac{1}{2}\right]\left[\left(K_1^H\right)^{\alpha}+\left(K_1^F\right)^{\alpha}\right]$$

$$=\frac{\beta}{1+\beta}\frac{2\left(K_1^H\right)^{\alpha}-\left(K_1^H\right)^{\alpha}-\left(K_1^F\right)^{\alpha}}{2\left[\left(K_1^H\right)^{\alpha}+\left(K_1^F\right)^{\alpha}\right]}\left[\left(K_1^H\right)^{\alpha}+\left(K_1^F\right)^{\alpha}\right]$$

$$= \frac{\beta}{1+\beta} \frac{\left(K_1^H\right)^\alpha - \left(K_1^F\right)^\alpha}{2} = B_2^H = -B_2^F$$

Hence:

$$Y_2^{GNI,H} = Y_2^{H,FI} + \underbrace{r^* \frac{\beta}{1+\beta} \frac{\left(1-\phi^\alpha\right)}{2} \left(K_1^H\right)^\alpha}_{>0}$$

$$Y_2^{GNI,F} = Y_2^{F,FI} + \underbrace{r^* \frac{\beta}{1+\beta} \frac{\left(\phi^\alpha - 1\right)}{2} \left(K_1^H\right)^\alpha}_{<0}$$

Note: r^* is endogenous (see part 3.5b). To avoid rather messy (and not really informative) expressions, we do not substitute the equilibrium world interest rate into the above result.

d) Under perfect competition, the real wage equals the marginal product of labor, i.e. $w_t^c = F_{L,t}(K_t^c, L_t^c)$.

Under autarky:

$$w_2^{H,a} = (1-\alpha)\left[\frac{\alpha\beta}{1+\alpha\beta}\left(K_1^H\right)^\alpha\right]^\alpha \underbrace{\left(L_2^H\right)^{-\alpha}}_{=1}$$

$$w_2^{F,a} = (1-\alpha)\left[\frac{\alpha\beta}{1+\alpha\beta}\left(K_1^H\right)^\alpha \phi^\alpha\right]^\alpha \underbrace{\left(L_2^F\right)^{-\alpha}}_{=1} \Rightarrow w_2^{H,a} > w_2^{F,a}$$

Note: Due to the higher second-period capital stock, the second-period wage is higher in country H than in country F.

Under financial integration, wages are equalized across countries:

$$w_2^{H,FI} = w_2^{F,FI} = (1-\alpha)\left[\frac{\alpha\beta}{1+\alpha\beta}\frac{\left(1+\phi^\alpha\right)}{2}\left(K_1^H\right)^\alpha\right]^\alpha$$

Note:

$$w_2^{H,FI} < w_2^{H,a}\,;\, w_2^{F,FI} > w_2^{F,a}$$

Hence, in country H, the real wage in period 2 is *lower* under financial integration than under financial autarky. In country F, the real wage in period 2 is *greater* under financial integration than under financial autarky.

e) To assess the welfare consequences for individual *i* in country H (dropping the country subscript to avoid cluttered notation), we derive its indirect utility by reproducing its intertemporal optimization, using the notation from pages 92 ff. Note that, for quite a while, we do not differentiate between financial autarky and financial integration.

In period 1, individual *i* maximizes:

$$U_1^i = \ln\left(C_1^i\right) + \beta \ln\left(C_2^i\right)$$

s.t.

$$C_1^i = w_1 + x_1^i\left(d_1 + V_1\right) - \underbrace{\left(x_2^i V_1 + B_2^i\right)}_{=\,S_1^i},$$

where $V_1 = \dfrac{d_2}{1+r}$ gives the value of the representative firm as of period 1, d_t are the dividends paid out by that firm, and x_t^i is the share of the representative firm owned by individual *i* at the start of period t. (Using the construct of a representative firm considerably simplifies the individual's portfolio choice since we do not have to analyze how the individual allocates its savings to the shares of different firms). Note that x_1^i defines an individual's exogenous initial wealth.

$$C_2^i = w_2 + (1+r)B_2^i + x_2^i d_2 = w_2 + (1+r)\underbrace{\left(x_2^i V_1 + B_2^i\right)}_{=\,S_1^i}$$

Ignoring the individual's choice between bonds and equity, we determine optimal savings S_1^i by deriving and solving the intertemporal Euler equation:

$$\frac{1}{C_1^i} = \beta(1+r)\frac{1}{C_2^i} \Leftrightarrow C_2^i = \beta(1+r)C_1^i$$

$$\Leftrightarrow w_2 + (1+r)S_1^i = \beta(1+r)\left[w_1 + x_1^i\left(d_1 + V_1\right) - S_1^i\right]$$

$$\Leftrightarrow S_1^i = \frac{\beta}{1+\beta}\left[w_1 + x_1^i\left(d_1 + V_1\right)\right] - \frac{w_2}{(1+\beta)(1+r)}$$

By substituting this expression into the utility function, we can derive the individual's indirect utility

$$\tilde{U}_1^i = \ln\left\{w_1 + x_1^i\left(d_1 + V_1\right) - \frac{\beta}{1+\beta}\left[w_1 + x_1^i\left(d_1 + V_1\right)\right] + \frac{w_2}{(1+\beta)(1+r)}\right\} +$$
$$+\beta\ln\left\{w_2 + (1+r)\frac{\beta}{1+\beta}\left[w_1 + x_1^i\left(d_1 + V_1\right)\right] - (1+r)\frac{w_2}{(1+\beta)(1+r)}\right\}$$

$$= \ln\left\{\frac{1}{1+\beta}\left[w_1 + x_1^i\left(d_1 + V_1\right) + \frac{w_2}{(1+r)}\right]\right\} + \beta\ln\left\{\frac{\beta}{1+\beta}(1+r)\left[w_1 + x_1^i\left(d_1 + V_1\right) + \frac{w_2}{(1+r)}\right]\right\}$$

where the term in squared brackets is the present value of individual *i*'s income.

<u>Note</u>:

$$d_1 = K_1^\alpha - w_1 - K_2 = K_1^\alpha - (1-\alpha)K_1^\alpha - K_2 = \alpha K_1^\alpha - K_2$$

$$V_1 = \frac{d_2}{1+r} = \frac{\alpha K_2^\alpha}{1+r}$$

$$w_2 = (1-\alpha)K_2^\alpha$$

$$K_2 = \left(\frac{\alpha}{1+r}\right)^{\frac{1}{1-\alpha}}$$

Substituting these results into the above expression yields

$$\tilde{U}_1^i = \ln\left\{\frac{1}{1+\beta}\left[w_1 + x_1^i\left(\alpha K_1^\alpha - K_2 + \frac{\alpha K_2^\alpha}{1+r}\right) + \frac{(1-\alpha)K_2^\alpha}{1+r}\right]\right\} +$$
$$+\beta\ln\left\{\frac{\beta}{1+\beta}(1+r)\left[w_1 + x_1^i\left(\alpha K_1^\alpha - K_2 + \frac{\alpha K_2^\alpha}{1+r}\right) + \frac{(1-\alpha)K_2^\alpha}{1+r}\right]\right\}$$

Further simplifying the present value of income yields

$$w_1+\underbrace{x_1^i\alpha K_1^\alpha}_{exogenous}-x_1^i\left(\frac{\alpha}{1+r}\right)^{\frac{1}{1-\alpha}}+x_1^i\frac{\alpha\left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}}}{1+r}+\frac{(1-\alpha)\left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}}}{1+r}$$

$$=w_1+x_1^i\alpha K_1^\alpha-x_1^i\left(\frac{\alpha}{1+r}\right)^{\frac{1}{1-\alpha}}+x_1^i\left(\frac{\alpha}{1+r}\right)^{\frac{1}{1-\alpha}}+(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{1+r}\right)^{\frac{1}{1-\alpha}}$$

$$=w_1+x_1^i\alpha K_1^\alpha+(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{1+r}\right)^{\frac{1}{1-\alpha}}$$

Hence,

$$\widetilde{U}_1^i=\ln\left\{\frac{1}{1+\beta}\left[w_1+x_1^i\alpha K_1^\alpha+(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{1+r}\right)^{\frac{1}{1-\alpha}}\right]\right\}+$$

$$+\beta\ln\left\{\frac{\beta}{1+\beta}(1+r)\left[w_1+x_1^i\alpha K_1^\alpha+(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{1+r}\right)^{\frac{1}{1-\alpha}}\right]\right\}$$

$$=\ln\left\{\frac{1}{1+\beta}\left[w_1+x_1^i\alpha K_1^\alpha+(1-\alpha)\alpha^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha-1}{1-\alpha}}\left(\frac{1}{1+r}\right)^{\frac{1}{1-\alpha}}\right]\right\}+$$

$$+\beta\ln\left\{\frac{\beta}{1+\beta}\left[(1+r)\left(w_1+x_1^i\alpha K_1^\alpha\right)+(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{1+r}\right)^{\frac{\alpha}{1-\alpha}}\right]\right\}$$

$$=\ln\left\{\frac{1}{1+\beta}\left[w_1+x_1^i\alpha K_1^\alpha+\frac{(1-\alpha)}{\alpha}\underbrace{\left(\frac{\alpha}{1+r}\right)^{\frac{1}{1-\alpha}}}_{\text{decreasing in (1+r)}}\right]\right\}+$$

$$+\beta\ln\left\{\frac{\beta}{1+\beta}\left[\underbrace{(1+r)}_{\text{increasing in (1+r)}}\left(w_1+x_1^i\alpha K_1^\alpha\right)+(1-\alpha)\underbrace{\left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}}}_{\text{decreasing in (1+r)}}\right]\right\}$$

Recall from part b) that, for country H, the gross interest rate (1+ r) is *increasing* as a result of moving from financial autarky to financial integration. Hence, the second part of the expression in squared brackets is decreasing: the higher interest rate depresses the future capital stock and thus future wages. Moreover, it results in a stronger discounting of future income. However, the higher interest rate also raises the remuneration on first-period savings. The latter effect is the stronger, the higher an individual's first-period income, which, in turn, depends on an indiviual's initial share of the representative firm (x_1^i). While the total effect is ambiguous, the positive effect of financial integration is likely to dominate if x_1^i is large. By contrast, the overall effect is possibly negative if x_1^i is close to zero. This suggests that (exogenous) first-period factor endowments determine individuals' relative gains and losses from financial integration: in country H, individuals who own a large share of the first-period capital stock ("capitalists") are more likely to benefit from financial integration than individuals whose first-period capital ownership is zero ("workers"). Note, however, that the latter group's net utility gains depend on the first-period wage and are not necessarily negative.

f) If there is a system of lump-sum transfers that guarantees that agents are compensated for the welfare losses at the individual level, the society as a whole – and, hence, the representative consumer – benefits from financial integration in this simple model.

Exercise 3.6: Time consistent behavior

a) As $\beta(1+r)=1$, it is optimal for RC to implement a constant level of consumption over the three periods. In order to achieve this constant consumption level despite the uneven time path of income, RC *borrows* in period 1 and *lends* in period 2. In period 3, RC collects the principal and interest on his second-period lending. As a consequence, the country exhibits a current account deficit in periods 1 and 2, and a current account surplus in period 2.

b) We start by deriving RC's optimal consumption path. From the assumption that $\beta(1+r)=1$ it follows that $C_t = C \;\; \forall t$. Hence,

$$\Rightarrow \sum_{s=1}^{3}\left(\frac{1}{1+r}\right)^{s-1} C = (1+r)B_1 + \sum_{s=1}^{3}\left(\frac{1}{1+r}\right)^{s-1} Y_s \Leftrightarrow C\cdot\frac{1-\left(\frac{1}{1+r}\right)^3}{1-\frac{1}{1+r}} = \sum_{s=1}^{3}\left(\frac{1}{1+r}\right)^{s-1} Y_s$$

<u>Note</u>: We are using the result that $\sum_{j=0}^{T} a^j = \frac{1-a^{T+1}}{1-a}$ for $|a|<1$

$$C\cdot\frac{1-\left(\frac{1}{1+r}\right)^3}{1-\frac{1}{1+r}}=\sum_{s=1}^{3}\left(\frac{1}{1+r}\right)^{s-1}Y_s \Leftrightarrow C\cdot\frac{\frac{(1+r)^3-1}{(1+r)^3}}{\frac{(1+r)-1}{(1+r)}}=\sum_{s=1}^{3}\left(\frac{1}{1+r}\right)^{s-1}\cdot 1+\left(\frac{1}{1+r}\right)\cdot 3$$

The last term on the RHS reflects the fact that RC's income in the second period is three units higher than his income in periods 1 and 3. Using the above rule, we arrive at

$$C\cdot\frac{(1+r)^3-1}{r(1+r)^2}=\frac{(1+r)^3-1}{r(1+r)^2}\cdot 1+\left(\frac{1}{1+r}\right)\cdot 3 \Leftrightarrow (C-1)\cdot\frac{(1+r)^3-1}{r(1+r)^2}=\frac{3}{1+r}$$

$$\Leftrightarrow (C-1)\cdot\frac{(1+r)^3-1}{r(1+r)}=3 \Leftrightarrow C-1=\frac{r(1+r)}{(1+r)^3-1}\cdot 3 \Leftrightarrow C=1+\frac{r(1+r)}{(1+r)^3-1}\cdot 3$$

To compute the first-period current account, we use

$$CA_1=Y_1-C_1=1-C=-\frac{r(1+r)}{(1+r)^3-1}\cdot 3=B_2$$

which is obviously negative. To compute the second-period current account, we use

$$CA_2=Y_2+rB_2-C_2=Y_2+rB_2-C=4-\frac{r^2(1+r)}{(1+r)^3-1}\cdot 3-1-\frac{r(1+r)}{(1+r)^3-1}\cdot 3$$

$$\Leftrightarrow CA_2=3\underbrace{\left(1-\frac{r(1+r)^2}{(1+r)^3-1}\right)}_{>/<0\,?}$$

