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Impact of Stock Price Jumps on Option Values

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1. Introduction

Many empirical papers document the fact that the distribution of stock returns exhibits fatter tails than would be expected from a normal distribution. This might explain some of the pricing biases of the Black/Scholes model, which is based on a normal return distribution. Given this result, alternative option pricing models should be based on one of the following three classes of return models: (1) a stationary process, such as a paretian stable or a student's t-distribution, (2) a mixture of stationary distributions, such as two normal distributions with different means or variances, or a mixture of a diffusion and a pure jump process, or (3) a distribution such as a normal distribution with time-varying parameters. Although any of these choices could improve on the fit of the normal distribution, only a few are economically as appealing as the mixed jump diffusion model. According to this model, the total change in the price of a stock is equal to the sum of two components: (1) the normal fluctuations in price due to new information that causes only marginal changes in stock's value ('*diffusion component*'), and (2) the abnormal price changes due to the infrequent arrival of new information that has more than a marginal effect on price ('*jump component*'). This mixed jump diffusion model was first studied by Press (1967) and incorporated into the theory of option valuation by Merton (1976a). Although a considerable number of papers report a statistically significant jump component in stock returns as well as in index returns (Jarrow/Rosenfeld 1984, Ball/Torous 1985, Akgiray/Booth/Loistl 1989, Beinert/Trautmann 1991), few papers have investigated the effect of jumps in the underlying stock price process on option values. In the first published empirical paper on this subject, Ball/Torous (1985) find no operationally significant pricing differences between the Black/Scholes model and Merton's *idiosyncratic* jump risk model when pricing options on NYSE stocks. By contrast, Bates (1991) finds that a *systematic* jump risk model fits the actual data markedly better than the Black/Scholes model in the case of the transaction prices of S&P 500 futures options.

The purpose of this paper is threefold. First, we provide an economic rationale for the differences between Black/Scholes values and jump model values when jump risk is systematic and idiosyncratic, respectively. Second, we use a comprehensive sample of stock options traded at the *Frankfurt Options Market*, between March 1983 and June 1990, and at the *Deutsche Terminbörse* between January 1990 and December 1991, to examine the historical stock price jump impact on option values. Third, we take the systematic jump risk model to infer the stock price distributions implicit in observed option prices before, during, and after crash periods. The paper is organized as follows. Section 2 presents different formulae for valuing stock options when the underlying stock process includes systematic and idiosyncratic jumps, respectively. We also compare the model values for representative parameters. Section 3 examines the impact of stock price jumps on option values when using historical parameter estimates. The stock price distributions implicit in option prices according to the systematic jump risk model of Bates (1991) are presented in section 4. Section 5 concludes the paper.

2. Option valuation when jump risk is present

Options are usually priced as the discounted expected value of their future payoffs where the expectation is taken over the risk-neutral, rather than the true, distribution of the underlying asset. As long as the option's payoff can be replicated by a dynamic trading strategy in the underlying asset and a riskless bond, the equivalent risk-neutral distribution can be derived via no-arbitrage conditions. Unfortunately such a replication is *not* possible if the stock price, S_t, follows a general jump-diffusion process of the form

$$\frac{dS_t}{S_{t-}} = \alpha_D\, dt + \sigma\, dB_t + L\, dN_t \,, \qquad (1)$$

where

- α_D — drift rate of the diffusion component,
- σ — volatility of the diffusion component, $\sigma>0$,
- B — $\{B_t;\ t\geq 0\}$ is a standard Brownian motion,
- N — $\{N_t;\ t\geq 0\}$ is a Poisson counting process (independent of B) with parameter $\lambda>0$, denoting the expected number of jumps per unit time,
- L — $(S_t - S_{t-})/S_{t-}$ represents the percentage change in the price of a stock due to a jump at time t, with $\ln(1+L) \sim N(\mu_J, \sigma_J^2)$, and $k \equiv E(L)=e^{\alpha J}-1$ with $\alpha_J = \mu_J + (1/2)\,\sigma_J^2$.

