

Sequential Warrant Exercise in Large Trader Economies

by

Tobias Linder¹

Siegfried Trautmann²

Abstract

It is well known that the sequential (premature) exercise of American-type warrants *may* be advantageous for large warrant holders, even in the absence of regular dividends, because using exercise proceeds to expand the firm's scale increases the riskiness of an equity share. We show that for realistic interest rate levels even large warrant holders are better off not to exercise prematurely. This result, however, does not justify in general the simplifying restriction that warrants or convertible securities are valued as if exercised as a block.

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Key words: Warrants, Large Trader, Sequential Exercise, Partial Exercise Option

¹ Tobias Linder, CoFaR Center of Finance and Risk Management, Johannes Gutenberg-Universität, D-55099 Mainz, Germany, E-mail: linder@finance.uni-mainz.de

² Professor Dr. Siegfried Trautmann (corresponding author), CoFaR Center of Finance and Risk Management, Johannes Gutenberg-Universität, D-55099 Mainz, Germany, E-mail: traut@finance.uni-mainz.de, phone: (+) 49 6131 39 23760

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Warrants, unlike call options, are issued by companies and when exercised new shares are created with the exercise proceeds increasing the firm's assets. Because of this, there is some dilution of equity and dividend when warrants are exercised. Therefore the value accruing to one warrant holder is not independent of what other warrant holders do. Under certain conditions, the premature exercise of a warrant can increase the value of the warrants that remain outstanding, because using exercise proceeds to expand the firm's scale increases the riskiness of an equity share. Emanuel (1983), Constantinides (1984) and Constantinides and Rosenthal (1984) demonstrate the potential advantage of a sequential exercise strategy assuming a firm without (senior) debt. All these papers compare a sequential exercise strategy with an exercise strategy, called block exercise, where all warrant holders completely exercise their warrants simultaneously or not at all. Emanuel (1983) studies the monopolistic case, whereas Constantinides and Rosenthal (1984) focus on pricetaking warrant holders. Constantinides (1984) shows that the warrant price in a competitive equilibrium is smaller than or equal to the warrant price under the block exercise constraint, if all projects of the firm have a zero net present value and the firm pays dividends and coupons. In the absence of dividend payments, Cox and Rubinstein (1985) and Ingersoll (1987) demonstrate that a sequential exercise policy is never optimal for a pricetaker, while it can be beneficial to a monopoly warrant holder. Spatt and Sterbenz (1988) generalize this result to oligopoly warrant holders and show that there are reinvestment policies of the firm for which sequential exercise is not beneficial to non-pricetaking warrant holders. Their analysis helps to justify the frequent simplifying restriction that warrants or convertible securities are valued as if exercised as a block. Articles on warrant valuation which rely on the reasonableness of block exercise include Ingersoll (1977), Brennan and Schwartz (1977, 1980), Schulz and Trautmann (1994), and Crouhy and Galai (1994).

In this paper we show that for realistic interest rate levels it is *not* optimal even for large warrant holders to exercise long-lived warrants sequentially, if the firm uses the exercise proceeds to rescale its investment. Therefore, it turns out that from a theoretical perspective the potential advantage of sequential exercise strategies is not the main obstacle against the use of the block exercise assumption. The latter assumption, however, is questionable on the ground that it may be advantageous not to exercise all warrants if they finish in the money.

The *existence of senior debt* causes a *positive value* for the option to exercise only a fraction of the outstanding warrants at maturity in *large trader economies*. For competitive markets, Bühler and Koziol (2002) have demonstrated that allowing senior debt in the capital structure causes a partial conversion of convertible bonds to be optimal. Koziol (2003) extends these results for convertible bonds with conversion strategies in a monopoly while Koziol (2006) examines exercise strategies for warrants in a competitive market. Kapadia and Willette (2005) analyze warrant

exercise strategies in some large trader economies. We complete the results to all large trader economies.

The paper is organized as follows: In Section 1 we specify the model and define the different exercise policies. Section 2 looks at the partial exercise policies of European-type warrants and Section 3 examines the optimality of sequential exercise strategies of American-type warrants under the firm policy that the exercise proceeds are used to rescale the firm's investment. Section 4 concludes the paper. All technical proofs are given in Appendix A.

1 Model

We consider a firm with value V_t at time t following a Geometric Brownian Motion.¹ The firm is financed by issuing equity, warrants and debt and pays no regular dividends. Exercise proceeds are used to *rescale the firm's investment*. Furthermore we assume throughout the paper that there are no taxes or transaction costs, and no arbitrage opportunities in the project market. The risk neutral probability measure is denoted by Q .

Capital structure

At time $t = 0$ the firm's equity consists of N outstanding shares and n warrants with maturity T and strike price K . Every warrant entitles its owner to get one share of common stock when exercising the warrant at times $0 = t_1, t_2, \dots, t_J = T$ (American-type warrant) or only at maturity T (European-type warrant). Senior debt is issued in the form of a zero coupon bond with a common face value of F and maturity T_D with $0 < T < T_D$. At $t \in [0, T_D]$ we denote the price of one stock by S_t , one warrant by W_t , the debt by D_t and the number of warrants exercised (before and at time t) by m_t . The number of warrants exercised at time $t_j \in \{t_1, \dots, t_J\}$ is denoted by $m'_{t_j} = m_{t_j} - m_{t_{j-1}}$ with $m_{t_0} = 0$. According to Modigliani and Miller (1958) we assume that the firm value is equal to the value of all shares, all warrants, and total debt:

$$V_t = (N + m_t)S_t + (n - m_t)W_t + D_t \quad \text{for all } t \in [0, T].$$

We denote the firm value immediately before time t by V_{t-} and the value of the firm's initial assets by A_t which follows the same Geometric Brownian Motion as

¹However, Examples 1 and 2 are given in a binomial setting.

the firm value. So before the warrant holders exercise m_0 warrants at time $t = 0$ the firm value equals the asset value $V_{0-} = A_0$ and thereafter we have

$$V_{t_k} = A_{t_k} + \sum_{j=1}^k m'_{t_j} K \frac{A_{t_k}}{A_{t_j}} \quad (1)$$

for all $k \in \{1, \dots, J\}$, because exercise proceeds are used to rescale the firm's investment. \bar{S}_t denotes the total value of common stock. After the maturity of the warrants the firm value equals

$$V_t = (N + m_T)S_t + D_t = \bar{S}_t + D_t \quad \text{for all } t \in [T, T_D).$$

If at time T_D the firm value is less than the face value of the debt (i.e. $V_{T_D} \leq F$), a default occurs and the stocks get worthless, i.e. $S_{T_D} = 0$ and $D_{T_D} = V_{T_D}$. Otherwise the common stock equals the firm value minus the face value of the debt, so we get the equation

$$V_{T_D} = \bar{S}_{T_D} + \min\{F; V_{T_D}\}. \quad (2)$$

According to equation (2) the value of the total common stock $\bar{S}_t \equiv \bar{S}_t(V_t)$ equals the value of a call option on the firm value V_t with maturity T_D and strike price F at time $t \in [T, T_D)$. Since V_t follows a geometric Brownian motion, \bar{S}_t behaves similarly as the *Black/Scholes*-value of a European call option does, where the firm value includes the exercise proceeds. For all $V \in \mathbb{R}_+$ we have $\Delta_T(V) = \partial \bar{S}_T(V) / \partial V \in (0, 1)$.