To show that the second-period capital stock is positive, we proceed as follows:

$$1-\frac{r(1+r)^2}{(1+r)^3-1}>0 \Leftrightarrow \frac{r(1+r)^2}{(1+r)^3-1}<1 \Leftrightarrow (1+r)^3-1>r(1+r)^2$$

$$\Leftrightarrow (1+r)(1+2r+r^2)-1>r(1+2r+r^2)$$

$$\Leftrightarrow 1+2r+r^2+r+2r^2+r^3-1>r+2r^2+r^3 \Leftrightarrow 3r+3r^2+r^3>r+2r^2+r^3$$

$$\Leftrightarrow 2r+r^2>0 \Rightarrow CA_2>0$$

We can use this result to derive the NIIP at the start of period 3:

$$B_3 - B_2 = CA_2 \Leftrightarrow B_3 = B_2 + CA_2 = -\frac{r(1+r)}{(1+r)^3 - 1}\cdot 3 + 3\left(1 - \frac{r(1+r)^2}{(1+r)^3 - 1}\right)$$

$$\Leftrightarrow B_3 = 3 - \frac{r(1+r)^2}{(1+r)^3 - 1}\cdot 3 - \frac{r(1+r)}{(1+r)^3 - 1}\cdot 3 = 3 - \frac{r(1+r)}{(1+r)^3 - 1}\cdot 3(1+r+1)$$

$$= 3 - \frac{r(1+r)(2+r)}{(1+r)^3 - 1}\cdot 3$$

The current account balance of period 3 is given by

$$CA_3 = Y_3 + rB_3 - C_3 = 1 + r\left(3 - \frac{r(1+r)(2+r)}{(1+r)^3 - 1}\cdot 3\right) - 1 - \frac{r(1+r)}{(1+r)^3 - 1}\cdot 3$$

$$= 3r\left(1 - \frac{r(1+r)(2+r)}{(1+r)^3 - 1} - \frac{(1+r)}{(1+r)^3 - 1}\right) = 3r\left(\frac{(1+r)^3 - 1 - r(1+r)(2+r) - (1+r)}{(1+r)^3 - 1}\right)$$

$$= 3r\left(\frac{(1+r)(1+2r+r^2) - 1 - (r+r^2)(2+r) - (1+r)}{(1+r)^3 - 1}\right)$$

$$= 3r\left(\frac{1+2r+r^2+r+2r^2+r^3-1-2r-r^2-2r^2-r^3-1-r}{(1+r)^3 - 1}\right) = 3r\left(\frac{-1}{(1+r)^3 - 1}\right) < 0,$$

which is obviously negative.

To double-check this result, we compute the NIIP at the start of period 4:

$$B_4 = B_3 + CA_3 = 3\left(1 - \frac{r(1+r)(2+r)}{(1+r)^3 - 1}\right) - 3\left(\frac{r}{(1+r)^3 - 1}\right)$$

$$= 3\left[\frac{(1+r)^3 - 1 - r(1+r)(2+r) - r}{(1+r)^3 - 1}\right] = 3\left[\frac{(1+r)(1+2r+r^2) - 1 - (r+r^2)(2+r) - r}{(1+r)^3 - 1}\right]$$

$$= 3\left[\frac{1+2r+r^2+r+2r^2+r^3-1-2r-r^2-2r^2-r^3-r}{(1+r)^3 - 1}\right] = 0$$

The fact that the NIIP at the start of the first "post-existence" period is zero is not surprising and confirms the above results.

c) At the start of period 2, RC's plans for the two remaining periods of his life:

$$U_2 = \ln C_2 + \beta \ln C_3$$

As before, we have

$$\beta(1+r)=1 \Rightarrow C_2 = C_3$$

Using the intertemporal budget constraint for the last two periods yields

$$C\left(1+\frac{1}{1+r}\right)=(1+r)B_2+Y_2+\frac{Y_3}{1+r}=(1+r)B_2+(1+3)+\frac{1}{1+r}$$
$$\Leftrightarrow C\left(\frac{2+r}{1+r}\right)=(1+r)B_2+\left(\frac{2+r}{1+r}\right)\cdot 1+3 \Leftrightarrow C=B_2\frac{(1+r)^2}{(2+r)}+1+3\cdot\frac{(1+r)}{(2+r)}$$

Note that B_2, which RC takes as given at the start of period 2, has been derived above. Substituting the above expression for B_2 into the result for the constant consumption path yields

$$\Leftrightarrow C=-\frac{3\cdot r(1+r)}{(1+r)^3-1}\cdot\frac{(1+r)^2}{(2+r)}+1+3\cdot\frac{(1+r)}{(2+r)}=1-3\cdot\frac{(1+r)}{(2+r)}\left[\frac{r(1+r)^2}{(1+r)^3-1}-1\right]$$
$$=1-3\cdot\frac{(1+r)}{(2+r)}\left[\frac{r(1+r)^2-(1+r)^3+1}{(1+r)^3-1}\right]=1-3\cdot\frac{(1+r)}{(2+r)}\left[\frac{-(1+r)^2+1}{(1+r)^3-1}\right]$$
$$=1-3\cdot\frac{(1+r)}{(2+r)}\left[\frac{-1-2r-r^2+1}{(1+r)^3-1}\right]=1-3\cdot\frac{(1+r)}{(2+r)}\left[\frac{-r(2+r)}{(1+r)^3-1}\right]=1+\frac{3r(1+r)}{(1+r)^3-1}$$

This is the same consumption level as before. This documents that, even if he re-optimizes at the start of period 2, RC has no incentive to deviate from his previous plans – unless, of course, he has received new information.

d) These results document that the behavior of RC is time-consistent, i.e. even if he re-optimizes, he has no incentive to deviate from his time path of consumption, investment etc.. There might, however, be an incentive to deviate from the original plan if RC's income in period 2 and 3 or the interest rate change unexpectedly.

Chapter 4

Exercise 4.1: Public debt, the current account, and welfare in a model with heterogeneous interest rates

a) As the government has to pay a lower interest rate on loans than RC, the private budget constraint does depend on the government's decision to finance its spending through taxes or debt, so the Ricardian Equivalence doesn't hold.

b) If the government raises taxes only in period 1, i.e. $T_2 = 0$, the constant time path of government spending implies that

$$T_1 = (1 + \frac{1}{1+r})G$$

For the RC's net assets, we have

$$B_2^{priv} = Y - \left(1 + \frac{1}{1+r}\right)G - C_1 < 0\,;\; B_3^{priv} = 0 = Y + (1 + r^P)B_2^{priv} - C_2$$

The fact that $B_2^{priv} < 0$ follows from our assumptions on Y and G. Hence, RC borrows in the first period to compensate for the large (and temporary) tax burden. This, in turn, implies that the borrowing interest rate r^P is relevant for his consumption decision. Rewriting the above conditions yields

$$C_1 = Y - (1 + \frac{1}{1+r})G - B_2^{priv}\,;\; C_2 = Y + (1 + r^P)B_2^{priv}$$

The first-order condition for a utility maximum is

$$\frac{1}{C_1} = \beta(1 + r^P)\frac{1}{C_2}$$

Substituting from above yields

$$Y - (1 + \frac{1}{1+r})G - B_2^{priv} = \frac{1}{\beta(1+r^P)}(Y + (1 + r^P)B_2^{priv})$$

$$Y - \frac{1}{\beta(1+r^P)}Y - (1 + \frac{1}{1+r})G = \underbrace{\left[1 + \frac{1+r^P}{\beta(1+r^P)}\right]}_{\frac{\beta+1}{\beta}}B_2^{priv}$$

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): *International Macroeconomics, 2nd edition*, Tübingen (Mohr Siebeck).
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

$$B_2^{priv} = \frac{\beta}{\beta+1}\left[\frac{\beta(1+r^P)-1}{\beta(1+r^P)}Y - (1+\frac{1}{1+r})G\right] = \frac{\beta(1+r^P)-1}{(\beta+1)(1+r^P)}Y - \frac{\beta(2+r)}{(\beta+1)(1+r)}G$$

$$C_1 = Y - \frac{2+r}{1+r}G - \frac{\beta(1+r^P)-1}{(\beta+1)(1+r^P)}Y + \frac{\beta(2+r)}{(\beta+1)(1+r)}G$$

$$= \frac{(\beta+1)(1+r^P)-\beta(1+r^P)+1}{(\beta+1)(1+r^P)}Y + \frac{\beta(2+r)-(2+r)(\beta+1)}{(\beta+1)(1+r)}G$$

$$= \frac{2+r^P}{(\beta+1)(1+r^P)}Y - \frac{2+r}{(\beta+1)(1+r)}G = \frac{1}{\beta+1}\left[\frac{2+r^P}{1+r^P}Y - \frac{2+r}{1+r}G\right]$$

Hence, $C_1(T_2=0) = \frac{1}{\beta+1}\left[\frac{2+r^P}{1+r^P}Y - \frac{2+r}{1+r}G\right]$.

For C_2, we have

$$C_2 = Y + (1+r^P)\left[\frac{\beta(1+r^P)-1}{(\beta+1)(1+r^P)}Y - \frac{\beta(2+r)}{(\beta+1)(1+r)}G\right]$$

$$= Y + \frac{\beta(1+r^P)-1}{\beta+1}Y - \frac{\beta(2+r)(1+r^P)}{(\beta+1)(1+r)}G = \frac{\beta+\beta(1+r^P)}{\beta+1}Y - \frac{\beta(2+r)(1+r^P)}{(\beta+1)(1+r)}G$$

Hence, $C_2(T_2=0) = \frac{\beta(1+r^P)}{\beta+1}\left[\frac{(2+r^P)}{(1+r^P)}Y - \frac{2+r}{1+r}G\right]$

c) If the government raises taxes only in period 2, i.e. $T_1 = 0$, the constant time path of government spending implies that
$T_1 = 0$; $T_2 = G + (1+r)G = (2+r)G$

As far, as the RC's net assets are concerned, we conjecture that $B_2^{priv} \geq 0$, i.e. RC saves in period 1 to make provisions for the large tax burden in period 2.

To prove that this is correct, we show that $B_2^{priv} < 0$ would result in a contradiction. Suppose that $B_2^{priv} < 0$, which would imply that $Y - C_1 < 0$. In this case, the relevant interest rate would be the (borrowing) interest rate r^P, and we would have

$$C_2 = Y - (2+r)G + (1+r^P)(Y - C_1)$$

Recall our assumption that $Y < \frac{2+r}{1+r}G$, which implies that $Y < (2+r)G$.

Hence, $C_2 = Y - (2+r)G + (1+r^P)(Y - C_1) < 0$ if $Y - C_1 < 0$. However, RC's period utility function implies that negative second-period consumption can be excluded. This demonstrates that $B_2^{priv} \geq 0$.

Since RC accumulates positive net assets in the first period, the relevant interest rate is the (lending) interest rate *r*. Utility maximization requires that

$$\frac{1}{C_1} = \beta(1+r)\frac{1}{C_2} \Rightarrow Y - B_2^{priv} = \frac{1}{\beta(1+r)}\left[Y - (2+r)G + (1+r)B_2^{priv}\right]$$

$$Y(1-\frac{1}{\beta(1+r)}) + \frac{2+r}{\beta(1+r)}G = B_2^{priv}(1+\frac{1}{\beta})$$

$$\frac{\beta(1+r)-1}{\beta(1+r)}Y + \frac{2+r}{\beta(1+r)}G = \frac{\beta+1}{\beta}B_2^{priv} \Leftrightarrow B_2^{priv} = \frac{\beta(1+r)-1}{(\beta+1)(1+r)}Y + \frac{2+r}{(\beta+1)(1+r)}G$$

$$C_1 = Y - \frac{\beta(1+r)-1}{(\beta+1)(1+r)}Y - \frac{2+r}{(\beta+1)(1+r)}G = \frac{2+r}{(\beta+1)(1+r)}Y - \frac{2+r}{(\beta+1)(1+r)}G.$$

Hence, $C_1(T_1 = 0) = \frac{2+r}{(\beta+1)(1+r)}(Y - G)$.

$$C_2 = Y - (2+r)G + (1+r)\frac{\beta(1+r)-1}{(\beta+1)(1+r)}Y + (1+r)\frac{2+r}{(\beta+1)(1+r)}G$$

$$= Y\left(\frac{(\beta+1)+\beta(1+r)-1}{\beta+1}\right) + G\left(\frac{(2+r-(2+r)(\beta+1)}{\beta+1}\right)$$

$$= Y\left(\frac{\beta(2+r)}{\beta+1}\right) + G\left(\frac{2+r-(2+r)(\beta+1)}{\beta+1}\right) = \frac{1}{\beta+1}\left[\beta(2+r)Y - \beta(2+r)G\right]$$

Hence, $C_2(T_1 = 0) = \frac{\beta(2+r)}{(\beta+1)}(Y - G)$

d) In order to analyze whether government lending $(T_2 = 0)$ or borrowing $(T_1 = 0)$ maximizes RC's welfare, we calculate the difference in consumption levels under the two policies.

For first-period consumption, we have

$$C_1(T_1=0)-C_1(T_2=0)=\frac{2+r}{(\beta+1)(1+r)}(Y-G)-\frac{1}{\beta+1}\left[\frac{2+r^P}{1+r^P}Y-\frac{2+r}{1+r}G\right]$$
$$=\frac{1}{\beta+1}\left(\frac{2+r}{1+r}-\frac{2+r^P}{1+r^P}\right)Y$$

It is easy to show that this is positive since

$$\frac{2+r}{1+r}>\frac{2+r^P}{1+r^P}\Leftrightarrow(2+r)(1+r^P)>(2+r^P)(1+r)\Leftrightarrow r^P>r$$

For second-period consumption, we have

$$C_2(T_1=0)-C_2(T_2=0)=\frac{\beta(2+r)}{\beta+1}(Y-G)-\frac{\beta(2+r)}{\beta+1}\left(\frac{2+r^P}{2+r}Y-\frac{1+r^P}{1+r}G\right)$$
$$=\frac{\beta(2+r)}{\beta+1}\left[\frac{(2+r)-(2+r^P)}{2+r}Y-\frac{(1+r)-(1+r^P)}{1+r}G\right]$$
$$=\frac{\beta(r-r^P)}{\beta+1}\left[Y-\frac{2+r}{1+r}G\right],$$

which is positive since $r^P>r$ and $Y<\frac{2+r}{1+r}G$ by assumption.

This shows that RC can consume more in both periods if the government raises taxes only in the second period and finances its first-period expenditures by taking out a loan. As RC's utility function is monotonically increasing in C_t, this raises RC's lifetime utility.

