Stock Price Jumps and Their Impact on Option Valuation

by

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First draft: July 1993

Current draft: March 1995

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The authors are grateful for comments and suggestions received from David Bates, Franz Palm, William Perraudin and Kaushik Amin. The latter did provide us with the computer program used to value American puts. We also thank participants in seminars at Mannheim, St. Gallen, Louvain-la-Neuve, HEC, and at the Annual Meeting of the EFA 1994 in Brussels. The financial support from the Deutsche Forschungsgemeinschaft (Project Tr 255/1 within the Schwerpunktprogramm 'Empirische Kapitalmarktforschung') is gratefully acknowledged.

Abstract

The purpose of this paper is threefold. First, we use ML-techniques to estimate a Poisson-type jump diffusion model that describes the return behavior of actively traded German stocks and the DAX stock index as a proxy of aggregate wealth, respectively. We find that jump risk is statistically significant and systematic.

Second, we compute option values according to Merton's idiosyncratic jump risk model and the more recent systematic jump risk model and compare them with Black/Scholes values. Using a comprehensive sample of stock options traded at the *Frankfurt Options Market* between April 1983 and June 1990 and at the *Deutsche Terminbörse* between January 1990 and December 1991, respectively, we find only in post-crash periods economically significant differences between Black/Scholes and systematic jump risk option values when using historical parameter estimates.

Third, we take the systematic jump risk model to infer the implicit stock price distributions from observed option prices before, during, and after periods of dramatic stock price changes in the sample period from April 1983 to December 1991. The implicit parameters reflect the different expectations of call and put market participants. Confirming the findings of Bates (1991) for the US-market, our implicit parameters estimated for pooled calls and puts indicate strong crash fears especially in July 1987 but not during the 2 months immediately preceding the October 1987 crash. While after the market crash the results for the US-market exhibit even stronger crash fears, our implicit parameters reflect mainly rebound hopes.

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1 Introduction

Not only because of the various stock market crashes in recent years, it seems natural to model stock prices for option valuation as continuous time stochastic processes with discontinuous sample paths. These processes constitute an important alternative to the standard diffusion model of Black/Scholes (1973) and were first studied by Press (1967) and incorporated into the theory of option valuation by Merton (1976a). More recent applications of such processes in the option valuation context include the papers of Jones (1984), Naik/Lee (1990), Jarrow/Madan (1991), Bates (1991), Ahn (1992), Amin/Ng (1993), and Amin (1993). The popularity of these jump-diffusion type continuous time stochastic processes stems from at least two facts. First, as distinguished from pure diffusion processes, these processes can explain the observed empirical characteristics of stock return distributions, such as high levels of kurtosis and skewness. Second, they are economically appealing because they allow that stock prices change by significant amounts in a very short time ('jumps') – a reasonable assumption for an efficient stock market – while the probability of such jumps is zero in diffusion processes.

Statistical investigations of such mixtures of a diffusion process and a compound jump process for American stock prices may be found in Press (1967), Beckers (1981), Ball/Torous (1983, 1985), Jarrow/Rosenfeld (1984) and Akgiray/Booth (1986). Akgiray/Booth/Loistl (1989) provide some evidence for the general form of the mixed process by examining the weekly returns of a portfolio consisting of 48 stocks actively traded on the Frankfurt Stock Exchange. However, few papers have investigated the effect of jumps in the underlying stock price process on stock option values. Ball and Torous (1985, p.155) point out that there were no 'operationally significant differences between the Black/Scholes and Merton model prices' in the context of pricing options on NYSE stocks. However, their jump-diffusion model restrict jump sizes to having zero mean and is therefore incapable a priori of eliminating the Black/Scholes model bias with respect to the call option's exercise price. More importantly, Merton's (1976a) option pricing model is based on the crucial assumption that jump risk is idiosyncratic and therefore diversifiable. By contrast, Bates (1991) finds that a jump-diffusion model allowing systematic jump risk fits the actual data markedly better than the Black/Scholes model when examining transaction prices of S&P 500 futures options over the period 1985-1987.

The main purpose of this paper is to investigate the relationship between Black/Scholes option values and the option values according to the idiosyncratic jump risk model of Merton (1976a) and the systematic jump risk model of Bates (1991) and Amin/Ng (1993), respectively. Using a comprehensive sample of stock options traded at the *Frankfurt Options Market* (FOM) between April 1983 and March 1991 and the *Deutsche Terminbörse* (DTB) between January 1990 and December 1991 we pose a similar question as Ball/Torous (1985, p.156): 'Can this more general specification eliminate the systematic biases of the Black/Scholes option pricing model?' However, this paper extends the methodology applied in Ball and Torous in at least three ways. First, we contrast the constant variance diffusion model (BS-model) of Black and Scholes with the *unrestricted* Poisson-type jump-diffusion model with *idiosyncratic* as well as *systematic* jump risk. Second, instead of valuing American call options on dividend paying stocks according to the pseudo-American valuation model of Black (1975) (where the pseudo-American call value is the maximum of the European call value assuming no premature exercise and the European call value with expiration date corresponding to the ex-dividend date), we use a discrete time model recently developed by Amin (1993). Third, we calculate American put values by the same efficient approximation scheme.

Furthermore, we use the systematic jump risk model of Bates (1991) to infer the implicit stock price distributions before, during, and after periods of dramatic stock price changes within the sample period from April 1983 to December 1991. We examine especially the period around the October 1987 crash and the period around the German reunification/Kuwait crisis in 1990.

The paper is organized as follows. Section 2 contains a description of the Poisson-type jump-diffusion process and the methodology of parameter estimation. Parameter estimates are presented for the Deutscher Aktienindex (DAX) and 5 actively traded German stocks for the period from January 1, 1981 to December 30, 1991. Section 3 presents different formulae and a discrete Markov chain model for valuing American stock options when the underlying stock process includes idiosyncratic and systematic jumps, respectively. Section 4 examines the impact of stock price jumps on option values based on hypothetical as well as historical process parameters while section 5 presents the parameter estimates implied in option prices around stock market crashes. Section 6 concludes the paper.

2 Modeling stock price jumps

2.1 The Poisson jump-diffusion model

Jump-diffusion processes are popular processes to model stock prices since they have an intuitive interpretation. The jump component is an attempt to incorporate the arrival of very important (*abnormal*) new information while the diffusion component models the arrival of less important (*normal*) new information. The most general jump-diffusion process with independent increments is a Brownian motion superimposed by a compound jump process of the Poisson-type. The Poisson process is assumed to be homogeneous (with respect to time and state) and independent of the Brownian motion. Letting S_t denote stock price at time t and S_{t-} the stock price an instant before time t, the dynamics of the stock price process $S \equiv \{S_t; t \geq 0\}$ can be represented by the following stochastic differential equation

$$\frac{dS_t}{S_{t-}} = \alpha_D dt + \sigma_D dB_t + I_t dN_t \ , \tag{1}$$

where

(i) $B \equiv \{B_t; t \geq 0\}$ is a standard Brownian motion,
α_D is the drift parameter and $\sigma_D > 0$ is the volatility parameter of the diffusion component of the Poisson-jump diffusion process,

(ii) $N \equiv \{N_t; t \geq 0\}$ is a Poisson counting process with parameter $\lambda \geq 0$, denoting the expected number of jumps per unit time,

(iii) $I \equiv \{I_t; t \geq 0\}$ is a process with left-continuous sample paths describing the stochastic size of the jump occuring next: $I_t = \sum_{n=1}^{\infty} L_n 1_{(T_{n-1},T_n]}(t)$ where $L = (L_1, L_2, \ldots)$ is an i.i.d. sequence of random variables with $L_n > -1$ representing the percentage change of S due to a jump (jump size) occuring at time T_n: $L_n \equiv (S_{T_n} - S_{T_n -})/S_{T_n -}$, $T_0 \equiv 0$ and $\{T_1, T_2, \ldots\} = \{t \geq 0 | N_t - N_{t-} = 1\}$ is the set of arrival times of the jumps. The expression $I_t dN_t$ symbolizes a compounded Poisson process,

(iv) B, N, I are independent,

(v) B, N and I are adapted to the filtration $\{\mathcal{F}_t; t \geq 0\}$, i.e., N_t, B_t and I_t are $\mathcal{F}_t$-measurable random variables. The filtration will be assumed to satisfy the usual conditions[1].

[1] See, e.g., Karatzas and Shreve (1988, p.10).

An equivalent representation of relationship (1) reads as follows:

$$dS_t = \alpha_D S_{t-} dt + \sigma_D S_{t-} dB_t + S_{t-} I_t dN_t \ . \tag{2}$$

Accordingly, the stock price change $dS_t = S_{t+dt} - S_{t-}$ is the sum of three components. The component $\alpha_D S_{t-} dt$ represents the instantaneous expected stock price change conditional on no arrivals of abnormal information. The $\sigma_D S_{t-} dB_t$ part describes the unanticipated part of the instantaneous stock price change due to the arrival of normal information, and the $S_{t-} I_t dN_t$ part describes the total instantaneous stock price change due to the arrival of *abnormal* information. Application of a fairly general version of Itô's lemma (see, e.g., Rogers and Williams (1987, p. 394)) to $\ln(S_t)$ delivers

$$\ln(S_t) = \ln(S_0) + (\alpha_D - \sigma_D^2/2)t + \sigma_D B_t + \sum_{n=1}^{N_t} \ln(1 + L_n) \ . \tag{3}$$

Defining $X_t \equiv \ln(S_t/S_0)$ to be the rate of return over the interval $[0, t]$, $\mu_D \equiv \alpha_D - \sigma_D^2/2$, and $J_n \equiv \ln(1 + L_n)$, we obtain

$$X_t = \mu_D t + \sigma_D B_t + \sum_{i=1}^{N_t} J_i \qquad (t \geq 0) \ . \tag{4}$$

In the special case when the $\{J_i\}$ are normally distributed with parameters μ_J and σ_J^2, we have $k \equiv E(L) = e^{\alpha_J} - 1$ with $\alpha_J = \mu_J + \sigma_J^2/2$ and the rate of return over the unit interval $[0, 1]$, X_1, is then distributed as

$$\begin{aligned} F(x) &= \mathrm{E}P[X_1 \leq x | N_1] \\ &= \mathrm{E}\Phi(x | \mu_D + N_1 \mu_J, \sigma_D^2 + N_1 \sigma_J^2) \\ &= \sum_{n=0}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} \ \Phi(x | \mu_D + n\mu_J, \sigma_D^2 + n\sigma_J^2) \ , \end{aligned} \tag{5}$$

where $\Phi(\cdot)$ denotes the cumulated normal density function. The corresponding density function is easily obtained as

$$f(x) = \sum_{n=0}^{\infty} \frac{e^{-\lambda} \lambda^n}{n!} \ \varphi(x | \mu_D + n\mu_J, \sigma_D^2 + n\sigma_J^2) \ , \tag{6}$$

where $\varphi(\cdot)$ denotes the normal density function. In a similar manner we can get the unconditional expected rate of return per unit time and the unconditional variance of the rate of return per unit time, respectively:

$$\mathrm{E}(X_1) \ = \ \mathrm{E}\mathrm{E}[X_1 | N_1]$$

$$= \mu_D + \lambda\mu_J \ , \tag{7}$$

$$\begin{aligned} \mathrm{Var}(X_1) &= \mathrm{EVar}[X_1|N_1] + \mathrm{VarE}[X_1|N_1] \\ &= \sigma_D^2 + \lambda(\sigma_J^2 + \mu_J^2) \ . \end{aligned} \tag{8}$$

Compared to the normal density function, the shape of $f(x)$ is always more peaked in the center (leptokurtic) and has thicker tails as long as $0 < \lambda < \infty$. The density function $f(x)$ is symmetric around μ_D if $\mu_J = 0$ and skewed otherwise[2]. Therefore a jump-diffusion process with $0 < \lambda < \infty$ might explain the observed leptokurtosis and skewness of stock return distributions.

2.2 Parameter estimation

In accordance with most of the preceding studies, we calculate the maximum likelihood estimates (MLEs) of the process parameters. Given a sample of (daily or weekly) stock returns $\mathbf{x} \equiv (x_1, x_2, \ldots, x_m)$, the logarithm of the corresponding likelihood function is defined as

$$\ln L(\mathbf{x}|\theta) = \sum_{i=1}^{m} \ln f(x_i|\theta) \ , \tag{9}$$

where $\theta \equiv (\mu_D, \sigma_D^2, \lambda, \mu_J, \sigma_J^2)$, and $f(\cdot|\theta)$ is the density function given in (6) resulting from a Poisson-type jump-diffusion process. Relying on the experimental evidence reported in Ball and Torous (1985, p. 160), we truncate the infinite sum in $f(x_i|\theta)$ at $N = 10$ and maximize instead of (9) the truncated log-likelihood function

$$\ln L_N(\mathbf{x}|\theta) = \sum_{i=1}^{m} \ln \Big(\sum_{n=0}^{N} \frac{e^{-\lambda}\lambda^n}{n!} \ \varphi(x_i|\mu_D + n\mu_J, \sigma_D^2 + n\sigma_J^2) \Big) \tag{10}$$

with $N = 10$. Necessary conditions for a maximum likelihood estimator θ^* become

$$\frac{\partial \ln L_N(\mathbf{x}|\theta^*)}{\partial \theta_i} = 0 \ , \qquad i = 1, \ldots, 5 \ , \tag{11}$$

[2] Akgiray and Booth (1986, p. 169) show graphs of various density functions $f(\cdot)$ and a standard normal density function $\varphi(\cdot)$ for comparison. However, when $\lambda \to \infty$, the jump component converges to a second standard Brownian motion. Even for a smaller λ , e.g. $\lambda = 10$, the Poisson distribution approximates the normal distribution quite good.

sufficient conditions require the positive definiteness of $-H(\mathbf{x}|\theta^*)$, the 5×5 Hessian matrix $H(\mathbf{x}|\theta)$ being defined by

$$H(\mathbf{x}|\theta)_{ij} = \frac{\partial^2 \ln L_N(\mathbf{x}|\theta)}{\partial\theta_i\,\partial\theta_j}\;, \quad i,j = 1,\ldots,5\;. \tag{12}$$

The MLEs of the process parameters are calculated by solving the nonlinear equation system (11) numerically. The employed quasi-Newton procedure[3] is known to converge quickly, provided the initial values of the algorithm are close to the final solution. Confirming Ball and Torous (1985), we found that the Bernoulli jump-diffusion[4] MLEs provide excellent starting values for the quasi-Newton algorithm. Therefore we computed first of all the MLEs for the simpler process by constraining the mean logarithmic jump size equal to zero, $\mu_J = 0$, and by taking arbitrary starting values for the other parameters to be estimated.

Since a diffusion-only model is nested within a combined diffusion and jump model, a likelihood ratio test can be used to test the null hypothesis H_0: stock return and stock index returns are normally distributed. We calculate the likelihood ratio statistic

$$\Delta = -2(\ln L(\mathbf{x}|\theta^*) - \ln L(\mathbf{x}|\theta^{\mathbf{0}}))\;, \tag{13}$$

where θ^* is the MLE under a jump-diffusion specification, and $\theta^{\mathbf{0}}$ is the MLE corresponding to the situation when no jump structure is present (i.e., $\lambda = \mu_J = \sigma_J = 0$). We assume that Δ is asymptotically χ^2-distributed with 3 degrees of freedom.[5] Estimates of the standard errors of θ^* are obtained from the main diagonal of the inverse of the Hessian evaluated at θ^*.

[3]We used a FORTRAN routine (E04JAF) available in the NAG program library. All calculations were done on the IBM 3090 mainframe of the Rechenzentrum der Universität Karlsruhe.

[4]If stock prices follow a Bernoulli jump-diffusion model then over a fixed period of time either no information impacts upon the stock price, or at most one significant information arrival occurs. Furthermore, if returns were computed for finer time intervals, the Bernoulli jump-diffusion model would converge to the Poisson jump-diffusion model.

[5]The null hypothesis can be rejected if $\Delta > \chi^2_{(3;\alpha)}$ for some significance level α. The critical values of Δ are 6.25, 7.81 and 11.35, for α =0.10, 0.05 and 0.01, respectively. Because $\lambda \geq 0$ and $\sigma_D^2 \geq 0$ the actual return distribution is a weighted sum of chi-squared ones. Since Δ is very large, see table 2, this approximation will be acceptable.

2.3 Parameter estimates for the period from 1981 to 1990

Parameter estimates of the Poisson jump-diffusion process were estimated for the DAX stock index[6] and 14 common stocks with DTB-traded options. The raw data consist of daily share prices (Kassakurse) quoted at the Frankfurt Stock Exchange spanning the 10-year period from January 1, 1981 through December 31, 1990. The data source is a DFDB [7] daily stock price file, wherein cash dividends, issue rights, stock dividends and splits are accounted for by adjusting previous prices downward. The stock's rate of return of trading day t is then defined as $X_t = \ln(S_t/S_{t-1})$ where S_{t-1} is the adjusted share price of the preceding trading day. Accordingly, weekend and holiday returns are treated as overnight returns. A weekly rate of return is defined as the difference between the logarithm of two successive Wednesday prices.

Table 1 summarizes the Poisson jump-diffusion parameter estimates for the DAX stock index returns across different subperiods. In addition to the five parameters to be estimated (instantaneous mean μ_D and variance σ_D^2 of the diffusion component, the mean number of abnormal information arrivals (jumps) per unit time λ, the mean μ_J and variance σ_J^2 of the (logarithmic) jump size) the table reports on the annualized total standard deviation (volatility) of the jump-diffusion process (VOLA)[8], the log-likelihood value and the likelihood ratio test statistic (Δ). Standard errors are given in parentheses. The resulting t-values indicate that the parameter estimates are statistically significant (at the 1%-level).

Based on the likelihood ratio test, in all cases considered here we have evidence implying the existence of a jump structure in DAX returns. The null hypothesis of a pure diffusion process is rejected at the 1% significance level. A comparison of the results for the daily returns in the two subperiods 1981-1985 and 1986-1990 shows that the likelihood ratio test statistic is in the second subperiod substantially larger than in the first subperiod. Furthermore, while the mean jump size is positive in the first subperiod it becomes ne-

[6] The DAX is a capital weighted index of 30 stocks actively traded on the Frankfurt Stock Exchange. Since December 30, 1987 the DAX is quoted continuously during the trading hours and is supposed to be the most important German stock index.

[7] DFDB (Deutsche Finanzdatenbank) is a German capital market data base maintained with the support from Deutsche Forschungsgemeinschaft (DFG).

[8] VOLA$\equiv \sqrt{Var(X_t)n} = \left[(\hat{\sigma}_D^2 + \hat{\lambda}(\hat{\sigma}_J^2 + \hat{\mu}_J^2))n\right]^{1/2} \cdot 100\%$ where $n = 52$ (weeks a year) and $n = 250$ (trading days per year) when using weekly and daily estimates, respectively.

gative in the second subperiod. The latter results from an empirical return distribution skewed to the left. This observation can be explained with the market crashes in the second subperiod (e.g., the October 1987 and October 1989 crashes).

A comparison of the results based on daily returns (panel A) and weekly returns (panel C) shows that the jump component is statistically more significant for daily data[9]. However, when eliminating Monday and Friday returns in daily return series (panel B), the statistical significance of the jump structure is even lower than with weekly data for the subperiod from 1981 to 1985.

Table 2 reports the MLEs of the five parameters for 14 common stocks based on daily return data from January, 1981 to December, 1985 (panel A) and from January, 1986 to December, 1990 (panel B). The standard errors of the estimates (not reported here) indicate that most of these estimates are statistically significant. The null hypothesis was rejected in all cases. Furthermore, we found that in the total period (the results are not reported here) the likelihood ratio test statistic is always significant for daily returns. For weekly returns, however, the null hypothesis is rejected for 71.4% and 92.9% of the stocks considered in the subperiods 1981-1985 and 1986-1980, respectively. In the total period the null hypothesis can be rejected for the weekly returns of all stocks considered in the sample[10].

Figure 1 visualizes the peakedness[11] and the thicker tails of the empirical density function of daily Deutsche Bank stock returns observed between January 1, 1981 and December 31, 1985. The two remaining density functions result from the parameter estimates of

[9]This observation confirms corresponding results for a value-weighted index including all stocks on the New York Stock Exchange and the American Stock Exchange as documented in Jarrow/Rosenfeld (1984).

[10]To compare our results with the results of earlier papers (e.g. Ball and Torous (1985)) the parameters were also estimated under the assumption $\mu_J = 0$. In comparison with the unconstrained model little of the explanatory power of the model was lost, but the values of all other parameters were influenced. Therefore the results obtained for the unconstrained model are more valuable.

[11]Peakedness is determined by the kurtosis and defined as KURT$\equiv E(x-\overline{x})^4/\sigma^4$ while skewness is defined as SKEW$\equiv E(x-\overline{x})^3/\sigma^3$, where x denotes the observations of the sample, $\overline{x}$ and σ denotes mean and volatility of the sample. For a jump diffusion model kurtosis and skewness specialize to KURT$\equiv \lambda[\mu_J^4+6\mu_J^2\sigma_J^2+3\sigma_J^2]/((\sigma_D^2+\lambda(\mu_J^2+\sigma_J^2))^2)$ and SKEW$\equiv \lambda\mu_J[\mu_J^2+3\sigma_J^2]/((\sigma_D^2+\lambda(\mu_J^2+\sigma_J^2))^{3/2})$, respectively. One can get annualized values for skewness and kurtosis by dividing SKEW by $\sqrt{n}$ and by dividing KURT by n, respectively, where for weekly returns $n=52$ (weeks a year) and for daily returns $n=250$ (trading days per year).

the normal distribution and of the distribution of the Poisson jump-diffusion process. Figure 2 shows the density functions for daily returns for the subperiod form January 1, 1986 to December 31, 1990. Obviously, during this subperiod the return volatility is higher than in the first subperiod. In both cases the density function of the Poisson jump-diffusion process approximates the peakedness of the empirical density function of the returns much better than the normal distribution. Furthermore, Figure 3 and 4 visualize the leptokurtosis of the empirical density functions of the daily DAX returns and Deutsche Bank returns observed between January 1, 1981 and December 30, 1990. Consequently, the density function of the Poisson jump-diffusion process approximates the empirical density function much better than the normal density function.

For the sake of convenience, we refer in the context of option pricing only to *annualized* jump intensities and diffusion variances. To annualize these parameters, we multiply the parameter estimates from daily returns by 250 (average number of trading days per year).

