External Performance Attribution with the Exponential Performance Measure

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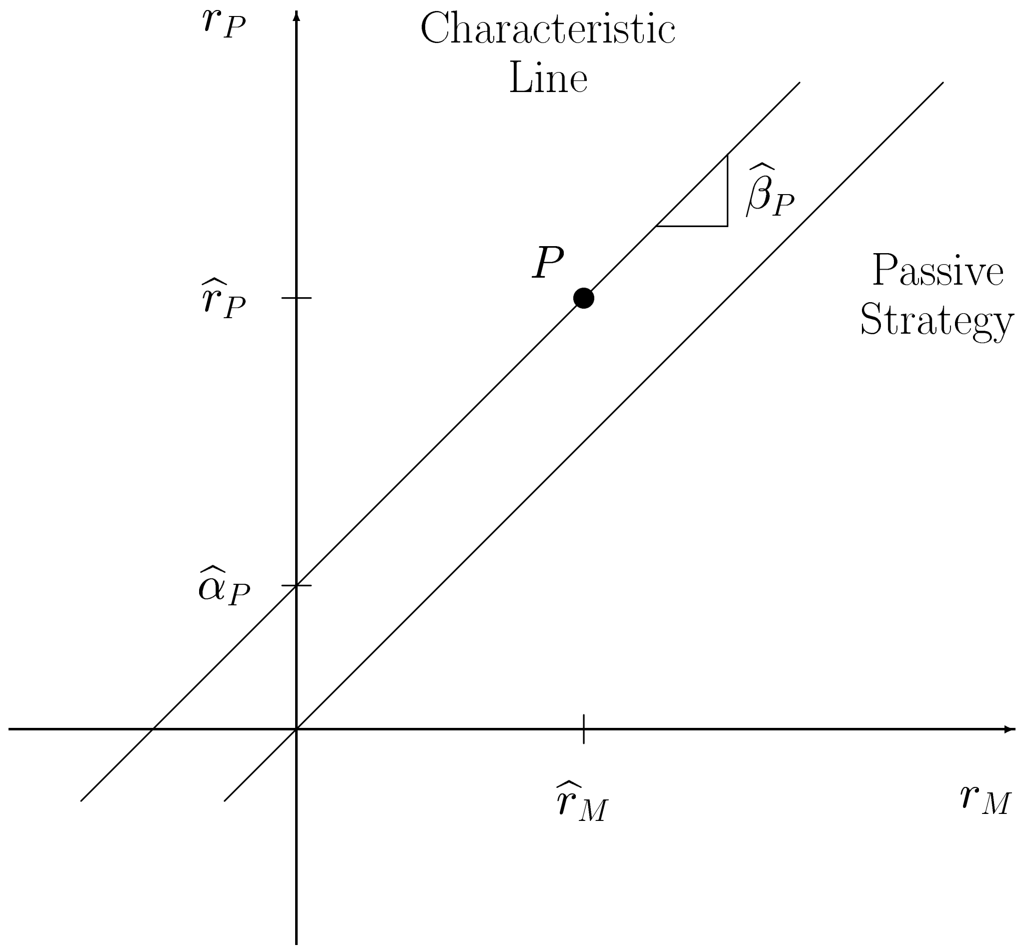
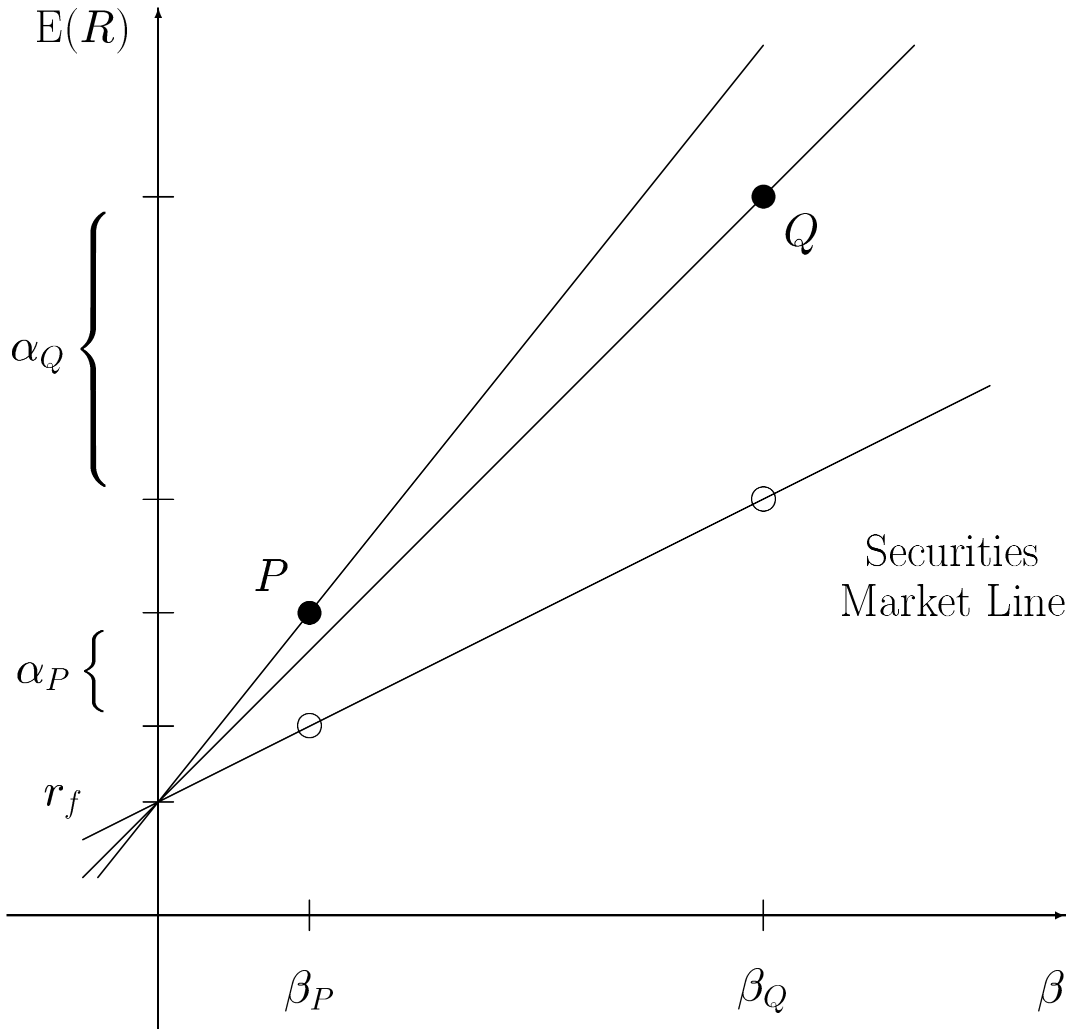
1 Traditional Performance Measurement

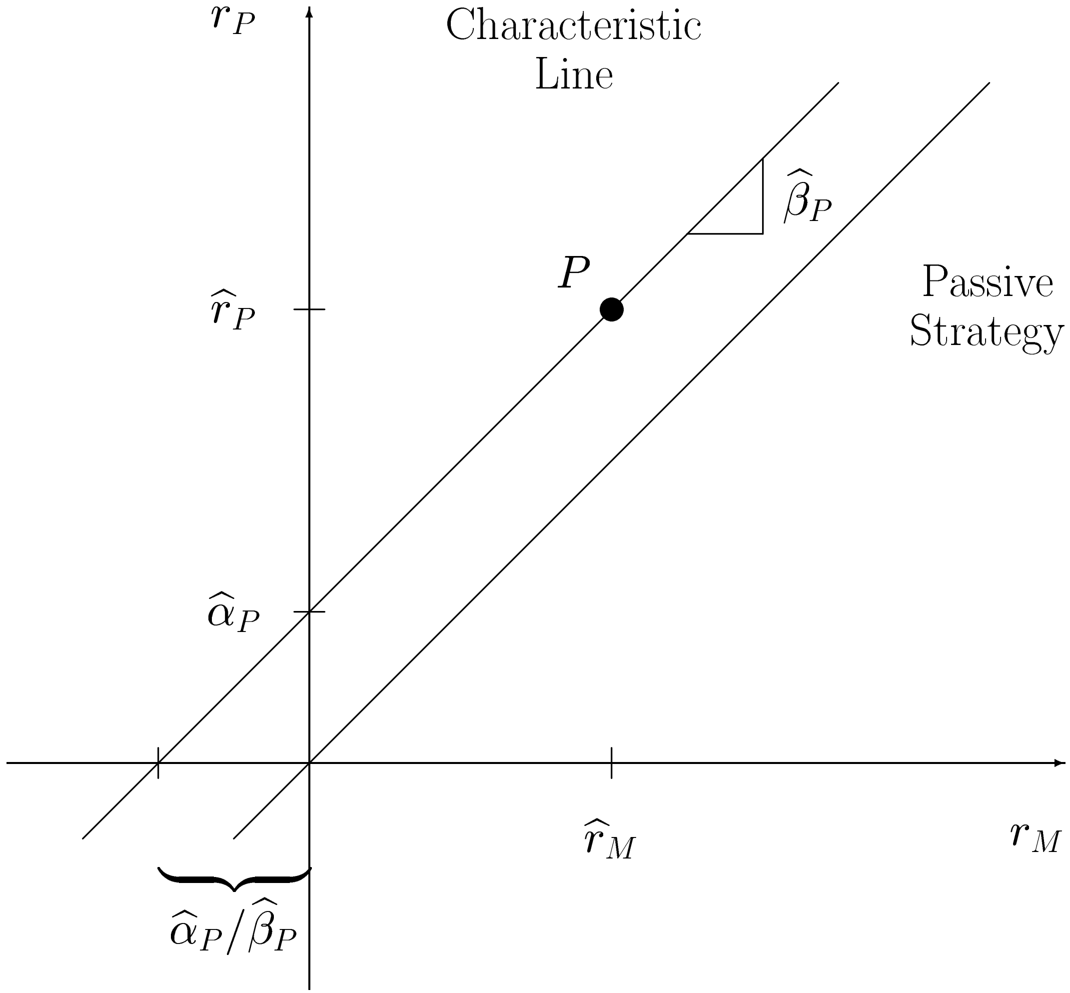
Characteristic Line and Jensen's Alpha.