The economic rationale behind this result is that the government pays a lower interest rate on its loans. As a consequence, a policy of government borrowing raises the present value of RC's lifetime disposable income:

$$PV(Y^{disp},T_2=0)=Y-\frac{2+r}{1+r}G+\frac{Y}{1+r^P}$$
$$PV(Y^{disp},T_1=0)=Y-\frac{2+r}{1+r}G+\frac{Y}{1+r}$$

Since $r^P>r$, the second expression is greater than the first one.

Exercise 4.2: Intertemporal elasticity of substitution, intratemporal elasticity of substitution, and the effect of anticipated price changes on consumption

The utility derived from consumption in period t is given by $u(C_t)=\dfrac{C_t^{1-\sigma}-1}{1-\sigma}$ where, according to (4.33), $C_t=\left[\gamma^{\eta}\left(C_t^T\right)^{1-\eta}+(1-\gamma)^{\eta}\left(C_t^N\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$.

Taking the derivative with respect to C_t^T yields the marginal utility of tradable goods consumption:

$$\frac{\partial u}{\partial C_t^T}=C_t^{-\sigma}\cdot\frac{1}{1-\eta}\cdot\left[\gamma^{\eta}\left(C_t^T\right)^{1-\eta}+(1-\gamma)^{\eta}\left(C_t^N\right)^{1-\eta}\right]^{\frac{\eta}{1-\eta}}\cdot(1-\eta)\cdot\gamma^{n}\cdot\left(C_t^T\right)^{-\eta}$$

$$=\left[\gamma^{\eta}\left(C_t^T\right)^{1-\eta}+(1-\gamma)^{\eta}\left(C_t^N\right)^{1-\eta}\right]^{\frac{-\sigma}{1-\eta}}\cdot\left[\gamma^{\eta}\left(C_t^T\right)^{1-\eta}+(1-\gamma)^{\eta}\cdot\left(c_t^N\right)^{1-\eta}\right]^{\frac{\eta}{1-\eta}}\cdot\gamma^{n}\cdot\left(C_t^T\right)^{-\eta}$$

$$=\left[\gamma^{n}\left(C_t^T\right)^{1-\eta}+(1-\gamma)^{\eta}\left(C_t^N\right)^{1-\eta}\right]^{\frac{\eta-\sigma}{1-\eta}}\cdot\gamma^{\eta}\cdot\left(C_t^T\right)^{-\eta}.$$

Taking the derivative of this expression with respect to C_t^N yields

$$\frac{\partial^2 u}{\partial C_t^T\,\partial C_t^N}$$

$$=\frac{\eta-\sigma}{1-\eta}\left[\gamma^{\eta}\left(C_t^T\right)^{1-\eta}+(1-\gamma)^{\eta}\left(C_t^N\right)^{1-\eta}\right]^{\frac{2\eta-\sigma-1}{1-\eta}}\cdot\gamma^{n}\cdot\left(C_t^T\right)^{-\eta}\cdot(1-\gamma)^{\eta}\cdot(1-\eta)\cdot\left(C_t^N\right)^{-\eta}$$

$$=(\eta-\sigma)\cdot\underbrace{\left[\gamma^{\eta}\left(C_t^T\right)^{1-\eta}+(1-\gamma)^{\eta}\left(C_t^N\right)^{1-\eta}\right]^{\frac{2\eta-\sigma-1}{1-\eta}}}_{>0}\cdot\underbrace{\gamma^{\eta}}_{>0}\cdot\underbrace{\left(C_t^T\right)^{-\eta}}_{>0}\cdot\underbrace{(1-\gamma)^{\eta}}_{>0}\cdot\underbrace{\left(C_t^N\right)^{-\eta}}_{>0}.$$

It is easy to show that changing the order of differentiation – i.e. computing $\dfrac{\partial^2 u}{\partial C_t^N\,\partial C_t^T}$ – results in the same expression.

The sign of this cross-derivative depends on the sign of $(\eta-\sigma)$, with $(\eta-\sigma)>0$ if $1/\sigma>1/\eta$, i.e. if the *inter*temporal elasticity of substitution is greater than the *intra*temporal elasticity of substitution. Hence, tradable goods and non-tradable goods are complements (substitutes) if $1/\sigma>1/\eta$ ($1/\sigma<1/\eta$).

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): ***International Macroeconomics, 2nd edition*****, Tübingen (Mohr Siebeck)**.
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

Equation (4.51) indicates that an anticipated increase in the price level, driven by an anticipated increase in non-tradable goods prices, raises the first-period current account if $1/\sigma < 1/\eta$ – i.e. if the two goods types are substitutes – and reduces the first-period current account if $1/\sigma > 1/\eta$, i.e. if they are complements. This has a straightforward intuition: the anticipated price increase of non-tradable goods induces the RC to shift consumption of non-tradable goods from period 2 into period 1. If the two goods are substitutes, he reduces first-period consumption of tradable goods, which has a positive effect on the current account. Conversely, if the two goods are complements, higher non-tradable goods consumption in period 1 raises tradable-goods consumption in that period, and this drags down the current account balance.

Exercise 4.3: Deregulation and the current account

The removal of barriers to entry in the (non-tradable) services sector lowers prices and raises output in that sector. The expansion of services production necessitates a re-allocation of both labor and capital. If the (tradable) manufacturing sector is capital intensive, the reduction of the capital stock in that sector is more pronounced than the increase in the capital stock employed by services firms. As a consequence, overall investment decreases, and this has a positive impact on the current account.

Exercise 4.4: Asset prices in a two-country model

The following information on the time path of the countries' GDP is given:

$$Y_1^H = 1, \quad Y_1^F = 1$$

$$Y_2^H = \begin{cases} 2 & \text{with } \pi = 0.5 \\ 1 & \text{with } 1 - \pi = 0.5 \end{cases} \qquad Y_2^F = \begin{cases} 0 & \text{with } \pi = 0.5 \\ 3 & \text{with } 1 - \pi = 0.5 \end{cases}$$

a) In the first state of nature, country H's second-period GDP is high ($Y_{2,1}^H = 2$), while country F's GDP is low ($Y_{2,1}^F = 0$). In the second state of nature, country H's second-period GDP is low ($Y_{2,2}^H = 1$), while country F's second-period GDP is high ($Y_{2,2}^F = 3$). We therefore expect a *negative* correlation of growth rates.

We can verify this conjecture by computing the correlation of growth rates:

We define the growth rate of country I as: $g_{c,t} = \dfrac{Y_t^c - Y_{t-1}^c}{Y_{t-1}^c}$. The expected growth rate is given by

$$\mathrm{E}(g_{c,2}) = \mathrm{E}\left[\frac{Y_2^c - Y_1^c}{Y_1^c}\right] = \mathrm{E}\left[\frac{Y_2^c}{Y_1^c} - 1\right] = \frac{1}{Y_1^c}\mathrm{E}\left(Y_2^c\right) - 1$$

Of course, Y_1^c is known at $t = 1$.

$$\mathrm{E}\left(Y_2^H\right) = 0.5 \cdot 2 + 0.5 \cdot 1 = 1.5 \quad \Rightarrow \mathrm{E}(g_{H,2}) = 1.5 - 1 = 0.5$$
$$\mathrm{E}\left(Y_2^F\right) = 0.5 \cdot 0 + 0.5 \cdot 3 = 1.5 \quad \Rightarrow \mathrm{E}(g_{F,2}) = 1.5 - 1 = 0.5$$

The correlation between the two countries‘ growth rates is the covariance of the growth rates divided by the product of the growth rates’ standard deviations:

$$r_{g_{H,2};g_{F,2}} = \frac{Cov(g_{H,2}, g_{F,2})}{\sqrt{Var(g_{H,2}) \cdot Var(g_{F,2})}}$$

Using the values given above, we can compute the variance of country c’s growth rate:

$$Var(g_{c,2}) = \mathrm{E}\left[(g_{c,2} - \mathrm{E}(g_{c,2}))^2\right] = \pi(g_{c,2}^1 - 0.5)^2 + (1-\pi)(g_{c,2}^2 - 0.5)^2$$
$$Var(g_{H,2}) = 0.5^2 = 0.25$$
$$Var(g_{F,2}) = 1.5^2 = 2.25$$

The covariance of the growth rates is given by

$$\begin{aligned} &Cov(g_{H,2}, g_{F,2}) = \mathrm{E}\left[(g_{H,2} - \mathrm{E}(g_{H,2}))(g_{F,2} - \mathrm{E}(g_{F,2}))\right] = \\ &= \pi\left[(g_{H,2}^1 - \mathrm{E}(g_{H,2}))(g_{F,2}^1 - \mathrm{E}(g_{F,2}))\right] + (1-\pi)\left[(g_{H,2}^2 - \mathrm{E}(g_{H,2}))(g_{F,2}^2 - \mathrm{E}(g_{F,2}))\right] \\ &= -0.75 \end{aligned}$$

Inserting this into the expression above yields the correlation

$$r_{g_{H,2};g_{F,2}} = \frac{-0.75}{\sqrt{0.5^2 \cdot 1.5^2}} = \frac{-0.75}{0.5 \cdot 1.5} = -1$$

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): *International Macroeconomics, 2nd edition*, Tübingen (Mohr Siebeck).
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

Hence, there is a perfectly negative correlation between the two growth rates. This confirms the above conjecture.

b) As shown in part a), the countries' expected growth rates are given by $\mathrm{E}(g_{H,2}) = 0.5$; $\mathrm{E}(g_{F,2}) = 0.5$

The growth rate of world GDP is given by $\mathrm{E}(g_{W,2}) = 0.5 \cdot 0 + 0.5 \cdot 1 = 0.5$.

c) To compute the asset prices V_1^H and V_1^F, we use variants of expression (4.91) from the main text:

$$V_1^H = \beta \cdot \mathrm{E}\left[\frac{u'(C_2^H)}{u'(C_1^H)} \cdot Y_2^H\right] = \beta \cdot \mathrm{E}\left[\frac{u'(C_2^F)}{u'(C_1^F)} \cdot Y_2^H\right]$$

For the CRRA utility function, this can be written as

$$V_1^H = \beta \cdot \mathrm{E}\left[\left(\frac{C_1^H}{C_2^H}\right)^{\sigma} Y_2^H\right] = \beta \cdot \mathrm{E}\left[\left(\frac{C_1^F}{C_2^F}\right)^{\sigma} Y_2^H\right]$$

Recall from the main text (page 56) that $C_t^c = \mu^c \cdot Y_t^W$. This can be substituted into the above expression to yield

$$V_1^H = \beta \cdot \mathrm{E}\left[\left(\frac{Y_1^W}{Y_2^W}\right)^{\sigma} Y_2^H\right] = \beta(Y_1^W)^{\sigma}\left[\pi\left(Y_{2,1}^W\right)^{-\sigma} Y_{2,1}^H + (1-\pi)\left(Y_{2,2}^W\right)^{-\sigma} Y_{2,2}^H\right]$$

Combining this with the given parameter values yields

$$V_1^H = 0.5 \cdot 2^2\left[0.5 \cdot 2^{-2} \cdot 2 + 0.5 \cdot 4^{-2} \cdot 1\right] = \frac{9}{16}$$

For country F, we have

$$V_1^F = \beta \cdot \mathrm{E}\left[\left(\frac{Y_1^W}{Y_2^W}\right)^{\sigma} Y_2^F\right] = \beta(Y_1^W)^{\sigma}\left[\pi\left(Y_{2,1}^W\right)^{-\sigma} Y_{2,1}^F + (1-\pi)\left(Y_{2,2}^W\right)^{-\sigma} Y_{2,2}^F\right] = \frac{3}{16}$$

Apparently, claims on country H's future output are traded at a higher price than claims on country F's future output. Note that this result emerges although both countries have the same output growth rates. The reason for this finding is that country H's output is a better insurance against low world output (and thus low consumption): in the first state of nature, when world output is low, country H's output is high while country F's output is low. Conversely, country H's output is low in the second state of nature, when world output is high.

Exercise 4.5: The current account in a model of international diversification

Equation (4.100) states that

$$(4.100) \qquad CA_1^H = \theta_1^W \, Y_1^H - \left(1 - \theta_1^W\right) V_1^H$$

with $\theta_1^W \equiv V_1^W / (V_1^W + Y_1^W)$. The first-period price of a claim on country H's future output is given by (4.94):

$$(4.94) \qquad V_1^H = \beta \, \mathrm{E}\left[\left(\frac{C_1^H}{C_2^H} \right)^{\sigma} Y_2^H \right]$$

Since domestic consumption is a fixed share of world output in both periods, and since future world output is non-random, we can combine both expressions to get

$$CA_1^H = \theta_1^W \, Y_1^H - \left(1 - \theta_1^W\right) \beta \left(1 + g^W\right)^{-\sigma} \mathrm{E}\left(Y_2^H\right)$$

where $1 + g^W \equiv Y_2^W / Y_1^W$. This can be transformed into

$$CA_1^H = \frac{v_1^W \, Y_1^H - \beta \left(1 + g^W\right)^{-\sigma} \mathrm{E}\left(Y_2^H\right)}{1 + v_1^W}$$

where $v_1^W \equiv V_1^W / Y_1^W$. We can use a „global" version of (4.94), taking into account that there is no uncertainty about future output, to compute

$$V_1^W = \beta \left(\frac{Y_1^W}{Y_2^W} \right)^{\sigma} Y_2^W$$

Combining this expression with the definition of v_1^W, we get

$$\frac{CA_1^H}{Y_1^H}=\frac{\beta\left(1+g^W\right)^{-\sigma}}{1+v_1^W}\left[\left(\frac{Y_2^W}{Y_1^W}\right)-\mathrm{E}\left(\frac{Y_2^H}{Y_1^H}\right)\right]$$

The sign of this expression depends on the sign of the difference $\left(\frac{Y_2^W}{Y_1^W}\right)-\mathrm{E}\left(\frac{Y_2^H}{Y_1^H}\right)$. If the (certain) growth rate of world output exceeds the (expected!) growth rate of country H's output, the value of the assets purchased by the RC of country H in the first period exceeds the value of the liabilities incurred. This is reflected by a current account surplus. Conversely, if $\left(\frac{Y_2^W}{Y_1^W}\right)<\mathrm{E}\left(\frac{Y_2^H}{Y_1^H}\right)$, the value of the liabilities incurred exceeds the value of the assets purchased, resulting in a current account deficit. The strength of the effect on the current account is affected by the (synthetic) "world interest rate", $1+r^W=\left(1+g^W\right)^{\sigma}/\beta$, which we can compute by invoking (4.88) or, in fact, (3.28). The higher this interest rate, the weaker the effect of the term in squared brackets on the current account.