Table 1

Poisson jump-diffusion parameter estimates for the DAX across different subperiods

(Standard errors in parentheses)

Panel A: Daily returns

Period	m	λ	$\sigma_D^2 \times 10^4$	$\sigma_J^2 \times 10^3$	$\mu_D \times 10^3$	$\mu_J \times 10^3$	VOLA[a]	$\ln L$	Δ
1981–1985	1249	0.658 (0.017*)	0.371 (0.004*)	0.052 (0.001*)	0.825 (0.011*)	0.0004 (0.174)	13.34	4207	30.98*
1986–1990	1247	0.060 (0.000*)	1.302 (0.002*)	1.497 (0.013*)	0.309 (0.010*)	−4.933 (0.145*)	23.52	3615	279.1*
1981–1990	2496	0.071 (0.000*)	0.867 (0.001*)	0.780 (0.004*)	0.759 (0.004*)	−4.750 (0.052*)	18.97	7731	514.7*

Panel B: Daily returns without Monday and Friday returns

Period	m	λ	$\sigma_D^2 \times 10^4$	$\sigma_J^2 \times 10^3$	$\mu_D \times 10^3$	$\mu_J \times 10^3$	VOLA	$\ln L$	Δ
1981–1985	753	0.677 (0.030*)	0.395 (0.007*)	0.053 (0.001*)	0.714 (0.022*)	0.395 (0.030*)	13.71	2512	16.02*
1986–1990	751	0.054 (0.001*)	1.347 (0.003*)	1.144 (0.017*)	0.276 (0.017*)	−0.683 (0.233*)	22.16	2185	94.88*
1981–1990	1505	0.067 (0.001*)	0.925 (0.002*)	0.639 (0.006*)	0.781 (0.007*)	−2.573 (0.083*)	18.40	4653	178.5*

Panel C: Weekly returns

Period	m	λ	$\sigma_D^2 \times 10^4$	$\sigma_J^2 \times 10^3$	$\mu_D \times 10^3$	$\mu_J \times 10^3$	VOLA	$\ln L$	Δ
1981–1985	251	2.407 (0.112*)	0.055 (0.005*)	0.011 (0.000*)	0.623 (0.020*)	−0.095 (0.009*)	12.98	652	31.70*
1986–1990	250	0.210 (0.013*)	0.520 (0.009*)	0.167 (0.007*)	0.480 (0.016*)	−2.310 (0.121*)	22.60	526 0	38.30*
1981–1990	501	0.167 (0.005*)	0.356 (0.003*)	0.172 (0.004*)	0.447 (0.006*)	−1.373 (0.036*)	18.74	1154	86.50*

[a] Annualized volatility of the Poisson jump diffusion process in percent.
* Indicates significance at 1% level.

Table 2

Poisson jump-diffusion parameter estimates based on daily stock returns

Panel A: Subperiod from 1.1.1981 to 30.12.1985

Stock	m	λ	σ_D^2 $\times 10^4$	σ_J^2 $\times 10^3$	μ_D $\times 10^3$	μ_J $\times 10^3$	VOLA[a]	ln L	Δ
BASF	1249	1.384	0.255	0.064	−0.259	0.782	16.95	3922	61.22
BAYER	1249	1.502	0.299	0.059	−0.174	0.745	17.27	3894	44.16
BMW	1249	0.384	0.814	0.230	0.175	2.857	20.79	3688	112.12
COBANK	1249	0.479	0.900	0.268	−0.608	3.138	23.62	3535	112.02
DBENZ	1249	0.151	0.649	0.367	0.516	6.387	17.78	3942	214.06
DREBA	1249	0.115	1.358	0.453	−0.430	11.723	22.57	3590	103.62
DTBANK	1249	0.426	0.455	0.154	−0.465	3.824	17.13	3955	161.00
HOECHS	1249	1.344	0.202	0.070	−0.427	1.026	17.01	3928	93.50
MANNES	1249	2.167	0.159	0.062	−1.062	0.851	19.48	3744	59.00
RWEST	1249	0.318	0.332	0.151	−0.104	1.472	14.31	4194	187.22
SIEMNS	1249	0.406	0.485	0.142	0.789	0.533	16.30	3991	93.04
THYSSN	1249	0.460	0.840	0.302	−1.015	2.771	23.79	1459	58.40
VEBA	1248	0.300	0.580	0.249	0.164	2.321	18.32	3876	172.78
VW	1249	0.577	0.961	0.228	−0.461	2.519	24.05	3495	105.18

Panel B: Subperiod from 1.1.1986 to 30.12.1990

Stock	m	λ	σ_D^2 $\times 10^4$	σ_J^2 $\times 10^3$	μ_D $\times 10^3$	μ_J $\times 10^3$	VOLA	ln L	Δ
BASF	1246	0.421	0.818	0.330	0.121	−0.518	23.50	3551	177.12
BAYER	1246	0.197	1.266	0.713	−0.123	0.419	25.84	3459	201.72
BMW	1246	0.376	1.079	0.629	−0.947	2.252	29.42	3316	280.04
COBANK	1246	0.538	1.474	0.317	−0.262	0.103	28.19	3280	82.02
DBENZ	1246	0.140	2.128	1.397	0.064	−3.327	32.01	3200	231.06
DREBA	1246	0.452	1.239	0.379	0.506	−1.134	27.19	3347	122.96
DTBANK	1246	0.093	1.855	1.335	−0.061	−1.687	27.84	3362	201.76
HOECHS	1246	0.279	0.923	0.426	−0.216	0.446	22.98	3589	194.04
MANNES	1246	0.301	2.007	0.817	0.212	−0.399	33.42	3110	187.48
RWEST	1246	0.183	1.638	0.849	0.032	3.596	28.35	3333	210.78
SIEMNS	1246	0.383	1.189	0.393	0.435	−1.410	25.99	3409	140.44
THYSSN	1246	0.314	2.016	0.511	0.835	−1.987	30.14	3201	79.48
VEBA	1246	0.256	1.179	0.593	0.079	0.294	25.97	3446	247.94
VW	1246	0.201	2.123	1.072	0.400	−2.773	32.76	3151	212.16

[a] Annualized volatility of the Poisson jump diffusion process in percent.
* Indicates significance at 1% level.

Figure 1
Distribution of daily Deutsche Bank returns
(Sample period 81/1 - 85/12)

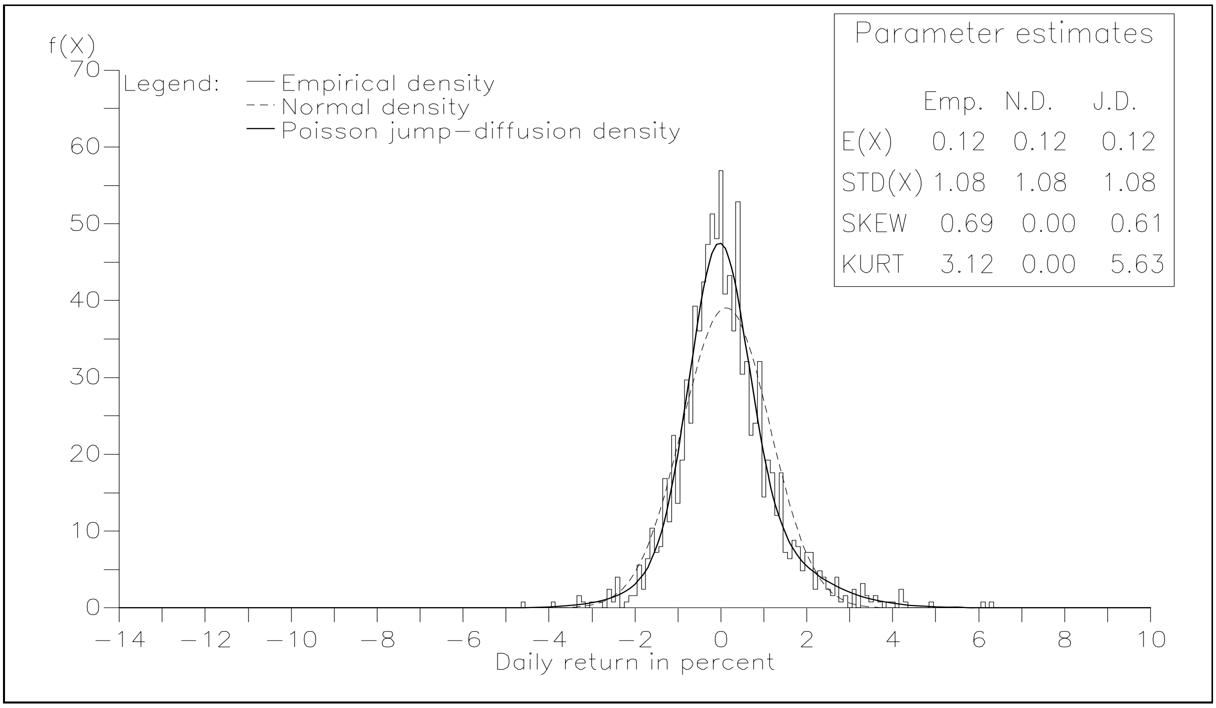


Figure 2
Distribution of daily Deutsche Bank returns
(Sample period 86/1 - 90/12)

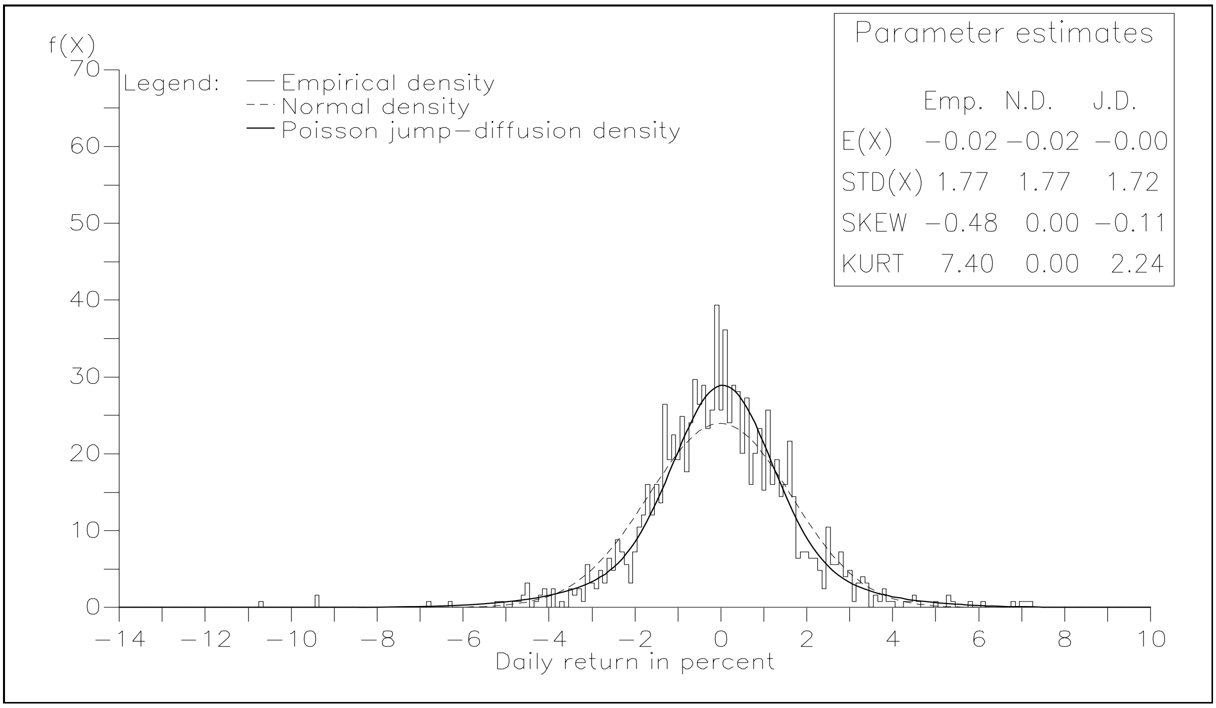


Figure 3
Distribution of daily DAX returns
(Sample Period 81/1 - 90/12)

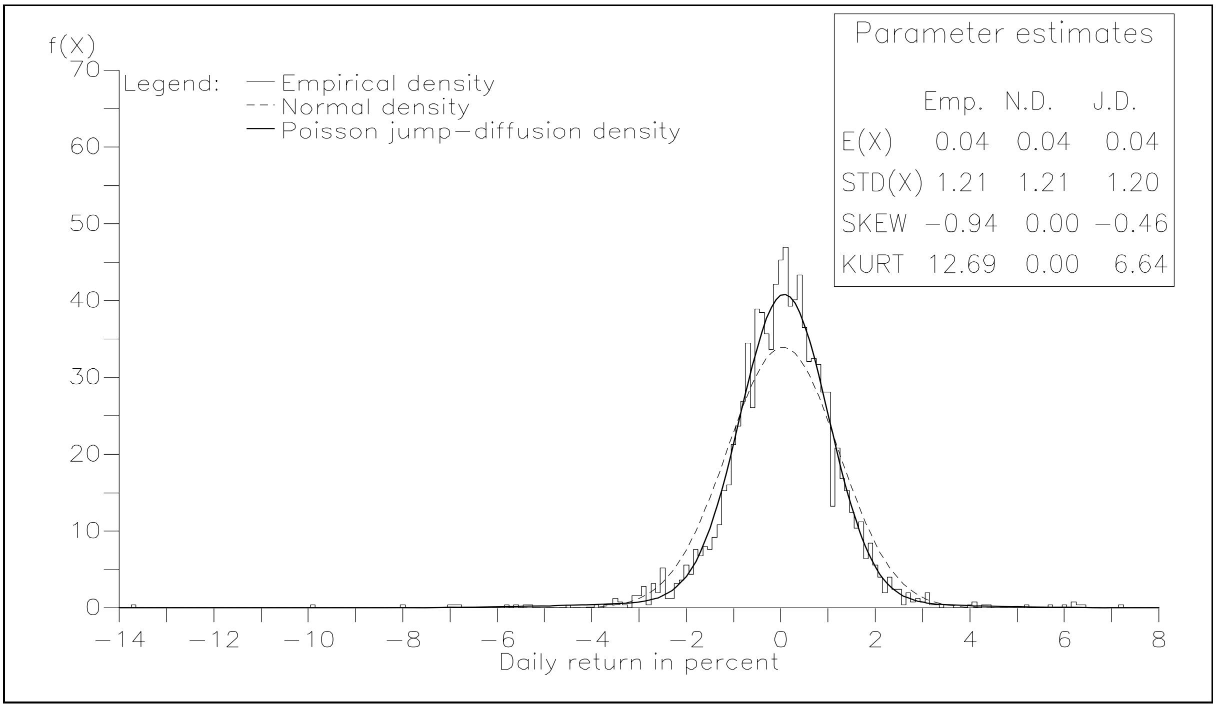
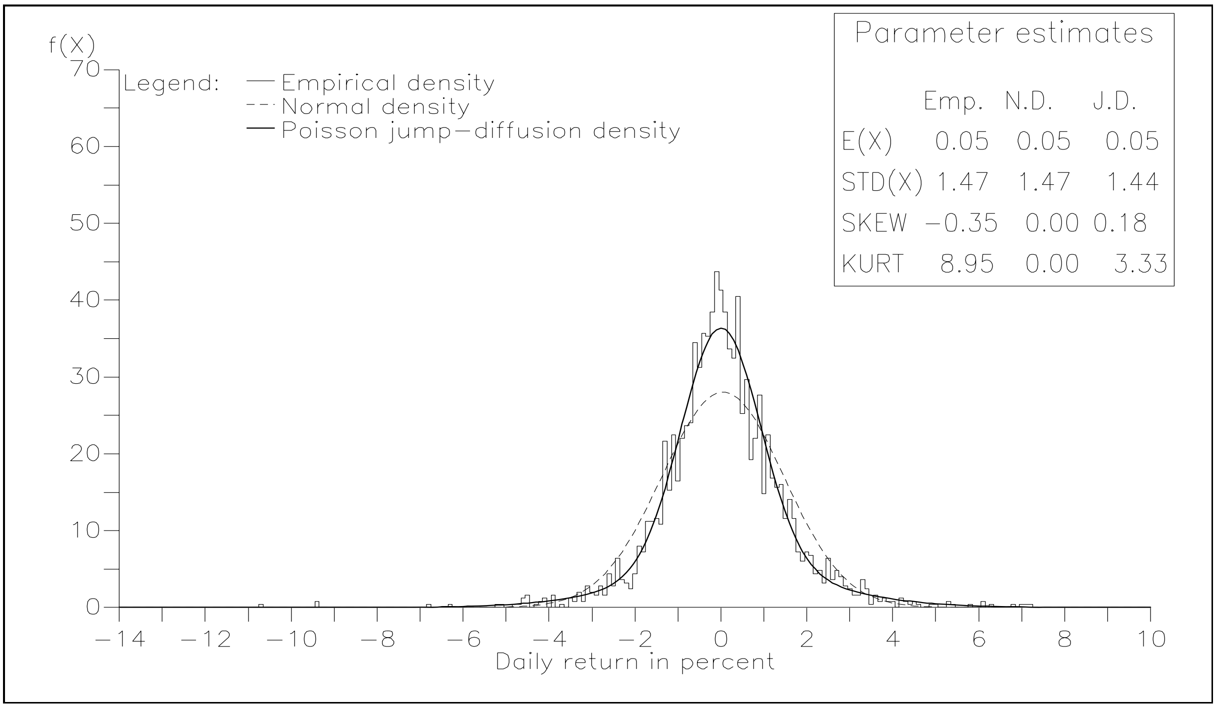


Figure 4
Distribution of daily Deutsche Bank returns
(Sample Period 81/1 - 90/12)



3 Option valuation when jump risk is present

Options are usually priced at the discounted expected value of their future payoffs where the expectation is taken over the risk neutral, rather than the true, return distribution of the underlying asset. A necessary condition for the risk neutral pricing methodology to be applicable is that the true and the risk neutral return distribution share a common support (i.e., are equivalent) and that the risk neutral return distribution summarizes the prices of relevant Arrow-Debreu state-contingent claims. As long as the option's payoff can be replicated by a dynamic trading strategy in the underlying asset and a riskless bond, the equivalent risk neutral return distribution can be derived via no-arbitrage conditions. Based on this methodology Black/Scholes (1973) derived their path-breaking option valuation model (henceforth *BS-model*). Unfortunately such a replication is *not* possible if the stock price follows a general jump-diffusion process. In this case deriving the appropriate risk neutral probability measure requires additional restrictions on distributions and/or on preferences. Merton's (1976a) idiosyncratic jump risk model (henceforth *IJD-model*), for instance, assumes that the jump risk is *idiosyncratic*, i.e., the jump component of a security's return is uncorrelated with the market return. More recently, Naik/Lee (1990), Bates (1991), Ahn (1992), and Amin/Ng (1993) assume the existence of a representative investor with time-separable power utility, so that Cox/Ingersoll/Ross (1985) and Rubinstein (1976) separability results, respectively, can be invoked to price the additional risk when stock jumps are *systematic*. Although the statistical significance of jumps in the DAX returns reported in section 2 indicates that Merton's simplifying assumption of diversifiable jump risk might not be fulfilled, we present first of all Merton's IJD-model. Afterwards we present Bates's version of the systematic jump risk model (henceforth *SJD-model*) and compare it with the IJD-model.

3.1 Diversifiable jump risk

The terminal payoff of a European call option maturing T years from now, given terminal asset price realization S_T and strike price K, is $\max(0, S_T - K)$. Under the standard assumption that the short-term interest rate r is constant over the lifetime of the option, the price of a European call, conditional upon a current stock price of $S \equiv S_0$, will be

$$C \quad = \quad e^{-rT} \widetilde{E}_0 \max(0, S_T - K) \tag{14}$$

$$= e^{-rT}\widetilde{E}_0(S_T - K|S_T \geq K)\widetilde{\Pr}(S_T \geq K)\ .$$

The call price is therefore the discounted expected payoff conditional upon finishing in-the-money times the probability of finishing in-the-money. Expectations and probabilities are calculated with respect to the risk neutral probability measure $\widetilde{Q}$. In the special case of a Geometric Brownian Motion governing the underlying stock price the above relationship specializes to the Black/Scholes formula for European calls:

$$\begin{aligned} C^{BS} &\equiv C^{BS}(S, K, T, \sigma^2, r) \\ &= S\Phi(d_1) - e^{-rT}K\Phi(d_2)\ , \end{aligned} \tag{15}$$

where

$$\begin{aligned} d_1 &= [ln(S/K) + (r + \sigma^2/2)T]/\sigma\sqrt{T}, \\ d_2 &= d_1 - \sigma\sqrt{T}, \\ \Phi(.) &\equiv \text{standard normal cumulative density function}, \\ r &\equiv \text{riskless rate of return}, \\ \sigma^2 &\equiv \text{variance of the stock's rate of return.} \end{aligned}$$

If the stock price follows the Poisson-type jump-diffusion dynamics described in (1) *and* if the jump risk is diversifiable, then relation (14) specializes to the Merton formula for European calls:[12]

$$\begin{aligned} C^{IJD} &= e^{-rT}\sum_{n=0}^{\infty}\widetilde{Pr}(n\ jumps)\widetilde{E}_0\left[\max(0, S_T - K)\mid n\ jumps\right] \\ &= \sum_{n=0}^{\infty}\left[e^{-\lambda T}(\lambda T)^n/n!\right]E_{0,X_n}\left[C^{BS}(SX_ne^{-\lambda kT}, K, T, \sigma_D^2, r)\right] \\ &= \sum_{n=0}^{\infty}\left[e^{-\lambda' T}(\lambda' T)^n/n!\right]\left[S\Phi(d_{1,n}) - e^{-r_nT}K\Phi(d_{2,n})\right]\ , \end{aligned} \tag{16}$$

where

[12] A detailed explanation of the following transformations can be found in the appendix.

X_n	$\equiv$	$\prod_{i=1}^{n}(L_n+1)$, random variable with the same distribution as the product of n independently and identically distributed random variables, each identically distributed to the random variable $1+L_n$, where it is understood that $X_0 \equiv 1$,
$E_{0,X_n}(.)$	$\equiv$	expectation operator with respect to the distribution of X_n,
λ'	$\equiv$	$\lambda(1+k) = \lambda e^{\mu_J+(1/2)\sigma_J^2}$,
v_n^2	$\equiv$	$\sigma_D^2 + n\sigma_J^2/T$,
r_n	$\equiv$	$r - \lambda k + n(ln(1+k))/T$,
$d_{1,n}$	$\equiv$	$\left[ln(S/K) + r_n T + 1/2(v_n^2 T)\right]/(v_n^2 T)^{1/2}$,
$d_{2,n}$	$\equiv$	$d_{1,n} - (v_n^2 T)^{1/2}$.