Warrant holders and their payoff functions

The set of the warrant holders is denoted by I and P is a measure on I . Every warrant holder $i \in I$ holds n^i warrants with $\int_I n^i dP = n$. Furthermore, we assume that warrant holders do not own shares of common stock of the firm at time $t = 0$ and that every warrant holder knows the number of warrants of each other warrant holder (complete information on the distribution of warrant ownership).

The set of strategies of warrant holder $i \in I$ are all possible exercise policies m_t^i , $t \in \{t_1, \dots, t_J\}$ with $m_t^i \in [0, n^i]$ and m_t^i increasing with respect to the time t . The number of warrants exercised by all warrant holders is $m_t = \int_I m_t^i dP \in [0, n]$, while m_t^{-i} denotes the number of warrants exercised by all warrant holders except i with $m_t = m_t^i P(\{i\}) + m_t^{-i}$.

We call warrant holder $i \in I$ a *pricetaker* if $P(\{i\}) = 0$, because the asset prices are independent of his trading and exercise policy (the latter does not affect the number of warrants exercised and therefore the asset prices). If $m_{t-}^i = 0$ the payoff function of a pricetaking warrant holder i at time $t < T$ is

$$\pi_t^i(m_t^i, m_t, V_t) = m_t^i (S_t(V_t) - K) + (n^i - m_t^i) W_t(V_t). \quad (3)$$

In contrast to a pricetaking warrant holder we call warrant holder $A \in I$ with $P(\{A\}) = 1$ a *non-pricetaker*. His exercise policy influences the asset prices, in particular the stock price $S_t(V_t)$. His payoff function is defined by

$$\pi_t^A(m_t^A, m_t^{-A}, V_t) = m_t^A (S_t(V_t) - K) + (n^A - m_t^A)W_t(V_t). \quad (4)$$

At time T the payoff of warrant holder i is defined as the exercise value of warrants exercised by warrant holder i if we have $m_{T-}^i = 0$.² Since non-exercised warrants at time T expire worthless, the payoff function of a pricetaking warrant holder $i \in I$ simplifies to

$$\pi_T^i(m_T^i, m_T, V_T) = m_T^i \left(\frac{\bar{S}_T(V_T)}{N + m_T} - K \right).$$

As the payoff function of each pricetaking warrant holder i is a function which is *linear* in the number of warrants exercised by himself, his payoff function is maximised at $m_T^i = 0$ or $m_T^i = n^i$. Only if we have $\bar{S}_T(V_T) - (N + m_T)K = 0$, every exercise policy of i maximises his payoff. The corresponding payoff function of a non-pricetaking warrant holder A reads now as follows:

$$\pi_T^A(m_T^A, m_T^{-A}, V_T) = m_T^A \left(\frac{\bar{S}_T(V_T)}{N + m_T^A + m_T^{-A}} - K \right).$$

Block exercise, partial exercise and sequential exercise

Stock prices rationally reflect anticipation of the number of warrants exercised and the assumed use of the exercise proceeds. We distinguish between three kinds of exercise policies:

Definition 1 *Warrant holders follow a so-called sequential exercise strategy if they exercise American-type warrants before maturity. Otherwise the warrant holders follow a so-called block exercise strategy if the number of warrants exercised at the maturity date is given by*

$$m_T = \begin{cases} 0 & \text{for } \frac{1}{N+n}\bar{S}_T(V_T) \in [0, K) \\ n & \text{for } \frac{1}{N+n}\bar{S}_T(V_T) \in [K, \infty), \end{cases}$$

or they follow a so-called partial exercise strategy.

²Since rational warrant holders will choose $m_T^i = 0$ if $\bar{S}_T(V_T)/(N + m_T) - K < 0$, it is not necessary to denote the exercise value of one warrant by the positive part of this function.

Definition 2 *The partial exercise option is the option to follow a partial exercise strategy instead of a block exercise strategy. The sequential exercise option is the option to follow a sequential exercise strategy instead of a partial exercise strategy.*

We model the warrant holders' exercise behavior as a noncooperative game and consider a Nash equilibrium as an optimal exercise strategy for the warrant holders. The noncooperative game is defined by the set of warrant holders, the exercise policies as the strategies sets, and the payoff functions. While Constantinides (1984) and other authors analyse a zero-sum game between the warrant holders and the stockholders (as passive players), our game is not zero-sum (like in Bühler and Koziol (2002) and Koziol (2003, 2006)), because there is a wealth transfer from the stockholders and the warrant holders to the debtholders by the exercise of a warrant.

Definition 3 *In case of European-type warrants the exercise strategy $(m_T^{i*})_{i \in I}$ is a Nash equilibrium if for every warrant holder $i \in I$*

$$\pi_T^i(m_T^{i*}, m_T^{-i*}, V_T) \geq \pi_T^i(m_T^i, m_T^{-i*}, V_T) \quad \text{holds for all } m_T^i \in [0, n^i].$$

In case of American-type warrants the exercise strategy $(m_t^{i})_{i \in I, t \in \{t_1, \dots, t_J\}}$ is a Nash equilibrium if for every warrant holder $i \in I$ and $t \in \{t_1, \dots, t_J\}$*

$$\pi_t^i(m_t^{i*}, m_t^{-i*}, V_t) \geq \pi_t^i(m_t^i, m_t^{-i*}, V_t) \quad \text{holds for all } m_t^i \in [m_{t-}^i, n^i].$$

In a Nash equilibrium each warrant holder takes the other warrant holders' exercise policy as given and maximises his payoff. We call a Nash equilibrium an optimal exercise strategy. Although the optimal exercise strategy may not be unique (e.g. if all warrant holders are pricetakers, the optimal exercise strategy is not unique), the stock price and warrant price is unique for all optimal exercise strategies. So the value of a partial exercise option and a sequential exercise option is well defined.

2 Partial exercise of European-type warrants

We start our analysis of optimal exercise strategies at the warrants' maturity with an example, where we (1) illustrate the optimal exercise policy in a competitive market, and (2) compare this policy to the optimal exercise policy in a monopolistic market. This example emphasizes the need for analysing the optimal exercise policies in large trader economies.

Example 1 We assume that in the interval $[T, T_D]$ the firm value V follows a simple binomial process where the firm value can increase or decrease by 50 % rather than a Geometric Brownian Motion. Furthermore, we assume an interest rate of zero percent such that the risk neutral probability for an increase or decrease of the firm value equals 0.5, respectively.

We assume that the firm has issued $N = 100$ shares of the common stock, $n = 100$ European-type warrants with a strike price of $K = 100$ and a zero coupon bond with a face value of $F = 53,950$. At time T^- the firm value equals $V_{T^-} = 50,000$. Then for the firm value V_{T_D} the following two realisations are possible:

$$\begin{array}{l}
 V_T = 50,000 + 100m_T \begin{cases} \nearrow \\ \searrow \end{cases} \\
 \begin{array}{l}
 V_{T_D}^u = 75,000 + 150m_T \\
 S_{T_D}^u = \frac{1}{N+m_T}[V_{T_D}^u - F]^+ \\
 \quad = \frac{1}{100+m_T}(21,050 + 150m_T) \\
 \\
 V_{T_D}^d = 25,000 + 50m_T < F \\
 S_{T_D}^d = \frac{1}{N+m_T}[V_{T_D}^d - F]^+ = 0
 \end{array}
 \end{array}$$

At time T the stock price equals $S_T = 0.5 \cdot S_{T_D}^u = (10,525 + 75m_T)/(100 + m_T)$. In a competitive economy the warrant holders exercise so many warrants that the stock price equals the strike price in a Nash equilibrium $S_T = K$. This results in $m_T^* = 21$, a stock price of $S_T = 100$ and an exercise value of the warrants of $W_T = 0$. If more warrants were exercised, the exercise value of the warrants would be negative, and if less warrants were exercised the exercise value of the warrants would be positive and every single pricetaking warrant holder would be better off exercising more warrants.