Chapter 5

Exercise 5.1: Poverty traps

a) Individuals have to satisfy basic consumption needs before they start saving. If the economy's output (and thus income) is lower than total subsistence consumption (= subsistence consumption per capita times population size), aggregate savings are zero. As soon as $Y > \tilde{c}L$, individuals save a constant share of the income that exceeds subsistence consumption.

b) The evolution of the capital stock per effective unit of labor is described by the following expressions (Note that the TFP parameter A_t is assumed to be constant):

$$K_{t+1} - K_t = S_t - \delta K_t = \begin{cases} s\left[K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha} - \tilde{c}L_t\right] - \delta K_t & \text{if } K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha} > \tilde{c}L_t \\ -\delta K_t & \text{if } K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha} \le \tilde{c}L_t \end{cases}$$

$$\Leftrightarrow K_{t+1} - K_t = \begin{cases} sK_t^{\alpha}\left(A_t L_t\right)^{1-\alpha} - s\tilde{c}L_t - \delta K_t & \text{if } K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha} > \tilde{c}L_t \\ -\delta K_t & \text{if } K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha} \le \tilde{c}L_t \end{cases}$$

$$\Leftrightarrow \frac{K_{t+1} - K_t}{A_t L_t} = \begin{cases} sK_t^{\alpha}\left(A_t L_t\right)^{-\alpha} - \dfrac{s\tilde{c}}{A_t} - \dfrac{\delta K_t}{A_t L_t} & \text{if } K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha} > \tilde{c}L_t \\ -\dfrac{\delta K_t}{A_t L_t} & \text{if } K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha} \le \tilde{c}L_t \end{cases}$$

$$\Leftrightarrow (1+n)\hat{k}_{t+1} - \hat{k}_t = \begin{cases} s\,\hat{k}_t^{\alpha} - \dfrac{s\tilde{c}}{A_t} - \delta\,\hat{k}_t & \text{if } \hat{k}_t > \left(\dfrac{\tilde{c}}{A_t}\right)^{\frac{1}{\alpha}} \\ -\delta\,\hat{k}_t & \text{if } \hat{k}_t \le \left(\dfrac{\tilde{c}}{A_t}\right)^{\frac{1}{\alpha}} \end{cases}$$

$$\Leftrightarrow (1+n)\hat{k}_{t+1} = \begin{cases} s\,\hat{k}_t^{\alpha} - \dfrac{s\tilde{c}}{A_t} + (1-\delta)\,\hat{k}_t & \text{if } \hat{k}_t > \left(\dfrac{\tilde{c}}{A_t}\right)^{\frac{1}{\alpha}} \\ (1-\delta)\,\hat{k}_t & \text{if } \hat{k}_t \le \left(\dfrac{\tilde{c}}{A_t}\right)^{\frac{1}{\alpha}} \end{cases}$$

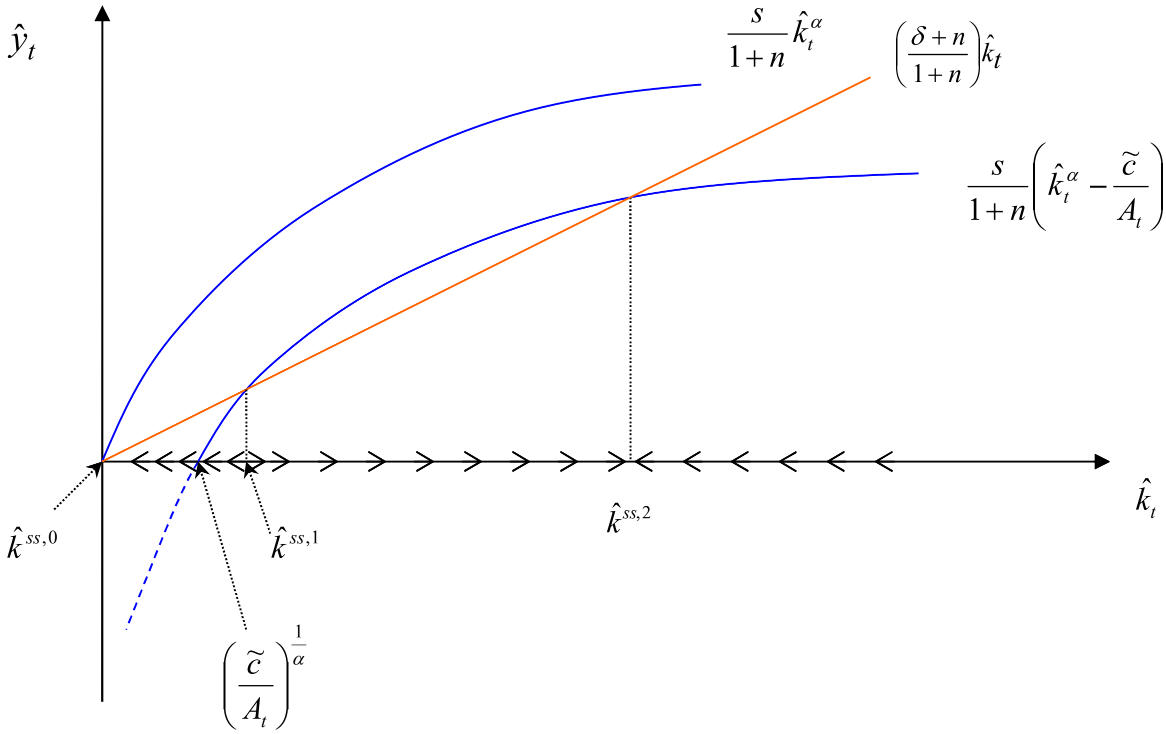
These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): *International Macroeconomics, 2nd edition*, Tübingen (Mohr Siebeck).
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

$$\Leftrightarrow \hat{k}_{t+1} \quad = \left\{ \begin{array}{ll} \dfrac{s}{1+n}\hat{k}_t^{\alpha} - \dfrac{s}{1+n}\dfrac{\tilde{c}}{A_t} + \dfrac{1-\delta}{1+n}\hat{k}_t & \text{if } \hat{k}_t > \left(\dfrac{\tilde{c}}{A_t}\right)^{\frac{1}{\alpha}} \\ \dfrac{1-\delta}{1+n}\hat{k}_t & \text{if } \hat{k}_t \leq \left(\dfrac{\tilde{c}}{A_t}\right)^{\frac{1}{\alpha}} \end{array} \right\}$$

$$\Leftrightarrow \hat{k}_{t+1} - \hat{k}_t = \left\{ \begin{array}{ll} \dfrac{s}{1+n}\hat{k}_t^{\alpha} - \dfrac{s}{1+n}\dfrac{\tilde{c}}{A_t} - \dfrac{\delta+n}{1+n}\hat{k}_t & \text{if } \hat{k}_t > \left(\dfrac{\tilde{c}}{A_t}\right)^{\frac{1}{\alpha}} \\ -\dfrac{\delta+n}{1+n}\hat{k}_t & \text{if } \hat{k}_t \leq \left(\dfrac{\tilde{c}}{A_t}\right)^{\frac{1}{\alpha}} \end{array} \right\}$$

$$\Leftrightarrow \hat{k}_{t+1} - \hat{k}_t = \left\{ \begin{array}{ll} \dfrac{s}{1+n}\left(\hat{k}_t^{\alpha} - \dfrac{\tilde{c}}{A_t}\right) - \dfrac{\delta+n}{1+n}\hat{k}_t & \text{if } \hat{k}_t > \left(\dfrac{\tilde{c}}{A_t}\right)^{\frac{1}{\alpha}} \\ -\dfrac{\delta+n}{1+n}\hat{k}_t & \text{if } \hat{k}_t \leq \left(\dfrac{\tilde{c}}{A_t}\right)^{\frac{1}{\alpha}} \end{array} \right\}$$

The following Figure is a modified version of the Solow diagram presented in Chapter V (Figure 5.1). The diagram indicates that there are two possible steady states with positive values of capital stock ($\hat{k}^{ss,1}$ and $\hat{k}^{ss,2}$) as well as one possible steady state with a zero capital stock ($\hat{k}^{ss,0}$). While $\hat{k}^{ss,0}$ and $\hat{k}^{ss,2}$ are stable steady states, $\hat{k}^{ss,1}$ is unstable.



c) If the economy's initial capital stock (per effective unit of labor) falls short of $\hat{k}^{ss,1}$, it eventually converges to zero. This is a "**poverty trap**": the low income is used for subsistence consumption. Absent savings, there is no investment, the capital stock depreciates, generating even lower output per capita in the subsequent periods. By contrast, if the initial capital stock (per effective unit of labor) exceeds $\hat{k}^{ss,1}$, the economy converges towards the Solow-type steady state $\hat{k}^{ss,2}$.

d) If the initial capital stock per effective unit of labor $\hat{k}_0$ is smaller than $\hat{k}^{ss,1}$, the economy suffers from a „capital shortage". One way to overcome the threshold value $\hat{k}^{ss,1}$ and to start growing – i.e. to escape from the poverty trap – would be to inject capital from abroad, either by attracting private foreign capital flows or by realizing aid-financed foreign investments. Note that even temporary capital inflows could be used to break the vicious circle of low income, low savings, and low investment. Alternatively, an increase of the saving rate s or of the TFP parameter A_t could lift the economy above $\hat{k}^{ss,1}$. To check the latter result, recall that – with a productivity parameter that does not exhibit trend growth, but that may jump to a higher level at a given point in time – the evolution of the capital stock *per capita* (!) may be written as

$$k_{t+1}-k_t=\begin{cases}\dfrac{s}{1+n}\left(A_t^{1-\alpha}k_t^{\alpha}-\tilde{c}\right)-\dfrac{\delta+n}{1+n}k_t & \text{if } A_t^{1-\alpha}k_t>\tilde{c}\\ -\dfrac{\delta+n}{1+n}k_t & \text{if } A_t^{1-\alpha}k_t<\tilde{c}\end{cases}$$

A one-time increase in A_t shifts the production function and thus the savings function upward, lowering the threshold steady state and thus (possibly) enabling an economy to escape from the poverty trap at a given level capital stock per capita.

Exercise 5.2: Distributional and growth effects of international investment

a) The production function of the representative firm in country c is given by $Y^c=\left(K^c\right)^{\alpha}\left(L^c\right)^{1-\alpha}$, with $c =$ H, F.

Per capita income is $\dfrac{Y^c}{L^c}=\dfrac{\left(K^c\right)^{\alpha}\left(L^c\right)^{1-\alpha}}{L^c}=\left(\dfrac{K^c}{L^c}\right)^{\alpha}=\left(k^c\right)^{\alpha}$, with $c = H, F$.

Since $k^H>k^F$, we have $\dfrac{Y^H}{L^H}=\left(k^H\right)^{\alpha}>\left(k^F\right)^{\alpha}=\dfrac{Y^F}{L^F}$, i.e. country H has the higher per-capita income.

The marginal productivity of capital is determined by taking the derivative of the production function w.r.t. capital : $\dfrac{\partial Y^c}{\partial K^c}=\alpha\left(K^c\right)^{\alpha-1}\left(L^c\right)^{1-\alpha}=\alpha\left(k^c\right)^{\alpha-1}$, with $c = H, F$.

Since $k^H>k^F$ and alpha<1, we have $\dfrac{\partial Y^H}{\partial K^H}<\dfrac{\partial Y^F}{\partial K^F}$, i.e. the marginal productivity of capital in country F is higher than in country H.

b) Unless the returns to capital are equal in the initial situation, removing barriers to international investment results in capital flows that allow a country's national savings to differ from its aggregate investment. More specifically, capital flows to the country with the higher marginal productivity of capital, i.e. with the lower initial capital stock per capita, until the returns to capital as well as the level of the capital stock (and the output) per capita are equalized.

c) As a consequence of the removal of barriers to international investment, the returns to capital in both countries equal the world interest rate: $\alpha\left(K^H\right)^{\alpha-1}\left(L^H\right)^{1-\alpha}=r^W=\alpha\left(K^F\right)^{\alpha-1}\left(L^F\right)^{1-\alpha}$. This, in turn, implies that the capital stocks per capita in both countries are equalized. Hence, country F accumulates and country H dismantles capital until $k^H = k^F$ and $\frac{\partial Y^H}{\partial K^H} = \frac{\partial Y^F}{\partial K^F}$. The increasing capital stock raises the real wage in country F, while the real wage in country H decreases.

d) After the removal of investment barriers, the capital stock per capita is equalized across countries and amounts to $k^W = \frac{K^H + K^F}{L^H + L^F}$. We want to show that the sum of national outputs divided by the global population after financial integration exceeds the sum of autarky outputs relative to the global population, i.e.

$$\left[\left(\frac{K^H + K^F}{L^H + L^F} L^H\right)^{\alpha} \left(L^H\right)^{1-\alpha} + \left(\frac{K^H + K^F}{L^H + L^F} L^F\right)^{\alpha} \left(L^F\right)^{1-\alpha}\right] / \left(L^H + L^F\right)$$

$$> \left[\left(K^H\right)^{\alpha} \left(L^H\right)^{1-\alpha} + \left(K^F\right)^{\alpha} \left(L^F\right)^{1-\alpha}\right] / \left(L^H + L^F\right)$$

This can be rewritten as

$$\left(\frac{K^H + K^F}{L^H + L^F}\right)^{\alpha} \left(\frac{L^H + L^F}{L^H + L^F}\right) > \left(\frac{K^H}{L^H}\right)^{\alpha} \left(\frac{L^H}{L^H + L^F}\right) + \left(\frac{K^F}{L^F}\right)^{\alpha} \left(\frac{L^F}{L^H + L^F}\right)$$

Define $\frac{L^F}{L^H + L^F} \equiv \theta$, we rephrase the previous inequality as

$$\left(\theta \frac{K^H}{L^H} + (1-\theta) \frac{K^F}{L^F}\right)^{\alpha} > \theta \left(\frac{K^H}{L^H}\right)^{\alpha} + (1-\theta) \left(\frac{K^F}{L^F}\right)^{\alpha}$$

The fact that this is correct follows from **Jensen's inequality** (see page 163).