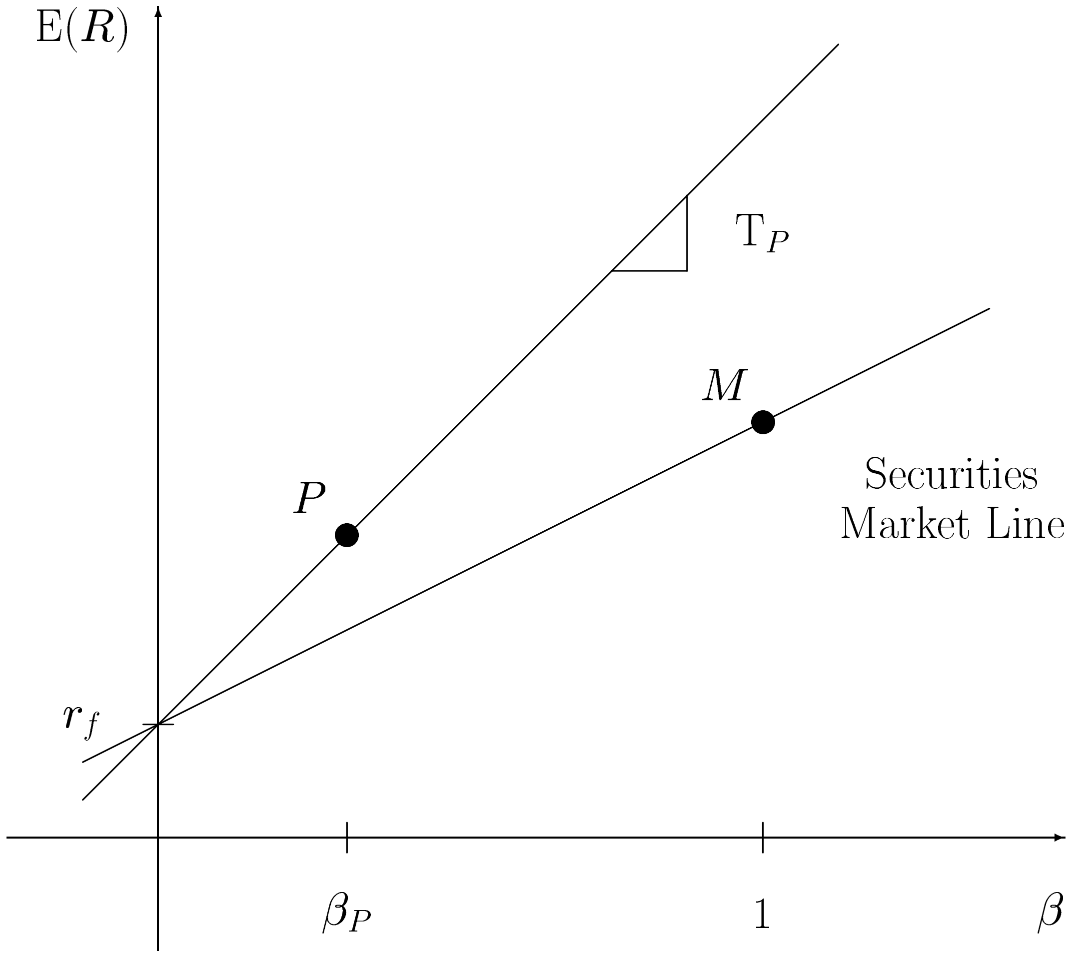
Ambiguous Ranking of Jensen's Alpha.



Jensen's Alpha and Treynor's Ratio.

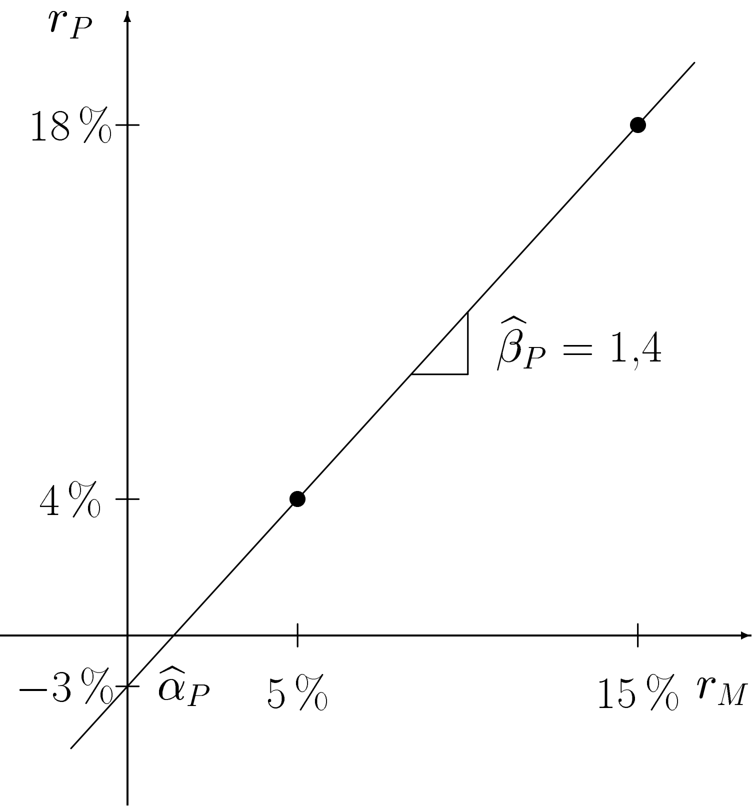


Treynor's Reward to Volatility Ratio.



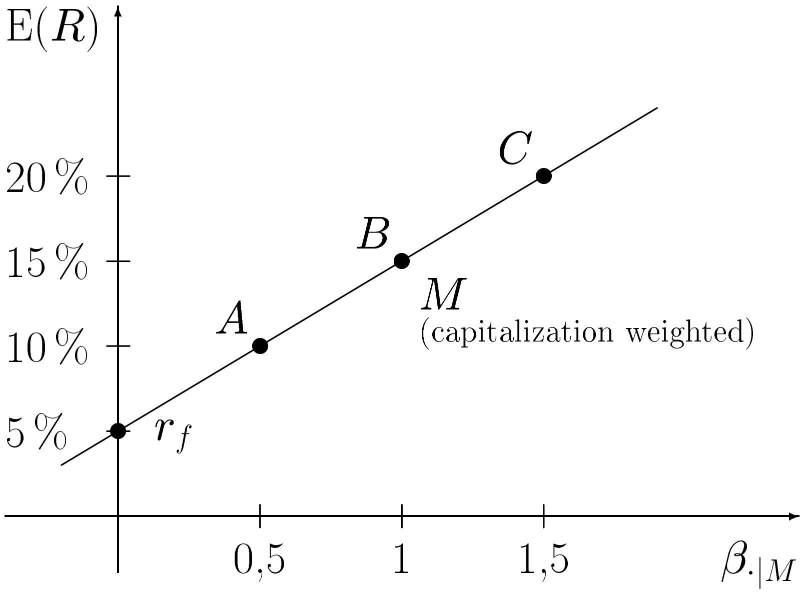
The Timing Bias in Jensen's Alpha.

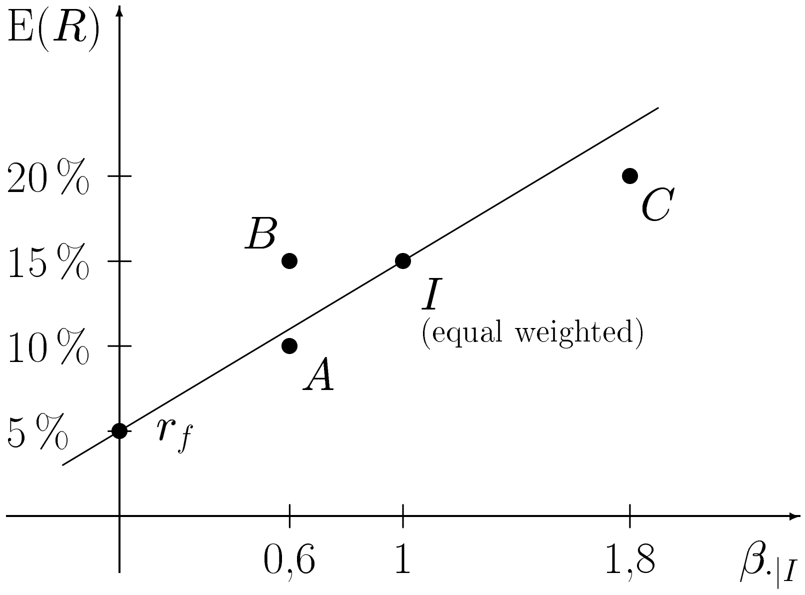
Period	Index Excess Return	Beta	Portfolio Excess Return
1	5 %	0,8	4 %
2	15 %	1,2	18 %



Roll's Critique: Inefficient Market Index.

	A	B	C
E(R)	10 %	15 %	20 %
σ^2	5 %	5 %	15 %
Capitalization	25 %	50 %	25 %





2 Admissible Performance Measures

- An admissible performance measure (APM), as introduced by Chen and Knez (1996), requires:

  (C1) Zero Performance of Passive Strategies,

  (C2) Linearity,

  (C3) Continuity,

  (C4) Non-Triviality,

  and has the representation:

$$\alpha(r_P) = \mathrm{E}(w \cdot r_P) \quad \text{with} \quad \mathrm{E}(w \cdot \boldsymbol{R}_P) = \boldsymbol{k} \in \mathbb{R}.$$

- A positive APM (PAPM) requires in addition:

  (C5) Positivity.