Equivalently, Merton's call value (relation (16)) can be represented by

$$C^{IJD} = e^{-rT} E\left(\max(0, \widetilde{S}_T - K)\right) \ , \tag{17}$$

where $\widetilde{S}_T$ is the terminal stock price resulting from the risk neutral stock price dynamics

$$\widetilde{S}_T = S_0 \ exp\left\{(r - \frac{1}{2}\sigma_D^2 - \lambda k)T + \sigma_D B_T + \sum_{i=1}^{N_T} J_i\right\}. \tag{18}$$

European puts have an analogous formula:

$$P^{IJD} \ = \ \sum_{n=0}^{\infty}\left[e^{-\lambda' T}(\lambda' T)^n/n!\right]\left[e^{-r_n T}K\Phi(-d_{2,n}) - S\Phi(-d_{1,n})\right]. \tag{19}$$

3.2 Systematic jump risk

Since the empirical results presented in section 2 indicate that jump risk is systematic, Merton's diversification argument is not valid. Therefore we use a general equilibrium approach to test for the impact of jumps on option values. While the model of Amin/Ng (1993) is based on the discrete time model of Rubinstein (1976), Bates (1991) derives a similar formula which is embedded in the Cox/Ingersoll/Ross (1985) equilibrium framework. While Rubinstein (1976) models a pure exchange economy, Cox/Ingersoll/Ross (1985) derive their results for a production economy. Therefore, instead of modeling the aggregate consumption process as in Amin/Ng (1993), Bates (1991) models the optimal invested wealth as a jump diffusion process:

$$\frac{dW_t}{W_{t-}} = (\alpha_{D,W} - \lambda k_W - Y_t/W_t)\, dt + \sigma_{D,W} B_t + L_W dN_t \ ,$$

where $\{Y\}$ represents the optimal consumption stream. The percentage wealth jump sizes plus one, $(1+L_W)$, are independently and identically log-normally distributed: $\ln(1+L_W) \sim N(\mu_{J,W}, \sigma^2_{J,W})$ with $Cov(\ln(1+L), \ln(1+L_W)) = \sigma_{J,SW}$, while the covariance between the stock returns and the change in the optimal invested wealth conditional on no jumps is given by $\sigma_{D,SW}$. By construction jump risk is systematic, i. e., stock prices and wealth jump simultaneously, albeit by possibly different amounts.

Like in the Cox/Ingersoll/Ross (1985) world, Bates assumes the existence of a representative investor who seeks to maximize his expected utility of lifetime consumption. This investor has an indirect utility of wealth function of the form $V(W,t) = \max_{\{Y_\tau\}} E_t \int_t^\infty e^{-\rho t} U(Y_\tau) d\tau$, where the direct utility function is given as $U(Y_t) = (1/(1-R))Y_t^{1-R}$, where Y_t is the consumption at date t, ρ is the time discount factor and R is a coefficient of relative risk aversion[13]. Within this equilibrium framework Bates derives the following 'risk-neutralized' valuation formula[14] for a European call:

$$\begin{aligned} C^{SJD} &= e^{-rT} \sum_{n=0}^{\infty} \widetilde{Pr}(n\ jumps) \widetilde{E}_0 \left[\max(0, S_T - K) \mid n\ jumps\right] \qquad (20) \\ &= e^{-rT} \sum_{n=0}^{\infty} \left[e^{-\lambda^* T} (\lambda^* T)^n / n!\right] \left[S e^{r_n T} \Phi(d_{1,n}) - K \Phi(d_{2,n})\right] , \end{aligned}$$

where

$$\begin{aligned} r_n &\equiv (r - \lambda^* k^*) + n\mu_J^*/T, \\ d_{1,n} &\equiv \left[\ln(S/K) + r_n T + (\sigma_D^2 T + n\sigma_J^2)/2\right] / \left[(\sigma_D^2 T + n\sigma_J^2)^{1/2}\right], \\ d_{2,n} &\equiv d_{1,n} - (\sigma_D^2 T + n\sigma_J^2)^{1/2} , \\ \lambda^* &\equiv \lambda \exp(-R\alpha_{J,W} + (1/2)R(1+R)\sigma^2_{J,W}), \\ \alpha_J^* &\equiv \alpha_J - R\sigma_{J,SW}, \\ k^* &\equiv \exp(\alpha_J^*) - 1 , \\ \mu_J^* &\equiv \alpha_J^* - 1/2\sigma_J^2 , \\ J^* &\equiv \text{is } N(\mu_J^*, \sigma_J^2) \text{ distributed.} \end{aligned}$$

The terminal stock price resulting from the 'risk neutral' stock price dynamics, $\widetilde{S}_T$, is given by the following relation:

$$\widetilde{S}_T = S_0 \exp\left\{(r - 1/2\sigma_D^2 - \lambda^* k^*)T + \sigma_D B_T + \sum_{i=1}^{N_T} J_i^*\right\} . \qquad (21)$$

[13] With $R=0$, one obtains a risk neutral investor, and with $R \to 1$, one obtains logarithmic preferences.

[14] Although formula (20) is written in a 'risk-neutralized' fashion, the call value depends via λ^* and k^* on the risk aversion parameter R.

Equation (20) is a specialization of the general equilibrium pricing formula:

$$C = E_0 \left(e^{-\rho T} \frac{V_W(T)}{V_W(0)} \left(S_T - K\right)^+ \right) , \tag{22}$$

where $V_W(T)$ denotes the marginal utility of wealth at time T. In this equilibrium framework the instantaneous drift of the stock price process α and the instantaneous riskless interest rate r are endogenously given by the following two conditions:[15]

$$\begin{aligned} \alpha - r &= -E_t \left(\frac{dV_W(t)}{V_W(t)} \frac{dS_t}{S_t} \right) / dt \\ &= R\sigma_{D,SW} - \lambda E \left\{ \left[(1 + L_W)^{-R} - 1 \right] L \right\} \\ &= R\sigma_{D,SW} - \lambda e^{-R\alpha_{J,W} + (R(1+R)/2)\sigma^2_{J,W}} (e^{\alpha_J - R\sigma_{J,SW}} - 1) + \lambda(e^{\alpha_J} - 1) , \end{aligned} \tag{23}$$

and

$$\begin{aligned} r &= -E_t \left(\frac{dV_W(t)}{V_W(t)} \right) / dt \\ &= \alpha_W - R\sigma^2_{D,W} + \lambda E \left\{ \left[(1 + L_W)^{-R} - 1 \right] L_W \right\} \\ &= \alpha_W - R\sigma^2_{D,W} - \lambda k_W + \lambda e^{-R\alpha_{J,W} + (R(1+R)/2)\sigma^2_{J,W}} (e^{\alpha_{J,W} - R\sigma^2_{J,W}} - 1) . \end{aligned} \tag{24}$$

According to equation (24), the riskless interest rate r is equal to minus the expected rate of change in the marginal utility of wealth. Therefore jumps that increase both, the stock price and wealth, will increase the expected rate of change in the marginal utility of wealth and therefore decrease the riskless interest rate. All known option formulae for jump-diffusion processes are special cases of Bates' formula. When an investor with logarithmic utility ($R = 1$) is assumed, then the above formula collapses the one presented by Ahn (1992). In the case of index options on a proxy of the market portfolio, i. e., $\sigma_{J,WS} = \sigma_{D,WS} = 1, \sigma_{D,W} = \sigma_D, \sigma_{J,W} = \sigma_J$ and $\mu_{J,W} = \mu_J$, we obtain the formula proposed by Naik/Lee (1990). Finally, when stock jumps are idiosyncratic, i. e., $\alpha_{J,W} = \sigma_{J,W} = \sigma_{J,SW} = 0$, then we have $r = \alpha_{D,W} - R\sigma^2_W$ and $\alpha_D = r + R\sigma_{D,SW} - \lambda(\exp\{\alpha_J\} - 1)$ and the call formula (20) collapses to Merton's call formula.

[15] The above representations of the riskless interest rate and the stock price drift are derived by a general version of the CAPM in a production economy. They differ therefore from the ones in Amin/Ng (1993) since in their model these are derived from the Euler conditions in an exchange economy. The equivalence of these two approaches is shown in Breeden (1986).

3.3 Valuing American Puts

Like in the pure diffusion environment, there is no known analytic solution for American puts when the underlying stock obeys a jump diffusion process. Finite difference methods can be applied to evaluate options accurately for the modeled jump diffusion setting but at a prohibitive cost in computer time. Therefore Bates (1991) generalizes McMillan's (1987) quadratic approximation to American option values for jump-diffusion processes. The partial differential equation to be solved for American puts is given by

$$P_t + \{r - \lambda^* k^*\} S P_S + \frac{1}{2}\sigma_D^2 S^2 P_{SS} + \lambda^* E[P(Se^{J^*}, T, K) - P] = rP, \tag{25}$$

where the subindices of P denote the partial differentials. Approximating this formula leads to the following formula for American puts

$$P^{SJD} = \begin{cases} p^{SJD}(S,T,K) + KA_1(\frac{S}{K}/y_p^*)^{q_1} & \text{for } S/K > y_p^* \\ K - S & \text{for } S/K \leq y_p^* \end{cases}, \tag{26}$$

where $p^{SJD}(y_p^*, T, 1)$ denotes the European put evaluated at the critical stock price/strike price ratio $y_p^* \equiv S/K \leq 1$. This ratio below which the put is exercised immediately is given implicitly by solving the equation

$$1 - y_p^* = p(y_p^*, T, 1) + (y_p^*/ - q_1)[1 + p_S(y_p^*, T, 1)],$$

where q_1 is the negative root of

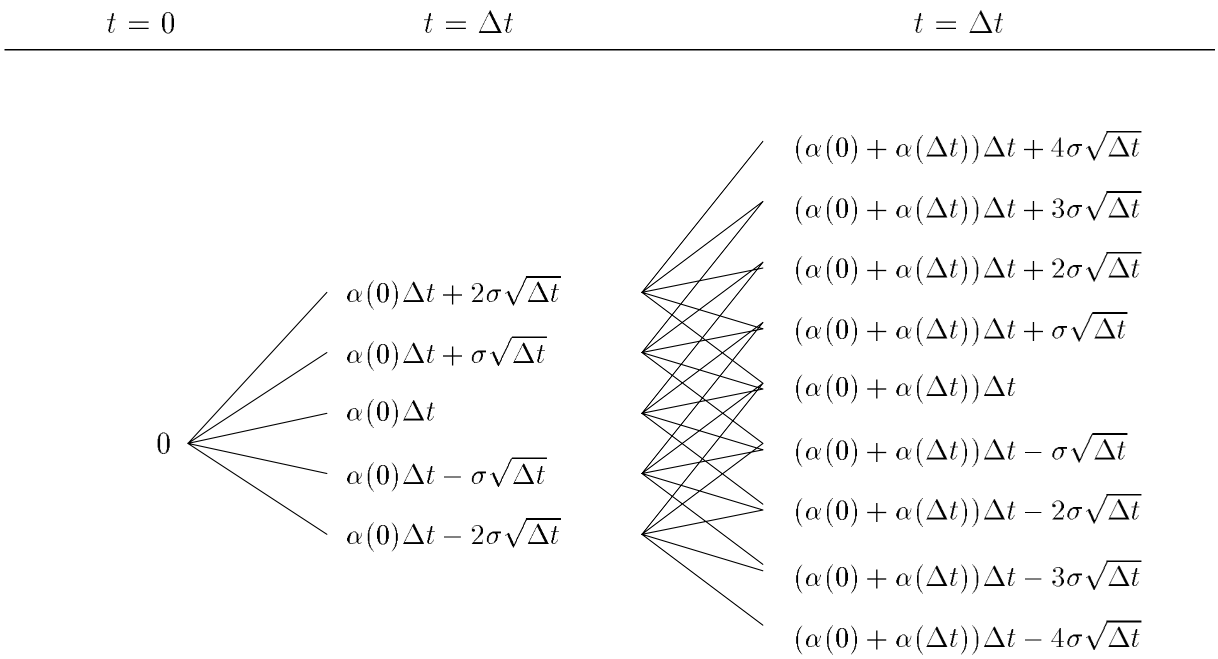
$$\frac{1}{2}\sigma_D^2 q^2 + (-\lambda^* k^* - \frac{1}{2}\sigma_D^2)q - \frac{r}{1 - e^{-rT}} + \lambda^*[e^{\alpha_J^* q + 1/2q(1-q)\sigma_J^2} - 1] = 0.$$

p_S gives the first partial differential of the European put and $A_1 \equiv (y_p^*/ - q_1)[1 + p_S(y_p^*, T, 1)]$. The parameters q_1 and y_p^* can be evaluated via Newton's method for a given parameter set and a specific option. This quadratic approximation is fast and inexpensive in computer resources. Therefore we use this method to estimate the parameters implicit in observed option prices.

In order to value puts based on historical parameter estimates,we use Amin's (1993) more accurate Markov chain model. Amin (1993) approximates the jump-diffusion process sufficiently accurate with a Markov chain. This enables him to carry out calculations of order roughly 20 to 40 times that for the ordinary binomial model of Cox/Ross/Rubinstein. More precisely, Amin (1993) includes rare event jumps by extending the recombining binomial model to a recombining multinomial model. The stock price at each date is determined as its time zero value times the exponential of the value of the state variable obtained from the grid of figure 7. Every transition from time $k\Delta t$ to time $(k+1)\Delta t$ is the sum of a drift component $\alpha(k\Delta t)\Delta t$, a binomial part $\pm\sigma\sqrt{\Delta t}$, and the jump with part $\pm j\sigma\sqrt{\Delta t}$ for some integer $j \leq k$.

Figure 7

Discrete approximation of the stock price distribution



4 Impact of stock price jumps on option values

We now suppose that the underlying stock price processes have systematic jumps of random amplitude. Therefore investors have to use the SJD-model in order to value options appropriately. As long as investors are risk neutral, the SJD-model collapses to Merton's IJD-model. Clearly, both the SJD-model and IJD-model collapse to the BS-model if $\lambda = 0$, $\mu_J = 0$, and $\sigma_J = 0$. Furthermore, for a fixed volatility of the jump component, the compound jump process converges to a Geometric Brownian motion for $\lambda \rightarrow \infty$. Therefore the BS-model approximates the SJD-model quite accurate for a high jump intensity, say, on average more than 100 jumps a year: $\lambda \geq 100$ (see, e. g., Merton (1976b)). Consequently, there is a significant difference between BS-values and SJD-values only if the jump component is statistically and economically significant. To illustrate the impact of systematic or idiosyncratic jumps on option value, we now assume that an investor believes that stock prices follow a pure lognormal diffusion while the stock price dynamics actually obey a Poisson jump diffusion process with idiosyncratic or systematic jump risks. Therefore the investor erroneously calculates the call values with the BS-formula, when the SJD-formula or the IJD-formula should be used. This may potentially bring about significant errors in option pricing. Furthermore, we will illustrate the valuation errors which risk averse investors using the IJD-formula take into account when jump risk is systematic and the SJD-formula with a risk aversion parameter $R > 0$ should be used. The same error occurs if *risk averse* investors erroneously value options as if they were *risk neutral* when recognizing that jump risk is systematic. To ensure a 'fair' comparison, the drift of aggregate wealth is adjusted such that the observable interest rate is equal to the 'endogenous' interest rate in the SJD-model.

4.1 Results based on hypothetical parameter values

In the following we analyze the differences between BS-values, IJD-values, and SJD-values for European calls based on hypothetical model parameters. The preference-dependent SJD-value is calculated for the risk aversion parameter $R = 5$ implying strong risk aversion. Due to the put-call parity, the differences in model values for calls are exactly the same for otherwise identical European puts. All simulations presented in this section refer to calls on a stock index (as a proxy for aggregate wealth). We consider the case were investors have sufficiently long time series of closing prices so that the volatility estimate of the pure diffusion process corresponds to the total volatility of the Poisson jump diffusion process, VOLA $\equiv \sqrt{[\sigma_D^2 + \lambda(\mu_J^2 + \sigma_J^2)] \cdot 250}$. The other model parameters are fixed as follows: the strike price is $K = 100$, the riskless interest rate is $r = 10\%$, the total volatility is VOLA $= 30\%$ and 80% of the total variance is due to the jump component, that is, $\gamma \equiv \lambda(\mu_J^2 + \sigma_j^2)/[\sigma_D^2 + \lambda(\mu_J^2 + \sigma_J^2)] = 0.8$. The three-dimensional plots presented in the following illustrate the absolute deviation between different model values for different times to maturity and different money ratios. We use the money ratio classification of table 3.

Table 3

Money ratio classes

Money ratio (S/K)	Class				
$S/K \leq 0.80$	V1	$\leftrightarrow$	DOTM	$\equiv$	deep out of the money
$0.80 < S/K \leq 0.85$	V2	$\leftrightarrow$	DOTM	$\equiv$	deep out of the money
$0.85 < S/K \leq 0.90$	V3	$\leftrightarrow$	OTM	$\equiv$	out of the money
$0.90 < S/K \leq 0.95$	V4	$\leftrightarrow$	OTM	$\equiv$	out of the money
$0.95 < S/K \leq 1.00$	V5	$\leftrightarrow$	ATM	$\equiv$	at the money
$1.00 < S/K \leq 1.05$	V6	$\leftrightarrow$	ATM	$\equiv$	at the money
$1.05 < S/K \leq 1.10$	V7	$\leftrightarrow$	ITM	$\equiv$	in the money
$1.10 < S/K \leq 1.15$	V8	$\leftrightarrow$	ITM	$\equiv$	in the money
$1.15 < S/K \leq 1.20$	V9	$\leftrightarrow$	DITM	$\equiv$	deep in the money
$1.20 < S/K$	V10	$\leftrightarrow$	DITM	$\equiv$	deep in the money

Figure 6 visualizes the absolute deviation between the IJD-values (i. e., the SJD-values for $R = 0$) and the BS-values for the jump intensity $\lambda = 1$ and the mean jump return $\mu_J = 0$ implying a *symmetric* return distribution. It confirms the wellknown v-shaped relationship as presented for the first time by Merton (1976b). As long as the time to maturity is short, for ATM options the BS-values are significantly larger than the IJD-values. For DOTM options as well as for DITM options the IJD-values exceed the BS-values. For OTM options the mean percentage difference[16] is about 100%. With increasing time to maturity the BS-value becomes more and more larger than the IJD-value such that for options with 42 weeks time to maturity the v-shaped relationship is less obvious and corresponds to a smile-shaped relationship. This relationship can be explained by

[16] The percentage difference between C^{BS} and C^{IJD} is defined as $[C^{IJD} - C^{BS}]/C^{IJD} \cdot 100$.

Figure 6
SJD-value versus BS-value for calls
(K=100, r=10%, VOLA=30%, γ=0.8, λ=1, $\mu_J = 0$)

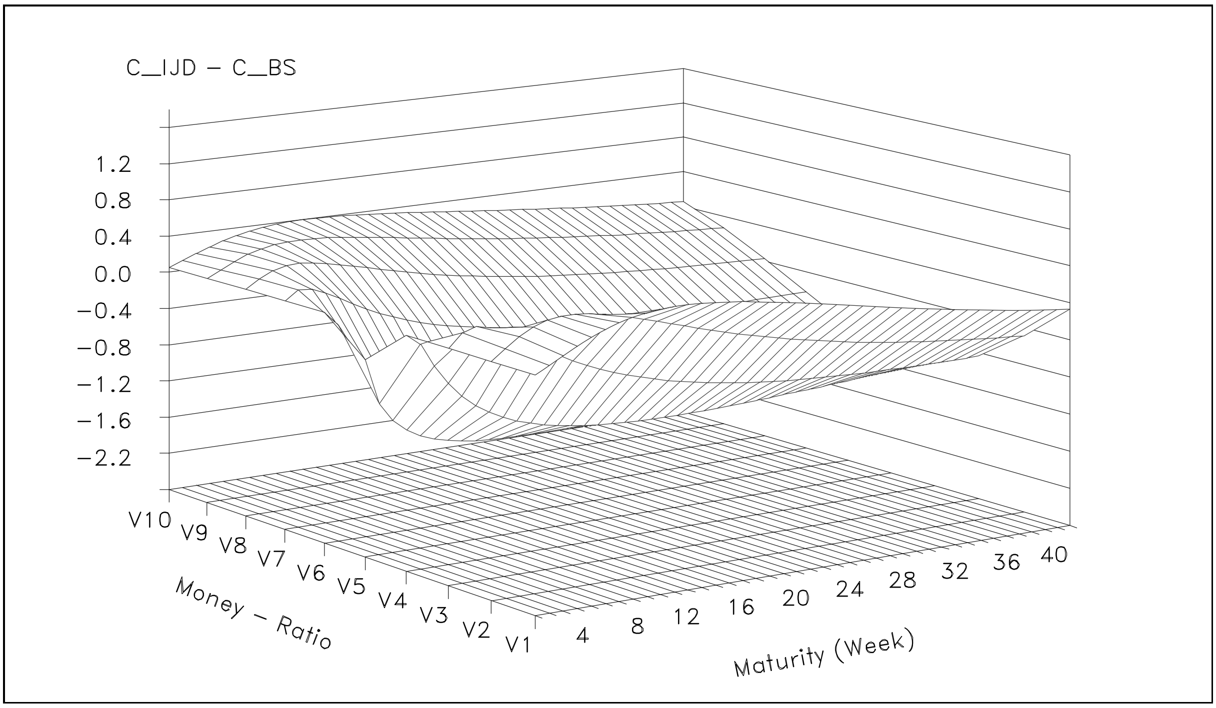
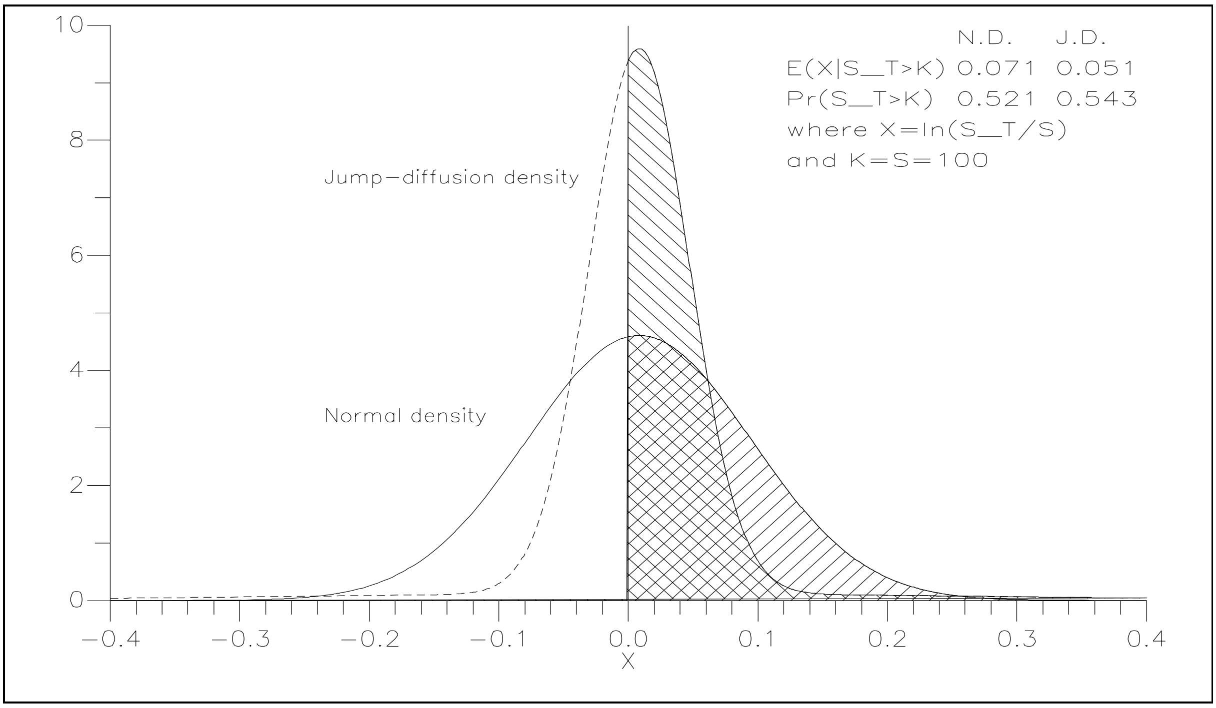


Figure 7
Risk neutral return distributions
(T=1/12, r=10%, VOLA=30%, γ=0.8, λ=1, $\mu_J = 0$)



the different shape of the models' risk neutral return distributions. For example, figure 7 visualizes the corresponding risk neutral density functions for options with one month time to maturity. The shaded area represents the risk neutral probability for an ATM option of ending up in the money. For an ATM option this probability is smaller in the BS-model than in the IJD-model. But the BS-model yields higher option values compared to the IJD-model since the expected return, conditional on the option being in the money at the expiration date, is larger in the BS-model. For ITM and OTM options these effects reverse each other. Therefore for these options the IJD-model value exceeds the BS-model value.