Exercise 5.3: Human capital and endogenous growth

a) $Y_t = K_t^{\alpha}\left(H_t L_t\right)^{1-\alpha} = K_t^{\alpha}\left(\gamma\, Y_t\, L_t\right)^{1-\alpha}$

$\Leftrightarrow \dfrac{Y_t}{Y_t^{1-\alpha}} = K_t^{\alpha}\left(\gamma L_t\right)^{1-\alpha} \;\Leftrightarrow Y_t^{\alpha} = K_t^{\alpha}\left(\gamma\, L_t\right)^{1-\alpha} \Leftrightarrow Y_t = K_t\left(\gamma\, L_t\right)^{\frac{1-\alpha}{\alpha}}$

$\dfrac{\partial Y_t}{\partial K_t} = \left(\gamma\, L_t\right)^{\frac{1-\alpha}{\alpha}} > 0\,;\; \dfrac{\partial^2 Y_t}{\partial {K_t}^2} = 0$

This production function is characterized by a constant positive marginal productivity of capital.

b) On the one hand, an increase of the capital stock, reduces the marginal productivity of capital. On the other hand, however, it results in a higher output and in this way contributes to an expansion of the human capital stock. As the weights of the physical and human capital in this specification add up to 1, these two effects of capital accumulation compensate each other, and the overall marginal product of capital is constant.

Exercise 5.4: Government spending and endogenous growth

Recall that, in the Barro (1991) model, $Y_t = K_t\left(\tau_t L_t\right)^{\frac{1-\alpha}{\alpha}}$. This seems to suggest that a higher tax rate unambiguously results in higher output. Note, however, that such a reasoning does not take into account that taxation possibly affects saving behavior and capital accumulation: if taxes depend on income, higher taxation reduces the after-tax interest rate. This depresses savings and thus the speed of capital accumulation. Hence, while a higher tax rate may enhance growth by financing productive government spending, it may depress growth by reducing the incentive to save, thus slowing down capital accumulation.

Exercise 5.5: Are the gains from international financial integration elusive?

Assuming structural symmetry across countries, Gourinchas and Jeanne (2006) show that financial integration generates a welfare gain for the individual economy by *accelerating* a country's convergence to the steady state. However, since financial integration does not alter the *position* of the steady state, this welfare gain turns out to be rather small.

One reason why the welfare gains from financial integration may be much higher is that, absent financial integration, there might be no convergence to the steady state at all – e.g. if countries are caught in a poverty trap (see Exercise 5.4). In this case, overcoming a capital shortage may start a growth process that would not have taken place in autarky. Another reason why the Gourinchas/Jeanne (2006) result should be qualified is that the infinite horizon/representative-consumer perspective adopted by these authors downplays the large welfare gains in the short run. If one replaces the assumption of an infinitely-lived representative consumer by an OLG model, in which individuals have finite time horizons, the total welfare gain from financial integration for an infinite sequence of generations may still be rather small. However, the generation currently alive may reap substantial benefits from a higher capital stock and higher wages.

Chapter 6

Exercise 6.1: Stable external debt

a) It follows from equation (3.2) that the evolution of the net international investment position is described by the following expression:

$$B_{t+1} = (1+r)B_t + Y_t - C_t$$

Dividing both sides of the equation by Y_{t+1}, and defining the (constant) growth rate of output $1+g \equiv Y_{t+1}/Y_t$ as well as $b_t \equiv B_t/Y_t$, we can rewrite this expression as

$$b_{t+1} = \frac{1+r}{1+g} b_t + \frac{Y_t - C_t}{(1+g)\,Y_t}$$

Considering the definition of the primary current account, we can turn this into

$$b_{t+1} = \frac{1+r}{1+g} b_t + \frac{ca_t^{pr}}{(1+g)}$$

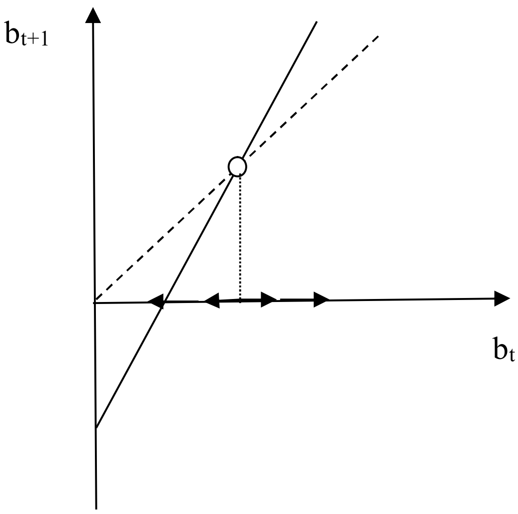
b) Assuming that the primary current account is constant, we can write the expression from part a) as

$$b_{t+1} = \frac{1+r}{1+g} b_t + \frac{ca^{pr}}{(1+g)}$$

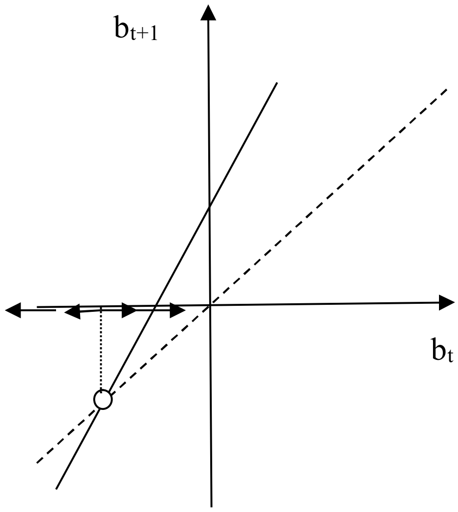
This is a linear function, relating the net international investment position (relative to GDP) in period t+1 to its value in period t and the primary current account balance divided by the growth rate. Due our assumption that $r > g$, the slope of this line is greater than one. Whether the intercept is positive or negative depends on the sign of ca^{pr}. In the following graph, we consider both cases, drawing both the above function and the (dashed) 45-degree line.

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): *International Macroeconomics, 2nd edition*, Tübingen (Mohr Siebeck).
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

$ca^{pr} < 0$



$ca^{pr} > 0$



c) The intersection of the linear function described above and the dashed 45-degree line gives the steady-state level b^{SS}. It can be computed by deleting the time subscripts on both sides of the difference equation:

$$b^{SS} = \frac{1+r}{1+g} b^{SS} + \frac{ca^{pr}}{(1+g)}$$

Solving this expression for b^{SS} yields

$$b^{SS} = \frac{ca^{pr}}{g-r}$$

This confirms the pattern suggested by the above graphs: if $ca^{pr} < 0$ ($ca^{pr} > 0$), the steady-state level of the net international investment position is positive (negative). This is intuitive: to keep the net international investment position at a constant (steady-state) level, a negative (positive) primary current account balance has to be compensated by positive (negative) primary income on the net international investment position.

d) The dynamic system characterized by the above difference equation is instable, i.e. if the initial value of b_t is greater than b^{SS}, the net international investment position grows infinitely large. If the initial value is smaller than b^{SS}, the NIIP grows towards minus infinity.

e) In the setup of this exercise, the primary current account is a constant share of income. This implies that consumption grows at the same rate as GDP. Conversely, the analysis of section VI.2 was based on the notion that consumption was fixed at a constant share of *initial* GDP. In that case, the primary current account deficit converges to zero as GDP grows infinitely large. Combining this with the assumption that the share of government spending in output is zero (i.e. $\phi = 0$) yields the expression b^{min}.

Note the similarities and differences between the two approaches to characterizing "sustainability": both approaches rest on the assumption of an exogenous time path of consumption (reflected by the primary current account). The analysis presented in the book defined a sustainable net international investment position as the level of the NIIP that was compatible with the intertemporal budget constraint. The analysis performed in this exercise defines sustainability as "stability": a sustainable NIIP is the steady state of an unstable dynamic system. Since the "explosion" (or "implosion") of the NIIP would violate the transversality condition (3.65), imposing stability amounts to making sure that the intertemporal budget constraint is respected.

Exercise 6.2: Empirical determinants of a default

a) Countries that are more open to trade have more to lose from the disruptions in trade that almost surely follow a default (see Rose 2005). Hence, this variable should have a negative effect on the likelihood of default.

b) The effect of government debt depends on the share of that debt that is owed to foreign creditors. If this share is high, it drives down the net international investment position and – according to all models presented in Chapter VI – raises the attractiveness of default. Conversely, if the share of domestic creditors in government debt is high, the (political) costs of default are much higher and the likelihood of a default lower.

c) If we link the default decision to countries' *ability* to repay, a high price volatility of the export commodity *raises* the likelihood of default, since net exports may be much lower than what is required to serve external liabilities.

However, if we follow the argument outlined in Subsection VI.3.2 – namely, that countries refrain default in order to maintain their access to international financial markets – a high price (and thus: income) volatility raises the value of capital market access and therefore reduces the incentive to default.

d) An institutional framework that guarantees the security of property rights is less likely to tolerate a default: first, because foreign creditors can rely on established mechanisms – most importantly, a functioning judiciary – to recuperate their claims. Second, because the political costs of default are likely to be high if formal and informal institutions support the enforcement of claims.

e) A greater inequality raises the likelihood of a default: first, because – as suggested in Subsection VI.3.5 – the costs from default may be distributed very unevenly, such that it would generate positive net gains for a substantial part of the population. Moreover, the government may revert of default in order to defuse a looming internal social conflict. However, it crucially depends on the properties of the political system whether a disenfranchised majority influences the government's decisions.

f) If the costs of default increase in a country's income, a higher income should reduce the likelihood of default. Conversely, if these costs are fixed, the harm imposed is lower for rich economies, making them more inclined to default. The first alternative seems more plausible: most rich economies are characterized by a sophisticated economic structure and developed financial markets where a default would wreak havoc.

Exercise 6.3: Adverse effects of debt relief?

While policymakers were careful to avoid the term "default" in the context of the 2012 Greek debt relief, the data of Beers and Nadeau (2015) characterize the country as being in default. It is quite likely that markets also adopted this assessment. As a consequence, potential creditors will be reluctant to lend money to the Greek government in the future – i.e. the Greek government is likely to be caught in a vicious circle à la Reinhart et al. (2003) (see Box 6.1): a tainted credit history, scant access to international financial markets, and the looming possibility of another default.

Chapter 7

Exercise 7.1: Relative purchasing power parity

Given the information in the exercise, we can write the CPI-based real exchange rate as follows:

$$Q_t^{CPI} = \frac{E_t \left(P_t^{A,F}\right)^{\gamma_F} \left(P_t^{B,F}\right)^{1-\gamma_F}}{\left(P_t^{A,H}\right)^{\gamma_H} \left(P_t^{B,H}\right)^{1-\gamma_H}}$$

Using the assumption that deviations from the law of one price are time-invariant, this expression can be rewritten as

$$Q_t^{CPI} = \frac{E_t \left(P_t^{A,F}\right)^{\gamma_F} \left(P_t^{B,F}\right)^{1-\gamma_F}}{\left(\varphi^A E_t P_t^{A,F}\right)^{\gamma_H} \left(\varphi^B E_t P_t^{B,F}\right)^{1-\gamma_H}},$$

which boils down to

$$Q_t^{CPI} = \frac{1}{\left(\varphi^A\right)^{\gamma_H} \left(\varphi^B\right)^{1-\gamma_H}} \left(\frac{P_t^{A,F}}{P_t^{B,F}}\right)^{\gamma_F-\gamma_H}$$

Regardless of the values γ_c and φ^j, this expression stays constant as long as the relative price $\frac{P_t^{A,F}}{P_t^{B,F}}$ does not vary. This illustrates that relative PPP is a much less restrictive concept than absolute PPP, since it neither hinges on the validity of the law of one price nor on the assumption of identical consumption baskets.

Exercise 7.2: The J-curve

In the Backus et al. (1994) model, countries are specialized in production, but not in consumption. A positive technology shock in the domestic economy has two effects: it raises the supply of the exported good and thus lowers its price (unless the price elasticity of foreign demand is infinite, which we exclude). This "worsening" of the terms trade is likely to be reflected by a real depreciation. On the supply side, the positive technology shock raises the marginal productivity of capital and thus provides an incentive to raise investment. Finally, if the technology shock is persistent, consumption increases on impact, anticipating future income growth. The increases in consumption and investment are likely to drag down the current account and thus net exports. Hence, in the short run, we observe a decline of net exports that is associated with a real depreciation. In the long run, investment declines since the capital stock has reached its higher level, and output exceeds consumption and investment. This is reflected by positive net exports. The important contribution of Backus et al. (1994) is to demonstrate that the observed "J-curve" need not be driven by delayed quantity reactions (as suggested on pages 296-297), but the equilibrium reactions of output, consumption, investment, and prices

Exercise 7.3: Price competitiveness and markups

a) The PPI-based and the ULC-based real exchange rates are defined as follows:

$$Q_t^{PPI} = \frac{E_t P_t^{PPI,F}}{P_t^{PPI,H}} \quad ; \quad Q_t^{ULC} = \frac{E_t\left(\dfrac{W_t^F L_t^F}{Y_t^F}\right)}{\left(\dfrac{W_t^H L_t^H}{Y_t^H}\right)} = \frac{E_t\left(\dfrac{W_t^F}{A_t^F}\right)}{\left(\dfrac{W_t^H}{A_t^H}\right)}$$

Moreover, prices are related to unit labor costs in the following fashion:

$$P_t^{PPI,c} = (1+\mu_t^c)\frac{W_t^c}{A_t^c}$$

Using the above expression to compute the logarithm of the PPI-based real exchange rate yields

$$q_t^{PPI} = e_t + \ln(1+\mu_t^F) + \ln\left(\frac{W_t^F}{A_t^F}\right) - \ln(1+\mu_t^H) - \ln\left(\frac{W_t^H}{A_t^F}\right)$$

$$\Leftrightarrow q_t^{PPI} + \ln(1+\mu_t^H) - \ln(1+\mu_t^F) = e_t + \ln\left(\frac{W_t^F}{A_t^F}\right) - \ln\left(\frac{W_t^H}{A_t^F}\right) = q_t^{ULC}$$

b) If all goods are tradable without frictions the LOP applies and we have $q_t^{PPI} = 0$. Substituting this into the above expression yields

$$q_t^{ULC} = \ln(1+\mu_t^H) - \ln(1+\mu_t^F)$$

This expression interprets movements of the ULC-based real exchange rate as a result of moving markups: the domestic currency depreciates in real, ULC-based, terms if the markup of domestic firms increases relative to the markup of foreign firms. Whether this variation should be interpreted as a gain in "cost competitiveness" could be a matter of debate.