To specify a (P)APM we look at an investor who follows a passive strategy:

$$R(x) \;=\; x \cdot R_M + (1-x)\cdot r_f = x \cdot r_M + r_f .$$

With the convention

$$w \;\equiv\; \mathrm{E}\left(\frac{\partial\, u(R)/\partial\, R}{\mathrm{E}\big(\partial\, u(R)/\partial\, R\big)}\right),$$

the benchmark performance is zero:

$$\mathrm{E}(w \cdot r_M) \quad = \quad \mathrm{E}\left(\frac{\partial\, u(R)/\partial\, R}{\mathrm{E}\big(\partial\, u(R)/\partial\, R\big)}\Big|_{x^*} \cdot r_M\right) \;=\; 0.$$

If the market model holds a performance measure based on the normalized marginal utility of a passive investor fulfills conditions (C1) – (C4).

Assumptions: (A1) Local Market Model: $r_{Pt} = \beta_{Pt} \cdot r_{Mt} + \epsilon_{Pt}$ with $\mathrm{Cov}(r_M, \epsilon_P) = 0$.

(A2) Portfolio beta and and benchmark return are jointly normally distributed.

A portfolio manager shows *timing ability*, if $\mathrm{Cov}(\beta_P, r_M) > 0$.
A portfolio manager shows *selectivity*, if $\mathrm{E}(\epsilon_P) > 0$.

With assumptions (A1) and (A2) a PAPM has the representation:

$$\alpha(r_P) \;=\; \left(1 + \mathrm{E}\left(\frac{\partial}{\partial r_M} w \cdot r_M\right)\right) \cdot \underbrace{\mathrm{Cov}(\beta_P, r_M)}_{\text{Timing}} + \underbrace{\mathrm{E}(\epsilon_P)}_{\text{Selectivity}} .$$

Proof:

$$\alpha(r_P) \;=\; \mathrm{E}(w \cdot r_P) = \mathrm{E}(w \cdot \beta_P \cdot r_M) + \underbrace{\mathrm{E}(w)}_{=1} \cdot \mathrm{E}(\epsilon_P) = \mathrm{Cov}(\beta_P, w \cdot r_M) + \mathrm{E}(\beta_P) \cdot \underbrace{\mathrm{E}(w \cdot r_M)}_{=0} + \mathrm{E}(\epsilon_P).$$

With Stein's lemma this yields:

$$\alpha(r_P) \;=\; \mathrm{E}\left(\frac{\partial}{\partial r_M}(w \cdot r_M)\right) \cdot \mathrm{Cov}(\beta_P, r_M) + \mathrm{E}(\epsilon_P) = \left(1 + \mathrm{E}\left(\frac{\partial}{\partial r_M} w \cdot r_M\right)\right) \cdot \mathrm{Cov}(\beta_P, r_M) + \mathrm{E}(\epsilon_P).$$

Special APMs:

1) Grinblatt and Titman's (1989) Positive Period Weighting Measure:

$$\widehat{\alpha}(r_P) \equiv \sum_{t=1}^{T} w_t \cdot r_{Pt} \quad \text{with} \quad \operatorname*{p\,lim}_{T\to\infty} \sum_{t=1}^{T} w_t \cdot r_{Mt} = 0, \qquad \sum_{t=1}^{T} w_t = 1, \qquad w_t > 0.$$

2) Jensen's (1968) Alpha:

$$\widehat{\alpha}(r_P) = \frac{1}{T} \sum_{t=1}^{T} w_t \cdot r_{Pt} = \widehat{r}_P - \widehat{\beta}_P \cdot \widehat{r}_M \quad \text{with} \quad w_t \equiv \frac{1}{T} \cdot \left(1 - \frac{\widehat{r}_M}{\widehat{\sigma}^2_M} \cdot (r_{Mt} - \widehat{r}_M)\right)$$

The timing bias in Jensen's alpha amounts to:

$$\mathrm{E}\left(\frac{\partial}{\partial r_M} w \cdot r_M\right) \cdot \mathrm{Cov}(\beta_P, r_M) \;=\; -\frac{\mathrm{E}^2(r_M)}{\sigma^2_M} \cdot \mathrm{Cov}(\beta_P, r_M).$$

3) Cumby and Glen's (1990) Performance Measure:

Based on the utility function $u(R) = \frac{1}{1-\theta} \cdot R^{1-\theta}$ with rate of return $R(x) = 1 + x \cdot r_M + r_f$ the timing bias amounts to:

$$\mathrm{E}\left(\frac{\partial}{\partial r_M} w \cdot r_M\right) \cdot \mathrm{Cov}(\beta_P, r_M) \;=\; -\theta \cdot \mathrm{E}\big(R(x^*) \cdot w \cdot r_M\big) \cdot \mathrm{Cov}(\beta_P, r_M).$$

3 The Exponential Performance Measure

We assume (A1), (A2), and

(A3) an uninformed passive investor with exponential utility: $u(R) = -\exp\{-a \cdot R\}$.

Then the optimal fraction invested in the tangency portfolio is:

$$x^* \; = \; \frac{\mathrm{E}(r_M)}{a \cdot \sigma_M^2}.$$

The normalized marginal utility represents the IMRS and reads as follows:

$$\frac{\partial\, u(R)/\partial\, R}{\mathrm{E}\big(\partial\, u(R)/\partial\, R\big)}\Big|_{x^*} \; = \; \exp\left\{-\frac{\mathrm{E}(r_M)}{\sigma_M^2} \cdot \left(r_M - \frac{\mathrm{E}(r_M)}{2}\right)\right\}.$$

Definition: A performance measure with state price density

$$w = \exp\left\{-\frac{\mathrm{E}(r_M)}{\sigma_M^2} \cdot \left(r_M - \frac{\mathrm{E}(r_M)}{2}\right)\right\}$$

is called *exponential performance measure* (EPM).

The EPM $\alpha^{\mathrm{eu}}(r_P)$ shows no timing bias. It has the representation:

$$\alpha^{\mathrm{eu}}(r_P) = \mathrm{Cov}(\beta_P, r_M) + \mathrm{E}(\epsilon_P).$$

Proof:

$$\mathrm{E}(w \cdot r_M) = \mathrm{E}\left(\exp\left\{-\frac{\mathrm{E}(r_M)}{\sigma_M^2} \cdot \left(r_M - \frac{\mathrm{E}(r_M)}{2}\right)\right\} \cdot r_M\right) = \frac{1}{\sqrt{2\pi} \cdot \sigma_M} \int_{-\infty}^{\infty} r_M \cdot \exp\left\{\frac{r_M^2}{2\sigma_M^2}\right\} d\, r_M = 0.$$

Therefore, the timing component is not biased:

$$\mathrm{E}\left(\frac{\partial}{\partial\, r_M} w \cdot r_M\right) = -\frac{\mathrm{E}(r_M)}{\sigma_M^2} \cdot \underbrace{\mathrm{E}(w \cdot r_M)}_{=0} = 0.$$

The EPM can be decomposed in an unbiased timing component and a selectivity component, based only on rate of return data.

The timing component is proportional to the difference of the EPM $\alpha^{\mathrm{eu}}(r_P)$ and Jensen's alpha $\alpha^{\mathrm{qu}}(r_P)$:

$$\mathrm{Cov}(\beta_P, r_M) \;=\; \big(\alpha^{\mathrm{eu}}(r_P) - \alpha^{\mathrm{qu}}(r_P)\big) \cdot \frac{\sigma_M^2}{\mathrm{E}^2(r_M)}.$$

The selectivity component computes as follows:

$$\mathrm{E}(\epsilon_P) \;=\; \alpha^{\mathrm{eu}}(r_P) - \big(\alpha^{\mathrm{eu}}(r_P) - \alpha^{\mathrm{qu}}(r_P)\big) \cdot \frac{\sigma_M^2}{\mathrm{E}^2(r_M)}.$$

Proof:

$$\begin{aligned}
\alpha^{\mathrm{eu}}(r_P) &= \mathrm{Cov}(\beta_P, r_M) + \mathrm{E}(\epsilon_P); \\
\alpha^{\mathrm{qu}}(r_P) &= \left(1 - \frac{\mathrm{E}^2(r_M)}{\sigma_M^2}\right) \cdot \mathrm{Cov}(\beta_P, r_M) + \mathrm{E}(\epsilon_P).
\end{aligned}$$

This yields:

$$\mathrm{Cov}(\beta_P, r_M) = \big(\alpha^{\mathrm{eu}}(r_P) - \alpha^{\mathrm{qu}}(r_P)\big) \cdot \frac{\sigma_M^2}{\mathrm{E}^2(r_M)} \;\;\Rightarrow\;\; \mathrm{E}(\epsilon_P) = \alpha^{\mathrm{eu}}(r_P) - \big(\alpha^{\mathrm{eu}}(r_P) - \alpha^{\mathrm{qu}}(r_P)\big) \cdot \frac{\sigma_M^2}{\mathrm{E}^2(r_M)}.$$

The EPM divided by the average beta,

$$\mathrm{E}(\beta_P) \;=\; \frac{\mathrm{E}(r_P) - \alpha^{\mathrm{eu}}(r_P)}{\mathrm{E}(r_M)},$$

allows a ranking of mutual fund performance.