Deriving the appropriate risk-neutral probability distribution requires additional restrictions on distributions and/or on preferences. Merton's (1976a) idiosyncratic jump risk model (henceforth *IJD-model*) assumes, for instance, that jump risk is *diversifiable*. Under this simplifying assumption Merton derives an option valuation formula that is independent of investors' preferences. Unfortunately, several papers report that Merton's assumption of diversifiable jump risk might not be realistic. For instance, Beinert/Trautmann (1991) report that DAX returns have a statistically significant jump component implying that jump risk must be deemed to be systematic in the German stock market. If jump risk is assumed to be systematic, then all stock prices and the market index jump simultaneously, albeit by possibly different amounts. To price the additional risk when jump risk is *systematic*, Bates (1991) extends the equilibrium-based model of Cox/Ingersoll/Ross (1985), while the alternative model of Amin/Ng (1993) is an extension of the pure exchange model proposed by Rubinstein (1976). Both of the resulting formulae depend on risk preferences. The following analysis is based on Bates' version of the systematic jump risk model (henceforth *SJD-model*). We first present the valuation formulae of Bates and Merton and then compare the two models with the Black/Scholes model (henceforth *BS-model*).

2.1 Systematic jump risk call formulae

Bates (1991) assumes the existence of a representative investor who seeks to maximize his expected utility of lifetime consumption and whose optimal invested wealth, W_t, follows a jump diffusion process

$$\frac{dW_t}{W_{t-}} = \left(\alpha_W - \lambda\, k_W - \frac{Y_t}{W_t} \right) dt + \sigma_W B_t + L_W\, dN_t\,, \qquad (2)$$

where Y_t represents the optimal consumption at time t. One plus the percentage wealth jump sizes, $(1+L_w)$, is log-normally distributed, $\ln(1+L_w) \sim N(\mu_{J,W}, \sigma^2_{J,W})$. Like in the IJD-model jump sizes are independent and identically distributed. The covariance between the stock returns and the change in the optimal invested wealth, conditional on no jumps, is given by $\sigma_{D,SW}$, while the covariance between jumps is defined as $Cov\left(\ln(1+L), \ln(1+L_W)\right) = \sigma_{J,SW}$. Under the assumption that the investor's direct utility function is given by $U(Y_t) = Y_t^{-R} / (1-R)$, with R as coefficient of relative risk aversion[1], Bates derives the following 'risk-neutral' valuation formula for European calls: [2]

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[1] Risk-neutrality obtains for R=0, and R→1 results in logarithmic preferences.

[2] Although formula (3) is written in a 'risk-neutralized' fashion, the call value depends via $\lambda*$ and $k*$ on the risk aversion parameter R. Like in the pure diffusion environment, there is no known analytic solution for American puts when the underlying stock obeys a jump diffusion process. Bates (1991) generalizes McMillan's (1987) quadratic approximation to

$$C^{SJD} = e^{-rT} \sum_{n=0}^{\infty} \frac{e^{-\lambda^* T}(\lambda^* T)^n}{n!} \quad [\, S\, e^{r_n T}\, \Phi(d_{1,n}) \; - \; K\, \Phi(d_{2,n}) \,] \qquad (3)$$

,

where

$$d_{1,n} \equiv \left[\ln(S/K) + r_n T + \left(\sigma^2 T + n\,\sigma_J^2\right)/2\right] / \left(\sigma^2 T + n\,\sigma_J^2\right)^{1/2},$$

$$d_{2,n} \equiv d_{1,n} - \left(\sigma^2 T + n\,\sigma_J^2\right)^{1/2},$$

$$\lambda^* \equiv \lambda \exp\left(-R\,\alpha_{W,J} + \left(R(1+R)\sigma_{J,W}^2\right)/2\right),$$

$$r_n \equiv r - \lambda^* k^* + \left(n\,\mu_J^*\right)/T,$$

$$k^* \equiv \exp\left(\alpha_J^*\right) - 1,$$

$$\alpha_J^* \equiv \alpha_J - R\,\sigma_{J,SW},$$

$\Phi(.) \equiv$ standard normal cumulative density function.