Contrary to the v-shaped relationship between BS-values and IJD-values, as depicted in figure 6, the SJD-values exceed the corresponding BS-values except for short-term ATM calls. Figure 8 visualizes the absolute differences between the SJD-value (for $R = 5$) and the BS-value for λ=1, i. e., for a low jump intensity. The difference with respect to volatility and skewness of the corresponding 'risk-neutralized' return distribution serves as an explanation for the higher SJD-value compared to the BS-value. For index calls with a mean jump return of $\mu_J = 0$, the risk-neutralized jump intensity and the squared risk-neutralized mean jump size increase with risk aversion: $\lambda < \lambda^* = \lambda \exp\{-R\mu_J + R^2\sigma_J^2/2\}$ and $\mu_J^2 < (\mu_J^*)^2 = (\mu_J - R\sigma_J^2)^2$ for $R > 0$. Therefore in the symmetric SJD-model (1) the 'risk-neutralized' volatility, $\text{VOLA}^* \equiv \sqrt{[\sigma_D^2 + \lambda^*((\mu_J^*)^2 + \sigma_J^2)] \cdot 250}$, exceeds the actual volatility, VOLA, used in the BS-model, and (2) the risk-neutralized skewness is negative $(\mu_J^* < 0)$[17]. While the first effect (*'volatility effect'*) leads to higher SJD-values, the second one (*'skewness effect'*) overcompensates the first effect for short-term OTM options. A comparison of figure 8 with figure 9 confirms the statement made earlier in this section that the BS-model approximates the SJD-model quite accurate for a high jump intensity (say, λ=100), given that the total volatility of the jump component is held fixed.

Table 4 presents a sample of corresponding model prices for European calls written on the market index. The column with the header 'BS' gives BS-value for short-term ($\tau = 1/12$, i. e., one month) and for long-term ($\tau = 1$, i. e., one year) OTM calls ($S = 80$), ATM calls ($S = 100$), and ITM calls ($S = 120$) calls, respectively. The numbers written in *bold face* correspond to *symmetric* return distributions as assumed in the foregoing comparisons illustrated in figures 6 to 9. Panel A contrasts Merton's IJD-values with BS-values while panel B does the same with respect to SJD-values with $R = 5$. While for the BS-model

[17]Recall that the sign of the mean jump return determines the sign of the skewness.

Figure 8
IJD-value versus BS-value for index calls
(K=100, r=10%, VOLA=30%, γ=0.8, λ=1, $\mu_J = 0$, R=5)

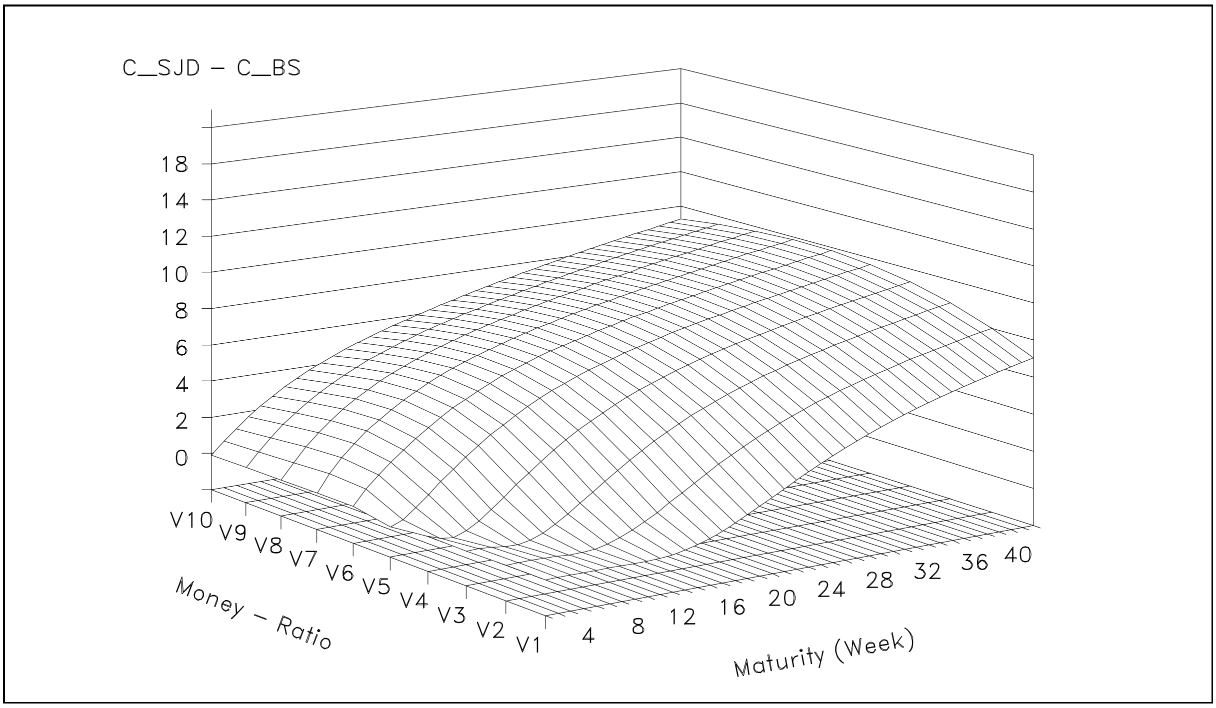


Figure 9
SJD-value versus BS-value for index calls
(K=100, r=10%, VOLA=30%, γ=0.8, λ=100, $\mu_J = 0$, R=5)

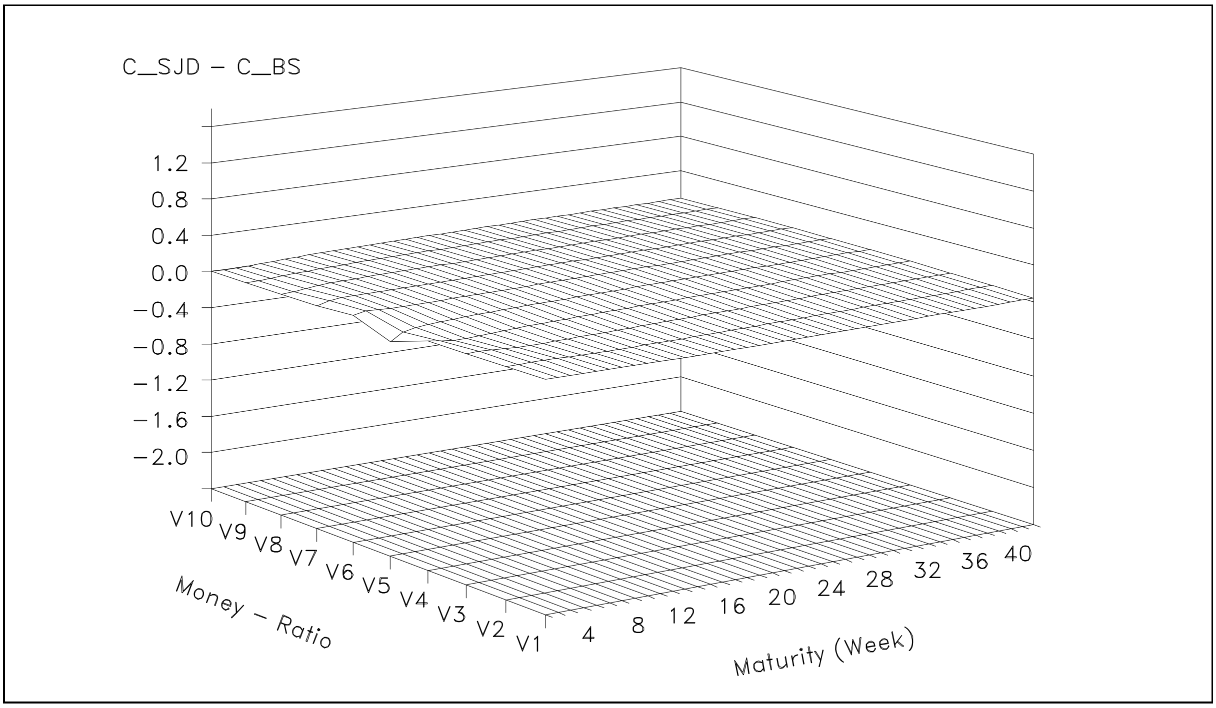


Table 4

Calls written on the market index: Influence of the jump component

(Fixed parameters:[a] $K = 100, r = 10\%$, VOLA=30%, $\gamma = 0.8$)

Panel A: Idiosyncratic jump risk model (Merton (1976a)) *or* risk neutrality ($R = 0$)

		BS	**IJD** ($\lambda = 1$)					**IJD** ($\lambda = 100$)				
	λ	**0**	1	1	**1**	1	1	100	100	**100**	100	100
	μ_J	**0**	−0.20	−0.10	**0.00**	0.10	0.20	−0.02	−0.01	**0.00**	0.01	0.02
	[b]λk	**0**	−0.17	−0.07	**0.03**	0.14	0.24	−1.96	−0.87	**0.03**	1.04	2.04
	VOLA	**30.0**	30.0	30.0	**30.0**	30.0	30.0	30.0	30.0	**30.0**	30.0	30.0
		$\mathbf{C^{BS}}$	Diff[c]	Diff	$\mathbf{C^{IJD}}$	Diff	Diff	Diff	Diff	$\mathbf{C^{IJD}}$	Diff	Diff
$S = 80$	$\tau = 1/12$	**0.02**	−0.30	−0.18	**0.31**	0.18	0.26	−0.02	−0.01	**0.02**	0.02	0.02
	$\tau = 1$	**5.76**	−1.40	−1.08	**5.52**	1.08	1.50	−0.13	−0.109	**5.75**	0.10	0.14
$S = 100$	$\tau = 1/12$	**3.87**	0.08	−0.01	**2.74**	0.07	0.19	0.02	0.01	**3.84**	0.01	0.03
	$\tau = 1$	**16.73**	0.48	0.15	**15.89**	0.29	0.79	−0.01	−0.01	**16.72**	0.02	0.03
$S = 120$	$\tau = 1/12$	**20.88**	0.26	0.15	**21.11**	−0.16	−0.27	0.04	0.03	**20.89**	−0.02	−0.04
	$\tau = 1$	**32.41**	0.70	0.50	**32.19**	−0.58	−0.81	0.06	0.05	**32.40**	−0.05	−0.07

Panel B: Systematic jump risk model *and* strong risk aversion ($R = 5$)

		BS	**SJD** ($\lambda = 1$)					**SJD** ($\lambda = 100$)				
	[d]λ^*	**0**	4.06	3.58	**2.46**	1.32	0.55	110.69	105.45	**100.90**	95.42	94.18
	[e]μ_J^*	**0**	−0.36	−0.41	**−0.36**	−0.21	0.04	−0.02	−0.01	**−0.00**	0.01	0.02
	$\lambda^* k^*$	**0**	−0.86	−0.61	**−0.33**	−0.12	0.02	−2.35	−1.34	**−0.32**	0.69	1.70
	[f]VOLA*	**30.0**	82.1	91.7	**71.7**	39.8	19.1	31.3	30.7	**30.1**	29.5	29.3
		$\mathbf{C^{BS}}$	Diff[c]	Diff	$\mathbf{C^{SJD}}$	Diff	Diff	Diff	Diff	$\mathbf{C^{SJD}}$	Diff	Diff
$S = 80$	$\tau = 1/12$	**0.02**	−0.04	−0.02	**0.05**	0.01	0.26	−0.01	−0.01	**0.02**	0.01	0.01
	$\tau = 1$	**5.76**	3.64	5.51	**15.25**	−8.79	−12.56	0.59	0.44	**5.79**	−0.38	−0.55
$S = 100$	$\tau = 1/12$	**3.87**	2.28	2.42	**5.86**	−2.58	−3.61	0.26	0.18	**3.87**	−0.18	−0.25
	$\tau = 1$	**16.73**	3.72	5.94	**28.73**	−9.48	−16.09	0.70	0.60	**16.78**	−0.54	−0.78
$S = 120$	$\tau = 1/12$	**20.88**	0.97	1.65	**23.89**	−2.09	−3.02	0.67	0.05	**20.90**	−0.04	−0.06
	$\tau = 1$	**32.41**	3.54	5.93	**44.29**	−8.94	−14.22	0.67	0.52	**32.45**	−0.44	−0.65

[a] the drift parameters α_D and $\alpha_{D,Y}$ are determined endogenously by the Euler conditions.

[b] The mean jump size per year is calculated as: $k \equiv \exp\{\mu_J + \sigma_J^2/2\} - 1$.

[c] Diff $\equiv C^{SJD}(\mu_J \neq 0) - C^{SJD}(\mu_J = 0))$.

[d] The 'risk neutral' jump intensity is defined as: $\lambda^* \equiv \lambda e^{-R\mu_J + R^2\sigma_J^2/2}$.

[e] The 'risk neutral' mean of $\ln(L_i + 1)$ is: $\mu_J^* \equiv \mu_J - R\sigma_J^2$.

[f] The 'risk neutral' volatility is given by: VOLA* $\equiv \sqrt{[\sigma_D^2 + \lambda^*((\mu_J^*)^2 + \sigma_J^2)] \cdot 250}$.

and IJD-model there is no difference between the shapes of the actual and the risk neutral return distributions, the shape of the risk neutral return distribution underlying the SJD-model depends on R as visualized in column 1 of table 5 for options with one year time to maturity. Since $\mu_J^* = \mu_J - R\sigma_J^2 < 0$ for $\mu_J = 0$ and $R > 0$, the risk neutral return distribution is skewed to the left. The representative sets of parameter values include not only situations where the actual return distribution is symmetric (when $\mu_J = 0.00$) but also situations where the actual return distribution is positively skewed (when $\mu_J = 0.20$ or $\mu_J = 0.10$) or negatively skewed (when $\mu_J = -0.20$ or $\mu_J = -0.10$). The columns with the header 'Diff' contain the differences between the corresponding jump diffusion model values according to the symmetric actual return distribution and the model values according to the skewed ones. Table 5 visualizes the differences between the BS-values and the IJD-values (middle column) as well as the differences between the BS-values and the SJD-values (for $R = 5$) (right hand side of the table) with respect to different money ratios, for options with one month and one year time to maturity, respectively. As distinguished from table 4, we consider beside a symmetric actual return distribution only one negatively skewed (when $\mu_J = -0.20$) and only one positively skewed (when $\mu_J = 0.20$) actual return distribution.

A comparison of panel A with panel B of table 4 shows that in the *symmetric case* the SJD-values exceed substantially the corresponding IJD-values except for short-term OTM calls. Again, the interaction of the volatility effect and the skewness effect explains this relationship. Another economic rationale for the relationship between IJD-values and SJD-values is given by Amin/Ng (1993). They compare the actual distributions rather than the 'risk-neutralized' ones. According to Amin/Ng (1993), the interaction of the so-called 'drift effect' and the so-called 'discounting effect' explains this difference. First, if there is a positive correlation between stock price jumps and wealth jumps, i.e. $\sigma_{J,SW} > 0$, then the stock return premium is higher under systematic jump risk relative to the diversifiable jump risk case. Therefore the stock price drifts upwards at a faster rate under systematic jump risk than under diversifiable jump risk and causes the call option value to be worth more (*'drift effect'*). Second, if a positive correlation between stock jumps and wealth jumps is assumed, then the expected rate of change of the marginal utility of wealth tends to jump with the stock return when jump risk is systematic but not under idiosyncratic jump risk. As shown in the former section this leads to a lower interest rate and therefore to lower call values (*'discounting effect'*). Since the direction of these effects depends on the assumption about the sign of the correlation between the consumption jumps and asset jumps, $\sigma_{J,SW}$, the above effects reverse itself when a negative correlation is assumed. The discounting and drift effect now influence the call price exactly in the opposite way.

Skewness of the actual return distribution changes the sign and magnitude of the differences between the BS-value and the alternative values.[18] Let us first of all look at the influence of skewness if jump risk is idiosyncratic. According to panel A of table 4, negative skewness of the actual return distribution ($\mu_J < 0$) decreases the IJD-value for OTM calls compared to the symmetric case, while a positive skewness of the return distribution ($\mu_J > 0$) decreases the IJD-value for ITM calls compared to the symmetric case. The IJD-value for ATM calls increase with skewness independently of the sign of the skewness. Therefore, according to the figures in the middle column of table 5, the v-shaped relationship between BS-value and IJD-value still exists for short-term options, but depending on the sign of the skewness the v-shape is lopsided to the right and to the left, respectively. For calls with one year time to maturity the difference in value increases (decreases) with the money ratio if the skewness is negative (positive). The shapes of the corresponding risk-neutralized return distributions, as visualized in the first column of table 5, explain this result.

In contrast to the idiosyncratic jump risk case, skewness of the actual return distribution influences the deviation from the BS-value significantly when jump risk is systematic. Unfortunately, the more realistic SJD model exhibits an even stronger smile effect when the actual return distribution is systematic or negatively skewed while the IJD model tends to reduce the smile effect when using the BS model.[19] According to panel B of table 4, the SJD-value decreases substantially compared to the symmetric case if the actual return distribution has a positive skewness. For an extremely positively skewed return distribution with $\mu_J = 0.20$, the BS-value exceeds the corresponding SJD-value, as illustrated in column 3 of table 5. For an extremely negatively skewed return distribution

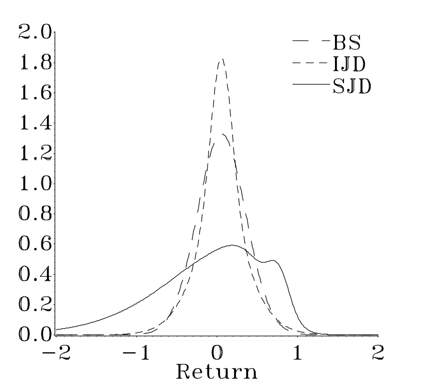
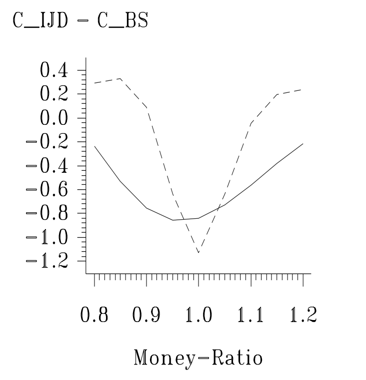
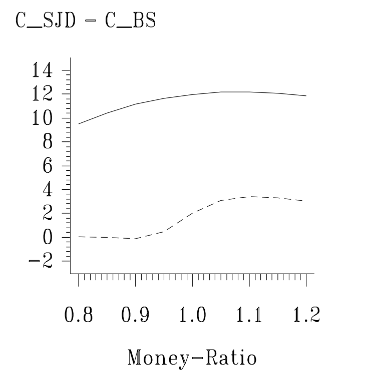
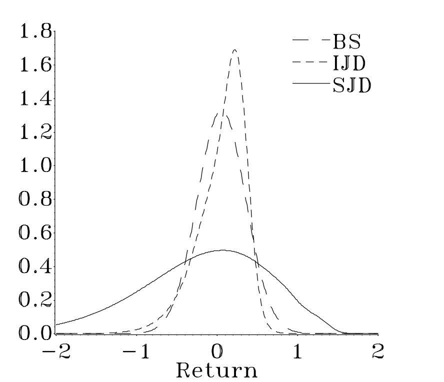
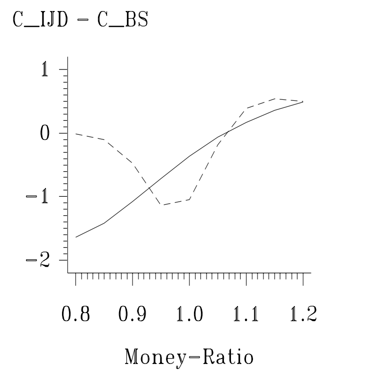
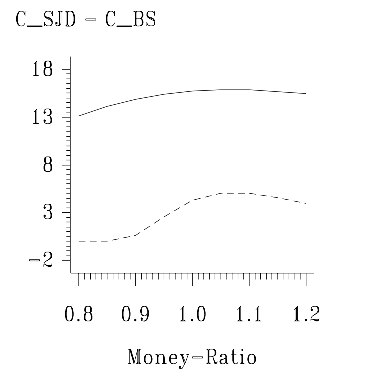
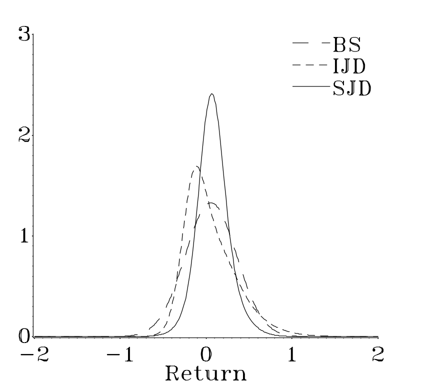
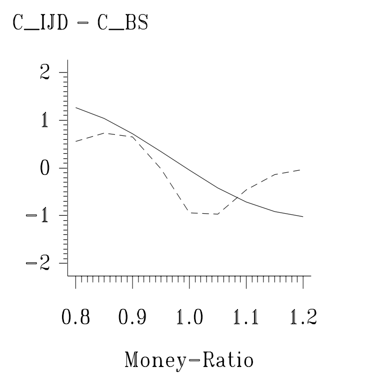
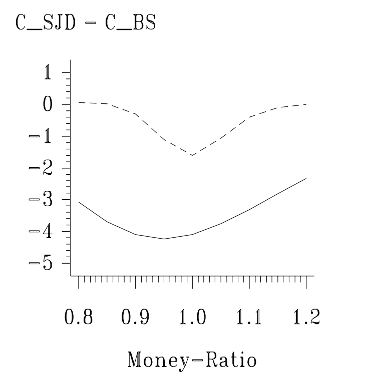
[18] Recall that the BS-model is based on a symmetric return distribution, while the jump diffusion-model allows for a positive or a negative skewness in the actual return distribution. Return distributions with a negative (positive) skewness exhibit a larger probability for returns far below (above) the mean than it is for returns far above (below) the mean. This implies that the mean lies below (above) the median if the return distribution has a negative (positive) skewness.