Proof:

$$\begin{aligned}
\mathrm{E}(r_P) &= \underbrace{\mathrm{E}(\beta_P)\cdot\mathrm{E}(r_M)}_{\text{Benchmark Return}} + \underbrace{\mathrm{Cov}(\beta_P, r_M)}_{\text{Timing}} + \underbrace{\mathrm{E}(\epsilon_P)}_{\text{Selectivity}} \\
&= \underbrace{\mathrm{E}(\beta_P)\cdot\mathrm{E}(r_M)}_{\text{Benchmark Return}} + \underbrace{\alpha^{\mathrm{eu}}(r_P)}_{\text{Performance}} .
\end{aligned}$$

This yields Treynor's ratio with average beta:

$$\frac{\alpha^{\mathrm{eu}}(r_P)}{\mathrm{E}(\beta_P)} \;=\; \frac{\mathrm{E}(r_P)}{\mathrm{E}(\beta_P)} - \mathrm{E}(r_M).$$

4 Empirical Performance Estimates

Traditional Performance Measurement (1975–1994)

Market Index				DAFOX			CDAX			DAX		
No.	Mutual Fund	Excess Return	Volatil.	Alpha	Beta	Degree of Divers.	Alpha	Beta	Degree of Divers.	Alpha	Beta	Degree of Divers.
1	Adifonds	3.03 %	15.78 %	0.05 %	0.93	96.51 %	1.06 %	1.05	94.93 %	1.86 %**	0.86	93.04 %
2	Adiverba	2.62 %	14.60 %	0.07 %	0.80	82.27 %	0.94 %	0.89	80.08 %	1.65 %	0.71	74.16 %
3	Fondak	2.41 %	16.04 %	−0.64 %	0.95	97.45 %	0.39 %	1.07	95.75 %	1.21 %	0.87	92.30 %
4	Fondra	1.23 %	11.20 %	−0.88 %	0.66	95.10 %	−0.17 %	0.74	93.61 %	0.40 %	0.60	91.33 %
5	Plusfonds	2.60 %	13.96 %	−0.01 %	0.82	94.16 %	0.86 %	0.92	94.03 %	1.57 %	0.75	90.60 %
6	Dekafonds	2.00 %	16.49 %	−1.13 %*	0.98	96.90 %	−0.07 %	1.10	95.31 %	0.77 %	0.89	92.97 %
7	Concentra	3.59 %	15.73 %	0.62 %	0.93	96.10 %	1.60 %**	1.05	96.01 %	2.40 %**	0.86	94.83 %
8	DIT-Fonds	3.28 %	12.74 %	1.04 %	0.70	82.87 %	1.76 %	0.80	85.45 %	2.40 %*	0.64	79.57 %
9	Thesaurus	2.78 %	15.56 %	−0.15 %	0.92	96.02 %	0.82 %	1.04	95.87 %	1.62 %*	0.85	93.35 %
10	Investa	3.68 %	15.18 %	0.80 %	0.90	96.72 %	1.77 %**	1.01	95.48 %	2.53 %**	0.83	94.96 %
11	FT Frankfurter	4.36 %	12.72 %	2.00 %**	0.74	93.11 %	2.79 %**	0.83	92.34 %	3.42 %**	0.68	91.59 %
12	MK Alfakapital	2.07 %	13.90 %	−0.47 %	0.80	90.25 %	0.38 %	0.90	89.63 %	1.06 %	0.73	87.67 %
13	Oppenheim Privat	0.58 %	13.88 %	−1.86 %	0.76	83.24 %	−1.06 %	0.87	84.50 %	−0.39 %	0.70	81.27 %
14	SMH-Special I	3.60 %	13.66 %	1.16 %	0.76	85.73 %	1.97 %*	0.86	85.92 %	2.65 %*	0.69	80.46 %
15	Unifonds	3.01 %	15.08 %	0.15 %	0.89	97.04 %	1.12 %	1.00	95.46 %	1.87 %**	0.83	95.26 %
16	Main I-Universal	1.94 %	14.94 %	−0.82 %	0.87	92.48 %	0.09 %	0.98	93.02 %	0.85 %	0.79	88.85 %
17	Universal-Effect	2.02 %	11.05 %	0.33 %	0.53	62.78 %	0.90 %	0.59	62.18 %	1.36 %	0.47	58.40 %
	Average	2.64 %	14.27 %	0.02 %	0.82	90.51 %	0.89 %	0.92	89.97 %	1.60 %	0.75	87.09 %
	DAFOX	3.20 %	16.61 %									
	CDAX	1.89 %	14.69 %									
	DAX	1.38 %	17.78 %									

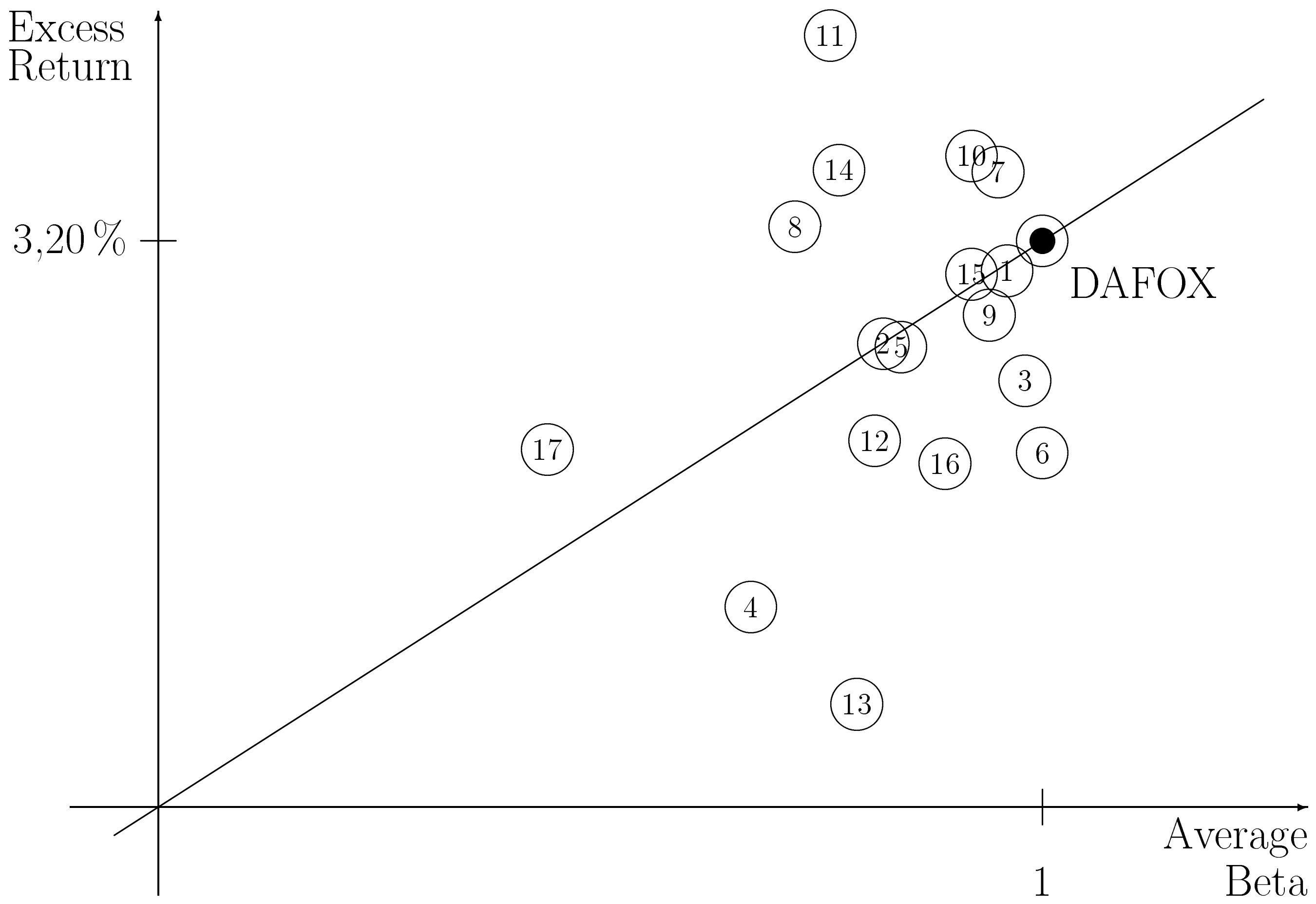
* Significance at the 10 % Level,
** Significance at the 5 % Level.

Performance Ranking According to the EPM (1975–1994)