Within this equilibrium framework the drift of the stock price process, α, and the riskless interest rate, r, are endogenously determined. The terminal stock price, $\widetilde{S}_T$, resulting from the 'risk-neutral' process is given by:

$$\widetilde{S}_T = S_0 \exp\left\{\left(r - \frac{1}{2}\sigma^2 - \lambda^* k^*\right)T + \sigma\, B_T + \sum_{i=1}^{N_T} \ln(1 + L^*)\right\}. \qquad (4)$$

Bates' option formula (3) can be specialized to all known option formulae proposed for a jump diffusion framework. Assuming an investor with logarithmic utility (R=1), the above formula collapses to that presented by Ahn (1992). If the call is written on a proxy of the market portfolio, we have $\sigma_{J,SW} = \sigma_{D,SW} = 1$, $\sigma_W = \sigma$, $\mu_W = \mu$, $\sigma_{J,W} = \sigma_J$ and $\mu_{J,W} = \mu_J$, and we obtain the formula proposed by Naik/Lee (1990). Furthermore, if the jump risk is *diversifiable* (idiosyncratic), i. e., $\alpha_{J,W} = \sigma_{J,W} = \sigma_{J,WS} = 0$, *or* the representative investor is risk-neutral, i. e., R=0, the call formula (3) collapses to Merton's idiosyncratic jump risk formula for European Calls:

$$C^{IJD} = \sum_{n=0}^{\infty} \frac{e^{-\lambda(1+k)T}\left(\lambda(1+k)T\right)^n}{n!} \left[S\,\Phi\left(d_{1,n}\right) - K e^{-rnT}\,\Phi\left(d_{2,n}\right)\right]. \qquad (5)$$

American option values for jump-diffusion process. Amin (1993) approximates the jump-diffusion processes with sufficient accuracy using a Markov chain.

Finally, in the special case of a Geometric Brownian Motion governing the underlying stock price, i.e., for $\lambda = 0$ or $\lambda \to \infty$ (while the jump coefficients tend to zero), we have $d_{1,n} = d_1 \equiv [\ln(S/K) + (r + \sigma^2/2)T]/\sigma\sqrt{T}$, and $d_{2,n} = d_2 \equiv d_1 - \sigma\sqrt{T}$ and the formulae (3) and (5) specialize to the Black/Scholes formula for European calls:

$$C^{BS} = S\,\Phi(d_1) - e^{-rT} K\,\Phi(d_2). \tag{6}$$

2.2 A comparison of Black/Scholes-values and jump-diffusion values

The impact of jumps on option values is illustrated for an investor who erroneously adopts the BS-formula, when either the SJD-formula or the IJD-formula should be used. We also demonstrate the valuation errors which risk averse investors using the IJD-formula take into account when jump risk is systematic and the SJD-formula with $R>0$ should be used.[3] We calculate model values for calls written on a stock index. The annualized volatility estimate of the pure diffusion process, VOLA, corresponds to the total volatility of the jump-diffusion process, VOLA=$\sqrt{\sigma^2 + \lambda(\mu_J^2 + \sigma_J^2)}$ = 30%, the strike price is K = 100, the riskless interest rate is r = 10%, and 80% of the total variance is due to the jump component, i. e., $\gamma \equiv \lambda(\mu_J^2 + \sigma_J^2) / (\sigma^2 + \lambda(\mu_J^2 + \sigma_J^2)) = 0.80$. Besides a symmetric distribution of actual stock returns (when $\mu_J = 0$), we allow for negative skewness (when $\mu_J = -0.20$ or $\mu_J = -0.10$) and positive skewness (when $\mu_J = 0.20$ or $\mu_J = 0.10$) in the actual return distribution.[4] Since substantial differences between the BS-value and the SJD-value occur only for a low jump intensity, we choose the rather low jump intensity of $\lambda = 1$, that is, on average one jump per year.[5]

In Table 1 the numbers in *bold face* correspond to a *symmetric* distribution of actual stock returns. The column headed 'BS' gives BS-values for short-term (τ = 1/12, i. e., one month) and long-term (τ = 1, i. e., one year) out-the-money (OTM) calls, at-the-money (ATM) calls, and in-the-money (ITM) calls,

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[3] The same error occurs if risk averse investors erroneously value options as if they were risk-neutral when recognizing that jump risk is systematic.