Now we assume that one monopolistic warrant holder A owns all warrants, so his payoff function and its first derivative with respect to the number of warrants exercised satisfy

$$\begin{aligned}
 \pi_T^A(m_T^A, V_T) &= m_T^A \left(\frac{10,525 + 75m_T^A}{100 + m_T^A} - K \right), \\
 \frac{\partial}{\partial m_T^A} \pi_T^A(m_T^A, V_T) &= \frac{1}{(100 + m_T^A)^2} \left(-25m_T^{A2} - 5,000m_T^A + 52,500 \right),
 \end{aligned}$$

respectively. Warrant holder A maximises his payoff by exercising $m_T^{A*} = 10$ warrants. Then the stock price equals $S_T = 102.5$ and the exercise value of the warrants equals $W_T = 2.5$.

Example 1 demonstrates that the exercise value in a monopoly can differ from the exercise value in a competitive economy. In the following we compare the exercise policies in large trader economies. We denote the subset of pricetaking

warrantholders by $I^p = \{i \in I | P(\{i\}) = 0\}$. The pricetaking warrant holders own $n^p = \int_{I^p} n^i dP$ and exercise $m_T^p = \int_{I^p} m_T^i dP$. The non-pricetakers $A_1, A_2, \dots, A_L \in I, L \geq 0$ own $n^{A_1} \leq n^{A_2} \leq \dots \leq n^{A_L}$ warrants and no shares. Extending the results of Koziol (2006) and Kapadia and Willette (2005) we get that the following strategy is a Nash equilibrium:

$$(m_T^{p*}, m_T^{A_1*}, m_T^{A_2*}, \dots, m_T^{A_L*}) = \begin{cases} (0, 0, 0, \dots, 0) & \text{for } V_{T-} \in [0, \underline{V}) \\ (x^*, 0, 0, \dots, 0) & \text{for } V_{T-} \in [\underline{V}, \underline{V}_1) \\ (n^p, x_1^*, x_1^*, \dots, x_1^*) & \text{for } V_{T-} \in [\underline{V}_1, \underline{V}_2) \\ (n^p, n^{A_1}, x_2^*, \dots, x_2^*) & \text{for } V_{T-} \in [\underline{V}_2, \underline{V}_3) \\ \vdots & \\ (n^p, n^{A_1}, \dots, n^{A_{L-1}}, x_L^*) & \text{for } V_{T-} \in [\underline{V}_L, \bar{V}_L) \\ (n^p, n^{A_1}, n^{A_2}, \dots, n^{A_L}) & \text{for } V_{T-} \in [\bar{V}_L, \infty) \end{cases}$$

where the critical firm values $\underline{V}, \underline{V}_1, \underline{V}_2, \underline{V}_3, \dots, \underline{V}_L$ and \bar{V}_L solve

$$\begin{aligned} \frac{1}{N} \bar{S}_T(\underline{V}) &= K \\ \frac{1}{N + n^p} \bar{S}_T(\underline{V}_1 + n^p K) &= K \\ \frac{\partial}{\partial m_T^{A_1}} \pi_T^{A_1}(n^{A_1}, n^p + (L-1)n^{A_1}, \underline{V}_2) &= 0 \\ \frac{\partial}{\partial m_T^{A_2}} \pi_T^{A_2}(n^{A_2}, n^p + n^{A_1} + (L-2)n^{A_2}, \underline{V}_3) &= 0 \\ &\vdots \\ \frac{\partial}{\partial m_T^{A_{L-1}}} \pi_T^{A_{L-1}}(n^{A_{L-1}}, n^p + n^{A_1} + \dots + n^{A_{L-2}} + n^{A_{L-1}}, \underline{V}_L) &= 0 \\ \frac{\partial}{\partial m_T^{A_L}} \pi_T^{A_L}(n^{A_L}, n^p + n^{A_1} + \dots + n^{A_{L-1}}, \bar{V}_L) &= 0 \end{aligned}$$

and the exercise policies $x^*, x_1^*, x_2^*, \dots, x_L^*$ are the solutions of

$$\begin{aligned} \frac{1}{N + x^*} \bar{S}_T(V_{T-} + x^* K) &= K \tag{5} \\ \frac{\partial}{\partial m_T^{A_1}} \pi_T^{A_1}(x_1^*, n^p + (L-1)x_1^*, V_T) &= 0 \\ \frac{\partial}{\partial m_T^{A_2}} \pi_T^{A_2}(x_2^*, n^p + n^{A_1} + (L-2)x_2^*, V_T) &= 0 \\ &\vdots \\ \frac{\partial}{\partial m_T^{A_L}} \pi_T^{A_L}(x_L^*, n^p + n^{A_1} + \dots + n^{A_{L-1}}, V_T) &= 0, \end{aligned}$$

respectively. If the firm has no senior debt in its capital structure (i.e. $F = 0$), we get $\underline{V} = \underline{V}_1 = \underline{V}_2 = \dots = \underline{V}_L = \overline{V}_L$ and the block exercise strategy is optimal. Furthermore, the optimal exercise strategy is not unique: Although equation (5) has a unique solution x^* , any exercise strategy $(x^{i*})_{i \in I^p}$ with $x^* = \int_{I^p} x^{i*} dP$ is a Nash equilibrium.

The optimal exercise policy of the pricetakers is to exercise all their warrants if the stock price exceeds the strike price. Although all pricetakers would benefit if they exercise less warrants, every pricetaking warrant holder wants to be a free rider and exercises as many warrants as possible without incurring a loss. Therefore the stock price can only be above the strike price if all pricetakers exercise all their warrants. This holds for all firm values $V_{T^-} > \underline{V}_1$. In contrast to pricetakers, non-pricetakers can increase the stock price through their exercise policies, increasing the exercise value of the warrants, and increasing their payoffs. If $V_{T^-} \in (\underline{V}_1, \overline{V}_L)$ the non-pricetakers are better off when exercising less warrants than pricetakers would in a competitive economy. The higher exercise value of the warrants exercised compensates the lower number of warrants exercised.

Surprisingly, warrant holders A_2, \dots, A_L exercise as many warrants as warrant holder A_1 if $V_{T^-} \in [\underline{V}_1, \underline{V}_2)$, although each of them owns more warrants than warrant holder A_1 . This is due to the fact that the payoff functions of non-pricetakers do not depend on the total number of warrants they hold. So if an optimal exercise policy is an inner solution for one warrant holder, the same exercise policy is optimal for another (non-pricetaking) warrant holder even if he holds a different number of warrants.