Exercise 7.4: The World Bank "Atlas method"

The "Atlas conversion factor" that is used to transform local-currency GDP figures into dollars is

$$E_t^{c,Atlas} = \frac{1}{3}\left[E_{t-2}\left(\frac{P_t^c}{P_{t-2}^c}\frac{P_{t-2}^{Ind}}{P_t^{Ind}}\right) + E_{t-1}\left(\frac{P_t^c}{P_{t-1}^c}\frac{P_{t-1}^{Ind}}{P_t^{Ind}}\right) + E_t\right]$$

This can be rewritten as

$$E_t^{c,Atlas} = \frac{E_t}{3}\left[\frac{E_{t-2}}{E_t}\left(\frac{1+\pi_{t,t-2}^c}{1+\pi_{t,t-2}^{Ind}}\right) + \frac{E_{t-1}}{E_t}\left(\frac{1+\pi_{t,t-1}^c}{1+\pi_{t,t-1}^{Ind}}\right) + 1\right]$$

where $\pi_{t,t-j}^c$ reflects the growth rate of the price level in country c between period *t-j* and period *t*.

If relative PPP holds, we have $\frac{E_t}{E_{t-j}} = \left(\frac{1+\pi_{t,t-j}^c}{1+\pi_{t,t-j}^{Ind}}\right)$. We can insert this into the above expression to show that, in this case, $E_t^{c,Atlas} = E_t$.

However, if $\left(\frac{1+\pi_{t,t-j}^c}{1+\pi_{t,t-j}^{Ind}}\right) > \frac{E_t}{E_{t-j}}$, the domestic inflation rate between period *t-j* and period *t* is not fully reflected by the evolution of the nominal exchange rate, and $\frac{E_{t-j}}{E_t}\left(\frac{1+\pi_{t,t-j}^c}{1+\pi_{t,t-j}^{Ind}}\right)$ is greater than one. This, in turn, results in $E_t^{c,Atlas} > E_t$. The „Atlas method" thus accounts for deviations of relative PPP by applying a higher exchange rate if, in the preceding two years, the currency appreciated in real terms.

There are (at least) two drawbacks of this method. First, it hinges on the notion that purchasing power parity is an appropriate theory of the equilibrium real exchange rate. Moreover – and more importantly – it corrects for deviations from relative PPP, but not for deviations from absolute PPP. That is, if a country's price level is *permanently* higher than other countries' price levels while the real exchange rate is not moving over time, the Atlas method suggests using the market nominal exchange rate.

Exercise 7.5: PPP adjustment in Luxembourg

Luxembourg is a country in the heart of Europe, it is a member of the European Union, and it is just a stone's throw to neighboring Germany, France and Belgium. As a consequence, there are neither physical nor administrative barriers which prevent the arbitrage transactions that sustain the law of one price. This, in turn, keeps the country's price level in check, such that the PPP adjustment does not strongly affect the assessment of this country's per-capita income.

Exercise 7.6: The Gerschenkron effect

a) An evaluation of countries' GDP levels at country H's prices yields

$$Y^H_{P^H} = 10 \cdot 200 + 20 \cdot 100 = 4000\,;\; Y^F_{P^H} = 10 \cdot 100 + 20 \cdot 200 = 5000 \;\Rightarrow Y^F_{P^H} > Y^H_{P^H}\,,$$

i.e. country F has the higher income if country H's prices are used as a basis.

An evaluation of countries' GDP levels at country F's prices yields

$$Y^H_{P^F} = 20 \cdot 200 + 10 \cdot 100 = 5000\,;\; Y^F_{P^F} = 20 \cdot 100 + 10 \cdot 200 = 4000 \Rightarrow Y^H_{P^F} > Y^F_{P^F}$$

i.e. country H has the higher income if country F's prices are used as a basis.

b) A higher supply of a good usually results in a lower relative price to other countries. If two countries' incomes are compared, the country whose ample supply of a good is evaluated at high prices that signal scarcity in the base country sees its GDP increase. To perform valid international income comparisons, one could evaluate goods at an "international price level", which can be computed as a weighted average of country-specific levels. As an alternative, we can use a Fisher index, which is a geometric average of income figures using different base countries, i.e. $\left(\frac{Y^F_{P^H}}{Y^H_{P^H}}\right)^{0.5}\left(\frac{Y^F_{P^F}}{Y^H_{P^F}}\right)^{0.5}$.

Chapter 8

Exercise 8.1: Siegel's paradox

a) From a foreign perspective, the uncovered interest rate parity condition is

$$1+i_t^F = \left(1+i_t^H\right)\cdot \frac{\mathrm{E}_t\left(1/E_{t+1}\right)}{1/E_t},$$

Note that E_{t+1} (in italics) denotes the nominal exchange rate – the price of the foreign currency in terms of the domestic currency – while the „straight" E_t denotes conditional expectations.

b) Uncovered interest rate parity from a domestic perspective can be rewritten as

$$\frac{1+i_t^H}{1+i_t^F}E_t = \mathrm{E}_t\left(E_{t+1}\right)$$

From a foreign perspective, the uncovered interest rate can be rewritten as

$$\frac{1+i_t^H}{1+i_t^F}E_t = \frac{1}{\mathrm{E}_t\left(1/E_{t+1}\right)}$$

However, $\mathrm{E}_t\left(E_{t+1}\right) \neq \frac{1}{\mathrm{E}_t\left(1/E_{t+1}\right)}$, due to Jensen's inequality (see page 163). Hence, the two equations cannot be satisfied simultaneously. Based on this observation, Siegel (1972) argued that the forward exchange rate cannot be an unbiased predictor of the future spot exchange rate.

Does Siegel's paradox imply that there is something fundamentally wrong about UIP? The answer is no. In fact, there is no reason why the specification of the UIP equation should be independent of the currency that agents are ultimately interested in. Once we account for the fact that agents focus on real returns, accounting for potential changes in prices in which their consumption baskets are denominated, and if we are willing to impose (relative) purchasing power parity, the paradox disappears. To demonstrate this, let us simplify matters by assuming that the domestic and the foreign nominal interest rates are

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): *International Macroeconomics, 2nd edition*, Tübingen (Mohr Siebeck).
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

zero, i.e. $i_t^H = i_t^F = 0$. In this case, UIP from a domestic perspective can be written as

$$\mathrm{E_t}\left(E_{t+1} - E_t\right) = 0,$$

while UIP from a foreign perspective can be written as

$$\mathrm{E_t}\left(\frac{1}{E_{t+1}} - \frac{1}{E_t}\right) = 0.$$

Domestic residents are interested in real returns in terms of domestic goods. Hence, they correct nominal returns for expected domestic inflation $\mathrm{E_t}\left(P_{t+1}^H / P_t^H\right)$. Inserting this into the above equation and accounting for the fact that P_t^H is known at time *t*, yields

$$\mathrm{E_t}\left(\frac{E_{t+1} - E_t}{P_{t+1}^H}\right) = 0$$

Dividing by E_{t+1} yields

$$\mathrm{E_t}\left(\frac{\frac{1}{E_{t+1}} - \frac{1}{E_t}}{\frac{P_{t+1}^H}{E_{t+1}}}\right) = 0$$

Invoking relative purchasing power parity, i.e. $E_{t+1} / E_t = \dfrac{P_{t+1}^H / P_t^H}{P_{t+1}^F / P_t^F}$, inserting this into the previous expression, and taking into account that all period-*t* variables cancel out, yields

$$\mathrm{E_t}\left(\frac{\frac{1}{E_{t+1}} - \frac{1}{E_t}}{P_{t+1}^F}\right) = 0$$

This is the „foreign“ version of the UIP condition, corrected for the fact that foreign agents are concerned about changes in their own price level. Hence, if

UIP holds in real terms for domestic consumers, it also holds in real terms for foreign consumers.

c) From a domestic perspective, the logarithmic transformation of the UIP condition reads

$$i_t^H = i_t^F + \mathrm{E}_t\left(e_{t+1}\right) - e_t$$

From a foreign perspective, it reads

$$i_t^F = i_t^H - \mathrm{E}_t\left(e_{t+1}\right) + e_t$$

Obviously, the two conditions are equivalent. This is because the logarithmic approximation eliminates the non-linearity which is at the heart of Siegel's paradox.

Exercise 8.2: Uncovered interest parity and the "Grexit"

We repeat the uncovered interest parity condition, treating Greece (GRC) as the domestic economy and the rest of the Euro area (R-EUR) as the foreign economy

$$i_t^{GRC} = i_t^{R-EUR} + \mathrm{E}_t\left(e_{t+1}\right) - e_t$$

As long as Greece is expected to remain in the Euro area, the term $\mathrm{E}_t\left(e_{t+1}\right) - e_t$ equals zero. However, a scenario of Greece leaving the Euro area possibly gives rise to exchange rate risk. Whether creditors are affected by exchange rate fluctuations crucially depends on the currency in which existing loans will be repaid. If this happens in Euro, there is still no exchange rate risk. However, if the loans' face values and returns are transformed into a new national currency – say, the drachma – at the exchange rate at which drachmae were originally converted into Euros, creditors face a sizeable exchange rate risk: the drachma is likely to depreciate substantially in the aftermath of the "Grexit", lowering the Euro value of the payments received by foreign creditors. Ex ante, this drives up $\mathrm{E}_t\left(e_{t+1}\right) - e_t$ and thus the interest rate that has to be paid by Greek borrowers.

Exercise 8.3: Money demand

Assuming that money does not yield a positive nominal return, we can write the expected real return on holding money as

$$r_t^{M,e} = -\pi_t^e$$

with π_t^e denoting the expected inflation rate. Conversely, we can write the expected real return on nominal bonds as

$$r_t^{B,e} = i_t^B - \pi_t^e$$

with i_t^B denoting the nominal interest rate on bonds.
Obviously, the difference between returns amounts to i_t^B, since inflation equally reduces the real return on both money and bonds.

Exercise 8.4: A monetary model with two large open economies

The following equations summarize the structure of the model:

$$\begin{aligned}
i_t^H &= i_t^F + \mathrm{E}_t e_{t+1} - e_t \\
p_t^H &= e_t + p_t^F \\
m_t^H &= p_t^H + \phi\, y_t^H - \lambda i_t^H \\
m_t^F &= p_t^F + \phi\, y_t^F - \lambda i_i^F
\end{aligned}$$

We assume that the foreign money supply is constant ($m_{t+1}^F = m_t^F$), and that output in country *c* grows at an exogenous rate ρ^c, i.e.

$$y_{t+1}^c = y_t^c + \rho^c \text{, } c = H, F.$$

Combining these expressions, iterating forward as in Subsection VIII.4.5, and excluding speculative bubbles yields

$$e_t = \left(\frac{1}{1+\lambda}\right) \sum_{s=t}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^{s-t} \left[m_s^H - m_s^F - \phi\left(y_s^H - y_s^F\right) \right]$$

Note that we can safely omit the expectations operator, since all exogenous variables are assumed to follow a certain time path.

For period t+1, we can write

$$e_{t+1} = \left(\frac{1}{1+\lambda}\right) \sum_{s=t+1}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^{s-(t+1)} \left[m_s^H - m_s^F - \phi\left(y_s^H - y_s^F\right) \right]$$

Computing the difference between the exchange rates in period t+1 and period t yields

$$e_{t+1} - e_t = \left(\frac{1}{1+\lambda}\right) \sum_{s=t+1}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^{s-t} \left[m_{s+1}^H - m_{s+1}^F - \phi\left(y_{s+1}^H - y_{s+1}^F\right) - m_s^H + m_s^F + \phi\left(y_s^H - y_s^F\right) \right]$$

Inserting the above assumptions on the time path of output and foreign money supply yields

$$e_{t+1} - e_t = \left(\frac{1}{1+\lambda}\right) \sum_{s=t}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^{s-t} \left[m_{s+1}^H - m_s^S - \phi\left(\rho^H - \rho^F\right) \right]$$

To keep the nominal exchange rate constant, the government has to make sure that the “present value” on the right-hand side is zero. The easiest way of implementing this is to keep the term in squared brackets equal to zero at every point in time, i.e. to adjust the time path of money supply such that

$$m_{s+1}^H - m_s^S = \phi\left(\rho^H - \rho^F\right)$$

In this case, the government adjusts the growth rate of the domestic money supply to the difference in national growth rates: if the domestic economy grows faster than the foreign one, the domestic money supply increases over time. If foreign output growth is faster than domestic growth, a shrinking money supply keeps the exchange rate constant.

Exercise 8.5: Fundamentals and exchange rates in the monetary model

The assumption that relative monetary fundamentals follow a stationary autoregressive process implies that they will eventually return to their initial level. The forward-looking nature of the nominal exchange rate implied by equation (8.41) accounts for this property, i.e. the current value of e_t reflects both the current shock and the anticipation of its long-run convergence. This dampens the volatility of the nominal exchange rate relative to the volatility of the relative monetary fundamentals.