[19] If option prices in the market are quoted according to the BS model, the implied volatility would be a constant function of the money ratio. In reality, this is not the case. Implied volatility 'smiles': see, for instance, Rubinstein (1985) or Trautmann (1986, 1989) for the Frankfurt Options Market.

Table 5

Calls written on the market index: Deviations from Black/Scholes-values

(Fixed parameters:[a] $K = 100, r = 10\%$, VOLA=30%, $\lambda = 1$, $\gamma = 0.8$)

<table>
<tr><td rowspan="2">Shapes of risk-neutralized return distributions (T=1)</td><td colspan="2">Deviations from Black/Scholes-values if jump risk is</td></tr>
<tr><td>idiosyncratic[b] (or R=0)</td><td>systematic[b] and R=5</td></tr>
<tr><td colspan="3">Panel A: $\mu_J = 0$ (i. e., the actual return distribution is symmetric)</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td colspan="3">Panel B: $\mu_J = -0.20$ (i. e., the actual return distribution is negatively skewed)</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td colspan="3">Panel C: $\mu_J = 0.20$ (i. e., the actual return distribution is positively skewed)</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
</table>

[a] Resulting risk-neutralized parameters of the SJD-model are the same as in table 4.

[b] Deviations are plotted only for options with one month (dashed line) and one year (solid line) time to maturity, respectively.

with $\mu_J = -0.20$ the difference in value increases for options with one year time to maturity for about 30% while for short-term OTM options the SJD-value decreases a little bit. The interaction between the volatility effect[20] and the skewness effect explains again this result.

4.2 Results based on historical parameter estimates

4.2.1 Sample description

The sample period starts on April 1, 1983 and ends on December 31, 1991 including the market crash periods around October 1987 and around October 1989 as well as the bearish period around August 1990 when Iraq invaded Kuwait. The option price data consist of more than 600,000 transaction prices quoted on the *Frankfurt Options Market* (FOM) in the period from April 1, 1983 to June 31, 1990 and on the *Deutsche Terminbörse* (DTB) in the period from January 26, 1990 to December 31, 1991, respectively, for calls and puts written on five actively traded stocks: *Daimler Benz, Deutsche Bank, Siemens, Thyssen, and VW.*[21] Henceforth this sample is called BIG5. More precisely, we examine only the subsample BIG5/NODIV since price observations are eliminated if dividends were paid or a stock split took place during the lifetime of the option. All option prices, stock prices, dividend data, and split data, as well as daily stock return were taken or generated from the *Karlsruher Kapitalmarktdatenbank* (Karlsruhe capital market data base). The FOM-sample consists of 76.721 call and 13.307 put transaction prices fixed between 11.30 and 13.30. Unfortunately, these option prices and the corresponding stock prices (Kassakurse, i. e., odd-lot prices) are not time-stamped. However, in the subperiod from January, 1990 to December, 1991, both, the option price and the price of the underlying stock are time-stamped, allowing more precise statements on option values. We use all available transaction prices quoted between 10.30 until 13.30 (333.467 call prices and 209.094 put prices)[22], since we have time-stamped stock prices from the Frankfurt Stock Exchange only

[20] For $\mu_J \leq 0$ we have VOLA $<$ VOLA* while for $\sigma_J^2 R/2 < \mu_J$ and $R > 0$ we have VOLA $>$ VOLA*.

[21] Although the older FOM still exists, options on these five underlyings (as well as several others 'blue chips' stocks) can only be traded on the DTB since its opening in January 1990. Options written on the DAX-index were not considered since trading started only in August 1991.

[22] The huge number of transaction prices observed on the DTB compared to the FOM is due to the different market structures. Although the FOM was designed as a *continuous* auction market, there was usually only *one* market call for a specific options series a day. In contrast, the DTB is a liquid screen

for this time interval. As shown in the foregoing section a substantial impact on option values can only be expected when the estimated model parameters reflect the statistical and economical significance of the jump risk. Therefore we present especially the values of calls and puts whose underlying parameter estimates are based on the extreme volatile stock returns around the crash period in October 1987 and October 1989. The riskless interest rate appropriate to each option was estimated by the interest rate on three-month inter-bank time deposits[23] (Geldmarktsätze für Dreimonatsgeld am Frankfurter Börsenplatz). These monthly data were compiled from various issues of the *Monatsberichte der Deutschen Bundesbank*.

4.2.2 Parameter estimation

The time-consuming parameter estimation for the Poisson jump-diffusion process was performed only once a month during the sample period, based on 250 daily returns preceding the estimation date. We use the DAX return as a proxy for the percentage change in aggregate wealth. Although the SJD-model requires a simultaneous estimation of the model parameters associated with all individual stock returns and the return of the market proxy, we did simplify the estimation procedure for computational reasons by the following two-step procedure. First, we estimate the historical parameters for the DAX-returns and the individual stock returns (except the correlation between jump returns) independently. Second, we identify the return of the DAX and an individual stock, respectively, of a certain day as a 'jump return' if either the DAX-return or the individual stock's return exceeds 3% or is less than –3%. The correlation between these 'jump returns' serves as a proxy of the true correlation between jumps in stock return and aggregate wealth.[24] Typically there is a strong positive correlation between our selected stock returns and the DAX-returns. This indicates that jump risk is indeed systematic and should therefore be valued.

Figures 10 and 11 show the time series of the estimated annualized jump intensity (λ), the mean jump return (μ_J), and the total annualized volatility (VOLA) of Deutsche Bank

based market where market maker quote bid-ask-spreads continuously during the trading hours.

[23] Recall that in the SJD-model the riskless interest rate is endogenously determined. In order to be able to compare the SJD-model with models whose r is exogenous given, we endogenize instead the drift rate of wealth: α_W is chosen such that the 'endogenous' r is equal to the observable r.

[24] The parameter of the pure diffusion process were estimated daily based on the 250 preceding daily stock returns and DAX-returns, respectively.

and DAX, respectively. Large estimates for the jump intensity are typical for the first 4-year subperiod from April 1983 to March 1987, and occur less frequently in the second 4-year subperiod from April 1987 to June 1991. This is consistent with the observation that in the first subperiod absolute DAX-returns, for example, exceed the 5% level only one time while this happens 20 times in the second subperiod. The average λ-estimate of the DAX-returns is 389 in the subperiod from April 1983 to March 1987 while for the crash period the corresponding value is only about 116.[25] Hence, the jump component tended to a Geometric Brownian motion, especially in the first subperiod until March 1987.

The figures 12 and 13 show the monthly reestimated parameter values of skewness and kurtosis of the jump diffusion process, based on 250 daily returns preceding the observation date. The historical distributions of DAX and Deutsche Bank are negatively skewed and leptokurtic, especially those estimated after the October 1989 crash and during the year 1992, reflecting the stock price decline in both periods.

4.2.3 Results for American calls

According to the foregoing discussion, a substantial stock price jump impact on option value can only be expected for a small jump intensity. As detected by figures 10 and 11, a *low* jump intensity is estimated especially in the post-crash periods[26] from November 1987 to January 1988 and from November 1989 to January 1990. Figure 14 visualizes the mean DM-differences between BS-values and IJD-values with respect to different money ratios (left scale) for the BIG5-calls traded in these periods. The right scale corresponds to the plotted frequency distribution of observed money ratios[27]. The deviation pattern between *BS-values and IJD-values* resembles the one when the return distribution is

[25]For $\lambda > 250$ the estimate suggests that there will be, on an average basis, more daily jumps than assumed price observations per day.

[26]Since there is only one estimate of the jump-diffusion parameter per month, the parameter estimates of November 1987 and 1989 are the first estimates considering the October 1987 and October 1989 crash returns, respectively.

[27]The frequency distribution of the money ratios is less leptokurtic in this FOM-subperiod than in the FOM-subperiod from April 1983 to June 1990 since we have relatively more observations in the ITM and OTM classes. This is due to the two stock market crashes in October 1987 and in October 1989. Furthermore, since the number of quoted transactions prices did steadily increase from 1983 to 1990, most observations considered in the figure are quoted in the period from November 1989 to January 1990.

Figure 10

Monthly reestimated annualized parameter estimates of Deutsche Bank

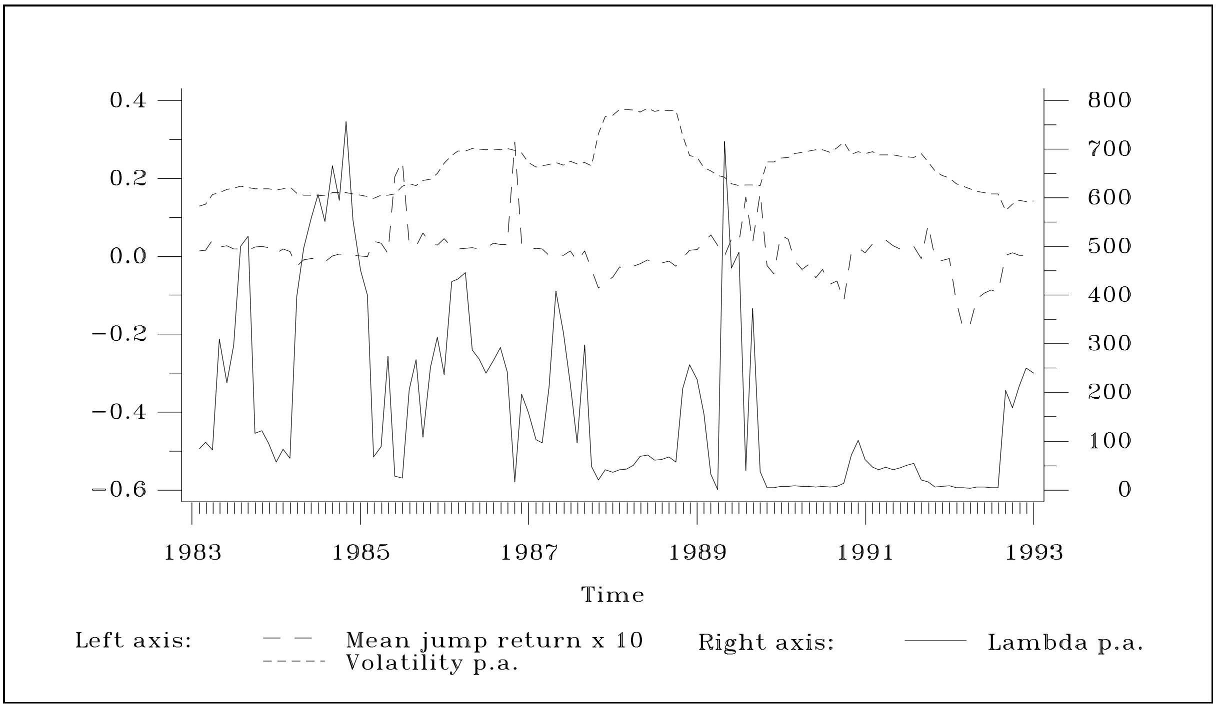


Figure 11

Monthly reestimated annualized Parameter estimates of DAX

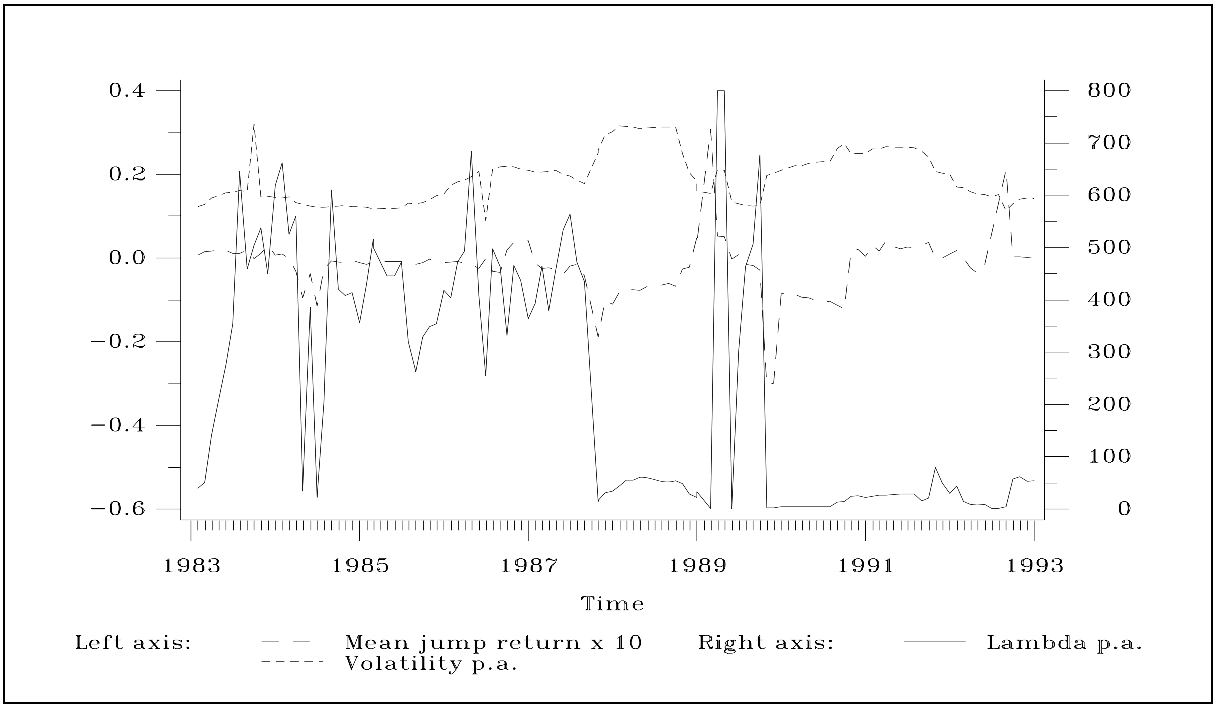


Figure 12

Annualized skewness of the jump diffusion process for Deutsche Bank and DAX

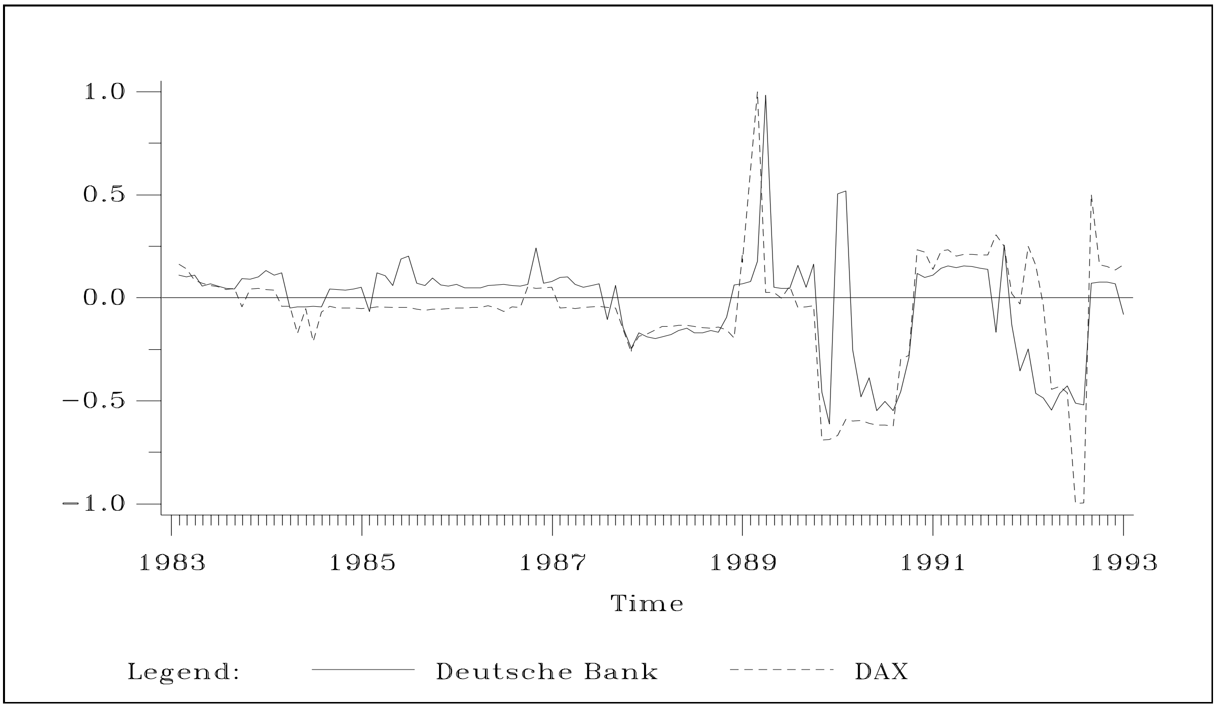
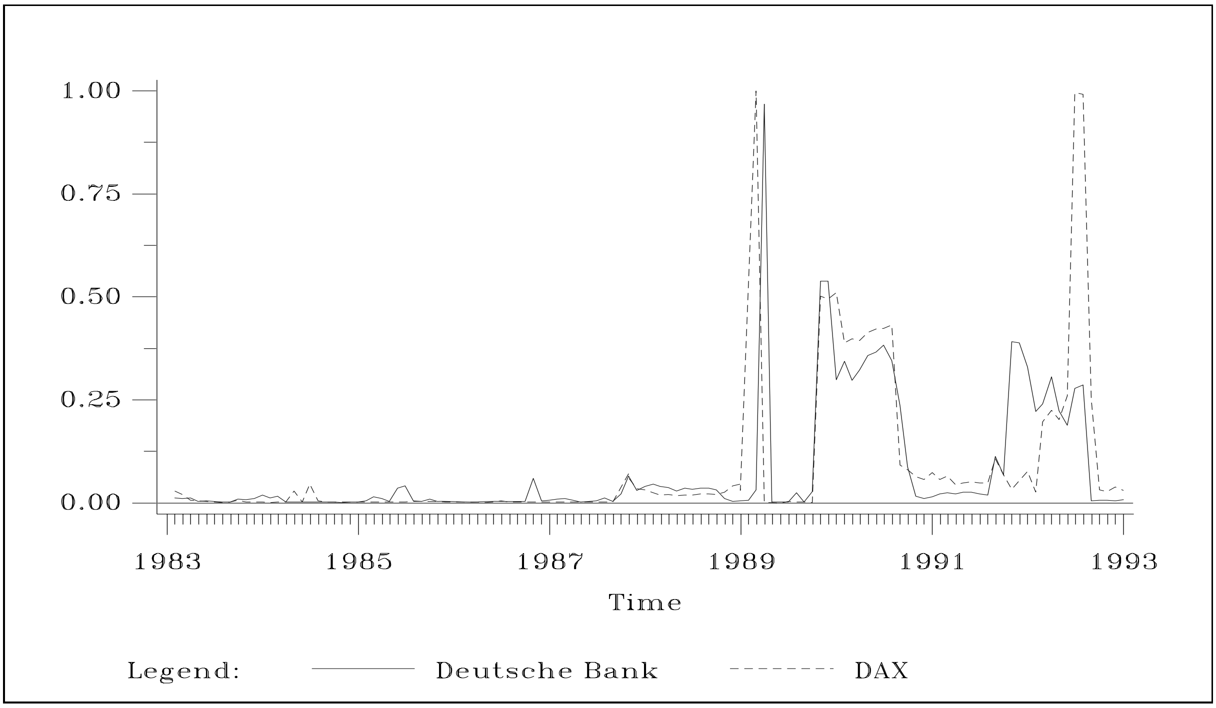


Figure 13

Annualized kurtosis of the jump diffusion process for Deutsche Bank and DAX



positively skewed, as visualized in table 5. This seems to be a quite *unexpected* result. But especially after the October 1989 crash large positive returns caused obviously positively skewed return distributions for some stocks during the three months considered. Figure 12 confirms the positive skewness of the Deutsche Bank return distribution after the October 1989 crash. While there is a v-shaped relationship for short-term and middle-term options, the IJD-value exceeds the BS-value especially for long-term OTM calls. The DM-difference (mean percentage difference) between the BS-value and IJD-value for OTM calls is for short-, middle-, and long-term calls DM 0.30 (9.5%), DM 0.61 (3.9%), and DM 0.76 (3.3%), respectively. In contrast to the OTM calls, for ATM calls the BS-value are DM 0.31 (1.2%), DM 0.10 (0.14%), and DM 0.21 (0.4%) for calls with a short, middle, and long time to maturity, respectively, higher than the corresponding IJD-value. For ITM calls the mean IJD-value exceeds the mean BS-value, but the percentage difference is negligible. This deviation pattern contradicts the one according to tables 4 and 5 when the return distribution is positively skewed. The answer to this puzzle reads as follows: the call value in the period from November 1987 to January 1888 and November 1989 to January 1990 is based on $5 \cdot 6 = 30$ different sets of parameter estimates. Unfortunately, some sets of parameter estimates imply a positively skewed return distribution while other sets result in a negatively skewed return distribution.