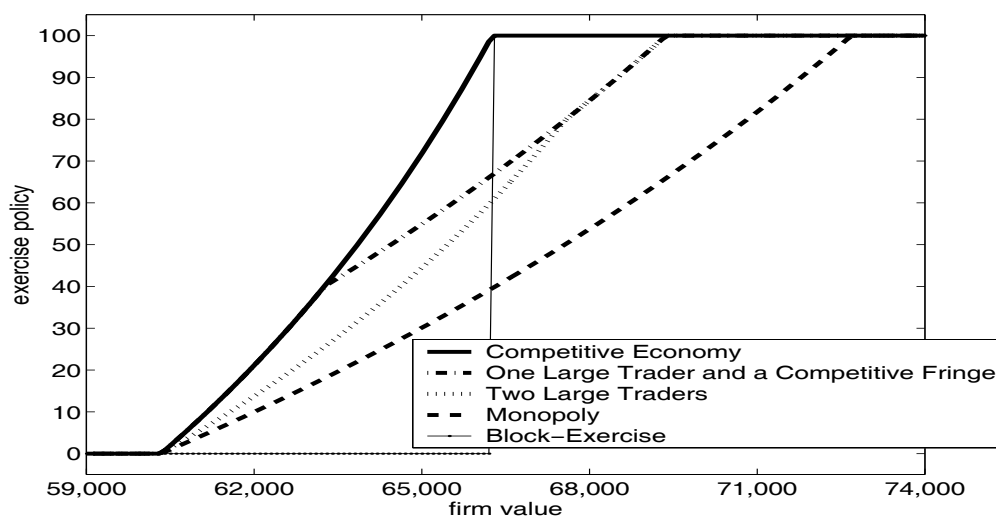
Figure 1 illustrates the differences of optimal exercise policies and their corresponding exercise values due to four different market structures. According to the figure in panel A, 100% of the outstanding warrants will be exercised in a competitive market at the critical firm value $\overline{V} = 66,258.47$ (the same percentage as with the block exercise strategy) while only a percentage between 40 and 66 will be exercised in the three large trader economies for the same firm value. The figure in panel B confirms, first of all, the well-known fact that there is no difference between warrant values in a competitive economy and a block exercise-constrained economy although the optimal exercise strategy in a competitive market deviates from the block exercise strategy. Moreover, this figure demonstrates that an increasing concentration of the warrant ownership distribution may lead to substantially higher exercise values of the outstanding warrants.

Now we look at a market structure with exactly one large warrant holder $A \in I$. Non-pricetaker A owns $n^A \in (0, n]$ warrants and the pricetaking warrant holders the remaining $n^{-A} < n$ warrants. Please note that the monopoly is a special case of this economy with $n^A = n$ and $n^{-A} = 0$. The number of warrants exercised by

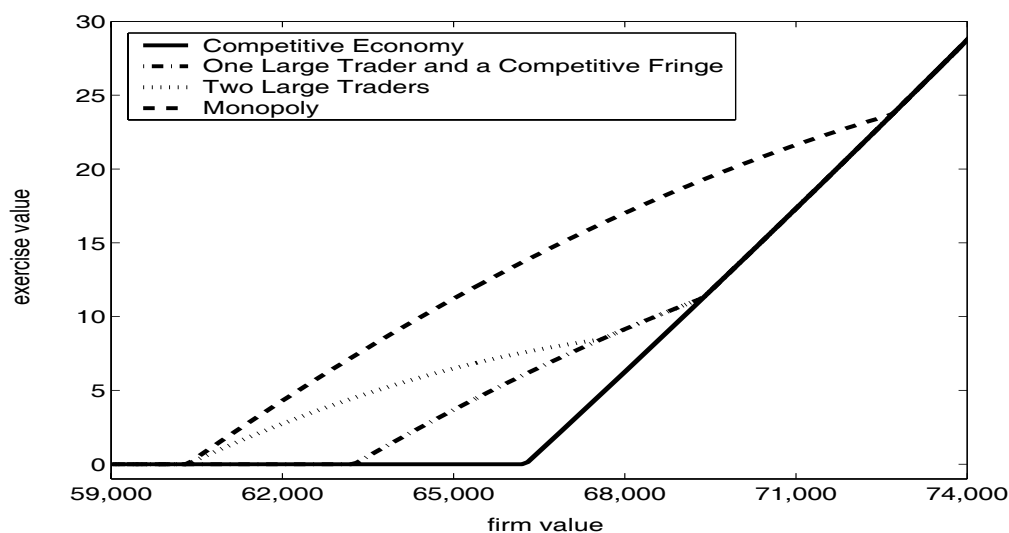
Figure 1: Exercise policies and exercise values

The figure shows the exercise rate of all players as a function of the firm value and the exercise value of a warrant as a function of the firm value at time T . We assume the parameters $r = 5\%$, $\sigma = 0.25$, $F = 80,000$, $T_D - T = 4$, $N = 100$, $n = 100$, $n_{-A} = n_b = 40$ and $K = 100$. The critical firm values are $\underline{V} = 60,330.53$ and $\bar{V} = 66,258.47$.

Panel A: Optimal exercise policies



Panel B: Exercise values of European-type warrants



all pricetakers (all warrant holders without A) is denoted by $m_T^{-A} = \int_{I \setminus \{A\}} m_T^i dP$ so that the total number of warrants exercised satisfies $m_T = m_T^A + m_T^{-A}$. Furthermore, we assume that non-pricetaker A owns $N^A \in [0, N)$ shares of the common stock. Then the following strategy is a Nash equilibrium:

$$(m_T^{-A*}, m_T^{A*}) = \begin{cases} (0, 0) & \text{for } V_{T-} \in [0, \underline{V}) \\ (x^*, 0) & \text{for } V_{T-} \in [\underline{V}, \underline{V}_A) \\ (n^{-A}, x_1^*) & \text{for } V_{T-} \in [\underline{V}_A, \bar{V}_A) \\ (n^{-A}, n^A) & \text{for } V_{T-} \in [\bar{V}_A, \infty) \end{cases}$$

where \underline{V} solves $\bar{S}_T(\underline{V}) = NK$, \underline{V}_A solves $\bar{S}_T(\underline{V}_A + n^{-A}K) = (N + n^{-A})K$ and \bar{V}_A solves $\partial \pi_T^A(n^A, n^{-A}, \bar{V}_A) / \partial m_T^A = 0$. The exercise policies x^* , x_1^* are the solutions of

$$\begin{aligned} \frac{1}{N + x^*} \bar{S}_T(V_{T-} + x^*K) &= K \\ \frac{\partial}{\partial m_T^A} \left[\frac{N^A + x_1^*}{N + n^{-A} + x_1^*} \bar{S}_T(V_{T-} + (n^{-A} + x_1^*)K) - x_1^*K \right] &= 0, \end{aligned}$$

respectively.