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): *International Macroeconomics, 2nd edition*, Tübingen (Mohr Siebeck).
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

Exercise 8.6: The Dornbusch model with government spending

We write down the modified version of equation (8.50) that describes the evolution of the price level, including government spending:

$$(8.50)^* \quad p_{t+1}^H - p_t^H = \phi\left[\,\delta\left(e_t + p_t^F - p_t^H\right) + g_t^H - y_t^H\,\right]$$

and combine it with the equation that describes the evolution of the nominal exchange rate:

$$(8.51) \quad e_{t+1} - e_t = \frac{1}{\lambda}\left[\phi\, y_t^H - \lambda\, i_t^F - m_t^H + p_t^H\,\right]$$

We continue by defining the demarcation lines of the phase diagram:

$$\Delta\, p_{t+1}^H = 0 \Leftrightarrow \phi\left[\delta\left(e_t + p_t^F - p_t^H\right) + g_t^H - y_t^H\right] = 0 \Leftrightarrow e_t + p_t^F - p_t^H = \frac{y_t^H - g_t^H}{\delta}$$

Note that this expression indicates that changes in government spending affect the real exchange rate. Since output is exogenous, an increase in government spending necessitates a (demand-reducing) real appreciation. The above expression gives the first demarcation line:

$$p_t^H = e_t + p_t^F - \frac{y_t^H - g_t^H}{\delta}$$

The second demarcation line along which the nominal exchange rate remains constant is easily found:

$$\Delta e_{t+1} = 0 \Leftrightarrow p_t^H = \lambda i_t^F + m_t^H - \phi y_t^H$$

Since the RHS of this equation is a function of exogenous variables, it can be used to derive the steady-state value for the domestic price level:

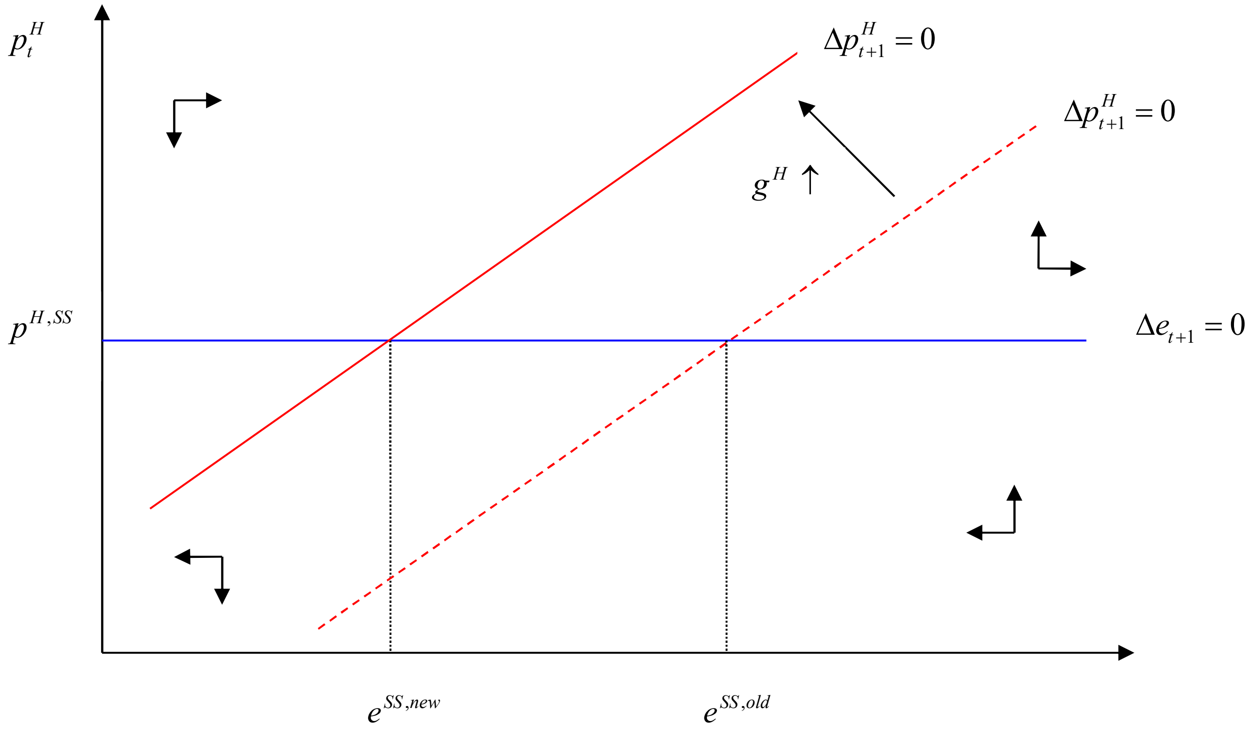
$$p^{H,SS} = \lambda i^F + m^H - \phi y^H$$

Combining this with the preceding demarcation line yields

$$e^{SS} = p^{H,SS} - p^{F} + \frac{y^{H} - g^{H}}{\delta}$$

Defining the real exchange rate in the steady state yields:

$$q^{SS} = e^{SS} + p^{F} - p^{H,SS} = \frac{y^{H} - g^{H}}{\delta}$$



A permanent increase of government spending shifts the $\Delta p_{t+1}^{H} = 0$ line upward. The adjustment of the nominal exchange rate to a lower value – i.e. a nominal and real appreciation – is immediate and monotonic. Hence, there is no overshooting. The reason is that the increase in government spending does not affect the money market and does not necessitate the variation in interest rates that gave rise to the overshooting result.

Exercise 8.7: The effect of discretionary foreign exchange interventions

If foreign-exchange interventions are discretionary, equation (8.61) has to be modified and turns into

(8.61*) $$e_t = \frac{\alpha \, \mathrm{E}_t \, e_{t+1} - \alpha \, \xi_t - \zeta_t - z_t + \eta_t}{\alpha\left(1+\delta\right)+\gamma}$$

By means of forward iteration, combining the information on the time series properties of the various shocks, adjusting for discretionary interventions, and imposing the no-bubble condition, we arrive at a modified version of (8.62):

$$e_t = -\sum_{s=t}^{\infty}\left(\frac{\alpha}{\alpha\left(1+\delta\right)+\gamma}\right)^{s-t}\left(\frac{\alpha \, \psi^{s-t}\xi_t + \varphi^{s-t}\zeta_t + \phi^{s-t} z_t}{\alpha\left(1+\delta\right)+\gamma}\right) + \frac{\eta_t}{\alpha\left(1+\delta\right)+\gamma}$$

Solving the above expression and assuming that $\eta_1 = \theta \, z_1$, we get

$$e_1 = -\frac{\alpha}{\alpha\left(1+\delta-\psi\right)+\gamma}\xi_1 - \frac{1}{\alpha\left(1+\delta-\varphi\right)+\gamma}\zeta_1 - \left(\frac{1}{\alpha\left(1+\delta-\phi\right)+\gamma} - \frac{\theta}{\alpha\left(1+\delta\right)+\gamma}\right) z_1$$

To identify the effects of replacing „rule-bound“ interventions (as in the main text) with discretionary interventions, we compare the last term in parentheses of the above expression to the last term in equation (8.63). It is easy to show that

$$\frac{1}{\alpha\left(1+\delta-\phi\right)+\gamma} - \frac{\theta}{\alpha\left(1+\delta\right)+\gamma} > \frac{1-\theta}{\alpha\left(1+\delta-\phi\right)+\gamma}$$

Hence, a „capital inflow shock“ results in a stronger nominal appreciation of the domestic currency if interventions are discretionary than if they are rule-bound. Put differently, the buffering effect of current foreign exchange interventions is stronger if market participants' expectations are shaped by a credible announcement that monetary authorities will continue to fight the consequences of capital inflow shock in subsequent periods.

Exercise 8.8: The problem of abandoning foreign exchange interventions

A central bank that has been intervening for a long time has accumulated a large volume of foreign-currency denominated assets. A nominal appreciation of the domestic currency would lower the (domestic-currency) value of these assets. To avoid such a negative valuation change, the central bank has an incentive to continue its policy of stabilizing the exchange rate.

Chapter 9

Exercise 9.1: The fiscal multiplier

According to equation (9.7), the marginal effect of an increase of government spending on domestic GDP – the “fiscal multiplier” – is given by the following expression:

$$\frac{d\,Y^H}{d\,G^H} = \frac{1}{1 - C_{Y^H} - NX_{Y^H}}$$

Hence, the multiplier increases in the marginal propensity to consume C_{Y^H} and decreases in marginal propensity to import, which is reinforcing the *negative* influence of domestic income on net exports. Both effects can be explained as follows: with a high marginal propensity to consume, an income increase has a strong effect on (consumption) demand. Additional demand results in additional output and income, and this further enhances consumption – even more so if the marginal propensity to consume is high. Conversely, if the marginal propensity to import is high, a large share of additional income is spent on imports, i.e. it creates demand that is not met by domestic supply and thus does not result in additional income. This slows down the multiplier effect.

Exercise 9.2: The Mundell-Fleming model without home bias

a) We rewrite the old and the new structural equations of the Mundell-Fleming model, already replacing the domestic interest rate by the foreign one, and using the assumption that producers fix their prices in domestic currency

(IS) $$y^H = \gamma\, y^H - \sigma\, i^F + g^H + \delta\, e + \varphi\, y^F + \xi^H$$

(LM*) $$m^H - (1-\theta)e = \phi\, y^H - \lambda\, i^F + \zeta^H$$

The nominal exchange rate that appears on the LHS of the LM* curve reflects the assumption that imported goods play a non-trivial role in the domestic

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): *International Macroeconomics, 2nd edition*, Tübingen (Mohr Siebeck).
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

consumption basket, such that variations in the nominal exchange rate affect the domestic price level despite the ex-ante fixed goods prices.

Intuitively, the presence of the exchange rate in the LM curve is likely to dampen the effect of an expansionary monetary policy: the induced depreciation of the domestic currency raises the domestic price level and dampens the increase of the real money supply. This, in turn, mutes the reaction of real output. Conversely, an expansionary fiscal policy may affect output despite perfect capital mobility, since the induced appreciation lowers the aggregate price level and thus raises the real money supply.

b) To save on notation, we eliminate all exogenous variables except for the money supply and government spending. This can be rationalized by implicitly setting their values equal to zero. For the comparative-static results, this assumption has no consequences. Our "mini version" of the Mundell-Fleming model consists of the following two equations:

$$y^H = \gamma\, y^H + g^H + \delta\, e$$

$$m^H - (1-\theta)e = \phi\, y^H$$

Solving the two equations for e yields

$$e = \frac{(1-\gamma)m^H - \phi\, g^H}{\delta\,\phi + (1-\theta)(1-\gamma)}$$

By substituting this expression into the IS curve (or the LM* curve) we get

$$y^H = \frac{\delta\, m^H + (1-\theta)g^H}{\delta\,\phi + (1-\theta)(1-\gamma)}$$

c) As in the benchmark Mundell-Fleming model, the domestic currency *depreciates* as a reaction to an expansionary monetary policy, while it *appreciates* as a reaction to increasing government spending. However, both reactions are muted if $\theta < 1$, i.e. if there is no complete home bias in consumption. As far as output y^H is concerned, an expansionary monetary policy has a positive effect which, however, is dampened if $\theta < 1$. Conversely, expansionary fiscal policy, which is ineffective if $\theta = 1$, has a positive impact on output if $\theta < 1$.

All these results are in stark contrast to the benchmark Mundell-Fleming model with $\theta = 1$. As suggested in part a), the reason is the impact of induced exchange rate variations on the price level and thus on the real money supply.

Exercise 9.3: The Mundell-Fleming model with imperfect capital mobility

a) The equation presented in the exercise replaces uncovered interest parity (combined with static exchange rate expectations) as an equilibrium condition for the foreign exchange market. In fact, the constellation considered in the benchmark model of perfect capital mobility is the special case of $\kappa \to \infty$. In this case, a minor deviation of the domestic interest rate from the foreign interest rate results in excess demand or supply on foreign exchange markets.

A finite value of κ signals some deviation from perfect and frictionless financial markets. In this case, equilibrium on the foreign exchange market is reflected by the balance-of-payments equilibrium condition, which requires that the sum of the current account and the capital account has to equal the financial account. In our model, the current account is described as a function of the nominal exchange rate and domestic income, with a nominal depreciation – an increase in e – raising net exports and an exogenous increase in domestic income reducing net exports by raising imports. To better understand the role of the financial account, we rewrite this equation as

$$\alpha\, e - \varphi\, y^H = \kappa\left(i^F - i^H\right)$$

The expression on the right-hand side represents net asset purchases by the domestic economy and illustrates that domestic residents accumulate more foreign assets if the foreign interest rate exceeds the domestic interest rate.

b) Note: Inadvertently, I have used the parameter φ both as a coefficient of foreign GDP in the IS curve (9.8) and in the new equation introduced in this exercise. Sorry! To avoid confusion (and without any loss of substance), I will set $y^F = 0$ when deriving the following results.

We start by re-stating the equilibrium conditions for the goods market (IS, equation (9.8)), and the money market (LM, equation (9.1)) – without replacing the domestic interest rate by the foreign interest rate ! – as well as the equilibrium condition for the foreign exchange market (FE). In the IS equation, we omit the effect of foreign income and in the IS and the LM equations, we set the domestic price level equal to zero. For the sake of simplicity, we also eliminate further shocks to goods demand $\left(\xi^H = 0\right)$ and money demand $\left(\zeta^H = 0\right)$. Hence, we have

(IS) $\quad y^H = \gamma\, y^H - \sigma\, i^H + g^H + \delta\, e$

(LM) $\quad m^H = \phi\, y^H - \lambda\, i^H$

(FE) $\quad \alpha\, e - \varphi\, y^H + \kappa\left(i^H - i^F\right) = 0$

The assumption of $\kappa = 0$ represents the extreme case of complete capital immobility. In such a situation, foreign exchange market equilibrium requires that the current account balance equals zero. Moreover, the domestic interest rate is completely isolated from the foreign interest rate. For $\kappa = 0$, the (FE) equation reads

$$\alpha\, e - \varphi\, y^H = 0$$

and can be solved for *e* to yield

$$e = \frac{\varphi}{\alpha}\, y^H$$

This expression has a straightforward interpretation: if domestic output increases, this raises import demand. To keep net exports and the current account at zero, the nominal exchange rate has to depreciate.