[4] Recall that in the BS-model the actual, as well as the risk-neutral, return distribution are symmetric. In the case of the Poisson-jump diffusion model, skewness specializes to [λ μJ (μJ2 + 3 σJ2)] / [(T)1/2 [(σ2 + λ(μJ 2 + σJ2)]3/2]; and, for R > 0 the risk-neutral return distribution will always be skewed.

[5] For a jump intensity of λ = 100, Trautmann/Beinert (1995) demonstrate that the deviations of the SJD-values from BS-values are negligible, even for short-term ATM options exhibiting the largest absolute differences in value.

respectively. The columns headed 'IJD' and 'SJD' contain the corresponding call values according to the IJD-model and to the SJD-model (for R = 3), respectively. While for the BS-model and IJD-model there is no difference between the shapes of the actual and the risk-neutral return distributions, the shape of the risk-neutral return distribution underlying the SJD-model depends on R as depicted in column 1 of Table 2 for options with one year to maturity. Since $\mu_J^* = \mu_J - R\,\sigma_J^2 < 0$ for $\mu_J = 0$ and $R > 0$, the risk-neutral return distribution is skewed to the left. The columns headed 'Diff' contain the difference between the corresponding jump diffusion model value according to the *symmetric actual* return distribution and the model value according to *skewed* distributions. Table 2 depicts the differences between the BS-values and the IJD-values (middle column) as well as the differences between the BS-values and SJD-values for R = 3 (right hand column) with respect to different money ratios, for options with one month and one year to maturity, respectively. As distinguished from Table 1, we consider in addition to a symmetric actual return distribution, only one negatively skewed (when $\mu_J = -0.20$) and only one positively skewed (when $\mu_J = 0.20$) actual return distribution.

Turning first to the *symmetric* return distributions; a comparison of BS-values and IJD-values shows that, for options with one month to maturity, the BS-value exceeds the IJD-value for ATM options, while the opposite is true for OTM options and for ITM options. But only in the case of the OTM options can the percentage difference[6] almost amount to 100%. This v-shaped relationship between the IJD-value and the BS-value was first documented by Merton (1976b). For options with one year to maturity, all BS-values exceed the IJD-values. Clearly, these effects are explained by the shape of the underlying risk-neutral return distributions, as depicted in column 1 of Table 2 .

In the case of a symmetric return distribution, the SJD-value exceeds the BS-value, except for short-term OTM options. The interaction between the so-called *volatility effect* and *skewness effect* may explain this. For index options, the risk-neutralized volatility[7], $\text{VOLA}^* \equiv \sqrt{\sigma^2 + \lambda^*\left((\mu_J^*)^2 + \sigma_J^2\right)}$, exceeds the actual volatility for $\mu_J < (1/2)\,R\sigma_J^2$ and $R > 0$. For $\mu_J > (1/2)\,R\,\sigma_J^2$ and $R > 0$ the opposite is true. Since we have $\text{VOLA}^* > \text{VOLA}$ for $\mu_J = 0$, the SJD-value exceeds the BS-value. The skewness effect describes the shift to the left of the risk-neutralized return distribution if $\mu_J < (1/2)\,R\sigma_J^2$, and the shift to the right for $\mu_J > (1/2)\,R\,\sigma_J^2$. Since we have $\mu_J^* < \mu_J = 0$, the negatively skewed risk-neutral return distribution causes the SJD-value of OTM calls to be smaller, while the SJD-value for ATM calls and ITM calls are higher by comparison with a symmetric risk-neutral return distribution. The skewness effect leads to a smaller SJD-

[6] The mean percentage difference is defined as the mean of [(CIJD - CBS) / CIJD] 100.
[7] Recall that $\lambda^* = \lambda\exp(-R\,\mu J + \sigma J2\ \ R2/2)$ and $\mu J^* = \mu J - R\,\sigma J2$.

value, compared to the BS-value for OTM calls with a short time to maturity, because of the smaller risk-neutral probability to end up in-the-money.