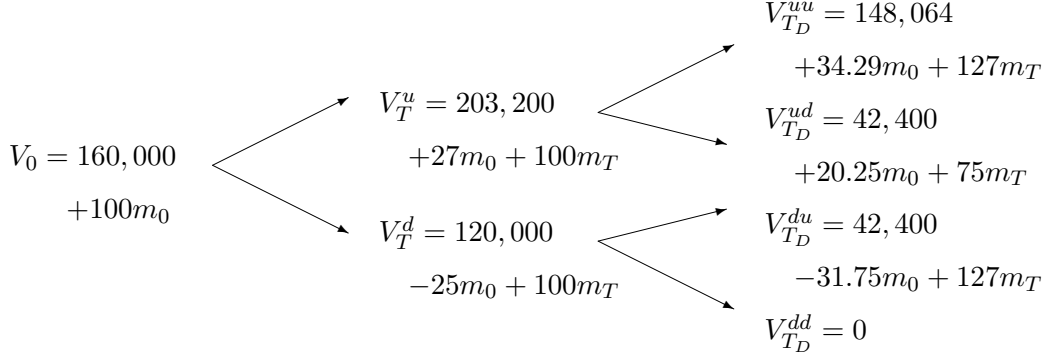
3 Sequential exercise of American-type warrants

Emanuel (1983) and Constantinides (1984) emphasize the potential advantage of sequential exercise strategies by warrant holders, even absent regular dividend payments. Cox and Rubinstein (1985), Ingersoll (1987) and Spatt and Sterbenz (1988) illustrate the potential optimality of sequential exercise based upon differing assumptions about the firm's policy regarding the use of warrant exercise proceeds and about the distribution of warrant ownership. All these examples disregard straight debt in the capital structure of the firm which is, however, considered in the following analysis. Without additional debt a wealth transfer from the stockholders to the warrant holders is possible when exercising warrants sequentially. The following analysis shows that in a model with additional debt the situation is more complex: The value of the debt can both increase and decrease due to the exercise of a warrant. Example 2 illustrates a wealth transfer from the debtholder to the stockholders and warrant holders.

Example 2 We assume that the firm value follows a binomial process with two periods starting in $t = 0$ and $t = T$. In each period the firm value can increase by 27% or decrease by 25%. The interest rate equals $r = 1\%$ so that the risk neutral probability for an increase of the firm value is $q = 0.5$. The current firm value equals $V_0 = 160,000$. Furthermore,

we assume that the firm has issued a zero coupon bond with a face value of 110,000, 100 stocks and 100 American-type warrants with a strike price of $K = 100$ and we assume that the firm pays no dividends.

The firm value is illustrated in the following figure including the redemption of the additional debt or the default at time T_D . Please note that the warrant holders exercise $m_T - m_0$ warrants at time T . This implies that the up-state firm value $V_T^u = V_0 \cdot 1.27 + (m_T - m_0)K$ satisfies.



A simple calculation as in Example 1 shows us that at time T , $V_T \in \{V_T^u, V_T^d\}$ all *pricetaking* and *non-pricetaking* warrant holders are better off to exercise all remaining warrants. Thus the optimal number of warrants exercised is $m_T^* = n - m_0$ warrants. Therefore the stock price, the warrant price, and the debt value satisfy

$$\begin{aligned}
S_T(V_T^u) &\approx \frac{1}{1+r} (526.66 + 0.14m_0) \\
W_T(V_T^u) &= S_T(V_T^u) - 100 \\
D_T(V_T^u) &= \frac{1}{1+r} 110,000 \\
S_T(V_T^d) &\approx \frac{1}{1+r} (137.75 - 0.08m_0) \\
W_T(V_T^d) &= S_T(V_T^d) - 100 \\
D_T(V_T^d) &\approx \frac{1}{1+r} (103,750 - 9.38m_0) ,
\end{aligned}$$

respectively. In time $t = 0$ the stock price, warrant price and the debt value satisfy

$$\begin{aligned}
S_0(V_0) &\approx \frac{1}{(1+r)^2} (332.21 + 0.03m_0) \\
W_0(V_0) &= S_0(V_0) - \frac{1}{1+r} 100 \\
D_0(V_0) &\approx \frac{1}{(1+r)^2} (106,875 - 4.69m_0) ,
\end{aligned}$$

respectively. Since $S_0(V_0) - K - W_0(V_0) < 0$ a pricetaking warrant holder is better off not to exercise warrants, i.e. in a competitive economy we get $m_0^* = 0$. In an economy with one large trader A with $n^A \in (0, n]$ and a competitive fringe the payoff function of warrant holder A and its first derivative with respect to the number of warrants exercised is equal to

$$\begin{aligned} \pi_0^A(m_0^A, 0, V_0) &= m_0^A(S_0(V_0) - K - W_0(V_0)) + n^A W_0(V_0) \\ &= m_0^A \left(\frac{1}{1+r} 100 - 100 \right) \\ &\quad + n^A \left(\frac{1}{(1+r)^2} 332.21 + \frac{0.03m_0}{(1+r)^2} - \frac{1}{1+r} 100 \right), \\ \frac{\partial}{\partial m_0^A} \pi_0^A(m_0^A, 0, V_0) &= \left(\frac{1}{1+r} 100 - 100 \right) + n^A \frac{0.03}{(1+r)^2}. \end{aligned}$$

The first derivative of the payoff function of warrant holder A is constant in the number of warrants exercised. Thus warrant holder A will exercise either all warrants or no warrant at all. The equation $\partial \pi_0^A(m_0^A, 0, V_0) / \partial m_0^A > 0$ is equivalent to

$$n^A > \frac{1}{0.03} (1+r)^2 \left(100 - \frac{1}{1+r} 100 \right) \approx 33.67.$$

If warrant holder A owns more than 33.67 warrants he exercises all his warrants, otherwise none. The following table shows the stock price, the warrant price and the debt value in an economy with one large trader holding different numbers of warrants and a competitive fringe.

	Competitive economy	One large trader ($n^A = 33$)	One large trader ($n^A = 66$)	Monopoly
Stock price	325.66	325.66	327.61	328,61
Warrant price	226.65	226.65	228.60	229,60
Debt value	104,769.14	104,769.14	104,465.70	104,309.38

In the foregoing example the assumed interest rate of $r = 1\%$ was mainly responsible for the optimality of a sequential exercise strategy, because exercising warrants prematurely is only beneficial if the interest rate is low. If we assume an interest rate of $r = 4\%$ a sequential exercise strategy is never optimal. Most examples of the related literature (e.g., Ingersoll, 1987, and Spatt and Sterbenz, 1988, proof of theorem 3) even assume an interest rate of $r = 0\%$. This leads to the question: Under which conditions is a sequential exercise beneficial to warrant holders?

It is well known that a rational pricetaker will never exercise a warrant before maturity in the absence of dividend payments. This results in

Proposition 1 *In the absence of dividend payments the sequential exercise option of a pricetaking warrant holder has zero value.*

The proof follows the same lines as the proof of the non-optimality of an early exercise of call options. Now we consider a non-pricetaking warrant holder A holding $n^A \in (0, n]$ warrants and a competitive fringe holding $n^{-A} = n - n^A$ warrants. We differ two targets of profit maximization: The paper wealth and the real wealth of warrant holder's A payoff function in the spirit of Jarrow (1992, p.312).

“Paper wealth is defined as the value of the [warrant holder's] position evaluated at the prices supported by the large trader. Real wealth, on the other hand, is the value of the large traders' position after liquidation (i.e., return to zero holdings). For a pricetaker, these values are identical; but for a large trader they are distinct.”

Real wealth maximization

If warrant holder A maximizes the real wealth of his position at time $t = 0$, he sells all stocks and warrants immediately after time $t = 0$ to pricetakers (if he sells his position to another non-pricetaker, his maximization problem equals the maximization of the paper wealth of his position). The payoff function of warrant holder A is defined by equation (4), where S_0 and W_0 are calculated as the stock and warrant price in a competitive economy.