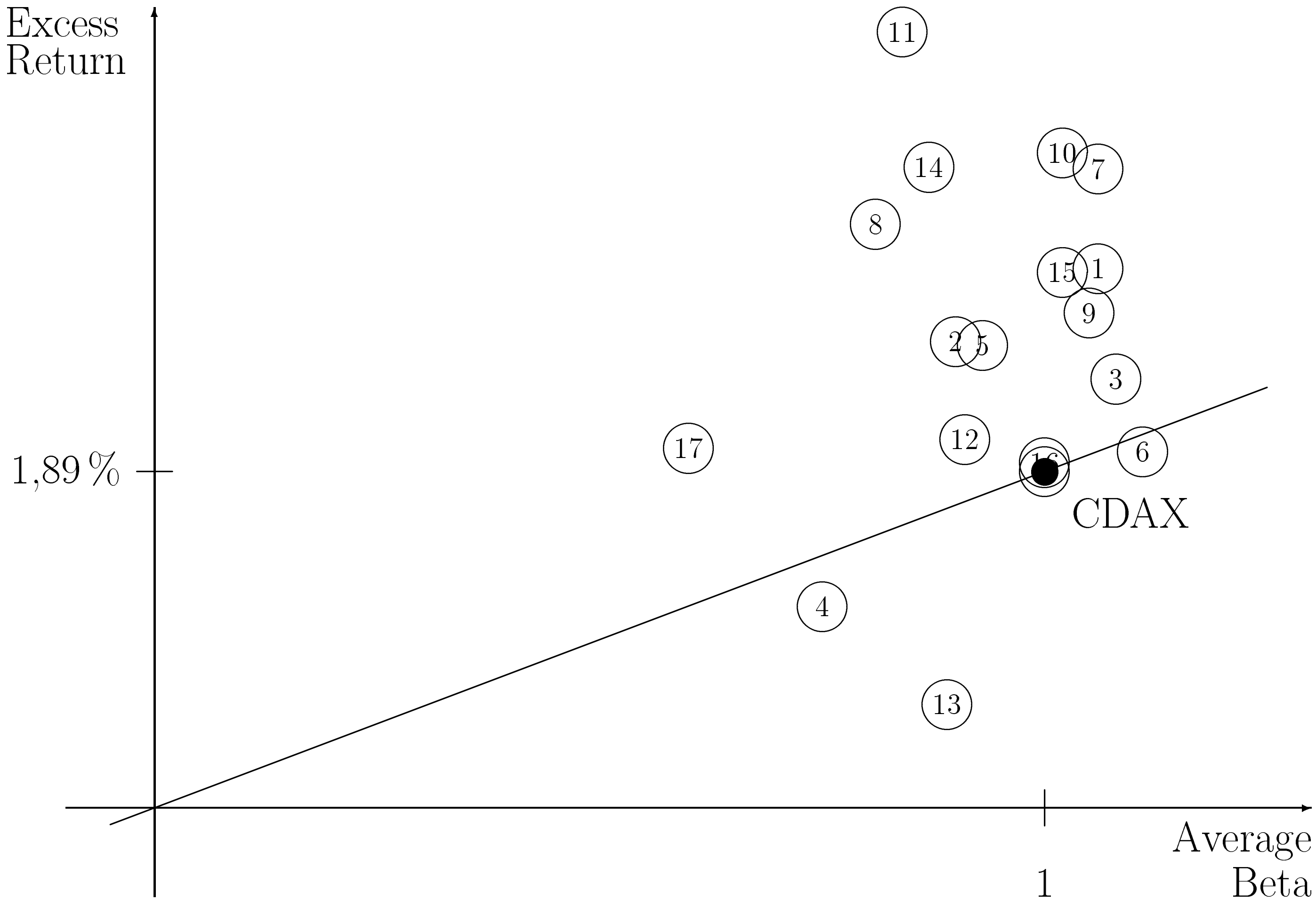
Index	DAFOX			CDAX			DAX		
No.	Performance	Average Beta	Rank	Performance	Average Beta	Rank	Performance	Average Beta	Rank
1	−0.02 %	0.96	9	1.03 %	1.06	9	1.85 %	0.86	9
2	0.00 %	0.82	8	0.91 %	0.90	8	1.64 %	0.71	7
3	−0.72 %	0.98	13	0.36 %	1.08	13	1.21 %	0.87	13
4	−0.93 %	0.67	16	−0.19 %	0.75	16	0.39 %	0.61	16
5	−0.08 %	0.84	10	0.83 %	0.93	10	1.56 %	0.75	10
6	−1.20 %	1.00	15	−0.10 %	1.11	15	0.76 %	0.90	15
7	0.55 %	0.95	5	1.58 %*	1.06	5	2.40 %**	0.87	6
8	0.99 %	0.72	3	1.74 %	0.81	3	2.39 %	0.64	3
9	−0.22 %	0.94	11	0.80 %	1.05	11	1.61 %	0.85	11
10	0.74 %	0.92	4	1.74 %*	1.02	4	2.53 %**	0.84	4
11	1.95 %**	0.76	1	2.77 %**	0.84	1	3.42 %**	0.69	1
12	−0.53 %	0.81	12	0.35 %	0.91	12	1.06 %	0.74	12
13	−1.94 %	0.79	17	−1.09 %	0.89	17	−0.40 %	0.71	17
14	1.13 %	0.77	2	1.96 %	0.87	2	2.65 %	0.69	2
15	0.09 %	0.92	7	1.09 %	1.02	7	1.87 %**	0.83	8
16	−0.90 %	0.89	14	0.06 %	1.00	14	0.84 %	0.80	14
17	0.28 %	0.54	6	0.88 %	0.60	6	1.36 %	0.48	5
Average	−0.05 %	0.84		0.87 %	0.94		1.60 %	0.76	

* Significance at the 10 % Level,
** Significance at the 5 % Level.

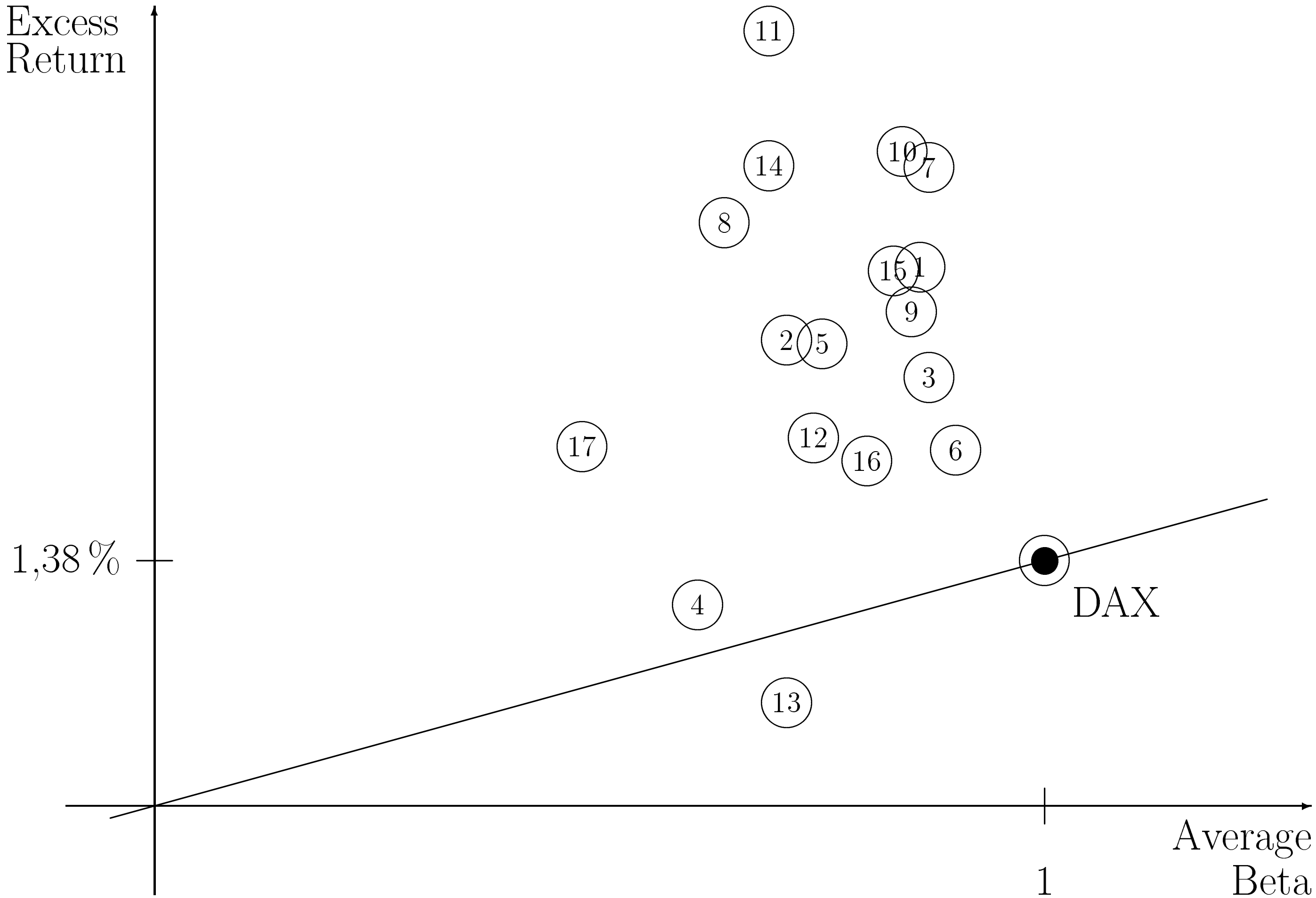
Treynor's Ratio with Average Beta Using the DAFOX.



Treynor's Ratio with Average Beta Using the CDAX.



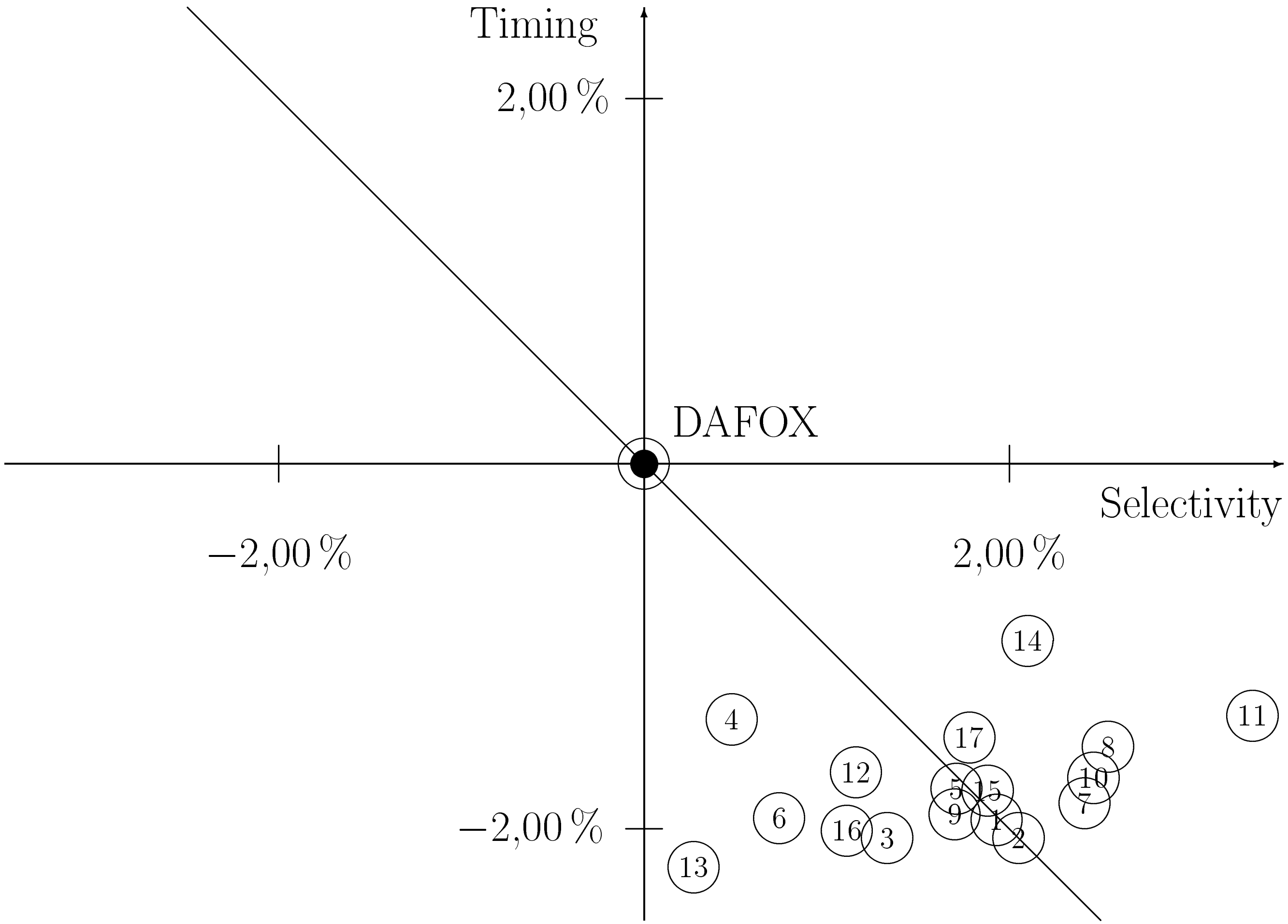
Treynor's Ratio with Average Beta Using the DAX.