Figure 15 depicts the mean DM-differences between BS-values and IJD-values of the BIG5-calls traded in the period from July 1990 to September 1990, that is shortly before and after Iraq's invasion into Kuwait. For long-term options the differences in value are larger than expected but still exhibit the expected v-shaped relationship. This might be due to the different sign of the skewness of the five individual stock's return estimated for the three months.[28] For short-term options the difference between BS-value and IJD-value is consistent with the expected v-shaped relationship.[29]

Figures 16 and 17 show the mean DM-differences between *BS-value and SJD-value* (for $R = 3$) for the same post-crash periods as in the figures 14 and 15, respectively, for

[28] Recall that the parameters of the 'pure' diffusion process were daily reestimated while the jump diffusion process parameter were monthly reestimated. Therefore the parameters of the jump diffusion process are adjusted in a slower way to new events than the 'pure' diffusion process ones.

[29] In contrast, in the whole DTB sample period (90/2 – 91/12) as well as in the FOM-subperiod 87/4 – 90/6, the v-pattern exists for options in all maturity classes considered (the figures are not presented in this paper). For OTM options the percentage differences are significant, while the DM-differences are not large.

different money ratios. According to the findings of section 4.1, the SJD-value exceeds the BS-value substantially, except for short-term OTM calls. This deviation pattern resembles the one for symmetric or negatively skewed return distributions, as visualized in table 5 in the foregoing section. In contrast to the observed pattern of the difference between the BS-value and the IJD-value, the estimated positive skewness of some of the five underlyings does not influence the expected result. Obviously, this is due to the fact that for symmetric as well as for negatively skewed return distributions the SJD-value exceeds the BS-value much more substantially than the BS-value exceeds the SJD-value in the case of positively skewed return distributions. In the post-crash period after the October 1987 and the October 1989 crash, the SJD-value is on average DM 0.66 (5.08%) higher than the BS-value. The largest mean percentage difference of 11.39% is observed for OTM calls while for ATM calls and ITM calls this difference is only 1.70% and 0.60%, respectively. As visualized in figure 17, the largest difference is observed for ATM calls with a long time to maturity. For long-term OTM calls the SJD-value is on average DM 0.26 (14.38%) higher than the BS-value. The percentage differences for ATM calls with a long, middle, and short time to maturity are 3.7%, 2.8%, and 3.8%, respectively.

The largest mean difference between the *SJD-values and the IJD-values* in the post-crash period from November 1987 to January 1988 and November 1989 to January 1990 is observed for long-term OTM calls. Figures are not presented here since this difference is a result from the differences discussed above. The mean difference is DM 1.30, while the mean percentage difference is about 4%. For ATM calls the mean difference is about DM 0.95, DM 1.22, and DM 0.88 for middle-, long-, and short-term options, respectively. The corresponding mean percentage differences are about 2.8%. Furthermore, since there is usually a *positive* correlation between asset jumps and wealth jumps, the 'discounting-effect' causes the short-term OTM calls to be worth more under idiosyncratic jump risk than under systematic jump risk. On the other side, the 'drift-effect' causes ITM calls and ATM calls with a longer maturity to be worth more under systematic jump risk than under idiosyncratic jump risk. This pattern can also be found in the period around the Kuwait crisis. Especially for ATM calls the SJD-value exceeds the IJD-value while for OTM calls this difference is much smaller and partly negative for short-term calls.

Figure 14

Mean differences (in DM) between model values in post-crash periods

(BIG5/NODIV Calls, 87/11 - 88/1 and 89/11 - 90/1)

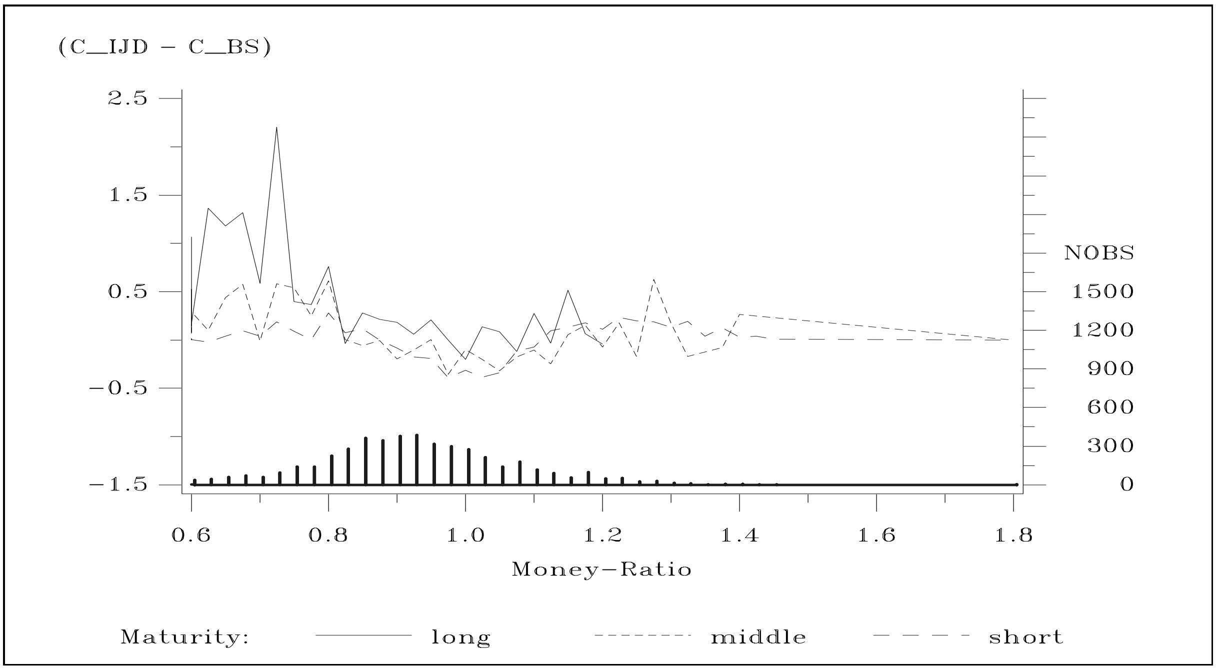


Figure 15

Mean differences (in DM) between model values at the beginning of the Kuwait crisis

(BIG5/NODIV Calls, 90/7 - 90/9)

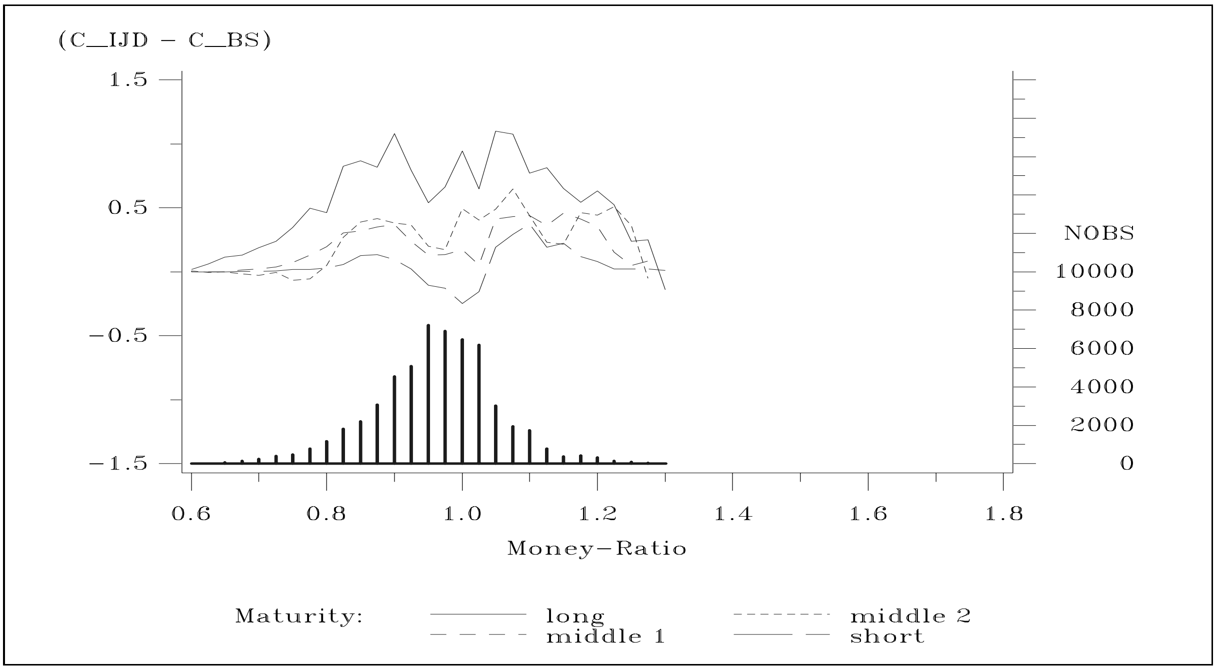


Figure 16

Mean differences (in DM) between model values in post-crash periods

(R=3, BIG5/NODIV Calls, 87/11 - 88/1 and 89/11 - 90/1)

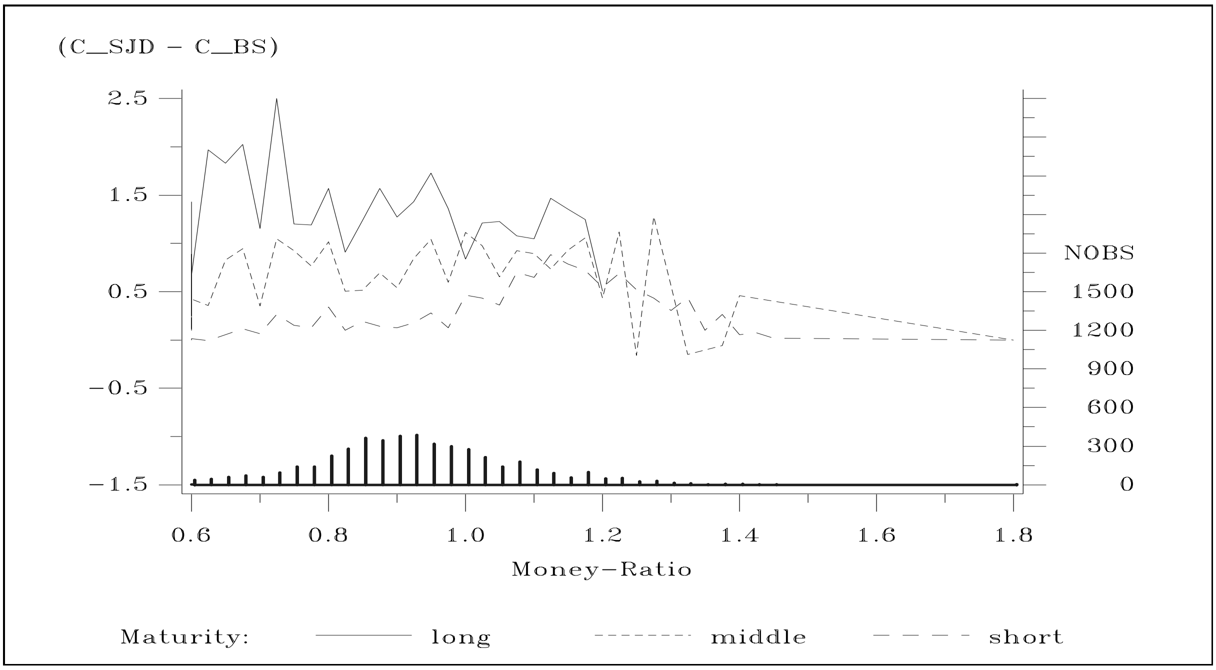
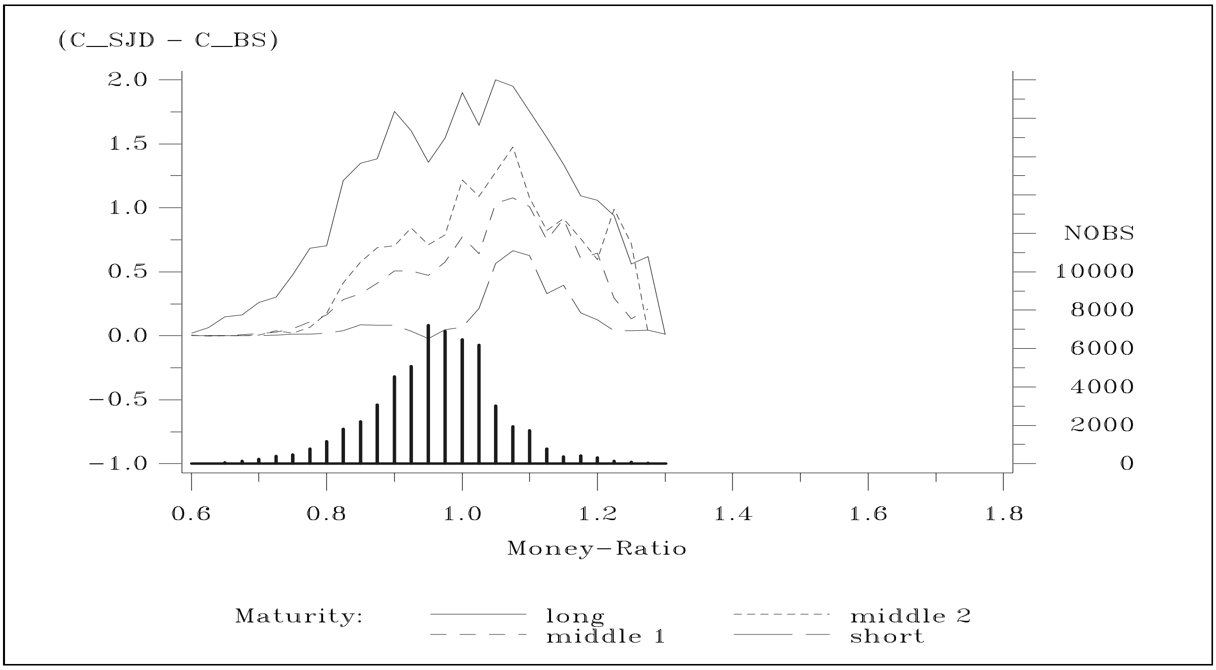


Figure 17

Mean differences (in DM) between model values at the beginning of the Kuwait crisis

(R=3, BIG5/NODIV Calls, 90/7 - 90/9)



4.2.4 Results for American puts

Put-call parity guarantees that the difference between the BS-value and an alternative value for a European call is the same as for an otherwise identical European put. But even if the American puts examined would be of the European type, we could not expect the same deviation pattern (with respect to S/K) unless there is for each call an otherwise identical put (and conversely) in the sample. Nonetheless, the deviations depicted in figures 18 and 19 are very similar to the ones plotted in figure 15 and 17, respectively.

Figure 18 visualizes the mean difference between *the BS-value and the IJD-value* for different money ratios in the subperiod from July 1990 to September 1990. Compared to the results for calls in the foregoing subsection (compare figure 15), the corresponding differences are somewhat larger for middle-term and long-term puts with a money ratio between 0.9 and 1.1. The mean DM-differences (mean percentage differences) for ATM puts with a short, middle, and long time to maturity are DM –0.09 (–0.01%), DM 0.50 (2.3%), and DM 1.03 (3.1%), respectively, while the corresponding differences for ATM calls are DM –0.26 (–0.2%), DM 0.30 (0.9%), and DM 0.94 (2.0%), respectively. In all other money ratio classes the mean DM-differences are smaller than the corresponding ones for calls.[30]

Figure 19 shows the mean differences (in DM) between *the BS-model and the SJD-model* for a risk-aversion parameter of $R = 3$ for the same subperiod as in figure 18. Compared to the calls, the differences are larger for puts with a money ratio between 0.9 and 1.1. As expected, the mean differences are larger for the 'bearish' period around August 1990 when Iraq invaded Kuwait compared to the sample period from January 1990 to December 1991.

Subtracting the difference in value as depicted in figure 18 from the difference in value as depicted in figure 19 gives the mean difference between *the IJD-model value and the SJD-model value*. Obviously, the SJD-value for middle-term and long-term ITM puts exceeds the IJD-value. This effect disappears for DITM puts.

[30]In the post-crash periods from October 1987 and October 1989 (the figures are not presented in the paper) the differences between BS-value and IJD-value for puts with a short, middle, and long time to maturity are DM –0.20 (–0.97%), DM –0.13 (–0.16%), and DM –0.24 (–1.10%), respectively. For calls the corresponding differences are DM –0.31 (–1.20%), DM –0.10 (–0.10%), and DM –0.20 (–0.40%).

Figure 18

Mean differences (in DM) between model values at the beginning of the Kuwait crisis
(BIG5/NODIV Puts, 90/7 - 90/9)

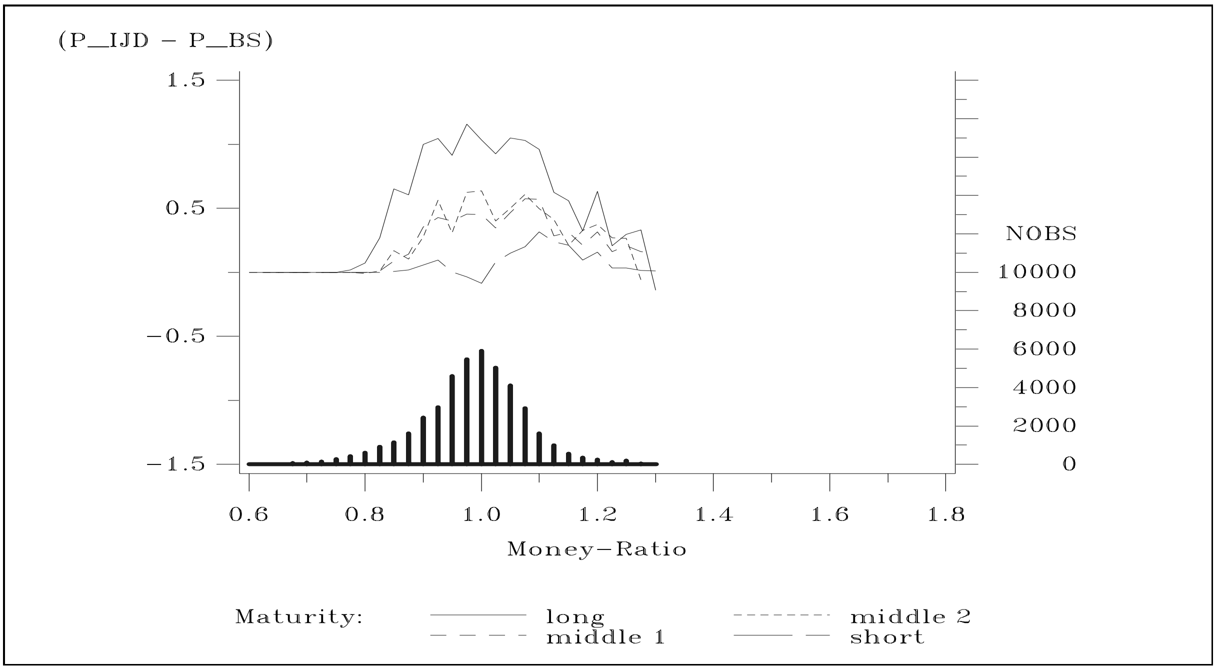
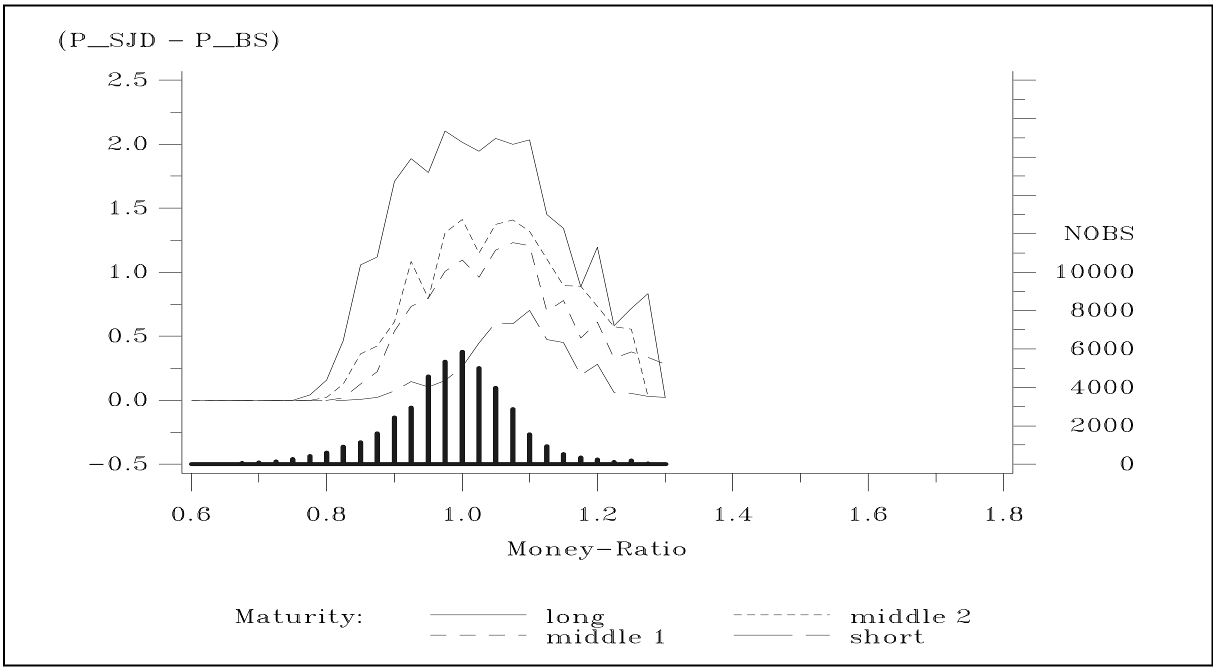


Figure 19

Mean differences (in DM) between model values at the beginning of the Kuwait crisis
(R=3, BIG5/NODIV Puts, 90/7 - 90/9)



5 Parameter estimates implied in option prices

We now present the parameters of the jump diffusion model *implicit* in call prices, put prices, as well as put and call prices. The reason for doing this is twofold:

(1) Historical process parameter estimates are in general not the best predictors for future process parameters. Furthermore, we have simplified the estimation procedure in the foregoing section for the sake of numerical tractability. Therefore the process parameter estimates may be biased.