Lemma 1 *If the firm uses the exercise proceeds to rescale the firm's investment and non-pricetaking warrant holder A maximizes the real wealth of his position, the marginal payoff of warrant holder A at time $t_1 = 0$ is bounded by*

$$\frac{\partial}{\partial m_0^A} \pi_0^A(m_0^A, m_0^{-A}, V_0) < K \left(\frac{n^A}{N + n^A} \frac{W_0^{am}(V_0)}{V_0} - (1 - e^{-rT}) \right) \quad (6)$$

for all (sequential) exercise strategies $(m_0^i)_{i \in I}$, where W_0^{am} is an at-the-money warrant on the firm value with maturity T .

The proof is given in Appendix A.

Paper wealth maximization

If warrant holder A maximizes the paper wealth of his position at time $t_1 = 0$, he holds his stocks and warrants at least until time T (if he sells a part of his position, this sale lowers the paper wealth of the remaining position). Of course, he is allowed to exercise more warrants at any time $t \in \{t_2, \dots, t_J\}$. Then A 's payoff function at time t_1 equals the discounted expected payoff at time T . According to Section 2 the first derivative of the payoff function at time T with respect to m_T^A is equal or below zero or the optimal number of warrants exercised satisfies $m_T^{A*} = n^A$. The latter and the fact that the optimal exercise strategy influences the firm values results in

Lemma 2 *If the firm uses the exercise proceeds to rescale the firm's investment and non-pricetaking warrant holder A maximizes the paper wealth of his position, the marginal payoff of warrant holder A at time $t_1 = 0$ is bounded by relation (6).*

The proof is given in Appendix A.

Lower bound of sequential exercise strategies

Lemma 1 and Lemma 2 represent the same upper bound of the marginal payoff of non-pricetaker A . Using the relation $W_0^{am}(V_0)/V_0 \leq 1$ we get a lower bound of interest rate levels which must be fulfilled if a large warrant holder can be better off to exercise prematurely:

Proposition 2 *In the absence of dividend payments the sequential exercise option has zero value if the interest rate satisfies*

$$r \geq \frac{1}{T} \ln \left(\frac{N + n_A}{N} \right). \quad (7)$$

Proof: If the upper bound for the marginal payoff (6) is negative a sequential exercise strategy is never optimal. Therefore if a sequential exercise strategy can only be optimal if the upper bound is positive, i.e. the following necessary condition must be satisfied:

$$\begin{aligned} 0 &\leq K \left(\frac{n_A}{N + n_A} \frac{W_0^{am}(V_0)}{V_0} - (1 - e^{-rT}) \right) \\ &\leq K \left(\frac{n_A}{N + n_A} - (1 - e^{-rT}) \right). \end{aligned} \quad (8)$$

Relation (8) is equivalent to the lower bound (7). □

Please note that the lower bound of Proposition 2 does not depend on the firm value V_0 , the distribution of the firm value process and the debt characteristics. Of course, this lower bound represents a tradeoff between the sharpness of the bound and the simplicity of its calculation. Nonetheless this bound is good enough to show that a sequential exercise policy is only optimal for warrants whose time to maturity is short.

The lower bound is plotted in Panel A of Figure 2. Please note that we do not need any information about the firm's capital structure except the maturity of the warrants, the non-pricetakers number of warrants and the number of stocks outstanding. Panel A of Figure 2 confirms that for relevant maturities of warrants and ownership concentration (measured by the ratio n^A/N) sequential exercise is not optimal for (non-pricetaking) warrant holders. If the non-pricetaking warrant holder owns $n^A = 10$ warrants with maturity $T = 10$ and $N = 100$ stocks are outstanding, the non-pricetaker does not exercise any warrant if the interest rate is above 1%.

Unfortunately, if the non-pricetaking warrant holder A holds many warrants whose time to maturity is short, Proposition 2 is not very useful. In this case we refer to relation (6) presenting a more precise lower bound on interest rate levels preventing sequential exercise.³ Panel B of Figure 2 shows for the same parameters (we assume that the firm value follows a Geometric Brownian Motion with volatility $\sigma \leq 0.25$) that a non-pricetaking warrant holder will not exercise his warrants if the interest rate is above 4%. Furthermore, for $n^A = 20$ and $T = 1$ non-pricetaker A does not exercise any warrant if the interest rate is above 1.8%. However, both lower bounds increase with a decreasing time to maturity T . Nonetheless, large warrant holders cannot increase their payoff substantially exercising short-lived warrants. According to relation (6) the upper bound of the marginal payoff of one more exercised warrant goes to zero if the time to maturity goes to zero.

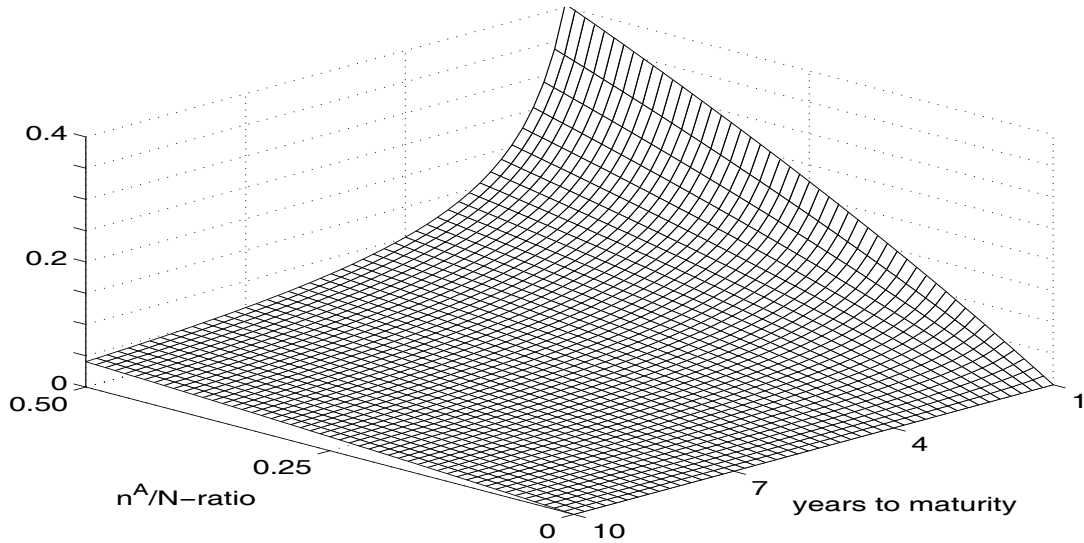
Proposition 2 justifies the assumption that warrants are not exercised prematurely if the exercise proceeds are used to expand the firm's investment. This result holds also for alternative reinvestment strategies, like those analysed in Spatt and Sterbenz (1988): reinvestment in riskless zero-coupon bonds or repurchase of shares plus issuance of new warrants.

³The upper bound for the marginal payoff (6) must be positive if a sequential exercise strategy is optimal. Solving relation (6) implicitly for the interest rate we get a lower bound on interest rate levels.