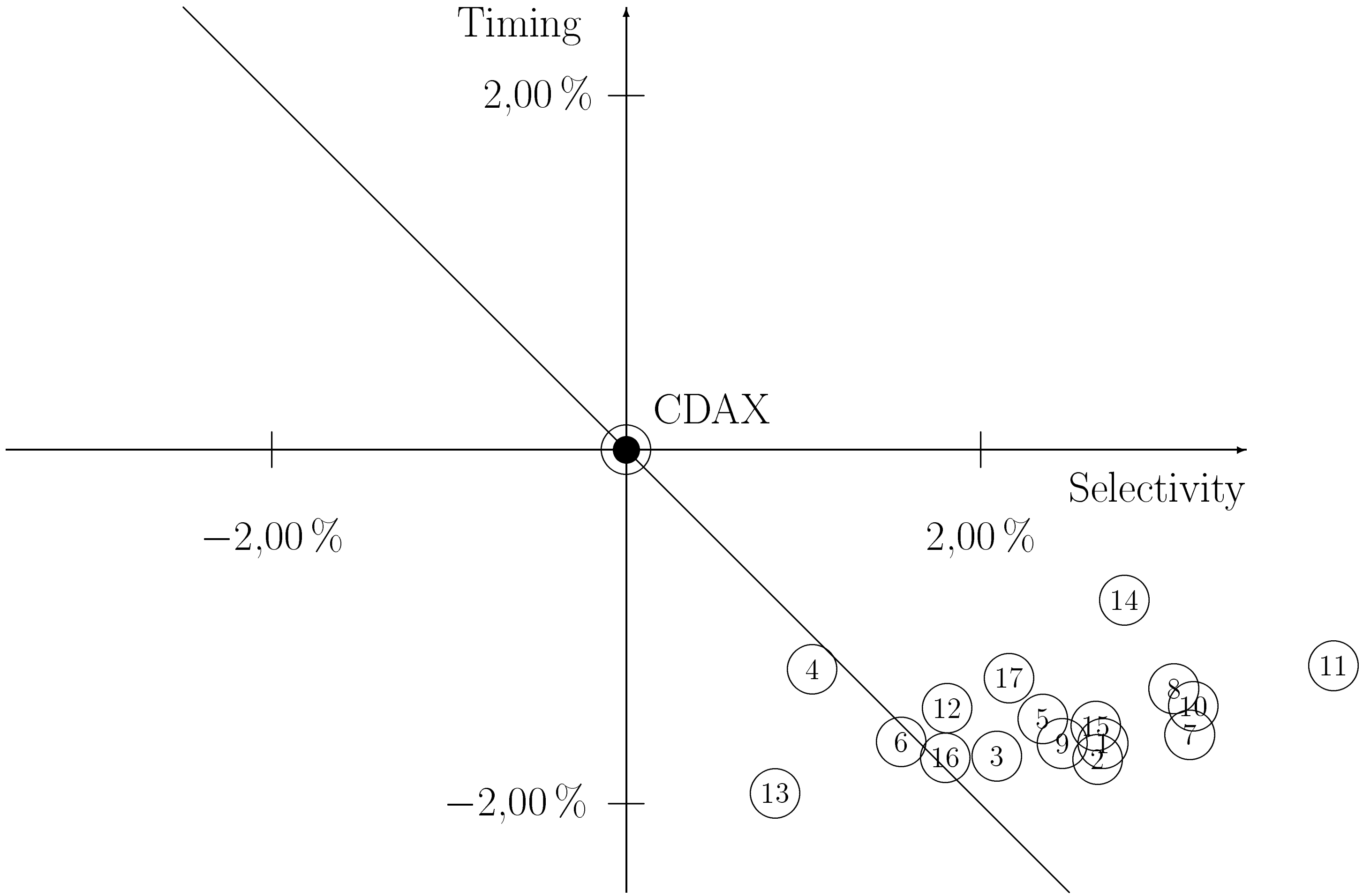
Performance Attribution (1975–1994): Timing and Selectivity

Index	DAFOX		CDAX		DAX	
No.	Selectivity	Timing	Selectivity	Timing	Selectivity	Timing
1	1.93 %	−1.95 %	2.69 %	−1.66 %	2.89 %	−1.04 %
2	2.05 %	−2.05 %	2.66 %	−1.75 %	2.90 %	−1.26 %
3	1.33 %	−2.05 %	2.09 %	−1.73 %	2.31 %	−1.11 %
4	0.47 %	−1.40 %	1.05 %	−1.24 %	1.17 %	−0.77 %
5	1.71 %	−1.78 %	2.35 %	−1.52 %	2.56 %	−0.96 %
6	0.74 %	−1.94 %	1.55 %	−1.65 %	1.76 %	−1.00 %
7	2.41 %	−1.86 %	3.18 %	−1.61 %	3.35 %	−0.95 %
8	2.54 %	−1.55 %	3.09 %	−1.35 %	3.25 %	−0.86 %
9	1.70 %	−1.92 %	2.46 %	−1.66 %	2.64 %	−1.03 %
10	2.46 %	−1.72 %	3.20 %	−1.45 %	3.38 %	−0.85 %
11	3.33 %	−1.38 %	3.99 %	−1.22 %	4.11 %	−0.69 %
12	1.16 %	−1.69 %	1.81 %	−1.46 %	1.98 %	−0.93 %
13	0.27 %	−2.21 %	0.84 %	−1.94 %	0.94 %	−1.33 %
14	2.10 %	−0.97 %	2.81 %	−0.85 %	3.02 %	−0.37 %
15	1.88 %	−1.79 %	2.65 %	−1.56 %	2.81 %	−0.94 %
16	1.11 %	−2.01 %	1.80 %	−1.74 %	1.99 %	−1.15 %
17	1.78 %	−1.50 %	2.16 %	−1.29 %	2.27 %	−0.91 %
Average	1.70 %	−1.75 %	2.38 %	−1.51 %	2.55 %	−0.95 %

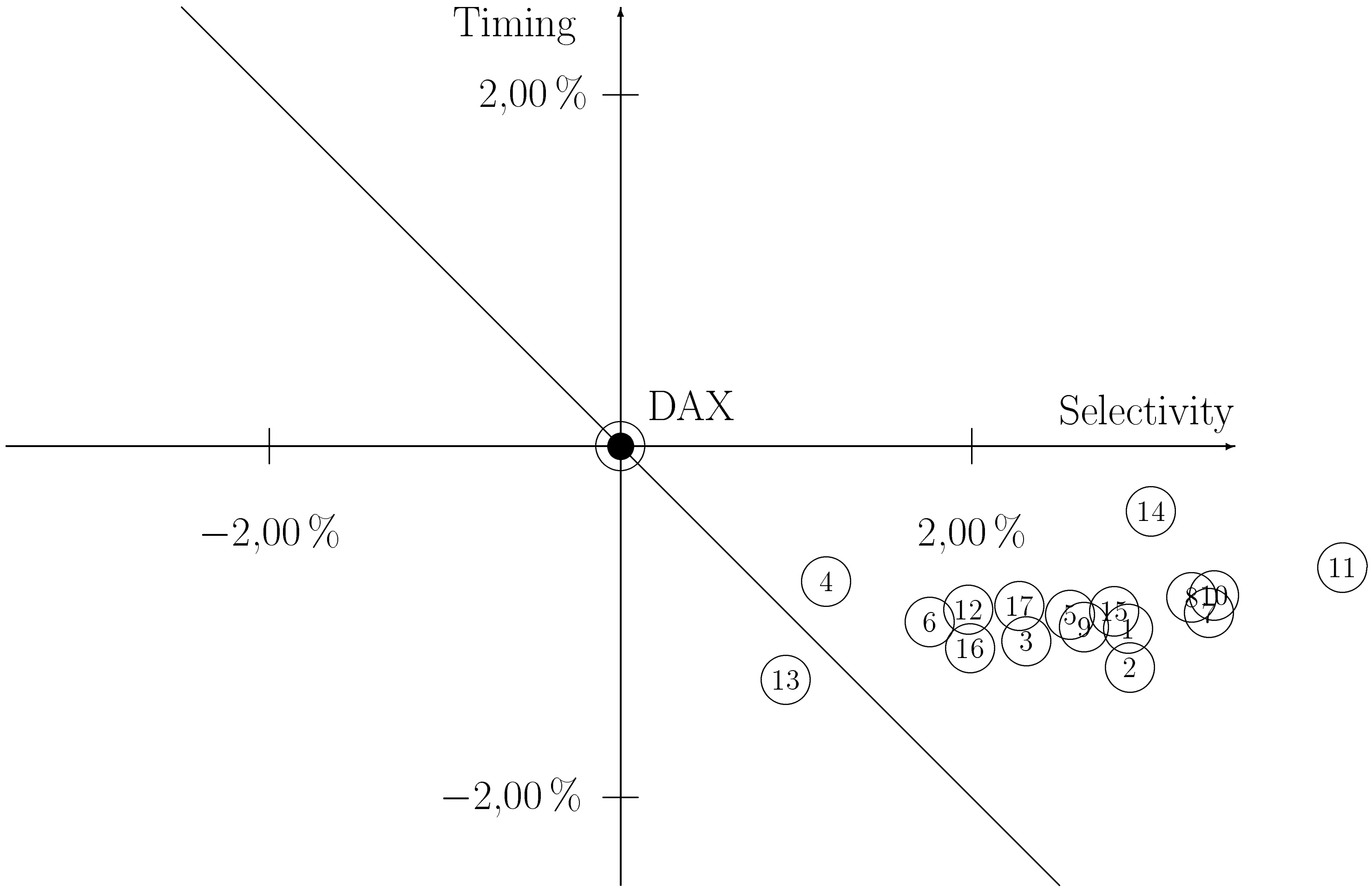
Performance Attribution Using the DAFOX: Timing and Selectivity.