(2) Since option prices offer a direct insight into the climate of expectations we can examine whether market participants expected extreme price movements. For instance, an assessed risk of a large downward movement in the market will result in higher OTM put prices compared to values calculated for a symmetric distribution. A chronology of implicit parameter estimates could thereby be generated, indicating market sentiments on a daily basis over the sample periods.

We take especially option prices observed around the October 1987 crash, the October 1989 crash, and around the Kuwait crisis in 1990 to examine whether these abnormal stock price movements were expected by the market participants .

The calculation of SJD-values requires *either* the risk aversion parameter R and the seven parameters $\sigma_D, \lambda, \alpha_J, \sigma_J, \alpha_{J,W}, \sigma_{J,W}$, and $\sigma_{J,WS}$ of the *true* stock price distribution and *true* wealth distribution, respectively, *or* the four parameters

$\lambda^* \equiv \lambda \exp(-R\alpha_{J,W} + (1/2)R(1+R)\sigma^2_{J,W})$, the implied risk neutral mean jump frequency (IJFrn),

$k^* \equiv \exp(\alpha_J - R\rho\sigma_J\sigma_{W,J}) - 1$, the implied risk neutral jump size

$\sigma_D \equiv$ the volatility of the diffusion component, and

$\sigma_J \equiv$ the jump size volatility,

characterizing the distribution of the risk neutral terminal stock price

$$\widetilde{S}_T = S_0 \exp\left\{(r - 1/2\sigma_D^2 - \lambda^* k^*)T + \sigma_D B_T + \sum_{i=1}^{N_T} J_i^*\right\} .$$

For the sake of numerical tractability, we restrict ourselves to infer only the parameters of the corresponding *risk neutral* (instead of the *true*) stock price distribution from observed option prices.

5.1 Estimation procedure

The risk neutral parameters λ^*, k^*, σ_D, and σ_J are estimated via nonlinear regression. We minimize the sum of the squared differences between market prices and corresponding SJD-values,

$$\sum_{j=1}^{n} \left[\frac{O_j}{S} - O^{SJD}(1, (K_j/S), T_j, \sigma_D, \lambda^*, k^*, \sigma_J) \right]^2,$$

where O_j denotes the market price of option $j = 1, \ldots n$. This minimization is done separately for every trading day during the observation period.[31] This procedure needs at least four price observations. If there are not sufficiently many option price observations for a given underlying stock (it happened only in the FOM market) we use in addition the corresponding price observations of up to four trading days preceding the trading day under consideration.

The implied parameters are inferred from transaction prices of two different sample periods. First, we examine all FOM-prices in the period around the October 1987 crash, and second, DTB-prices of the most liquid maturity class in the period from January 26, 1990 to December 30, 1991. Options in whose time to maturity dividends or other rights were paid are eliminated from the sample.

[31] We first transformed the problem to a three dimensional problem as proposed by Bates (1991) and used the FORTRAN routine BCLSF available in the IMSL program library. Furthermore, since this nonlinear optimization problem has usually many local minima, we started the optimization procedure with four different sets of starting values to improve the probability to arrive at the global minimum.

5.2 Results

We now present the time series of implicit risk neutral jump frequency per year (λ^*), the implicit risk neutral jump size per year ($\lambda^* k^*$), the implicit risk neutral skewness (SKEW*), and the implicit risk neutral volatility (VOLA*) of Deutsche Bank options. Positive and negative values for $\lambda^* k^*$ imply positive and negative skewness of the implicit risk neutral stock price distribution, respectively. According to our analysis in section 4.1, skewness will be large if

- the λ^* is sufficiently small and the absolute value of the mean jump size ($|\mu_J^*|$) is large, or
- the λ^* is sufficiently small and the mean jump size is close to zero ($|\mu_J^*| \approx 0$) when the volatility of the jump size is large ($\sigma_J \gg 0$).

In the first case negative skewness indicates strong crash fears while positive skewness indicates hopes of a trend reversion. In the second case there is a large uncertainty about the size of the jump that might occur. The figures 20 – 24 visualize the implied parameter estimates for *Deutsche Bank* calls, puts, and pooled calls and puts, respectively, for all trading days in the period from July 1, 1987 to December 30, 1987.

Figures 20 and 22 visualize the λ^* (left scale) and the $\lambda^* k^*$ (right scale) of Deutsche Bank calls and Deutsche Bank puts, respectively. The figures 21 and 23 show SKEW* (left scale) and VOLA* (right scale) of Deutsche Bank calls and Deutsche Bank puts, respectively. The plotted implicit parameters estimated for calls differ substantially from the corresponding ones for puts. This indicates different market sentiments of call and put market participants. We found that the implicit parameters estimated for Deutsche Bank *calls* indicate (1) significant crash fears in August 1987 and in the beginning of October 1987, and (2) a positively skewed implicit return distribution after the crash. This dramatic change in implicit stock price distributions after the crash is evinced in figures 20 and 21. Compared to the absolute value of the historical skewness (see figure 12), the absolute value of the implicit 'risk neutral' skewness is significantly larger. Figures 22 and 23 show the risk neutral parameters implicit in Deutsche Bank *puts*. While the call market reflected crash fears in July 1987, the put market did not show any signs of crash fears until late August 1987 and September 1987. Hopes of a trend reversion after the

Figure 20

Risk neutral jump frequency (IJFrn $\equiv \lambda^*$) and risk neutral jump size per year (IJSYrn $\equiv \lambda^* k^*$) implied in *Deutsche Bank* **calls** observed around October 1987

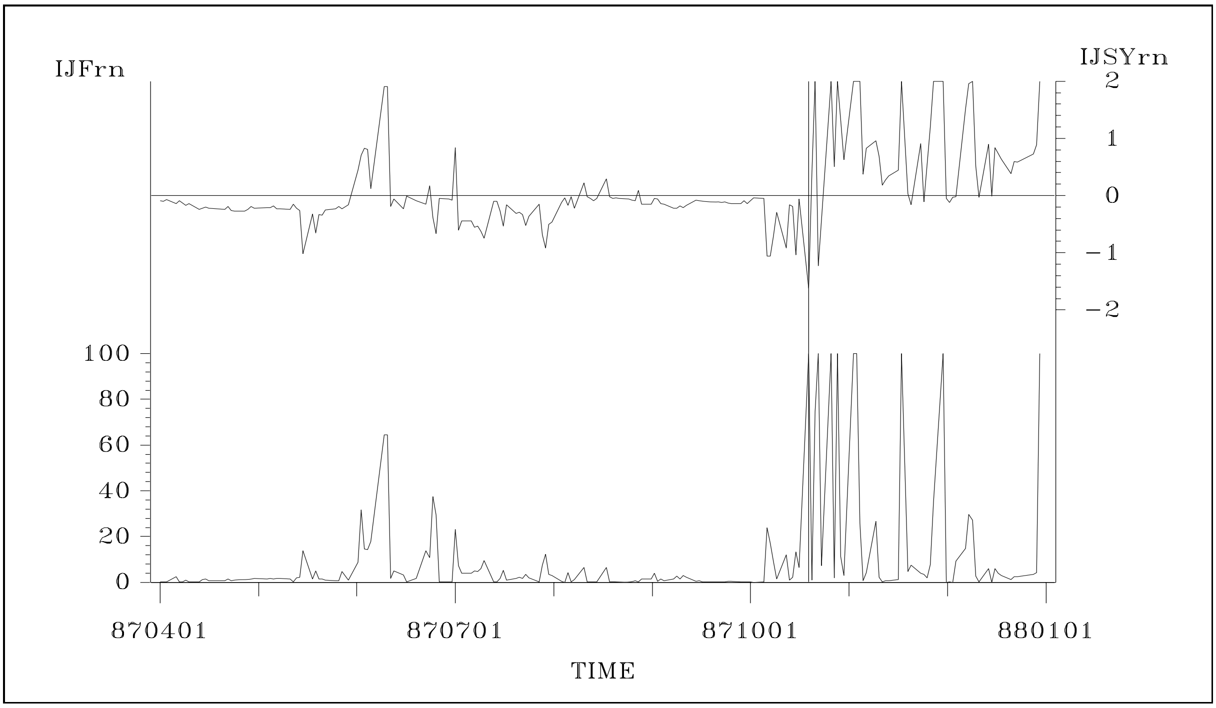


Figure 21

Risk neutral volatility (VOLArn $\equiv$ VOLA*) and risk neutral skewness (SKEWrn) implied in *Deutsche Bank* **calls** observed around October 1987

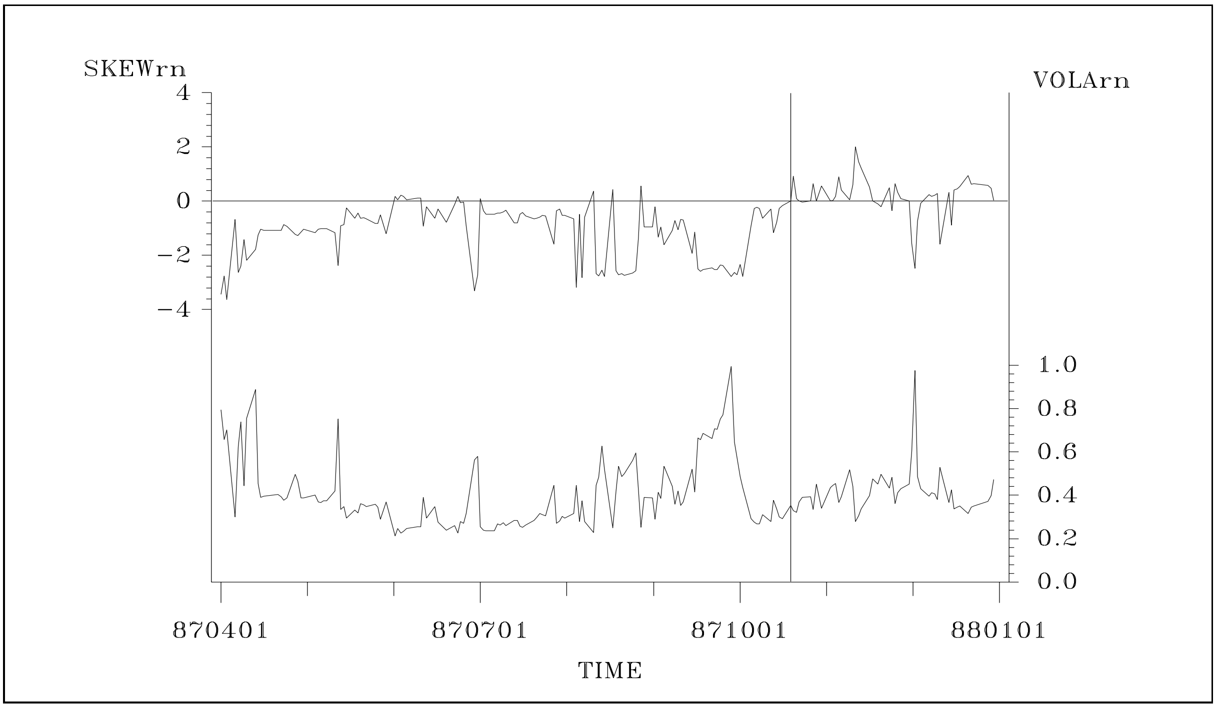


Figure 22

Risk neutral jump frequency (IJFrn $\equiv \lambda^*$) and risk neutral jump size per year (IJSYrn $\equiv \lambda^* k^*$) implied in *Deutsche Bank* **puts** observed around October 1987

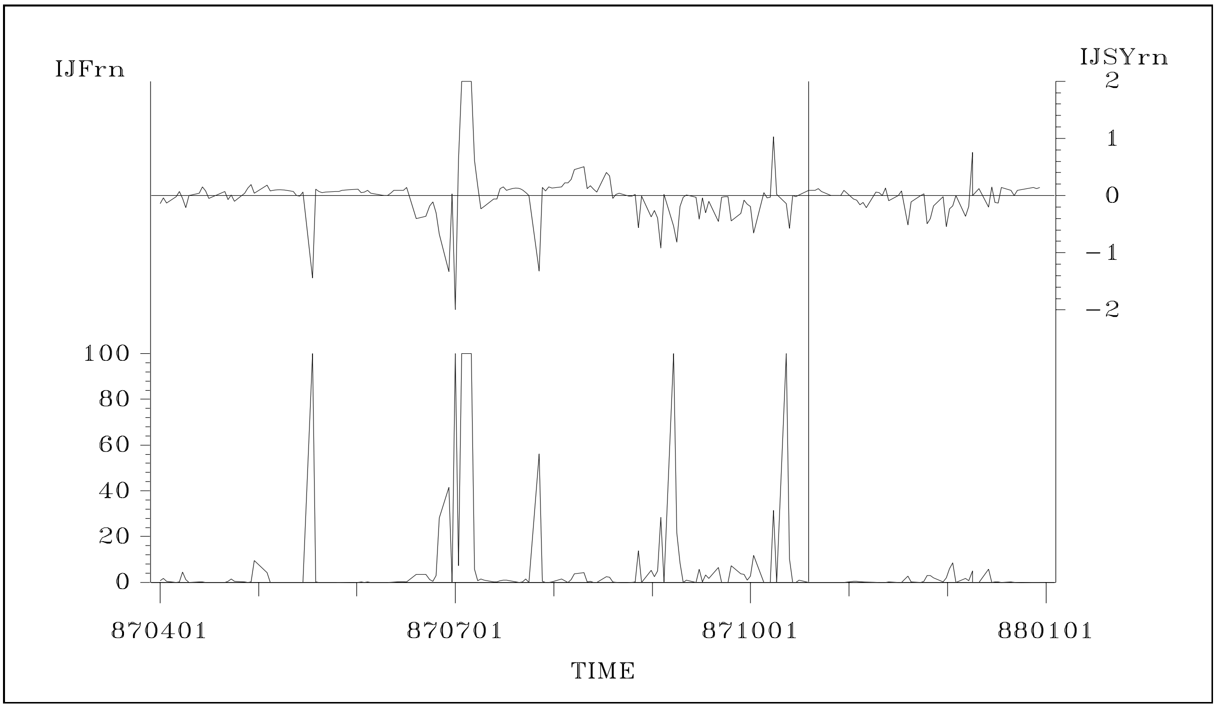
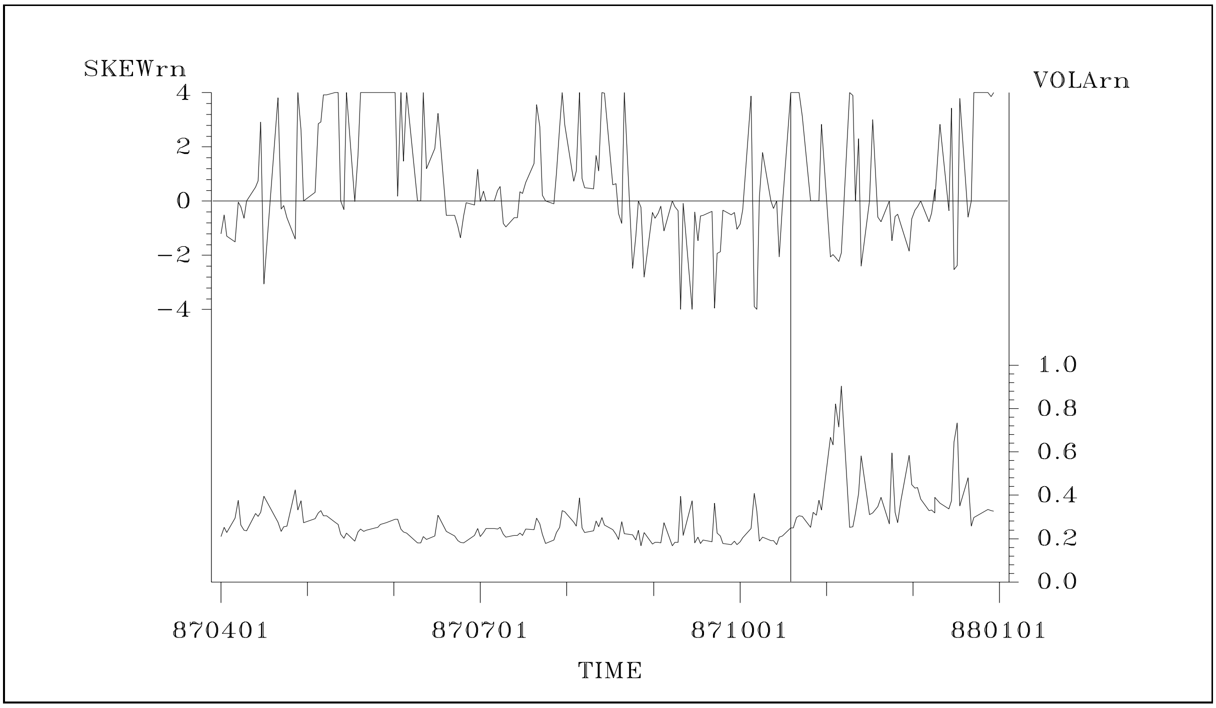


Figure 23

Risk neutral volatility (VOLArn $\equiv$ VOLA*) and risk neutral skewness (SKEWrn) implied in *Deutsche Bank* **puts** observed around October 1987



October 1987 crash are not observable in the time series of implicit parameters estimated for Deutsche Bank puts.

Especially figure 24 confirms the findings of Bates (1991) when examining the implicit return distribution of the S&P 500 futures price. There were strong crash fears in July 1987: $|\lambda^* k^*|$ is quite large while λ^* is small. Therefore the market participants expected a rare jump with a large negative amplitude (a crash). In the two months preceeding the crash, the S&P 500 futures options reflect a negatively skewed return distribution but during this time the estimates of the implicit jump size per year were low indicating no strong crash fears. In contrast to these findings, Deutsche Bank options reflect mostly a positively skewed distribution. While the crash fears reflected by the S&P 500 futures options prices returned after the stock market crashed around October 19, 1987, German options market participants expected an upward stock price correction. The graphs evince these changes in implicit stock price distributions after the crash. The different developments of US and German stock return distributions implicit in option prices can be explained by the corresponding historical stock price movement. While the US market peaked in August 1987 after a dramatic upward movement during the preceding twelve months, the German stock market peaked already in December 1985 and declined during the years 1986 and 1987. The October 1987 crash left the US stock market at year-end essentially unchanged from its level in January 1987, while the German one fell back to the level of January 1985. Therefore US options market participants feared a further drop while the German price level was so low that a further price drop was not expected by the options market participants. As distinguished from the US situation, option prices quoted at the FOM market reflected even strong rebound hopes.