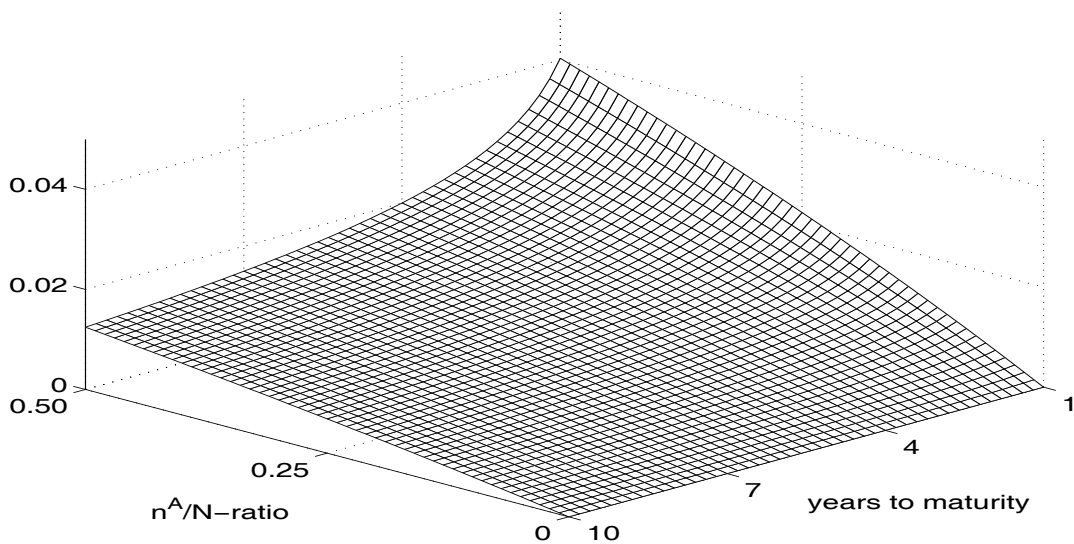
Figure 2: Lower bounds on interest rate levels preventing sequential exercise

The figures shows lower bounds for the interest rate. For any interest rate above these lower bounds a sequential exercise policy is not optimal. We assume in panel B an asset return volatility of $\sigma \leq 0.25$.

Panel A: Lower bound according to Proposition 2



Panel B: Lower bound according to Lemma 1 and Lemma 2



4 Conclusion

This paper investigates the impact large traders have on the optimal exercise strategies for convertibles and their corresponding market values. As distinguished from the existing literature, our analysis considers a firm that issues (additional) senior debt besides shares of common stock and warrants. We show that a sequential exercise can only be beneficial to a non-pricetaking warrant holder if the interest rate is below a critical lower bound. However, for a realistic parameter setting the interest rate is above the lower bound and a premature exercise of long-lived warrants is not beneficial. Hence, it turns out that from a theoretical perspective the potential advantage of sequential exercise strategies is not the main obstacle against the use of the block exercise condition in the absence of dividend payments. The latter condition is however questionable on the ground that it may be advantageous not to exercise all warrants if they finish in the money (partial exercise option).

This investigation can be extended in several directions. For instance, there is at least one fact not considered in the model: The pricetakers do not know the distribution of the warrant ownership. Perhaps less interesting is the analysis of a situation where warrant holders own in addition shares of the common stock of the firm: Non-pricetaking warrant holders will exercise less warrants at maturity, because the stock price is decreasing in the number of warrants exercised.

A Proofs

Proof of Lemma 1:

Assuming that all $(n - m_0^A)$ warrants that are not exercised at $t_1 = 0$ are sold to pricetakers, we get two critical asset values $\underline{A}(m_0)$ and $\overline{A}(m_0)$ at time $t = T$ with

$$\begin{aligned} \overline{S}_T \left(\frac{A_0 + m_0 K}{A_0} \underline{A}(m_0) \right) &= (N + m_0)K \\ \text{and } \overline{S}_T \left(\frac{A_0 + m_0 K}{A_0} \overline{A}(m_0) + (n - m_0)K \right) &= (N + n)K. \end{aligned}$$

If the asset value A_T is less than $\underline{A}(m_0)$, no warrant is exercised and the stock price is less than the strike price, if $A_T \in [\underline{A}(m_0), \overline{A}(m_0)]$ as many warrants are exercised as necessary to equalize stock price and strike price, whereas if $A_T \geq \overline{A}(m_0)$ all remaining warrants are exercised in a competitive market. The relation between firm value V_T and asset value A_T is given by equation (1). Since the pricetakers exercise no warrant before time T we have $S_T(V_T) - K = W_T(V_T)$ for all $A_T \geq \underline{A}(m_0)$, $W_T(V_T) = 0$ for all $A_T < \underline{A}(m_0)$ and

$$\begin{aligned} & \frac{\partial}{\partial m_0^A} \left[m_T^A \left(\frac{\overline{S}_T(V_T)}{N + m_T} - K - W_T(V_T) \right) \right] \\ &= \begin{cases} \frac{\partial}{\partial m_0^A} \left[m_0^A \left(\frac{\overline{S}_T(V_T)}{N + m_0} - K \right) \right] & \text{for } A_T \in [0, \underline{A}(m_0)) \\ 0 & \text{for } A_T \in [\underline{A}(m_0), \infty) \end{cases} \\ &= \begin{cases} \frac{N + m_0^{-A}}{(N + m_0)^2} \overline{S}_T(V_T) + \frac{m_0^A}{N + m_0} K \frac{A_T}{A_0} \Delta_T(V_T) - K & \text{for } A_T \in (0, \underline{A}(m_0)) \\ 0 & \text{for } A_T \in [\underline{A}(m_0), \infty) \end{cases} \\ &\leq \begin{cases} \frac{m_0^A}{N + m_0} K \left(\frac{A_T}{A_0} \Delta_T(V_T) - 1 \right) & \text{for } A_T \in (0, \underline{A}(m_0)) \\ 0 & \text{for } A_T \in [\underline{A}(m_0), \infty) \end{cases} \\ &\leq \begin{cases} \frac{n^A}{N + n^A} K \left(\frac{A_T}{A_0} - 1 \right) & \text{for } A_T \in [A_0, \max\{A_0, \overline{A}(m_0)\}) \\ 0 & \text{for } A_T \notin [A_0, \max\{A_0, \overline{A}(m_0)\}) \end{cases} \quad (\text{A1}) \end{aligned}$$

The warrant price and its first derivative with respect to the number of warrants exercised at time $t_1 = 0$ (m_0^A) read as

$$\begin{aligned} W_T(V_T) &= \begin{cases} 0 & \text{for } A_T < \overline{A}(m_0) \\ \frac{1}{N + n} \overline{S}_T(V_T) - K & \text{for } A_T \geq \overline{A}(m_0) \end{cases} \\ \frac{\partial}{\partial m_0^A} W_T(V_T) &= \begin{cases} 0 & \text{for } A_T < \overline{A}(m_0) \\ \frac{1}{N + n} K \left(\frac{A_T}{A_0} - 1 \right) \Delta_T(V_T) & \text{for } A_T \geq \overline{A}(m_0) \end{cases} \end{aligned}$$

This implies

$$\begin{aligned} \frac{\partial}{\partial m_0^A} W_0(V_0 + m_0 K) &= e^{-rT} \int_{\bar{A}(m_0)}^{\infty} \frac{\partial}{\partial m_0^A} W_T(V_T) dQ \\ &\leq e^{-rT} \int_{\max\{A_0, \bar{A}(m_0)\}}^{\infty} \frac{1}{N + n^A} K \left(\frac{A_T}{A_0} - 1 \right) dQ \end{aligned} \quad (\text{A2})$$

since $\Delta_T < 1$ and $n^A \leq n$.