Performance Attribution Using the CDAX: Timing and Selectivity.



Performance Attribution Using the DAX: Timing and Selectivity.

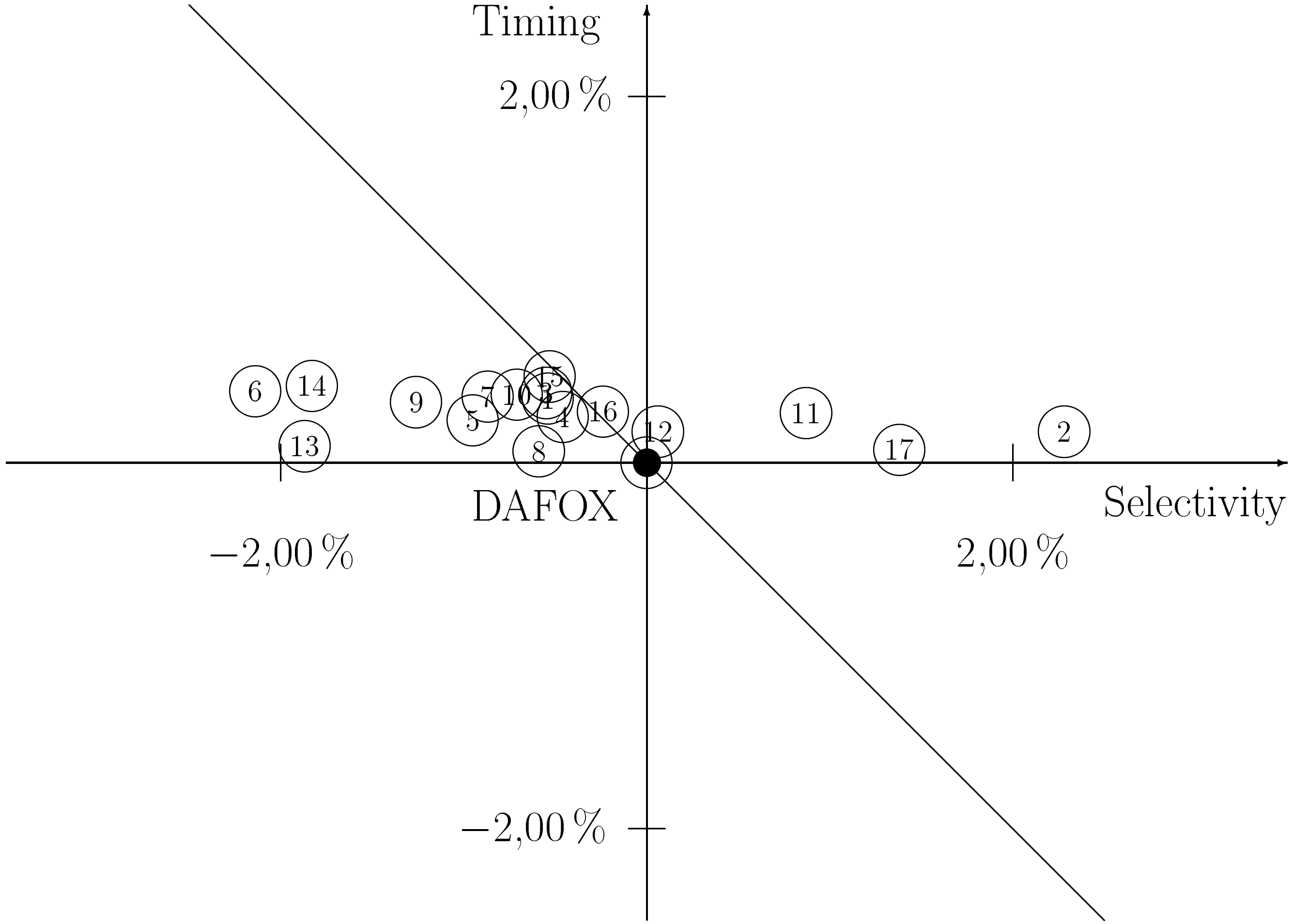


Performance (1975–1984 and 1985–1994)

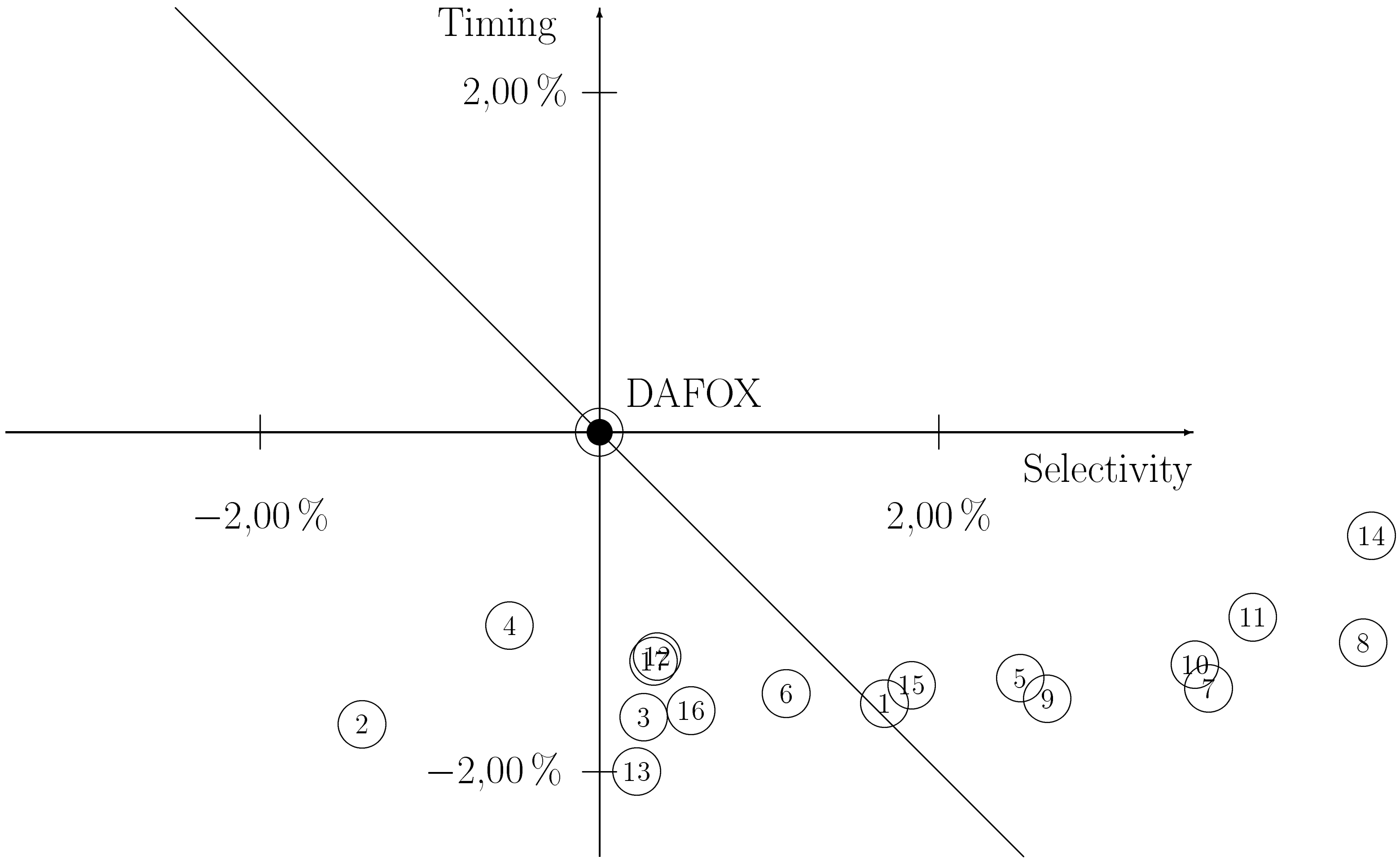
Period	1975–1984					1985–1994				
No.	Performance	Aver. Beta	Rank	Selectiv.	Timing	Performance	Aver. Beta	Rank	Selectiv.	Timing
1	−0.19 %	0.95	8	−0.54 %	0.35 %	0.08 %	0.93	9	1.68 %	−1.60 %
2	2.45 %*	0.76	1	2.28 %	0.17 %	−3.12 %	0.81	17	−1.40 %	−1.72 %
3	−0.16 %	0.94	7	−0.55 %	0.38 %	−1.42 %	0.96	13	0.26 %	−1.68 %
4	−0.21 %	0.67	9	−0.46 %	0.25 %	−1.67 %**	0.66	16	−0.53 %	−1.14 %
5	−0.72 %	0.73	13	−0.95 %	0.23 %	1.02 %	0.85	6	2.48 %	−1.45 %
6	−1.75 %**	0.93	15	−2.14 %	0.39 %	−0.44 %	1.00	10	1.10 %	−1.54 %
7	−0.51 %	0.90	11	−0.87 %	0.36 %	2.08 %**	0.95	5	3.59 %	−1.51 %
8	−0.53 %	0.55	12	−0.59 %	0.06 %	3.26 %**	0.76	2	4.50 %	−1.24 %
9	−0.96 %	0.86	14	−1.26 %	0.33 %	1.06 %	0.95	7	2.64 %	−1.57 %
10	−0.33 %	0.87	10	−0.71 %	0.37 %	2.15 %**	0.92	4	3.51 %	−1.37 %
11	1.13 %	0.79	3	0.87 %	0.27 %	2.76 %**	0.73	3	3.85 %	−1.09 %
12	0.23 %	0.76	4	0.06 %	0.17 %	−0.98 %	0.81	12	0.34 %	−1.32 %
13	−1.78 %	0.70	17	−1.87 %	0.09 %	−1.78 %	0.79	15	0.22 %	−2.00 %
14	−1.41 %	0.74	16	−1.83 %	0.42 %	3.94 %**	0.77	1	4.55 %	−0.61 %
15	−0.06 %	0.93	6	−0.53 %	0.47 %	0.35 %	0.89	8	1.84 %	−1.49 %
16	0.04 %	0.68	5	−0.24 %	0.28 %	−1.10 %	0.94	11	0.54 %	−1.64 %
17	1.45 %	0.58	2	1.38 %	0.07 %	−1.03 %	0.52	14	0.32 %	−1.35 %
Average	−0.19 %	0.78		−0.47 %	0.27 %	0.30 %	0.84		1.73 %	−1.43 %