Figure 25 visualizes the estimated implied risk neutral jump size per year and the implied risk neutral jump frequency in the period from January 1990 to December 1991 of Deutsche Bank for the *pooled* sample. Fears of a 'bearish' market are characteristic for almost the whole year 1990. A trend reversion was especially expected in the end of July 1990. But with the beginning of the Kuwait crisis in August 1990 these hopes disappeared. Substantial crash fears did not exist in this observation period. After the end of the Gulf war in March 1991, the level of the mean jump size per year increased. But since the jump frequency was also high, the implicit parameters indicate no strong crash fears or hopes of a trend reversion. Afterwards no crash fears or rebound hopes are reflected in option prices of Deutsche Bank. This might be due to the low price volatility in this period. The

Figure 24

Risk neutral jump frequency (IJFrn $\equiv \lambda^*$) and risk neutral jump size per year (IJSYrn $\equiv \lambda^* k^*$) implied in *Deutsche Bank* **calls and puts** observed around October 1987

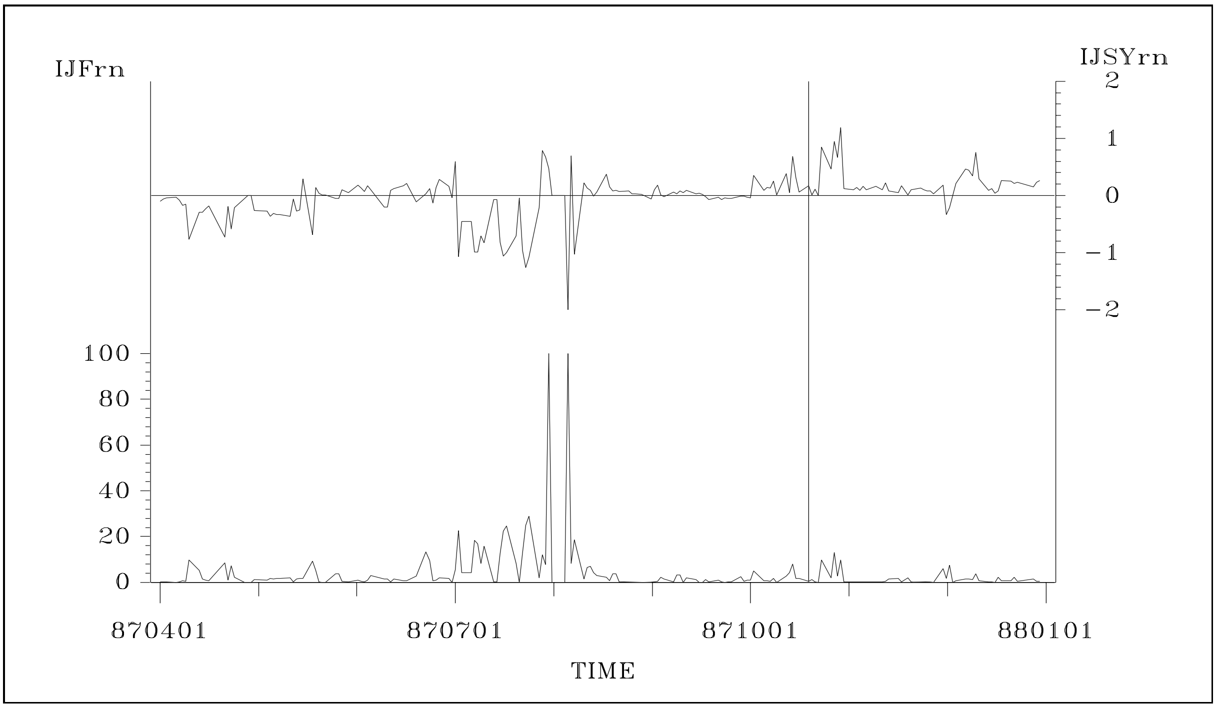
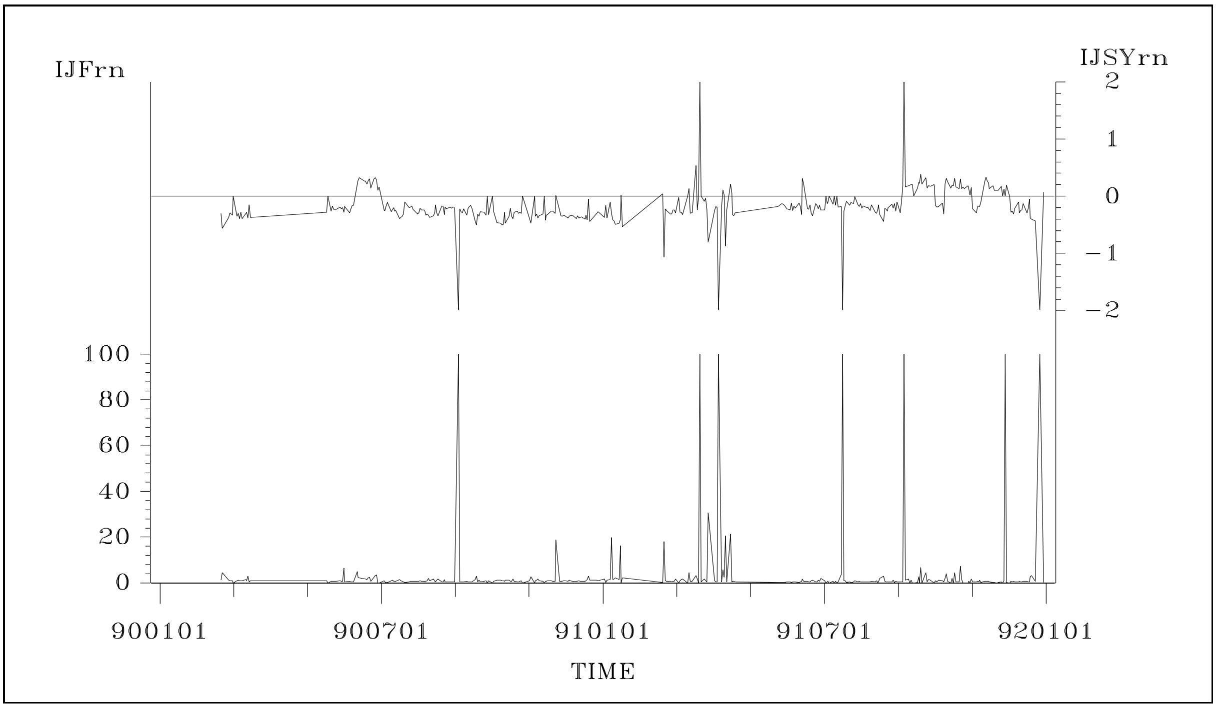


Figure 25

Risk neutral jump frequency (IJFrn $\equiv \lambda^*$) and risk neutral jump size per year (IJSYrn $\equiv \lambda^* k^*$) implied in *Deutsche Bank* **calls and puts** observed in 1990 to 1991



implicit parameters of Siemens (the figures are not presented in the paper) confirm these findings.

Figures 26 and 27 show the differences in root mean squared errors (RMSE) between the BS-model and SJD-model (upper scale) for the pooled sample of Deutsche Bank calls *and* puts. The lower scale of figures 26 and 27 visualizes the differences in RMSE for the SJD-model when the parameter estimation is based on the pooled sample of Deutsche Bank calls and puts compared to the situation when the parameter estimation is done separately for puts and calls written on Deutsche Bank stocks. Figure 26 is based on prices of calls and puts written on Deutsche Bank stocks quoted on the FOM in the period from April 1, 1987 to June 31, 1988. The root mean squared errors observed for Deutsche Bank options quoted on the FOM are quite large. This is obviously due to the fact that (1) all transaction prices of one day are used to estimate the implied parameters, and (2) price data for this period are not time-stamped. Figure 26 shows that the SJD-model does not yield a substantially better fit of the market prices compared to the BS-model, except *after* the October 1987 crash: in this period the SJD-model fits the data much better than the BS-model. Furthermore, the difference between the parameter estimates implied in Deutsche Bank calls and the parameter estimates implied in Deutsche Bank puts (compare figures 20 and 21 with figures 22 and 23) is reflected by the increase of the RMSE between late August 1987 and October 1987 when the pooled sample is used.

Figure 27 is based on Deutsche Bank options of the most liquid maturity class traded on the DTB from January 26, 1990 to December 30, 1991. Sampling only option prices of the most liquid maturity class reduces the RMSE of the SJD-model as well as the BS-model significantly. But this is not true for the the pooled sample. In accordance with the observed RMSE for FOM options, figure 27 shows only a slightly better fit of the market prices by the SJD-model compared to the BS-model for DTB options. The decrease in RMSE from relaxing the constraint of a pooled sample implied parameter estimation is visualized on the lower scale of figure 27. The difference between the parameter estimates implied in Deutsche Bank calls and the parameter estimates implied in Deutsche Bank puts explains this result.

Figure 26
Differences in RMSE as % of the stock price for Deutsche Bank
(Upper scale: RMSE(BS-model) minus RMSE(SJD-model) for calls and puts pooled)
(Lower scale: RMSE(SJD-model calls and puts pooled) minus RMSE(SJD-model unpooled))

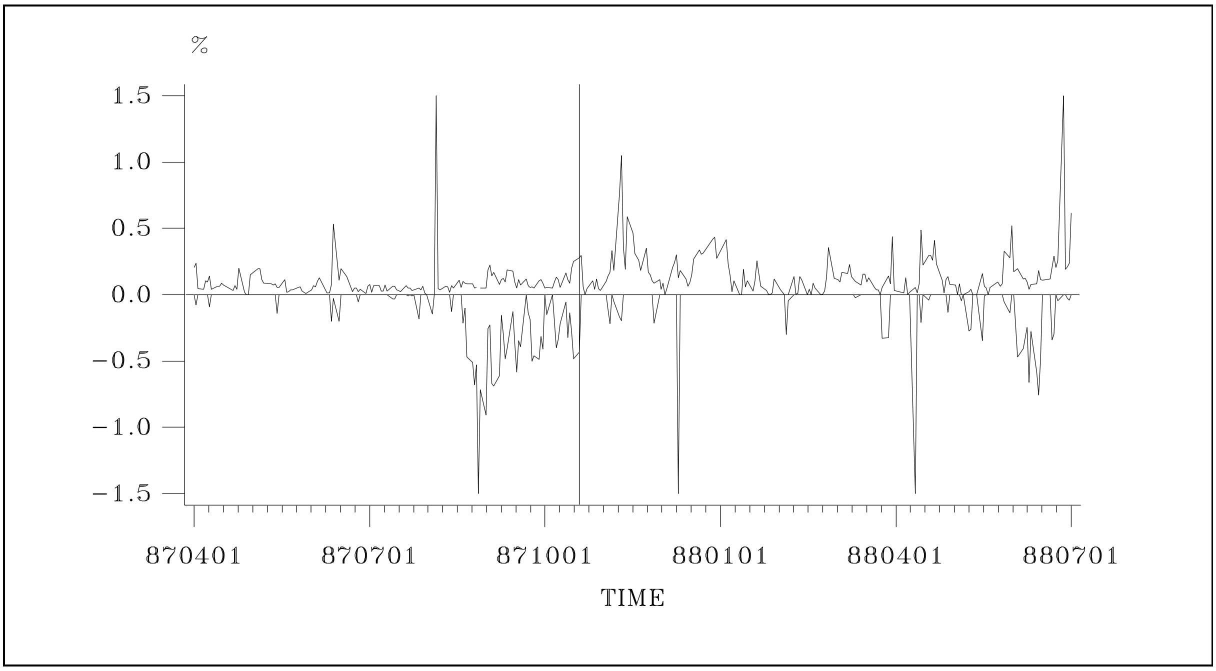
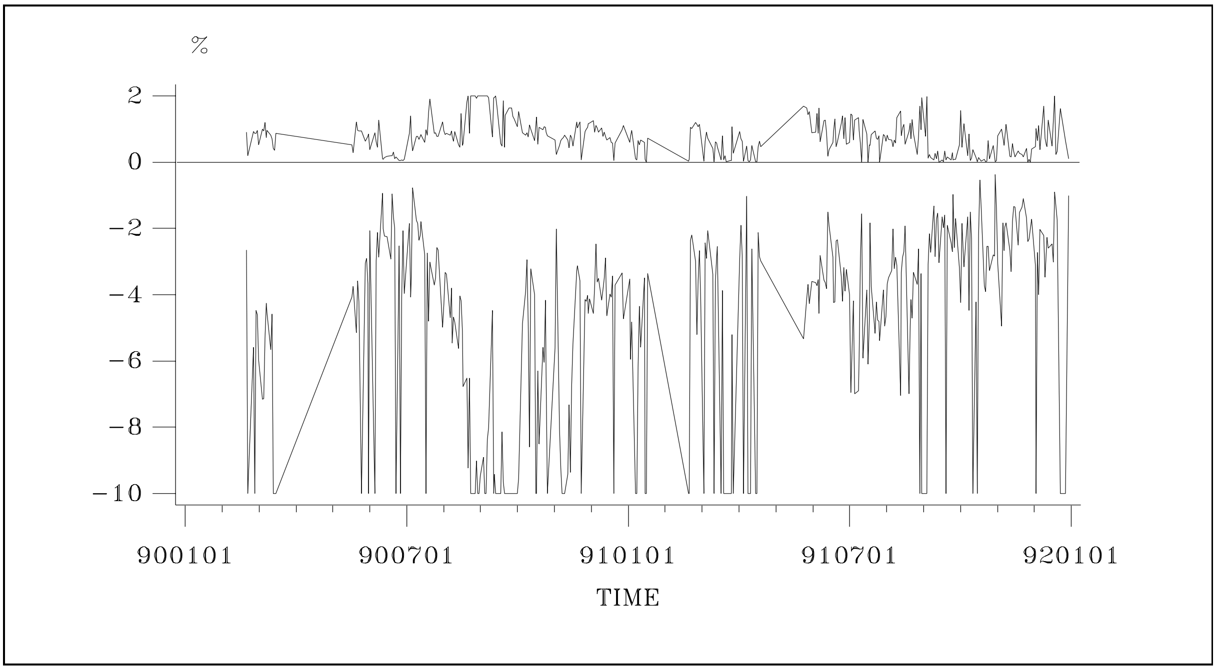


Figure 27
Differences in RMSE as % of the stock price for Deutsche Bank
(Upper scale: RMSE(BS-model) minus RMSE(SJD-model) for calls and puts pooled)
(Lower scale: RMSE(SJD-model calls and puts pooled) minus RMSE(SJD-model unpooled))



6 Conclusions

The classical Black/Scholes model assumes that stock price movements can be modeled by a pure diffusion process while Merton's jump diffusion model assumes that jump risk is diversifiable. When using daily and weekly return data we found, however, that German stocks and stock indices (especially the DAX) contain a *statistically* significant jump component. Since the DAX is supposed to be a good proxy for the market portfolio, the economic implication is that jump risk is *not* diversifiable. Consequently, this paper concentrates on the impact of *systematic* stock price jumps on option value. In conclusion we list several contributions of this study.

In the first place, we have presented a detailed analysis of the impact of stock price jumps on option values for representative model parameters. The shapes of the risk neutral return distributions plotted in figure 7 and table 5 help to explain the deviation of jump diffusion values from Black/Scholes values (BS-values). According to Amin/Ng (1993), the difference between the systematic jump risk model values (SJD-values) and the idiosyncratic jump risk model values (IJD-values) relies on the interaction between the *drift* effect and *discounting* effect. While the drift effect causes call options to be worth more under systematic jump risk relative to a model with idiosyncratic jump risk, the discounting effect leads to the opposite result. For longer-term calls and for in-the-money and at-the-money calls the drift effect dominates. Hence, these calls are worth more under systematic jump risk since the stock price drifts upwards at a faster rate than under diversifiable jump risk. However, for short-term out-the-money calls the discounting effect dominates. Therefore the option values given by the SJD formula are lower than the option values given by the IJD formula. The difference between the BS-value and the SJD-value can be explained by the interaction between the *volatility* effect and the *skewness* effect. As long as the representative investor is risk averse and the actual return distribution is symmetric, the risk-neutralized volatility exceeds the actual one. On the other side, the negatively skewed risk-neutralized return distribution causes out-the-money calls with a short time to maturity to be worth more under the BS-model than under the SJD-model.

In the second place, we have examined the historical stock price jump impact on option values. Based on historical parameter estimates the mean differences between BS-values and jump diffusion-values are surprisingly small for the *total* sample period from April 1983 to December 1991. Consequently, neither Merton's (1976a) IJD-model nor Bates

(1990) SJD-model are able to remove the wellknown smile effect of the Black/Scholes model as documented, e. g., in Trautmann (1986,1989) for the Frankfurt Options Market. This confirms the findings of Ball/Torous (1985). For options written on 30 NYSE listed common stocks Ball/Torous (1985) find no operational significant differences between BS-value and IJD-value although statistically significant jumps were present in the underlying stock returns. In their sample the mean percentage deviation from OTM calls is only 2.98%. When using the IJD-model in the *post-crash* period from November 1987 to January 1988 and from November 1989 to January 1990, we found a mean percentage difference between the BS-model and the IJD-model of 6.3% for OTM calls (–0.81% for ATM calls and 0.19% for ITM calls). However, the mean percentage difference between the SJD-value (for $R = 3$) and the BS-value is more substantial: 7.62% for long-term calls in general and even 11.39% for OTM calls, respectively. For puts the differences in values are of comparable magnitude.

In the third place, we have inferred the risk neutral jump intensity, jump size and the risk neutral skewness as well as the risk neutral overall volatility from transaction prices for calls and puts. The time pattern of the magnitude of the implied risk neutral jump size per year suggests that the market participants' hopes of stock market rebounds after dramatic drops are obviously reflected in option prices. Our implicit parameters estimated for the pooled sample including all quoted call and put prices, confirm the findings of Bates (1991) for the US-market. The parameters indicate strong crash fears especially in July 1987 but not during the 2 months immediately preceding the October 1987 crash. While after the market crash the results for the US-market exhibit even stronger crash fears, our implicit parameters reflect mainly rebound hopes. The more recent prices of the early Nineties for options written on German stocks indicate, however, a slight negative skewness of implicit stock return distributions.

Appendix:

Alternative derivation of jump-diffusion option formulae

We use two properties contained in the following lemma to derive jump-diffusion option formulae like the ones of Merton (1976a), Bates (1991) and Amin/Ng (1993).

Lemma:[1] *If the random variable X is normally distributed with parameters μ and σ^2, respectively, and α, β are real constants with $\beta > 0$, then the following properties hold:*

$$E\Phi(X) = \Phi\left(\mu/\sqrt{1+\sigma^2}\right). \tag{A.1}$$

$$Ee^X\Phi\left((X-\alpha)/\beta\right) = e^{\mu+\sigma^2/2}\Phi\left((\sigma^2+\mu-\alpha)/\sqrt{\sigma^2+\beta^2}\right). \tag{A.2}$$

Since Merton's formula (16) and the formula of Bates (24) are equivalent in a formal sense (you need only substitute λ and k in Merton's formula by λ^* and k^*, respectively, to get the SJD-formula of Bates), their derivations are analogous. Therefore we restrict ourselves to explain the transformations of relationship (16) in a more detailed way. The first two rows of relation (16) are identical because of the assumed independence of the Brownian motion from the Poisson process:

$$\begin{aligned} C^{Me} &= e^{-rT}\sum_{n=0}^{\infty} Pr(n\ jumps)\widetilde{E}_0\left[\max(0, S_T - K) \mid n\ jumps\right] && \text{(A.3)}\\ &= e^{-rT}\sum_{n=0}^{\infty}\left[e^{-\lambda T}(\lambda T)^n/n!\right]\widetilde{E}_0\left[\max(0, V_T X_n e^{-\lambda kT} - K)\right] && \text{(A.4)}\\ &= \sum_{n=0}^{\infty}\left[e^{-\lambda T}(\lambda T)^n/n!\right]E_{0,X_n}\left[e^{-rT}\widetilde{E}_0(\max(0, V_T X_n e^{-\lambda kT} - K)\right] && \text{(A.5)}\\ &= \sum_{n=0}^{\infty}\left[e^{-\lambda T}(\lambda T)^n/n!\right]E_{0,X_n}\left[C^{BS}(SX_n e^{-\lambda kT}, K, T, \sigma_D^2, r)\right]. && \text{(A.6)} \end{aligned}$$

where $V_T \equiv S\ exp\{(r-(1/2)\sigma_D^2)T + \sigma_D B_T\}$ denotes the risk neutral terminal stock price in the Black/Scholes world.

[1] The proof of this lemma can be obtained from the authors upon request.

In order to verify the equivalence of the second row of relation (16) with the idiosyncratic jump risk formula, we have now to prove the relationship

$$\begin{aligned} & \left[e^{-\lambda T}(\lambda T)^n/n!\right] E_{0,X_n}\left[C^{BS}(SX_n e^{-\lambda kT}, K, T, \sigma_D^2, r)\right] \\ = & \left[e^{-\lambda' T}(\lambda' T)^n/n!\right]\left[S\Phi(d_{1n}) - e^{-r_n T}K\Phi(d_{2n})\right] \end{aligned} \quad (A.7)$$

for all $n = 0, 1, 2, \ldots$. Since the random variable X_n is defined as $X_n = e^J$, where J is normally distributed, $J \sim N(n\mu_J, n\sigma_J^2)$, we have:

$$\begin{aligned} & E_{0,X_n}\left[C^{BS}(Se^J e^{-\lambda kT}, T, K, \sigma_D^2, r)\right] \\ = \; & E_0\left\{\left[Se^J e^{-\lambda kT}\Phi\left(ln(S/K) + J - \lambda kT + (r + \sigma_D^2/2)T\right)\right]/\sigma_D\sqrt{T}\right\} \quad (A.8) \\ - \; & E_0\left\{\left[Ke^{-rT}\Phi\left(ln(S/K) + J - \lambda kT + (r - \sigma_D^2/2)T\right)\right]/\sigma_D\sqrt{T}\right\} . \end{aligned}$$

We shall calculate these two terms (henceforth termed H_1 and H_2, respectively) one by one. Using (A.1), the second term can be written as

$$\begin{aligned} H_2 \quad & = \; Ke^{-rT}\Phi\left(\frac{\overline{\mu}}{\sqrt{1+\overline{\sigma}_D^2}}\right) \\ \text{with} \quad \overline{\mu} & = \; \left[ln(S/K) + n\mu_J + (r - \lambda k - \sigma_D^2/2)T\right]/\sigma_D\sqrt{T} \\ \text{and} \quad \overline{\sigma}^2 & = \; n\sigma_J^2/\sigma_D^2 T \quad . \end{aligned}$$

Using (A.2), the first term can be written as

$$\begin{aligned} H_1 \quad & = \; Se^{-\lambda kT}e^{n\mu_J + n\sigma_J^2/2}\Phi\left(\frac{n\sigma_J^2 + n\mu_J - \alpha}{\sqrt{n\sigma_J^2 + \beta^2}}\right) \\ \text{with} \quad \alpha & = \; \lambda kT - ln(S/K) - (r + \sigma_D^2/2)T \\ \text{and} \quad \beta & = \; \sigma_D\sqrt{T} \quad . \end{aligned}$$

Finally, putting the pieces back into equation (A.8), we get

$$\begin{aligned} & E_0\left[C^{BS}(Se^J e^{-\lambda kT}, T; K, \sigma_D^2, r)\right] \\ = \; & H_1 - H_2 \end{aligned}$$

$$= Se^{-\lambda kT} e^{n\mu_J + n\sigma_J^2/2} \Phi\left(\frac{1}{\sqrt{n\sigma_J^2 + \sigma_D^2 T}} \left[ln(S/K) + (r - \lambda k + n\mu_J/T + n\sigma_J^2/T + \sigma_D^2/2)T \right] \right)$$

$$-Ke^{-rT} \Phi\left(\left[ln(S/K) + (r - \lambda k + n\mu_J/T - \sigma_D^2/2)T \right] / \sqrt{\sigma_D^2 T + \sigma_J^2} \right)$$

$$= Se^{-\lambda kT} e^{n\mu_J + n\sigma_J^2/2} \Phi\left(\frac{1}{\sqrt{n\sigma_J^2 + \sigma_D^2 T}} \left[ln(S/K) + (r - \lambda k + (n\mu_J + n\sigma_J^2/2)/T + \right. \right.$$

$$\left. \left. (\sigma_D^2 + n\sigma_J^2/T)/2)T \right] \right)$$

$$-Ke^{-rT} \Phi\Big(\left[ln(S/K) + (r - \lambda k + (n\mu_J + n\sigma_J^2/2)/T - (\sigma_D^2 + n\sigma_J^2/T)/2)T \right] /$$

$$\sqrt{\sigma_D^2 T + \sigma_J^2} \Big)$$

$$= Se^{-\lambda kT} e^{n\mu_J + n\sigma_J^2/2} \Phi\left(\frac{1}{\sigma_n \sqrt{T}} \left[ln(S/K) + (r_n + \sigma_n^2/2)T \right] \right)$$

$$-Ke^{-rT} \Phi\left(\left[ln(S/K) + (r_n - \sigma_n^2/2)T \right] / \sigma_n \sqrt{T} \right)$$

$$= e^{-\lambda kT} e^{n\mu_J + n\sigma_J^2/2} \left\{ S\Phi(d_{1n}) - Ke^{-r_n T} \Phi(d_{2n}) \right\}$$

$$= e^{-\lambda kT} (1 + k)^n \left\{ S\Phi(d_{1n}) - Ke^{-r_n T} \Phi(d_{2n}) \right\} \ .$$

This proves the desired relationship (A.7).

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