Since A sells all his stocks and warrants to pricetakers, no additional warrant is exercised before time T according to Proposition 1. Thus we can rewrite the payoff function in the following way:

$$\begin{aligned} \pi_0^A(m_0^A, m_0^{-A}, V_0) &= e^{-rT} \int_0^{\infty} m_0^A (S_T(V_T) - K - W_T(V_T)) dQ \\ &\quad + e^{-rT} \int_{\bar{A}(m)}^{\infty} n^A W_T(V_T) dQ - m_0^A K (1 - e^{-rT}). \end{aligned}$$

Using relations (A1) and (A2) we get an upper bound for the marginal payoff of warrant holder A :

$$\begin{aligned} \frac{\partial}{\partial m_A} \pi_0^A(m_0^A, m_0^{-A}, V_0) &\leq e^{-rT} \int_{A_0}^{\max\{A_0, \bar{A}(m_0)\}} \frac{n^A}{N + n^A} K \left(\frac{A_T}{A_0} - 1 \right) dQ \\ &\quad + e^{-rT} \int_{\max\{A_0, \bar{A}(m_0)\}}^{\infty} \frac{n^A}{N + n^A} K \left(\frac{A_T}{A_0} - 1 \right) dQ \\ &\quad - K(1 - e^{-rT}) \\ &= \frac{n^A}{N + n^A} K \frac{W_0^{am}(A_0)}{A_0} - K(1 - e^{-rT}). \end{aligned}$$

This completes the proof since $W_0^{am}(A_0)/A_0 = W_0^{am}(V_0)/V_0$. \square

Proof of Lemma 2:

The payoff function of non-pricetaker A at time T satisfies

$$\pi_T^A(m_T^A, m_T^{-A}, V_T) = \frac{m_T^A}{N + m_T} \bar{S}_T(V_T) - \sum_{j=1}^T m_{t_j}^A K e^{r(T-t_j)}$$

$$\begin{aligned}
&= \frac{\sum_{j=1}^T m'_{t_j}{}^A}{N + m_T^{-A} + \sum_{j=1}^T m'_{t_j}{}^A} \bar{S}_T \left(A_T + \sum_{j=1}^T m'_{t_j}{}^A K \frac{A_T}{A_{t_j}} \right) \\
&\quad - \sum_{j=1}^T m'_{t_j}{}^A K e^{r(T-t_j)}.
\end{aligned}$$

Please remember that we use the convention $\Delta_T(V) = \partial \bar{S}_T(V) / \partial V$. Let the number of warrants exercised satisfy $m_T^A < n^A$. This implies $\partial \pi_T^A / \partial m_T^A \leq 0$. The first derivative of the payoff function with respect to m_0^A satisfy

$$\begin{aligned}
\frac{\partial}{\partial m_0^A} \pi_T^A(m_T^A, m_T^{-A}, V_T) &= \frac{N + m_T^{-A}}{N + m_T^A} \bar{S}_T(V_T) + \frac{m_T^A}{N + m_T} K \frac{A_T}{A_0} \Delta_T(V_T) - K e^{rT} \\
&= \frac{N + m_T^{-A}}{N + m_T^A} \bar{S}_T(V_T) + \frac{m_T^A}{N + m_T} K \Delta_T(V_T) - K \\
&\quad + \frac{m_T^A}{N + m_T} K \left(\frac{A_T}{A_0} - 1 \right) \Delta_T(V_T) - K e^{rT} + K \\
&\leq \frac{\partial}{\partial m_T^A} \pi_T^A(m_T^A, m_T^{-A}, V_T) - K (e^{rT} - 1) \\
&\quad + \frac{m_T^A}{N + m_T} K \left(\frac{A_T}{A_0} - 1 \right) \Delta_T(V_T) \cdot 1_{\{A_T \geq A_0\}} \\
&\leq \frac{m_T^A}{N + m_T} K \left(\frac{A_T}{A_0} - 1 \right) \cdot 1_{\{A_T \geq A_0\}} - K (e^{rT} - 1) \\
&\leq K \left(\frac{n^A}{N + n^A} \left(\frac{A_T}{A_0} - 1 \right) \cdot 1_{\{A_T \geq A_0\}} - (e^{rT} - 1) \right).
\end{aligned}$$

Let $m_T^A = n^A$. Since all pricetakers exercise all their warrants at maturity if a non-pricetaker exercise a fraction or all of his warrants, we get $m_T = n$. Thus the payoff function and its first derivate read as

$$\begin{aligned}
\pi_T^A(n^A, n^{-A}, V_T) &= \frac{n}{N + n} \bar{S}_T \left(A_T + \sum_{j=1}^T m'_{t_j}{}^A K \frac{A_T}{A_{t_j}} \right) - \sum_{j=1}^T m'_{t_j}{}^A K e^{r(T-t_j)} \\
\frac{\partial}{\partial m_0^A} \pi_T^A(n^A, n^{-A}, V_T) &= \frac{n}{N + n} K \frac{A_T}{A_0} \Delta_T(V_T) - K e^{rT} \\
&< \frac{n^A}{N + n^A} K \left(\frac{A_T}{A_0} - 1 \right) \cdot 1_{\{A_T \geq A_0\}} - K e^{rT} + K \\
&= K \left(\frac{n^A}{N + n^A} \left(\frac{A_T}{A_0} - 1 \right) \cdot 1_{\{A_T \geq A_0\}} - (e^{rT} - 1) \right).
\end{aligned}$$

Hence, if the non-pricetaking warrant holder A maximizes the paper wealth of his position the first derivative of his payoff function with respect to the number of warrants exercised at time $t_1 = 0$ is bounded by

$$\frac{\partial}{\partial m_0^A} \pi_T^A(m_T^A, m_T^{-A}, V_T) < K \left(\frac{n^A}{N + n^A} \left(\frac{A_T}{A_0} - 1 \right) \cdot 1_{\{A_T \geq A_0\}} - (e^{rT} - 1) \right).$$

Since non-pricetaker A holds all warrants and all stocks until maturity the first derivative of the payoff function with respect to m_0^A can be written as

$$\begin{aligned} \frac{\partial}{\partial m_0^A} \pi_0^A(m_0^A, m_0^{-A}, V_0) &= e^{-rT} \int \frac{\partial}{\partial m_0^A} \pi_T^A(m_T^A, m_T^{-A}, V_T) dQ \\ &\leq e^{-rT} \int_{A_0}^{\infty} K \left(\frac{n^A}{N + n^A} \left(\frac{A_T}{A_0} - 1 \right) - (e^{rT} - 1) \right) dQ \\ &= K \left(e^{-rT} \frac{n^A}{N + n^A} \frac{1}{A_0} \int_{A_0}^{\infty} (A_T - A_0) dQ - (1 - e^{-rT}) \right) \\ &= K \left(\frac{n^A}{N + n^A} \frac{W_0^{am}(A_0)}{A_0} - (1 - e^{-rT}) \right). \end{aligned}$$

This completes the proof since we have $W_0^{am}(A_0)/A_0 = W_0^{am}(V_0)/V_0$. \square

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