Substituting this result into the (IS) and (LM) equations and solving for the endogenous variables y^H and i^H yields

$$y^H = \frac{\alpha\lambda}{\Theta} g^H + \frac{\alpha\sigma}{\Theta} m^H$$

$$i^H = \frac{\alpha\phi}{\Theta} g^H - \frac{\alpha(1-\gamma) - \delta\varphi}{\Theta} m^H$$

with $\Theta \equiv \alpha\,\lambda\left(1-\gamma\right) + \alpha\,\sigma\,\phi - \delta\,\lambda\,\varphi$

By substituting the solution for y^H into the expression that links *e* and y^H we get

$$e = \frac{\varphi \lambda}{\Theta} g^H + \frac{\varphi \sigma}{\Theta} m^H$$

Several aspects of these results are noteworthy: first, the term Θ may be negative – especially, if the marginal propensity to import (φ) is very large. We can, however, exclude such a constellation on intuitive grounds. First, it is plausible to argue that the two parameters α and δ are close to each other. Moreover, the parameter γ should be interpreted as a combination of the marginal propensity to consume (γ^c) and the marginal propensity to import (φ), i.e. $\gamma \equiv \gamma^c - \varphi$. Given these two observations, it is rather implausible that Θ is negative. For the same reason, we can exclude that $\alpha(1-\gamma) - \delta\varphi < 0$. Hence, an increase in the domestic money supply lowers the domestic interest rate.

c) To better understand the above results, we relate them to the properties of the opposite extreme of perfect capital mobility, i.e. the equilibrium of the Mundell-Fleming model with $\kappa \to \infty$: in this case, we have

$$y^H = \frac{m^H}{\phi} - \frac{\lambda}{\phi} i^F$$

$$e = \frac{1}{\delta}\left[(1-\gamma)\frac{m^H}{\phi} - \left(\frac{(1-\gamma)\lambda}{\phi} - \sigma\right) i^F - g^H\right]$$

There are (at least) three important differences between the two extreme cases:

- With perfect capital mobility and static exchange rate expectations, the domestic interest rate of a small open economy is fully determined by the foreign interest rate. If capital mobility is restricted (to the extent of being fully excluded), the domestic interest rate is an endogenous variable that is raised by domestic government spending and reduced by the domestic money supply.

- With perfect capital mobility, variations in fiscal policy have no impact on output since the increase (reduction) of government spending is compensated by a reduction (increase) in export demand, resulting from an appreciation (depreciation) of the domestic currency. If capital mobility is restricted, this is not the case, and there is a role for government spending in affecting output.

- It is easy to show that $0<\frac{\alpha\sigma}{\Theta}<\frac{1}{\phi}$ if $\alpha(1-\gamma)-\delta\varphi>0$, which we can expect to be satisfied (see our argument above). Hence, relative to the situation of perfect capital mobility, the effects of an expansionary monetary policy are dampened. Conversely, the effect on the nominal exchange rate is stronger. The key to understanding these results is that, with imperfect capital mobility, the domestic interest rate is able to adjust to a monetary expansion. As a result, the increase in output that is necessary to keep the money market in equilibrium is smaller. The stronger reaction of the nominal exchange rate reflects the necessity to keep the current account balanced in the face of increasing import demand.

What makes the case of imperfect capital mobility so complicated is the fact that – unlike in the case of perfect capital mobility – there are three endogenous variables: output, the nominal exchange rate, and the domestic interest rate.

Exercise 9.4: Government debt in the CP model

In a model without financial frictions in which the government raises taxes in a lump-sum fashion, it is unlikely that the government's financing decision (taxation vs. debt) affects the allocation. In fact, Ricardian equivalence implies that agents compensate variations in the government's budget deficit by adjusting their own savings.

Exercise 9.5: Producers' choice of currency

Using the information on the demand and cost functions, we start by defining a firm's expected profit for the case of producer currency pricing (PCP) and local currency pricing (LCP), respectively

$$\mathrm{E}_t\left(\Pi_{t+1}^{PCP}\right)=\left(\widetilde{P}_t^{HF}-1\right)\mathrm{E}_t\left[\left(\frac{\widetilde{P}_t^{HF}}{E_{t+1}}\right)^{-\frac{1}{\nu}}\right]$$

$$\mathrm{E}_t\left(\Pi_{t+1}^{LCP}\right)=\mathrm{E}_t\left(E_{t+1}P_t^{HF}-1\right)\left(P_t^{HF}\right)^{-\frac{1}{\nu}}$$

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): *International Macroeconomics, 2nd edition*, Tübingen (Mohr Siebeck).
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

(Recall that the E in italics reflects the nominal exchange rate while the „straight“ E denotes the expectations operator.)

The crucial difference between the two pricing decisions is that in case of PCP, the uncertainty stems from the effect of exchange rate fluctuations on demand. By contrast, there is no uncertainty about demand in the case of LCP, since the price is fixed ex ante. However, exchange rate fluctuations affect the difference between revenues (in terms of domestic currency) and costs.

In period t, a firm chooses its optimal price, accounting for the specific type of uncertainty that it is exposed to. The first-order condition that implicitly defines the optimal price chosen by a PCP firm is

$$\mathrm{E}_t\left[\left(\frac{\tilde{P}_t^{HF}}{E_{t+1}}\right)^{-\frac{1}{\nu}}\right]-\frac{1}{\nu}\frac{\left(\tilde{P}_t^{HF}-1\right)}{\tilde{P}_t^{HF}}\mathrm{E}_t\left[\left(\frac{\tilde{P}_t^{HF}}{E_{t+1}}\right)^{-\frac{1}{\nu}}\right]=0$$

Solving for the optimal price yields

$$\tilde{P}_t^{HF,opt}=\frac{1}{1-\nu}$$

Substituting this result into the definition of a PCP's firm's expected profits yields

$$\mathrm{E}_t\left(\Pi_{t+1}^{PCP,opt}\right)=\left(\frac{\nu}{1-\nu}\right)(1-\nu)^{\frac{1}{\nu}}\mathrm{E}_t\left[\left(E_{t+1}\right)^{\frac{1}{\nu}}\right]$$

Performing the same steps for a LCP firm yields the first-order condition

$$\mathrm{E}_t\left(E_{t+1}\right)\left(P_t^{HF}\right)^{-\frac{1}{\nu}}+\mathrm{E}_t\left(E_{t+1}P_t^{HF}-1\right)\left(-\frac{1}{\nu}\right)\left(P_t^{HF}\right)^{-\frac{1}{\nu}-1}=0$$

Solving for the optimal price yields

$$P_t^{HF,opt}=\frac{1}{1\text{-}\nu}\frac{1}{\mathrm{E}_t\left(E_{t+1}\right)}$$

Using this result, we can compute an LCP firm's expected profit:

$$\mathrm{E}_{\mathrm{t}}\left(\Pi_{t+1}^{LCP,opt}\right)=\left(\frac{\nu}{1-\nu}\right)(1-\nu)^{\frac{1}{\nu}}\left[\frac{1}{\mathrm{E}_{\mathrm{t}}\left(E_{t+1}\right)}\right]^{-\frac{1}{\nu}}$$

Obviously, the two expressions for expected profits are not the same, and it is easy to show that $\mathrm{E}_{\mathrm{t}}\left(\Pi_{t+1}^{PCP,opt}\right)>\mathrm{E}_{\mathrm{t}}\left(\Pi_{t+1}^{LCP,opt}\right)$ if

$$\mathrm{E}_{\mathrm{t}}\left[\left(E_{t+1}\right)^{\frac{1}{\nu}}\right] > \left[\mathrm{E}_{\mathrm{t}}\left(E_{t+1}\right)\right]^{\frac{1}{\nu}}$$

Given the (standard) assumption that $\nu<1$ – Note that, otherwise, prices charged would be negative! – we can use Jensen's inequality (see page 163) – to show that the above inequality is, in fact, satisfied. Hence, firms choose to set prices in their own currency, i.e. they select PCP.

The intuition behind this result is straightforward: since the demand elasticity is greater than one, demand is a convex function of prices and – in the case of PCP – of the exchange rate. As a consequence, a depreciation raises demand by more than an appreciation lowers demand, and firms prefer this type of risk to the risk arising from fluctuations of the exchange rate that "linearly" transforms foreign-currency revenues into domestic-currency units.

Exercise 9.6: Determinants of exchange rate passthrough

The dampening effect of *per-capita income* on exchange rate passthrough can be explained as follows: richer countries have a larger services sector – including a retail sector that delivers imported goods to the customer. This retail sector acts like a buffer to exchange rate fluctuations since any retail price consists of a tradable and a non-tradable (retail) component.

The effect of the *inflation rate* can be related to firms' strategies with respect to price setting and price revision: in a high-inflation environment, firms are likely to develop techniques and organizational structures that allow them to quickly and frequently adjust prices. They are thus more likely to react to a variation of the nominal exchange rate. Conversely, prices are likely to change less frequently in low-inflation environments, since the opportunity costs of keeping prices constant are usually lower.

Chapter 10

Exercise 10.1: Seigniorage and inflation bias

Calvo (1978) identifies another potential source of time inconsistency: the government has an incentive to announce low inflation ex ante and to thus keep the nominal interest rate at a low level. This raises money demand and allows the government (which is interpreted as a combination of fiscal and monetary authorities) to finance its expenditure by issuing money. Ex post, however there is an incentive to increase the inflation rate, collecting an "inflation tax" that allows the government to avoid distortionary (output-reducing) taxes. As in the time inconsistency model presented in the book, rational agents understand the government's optimization problem and adjust their expectations.

While Calvo focuses on the role of money as a nominal asset, the implications of his model are, in fact, much broader: if agents hold nominal public debt, the government has an incentive to reduce its real debt burden by surprising its creditors with a higher inflation rate.

Exercise 10.2: Original sin

If a borrowing country's monetary authorities cannot commit to price stability, foreign creditors may be reluctant to lend in domestic currency: relative PPP implies that the high inflation rate will eventually be reflected by a depreciation of the domestic currency. If debt contracts were denominated in domestic currency, this would imply a capital loss. While this could be compensated by raising the interest rate on loans, a wide-spread way of tackling the problem is to denominate loans in foreign currency.

This, however, implies that domestic borrowers are exposed to a substantial exchange-rate risk: a depreciation of the domestic currency raises their effective debt burden. This may give rise to currency crises of the type described in Section X.3.4.

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): *International Macroeconomics, 2nd edition*, Tübingen (Mohr Siebeck).
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

Exercise 10.3: Expansionary and contractionary devaluations

Third-generation models of currency crises as described in Section X.3.4 highlight the adverse effects of depreciations on banks' net worth and, more generally, countries' net international investment positions. If these effects are strong, the expansion in demand that is associated with a depreciation of the domestic currency may be dominated by a contraction of demand resulting from a financial crisis. This is particularly likely if a large share of a country's liabilities is denominated in foreign currency. In this respect, the UK as well as the Scandinavian countries differed from most emerging markets: while the EMS crisis had a detrimental impact on the financial sectors of the affected economies, it was possible to quickly repair financial institutions' balance sheets. By contrast, adverse balance sheet effects were by orders of magnitude larger in most emerging economies, and the negative impact of the depreciation was dominant for a protracted period of time.

Exercise 10.4: Gold standard and balance of payments equilibrium

The Bretton Woods system of fixed exchange rate tied countries' currencies to the US dollar, whose value was supported by the Fed's gold reserves. However, as it turned out, the US eventually conducted a monetary policy according to its own priorities and needs, thus exploiting its hegemonic status in the system. By contrast, a gold standard does not hinge on the behavior of the country that issues the "reserve currency" since all national currencies are fully (or at least partially) backed by national gold reserves – i.e. monetary authorities fix a price of gold in terms of their own currency and are willing to sell gold at this price. As a result, gold implicitly acts as a medium of exchange on international financial markets since all national currencies are essentially claims on a given amount of that precious metal. Fixed exchange rates result as a ratio of national gold prices.

Ideally, the mechanism that establishes balance of payments equilibrium in such a system works as follows: if a country runs a current account deficit that is not matched by capital inflows – i.e. if $CA + KA < FA^{NR}$ – this results in a reduction of the country's gold reserves and hence of its money supply. The lower money supply, in turn, pushes down prices. Lower prices feed into the terms of trade, enabling the country to increase its net exports and thus its current account balance. This so-called ***price-specie flow mechanism***, which was first described by the Scottish philosopher David Hume (1711-1776), ensures the stability of a gold-standard system of fixed exchange rates.

These are solution sketches for the end-of-chapter exercises of the textbook
Harms, Philipp (2016): *International Macroeconomics, 2nd edition*, Tübingen (Mohr Siebeck).
This version: April 4, 2017. **Note**: All solutions may be subject to changes.

Exercise 10.5: Sudden stops and sectoral structure

A "sudden stop" is a situation in which financial markets are no longer willing to finance a country's current account deficit, forcing it to raise its current account balance. As we have seen in Chapter VII, a country's net exports – and thus its current account balance – increase in the real exchange rate. How much of a real depreciation is needed to bring about the necessary reduction of the current account deficit depends on a country's economic structure: if it is easy to relocate factors of production into exporting industries, and if demand for exportables is very price sensitive, a small depreciation is sufficient. In Figure 10.12, this would be depicted by a "flat" *CA(Q)*-curve. By contrast, if it is very difficult to shift resources into exporting industries – e.g. because a large share of the labor force is tied up in nontraded goods industries – the *CA(Q)*-curve in Figure 10.12 is very steep. As a consequence, the real depreciation that shifts the current account towards the level tolerated by the international capital market is very strong.

Exercise 10.6: Development aid and the Dutch disease

While the original argument about the causes and consequences of the Dutch disease focused on resource discoveries, foreign aid may play a similar role: as an exogenous increase in income, it shifts resources into non-tradable goods industries (e.g. the construction sector or an inflated bureaucracy). The real appreciation that is associated with this structural change harms the competitiveness of exporting industries, slowing down technological progress and making the country even more dependent on foreign transfers.

The argument brought forward by Rajan and Subramanian (2005) is plausible and supported by empirical evidence. The authors demonstrate that countries that received larger aid flows in the past saw their currencies appreciate in real terms and their manufacturing industries shrink. Does this imply that aid is more of a burden than a boon? Not necessarily: the Dutch-disease argument is based on the notion that only a small share of incoming aid flows is used to enhance a country's productive capacity. If, instead, foreign funds are used for productive purposes – e.g. by improving the infrastructure or educating the labor force – the negative "Dutch disease-like" effects do not have to materialize.