* Significance at the 10 % Level,
** Significance at the 5 % Level.

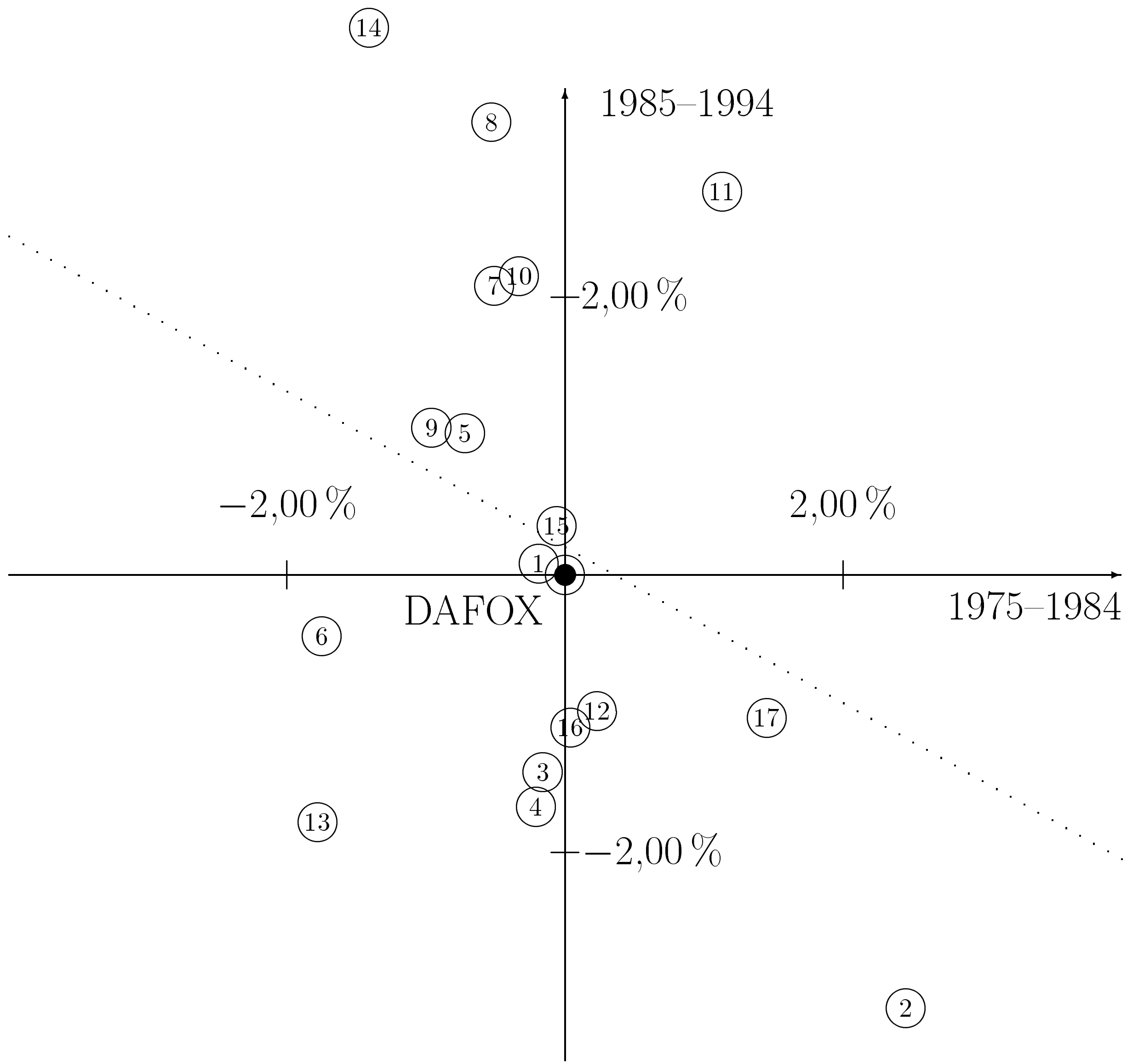
Timing and Selectivity (1975–1984).



Timing and Selectivity (1985–1994).



Performance (1975–1984 Vs 1985–1994).



5 Internal Versus External Performance Attribution

Local Market Model:

$$r_{it} \ = \ \beta_i \cdot r_{Mt} + \epsilon_{it}; \quad i = 1, \ldots, N; \quad t = 1, \ldots, T.$$

Portfolio Excess Return:

$$\widehat{r}_P \ = \ \frac{1}{T} \cdot \sum_{t=1}^{T} r_{pt} = \frac{1}{T} \cdot \sum_{t=1}^{T} \sum_{i=1}^{N} x_{it} \cdot r_{it} = \frac{1}{T} \cdot \sum_{t=1}^{T} \sum_{i=1}^{N} x_{it} \cdot (\beta_i \cdot r_{Mt} + \epsilon_{it}).$$

Portfolio Beta and Residual:

$$\beta_{Pt} = \sum_{i=1}^{N} x_{it} \cdot \beta_i \ \text{ and } \ \epsilon_{Pt} = \sum_{i=1}^{N} x_{it} \cdot \epsilon_{it}.$$

Assume the local market model holds. Then the performance consisting of timing and selectivity (i.e. external performance attribution) equals the Grinblatt and Titman (1993) internal performance measure:

$$\underbrace{\widehat{\mathrm{Cov}}(\beta_P, r_M)}_{\text{Timing}} + \underbrace{\widehat{\epsilon}_P}_{\text{Selectivity}} = \underbrace{\sum_{i=1}^{N} \widehat{\mathrm{Cov}}(x_i, r_i)}_{\text{Internal Performance Measure}} .$$

The Grinblatt and Titmans (1993) internal performance measure uses a passive strategy with average portfolio weights as a benchmark:

$$\underbrace{\sum_{i=1}^{N} \widehat{\mathrm{Cov}}(x_i, r_i)}_{\text{Internal Performance Measure}} = \widehat{r}_P - \underbrace{\widehat{\beta}_P \cdot \widehat{r}_M}_{\text{Benchmark Return}} .$$

Proof:

$$\begin{aligned} \widehat{r}_P - \widehat{\beta}_P \cdot \widehat{r}_M &= \widehat{\mathrm{Cov}}(\beta_P, r_M) + \widehat{\epsilon}_P = \frac{1}{T} \cdot \sum_{t=1}^{T} \left((\beta_{Pt} - \widehat{\beta}_P) \cdot r_{Mt} + \epsilon_{Pt} \right) = \frac{1}{T} \cdot \sum_{t=1}^{T} \left(\sum_{i=1}^{N} x_{it} \cdot (\beta_i \cdot r_{Mt} + \epsilon_{it}) \right) - \widehat{\beta}_P \cdot \widehat{r}_M \\ &= \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} x_{it} \cdot r_{it} - \sum_{i=1}^{N} \widehat{x}_i \cdot \underbrace{\beta_j \cdot \widehat{r}_M}_{\widehat{r}_i} = \sum_{i=1}^{N} \widehat{\mathrm{Cov}}(x_i, r_i). \end{aligned}$$

6 Summary

- Admissible performance measures in general show a timing bias (in a market model setting).
- The exponential performance measure (EPM) proposed equals the sum of timing and selectivity.
- The EPM allows performance attribution relying only on on return data.
- The EPM divided by the average beta allows a ranking of mutual fund performance.
- Return data of German mutual funds from 1975 to 1994 indicate that portfolio managers were good stock pickers and not that good market timers.
- Within the local market model external performance attribution gives the same information on timing and selectivity as Grinblatt and Titman's (1993) internal performance measure.