Table 1: Calls written on the market index: Influence of the jump component (Fixed parameters:[a] K=100, r=10%, λ=1, γ=0.8, VOLA=30%)

Parameters of the RN[b] distribution	BS	LJD (equals SJD-value for R = 0)					SJD for R=3 (Strong risk aversion)				
[c]λ^*	**0**	1.00	1.00	**1.00**	1.00	1.00	2.10	1.78	**1.38**	0.98	0.63
[d]μ_J^*	**0**	−0.20	−0.10	**0.00**	0.10	0.20	−0.30	−0.29	**−0.22**	−0.09	0.10
[e]$\lambda^* k^*$	**0**	−0.17	−0.07	**0.03**	0.14	0.24	−0.52	−0.41	**−0.23**	−0.06	0.08
[f]VOLA*	**0.30**	0.30	0.30	**0.30**	0.30	0.30	0.52	0.53	**0.43**	0.29	0.21
Option specification	C^{BS}	Diff[g]	Diff	C^{LJD}	Diff	Diff	Diff[g]	Diff	C^{SJD}	Diff	Diff
OTM τ=1/12	**0.02**	−0.30	−0.18	**0.31**	0.18	0.26	−0.09	−0.05	**0.09**	0.04	0.08
(S=80) τ=1	**5.76**	−1.40	−1.08	**5.52**	1.08	1.50	3.21	2.90	**7.19**	−2.80	−3.52
ATM τ=1/12	**3.87**	0.08	−0.01	**2.74**	0.07	0.19	1.22	0.80	**3.41**	−0.72	−1.05
(S=100) τ=1	**16.73**	0.48	0.15	**15.89**	0.29	0.79	3.12	3.01	**20.10**	−4.29	−6.88
ITM τ=1/12	**20.88**	0.26	0.15	**21.11**	−0.16	−0.27	0.71	0.64	**21.94**	−0.71	−1.08
(S=120) τ=1	**32.41**	0.70	0.50	**32.19**	−0.58	−0.81	2.69	2.70	**36.12**	−3.64	−6.02

[a] The drift parameters α_D and α_{DY} are determined endogenously by the Euler conditions.

[b] RN denotes the *risk-neutral* return distribution.

[c] The 'risk-neutral' jump intensity is defined as $\lambda^*=\lambda\ \exp(-R\mu_J + R\ \sigma_J^2/2)$.

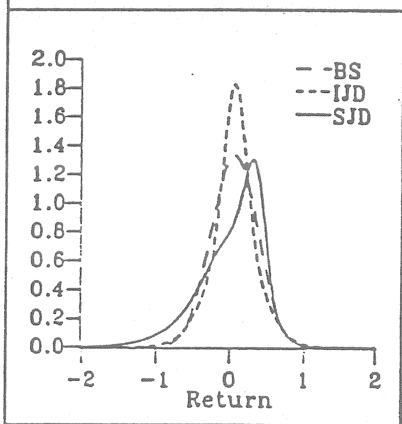
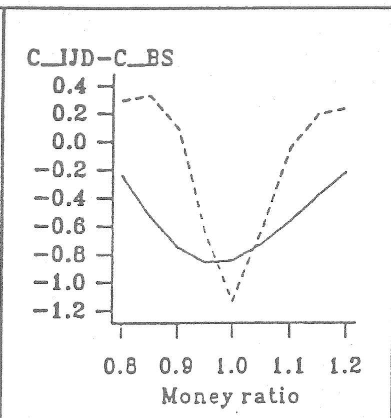
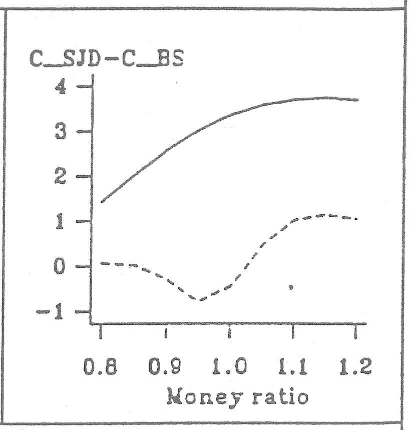
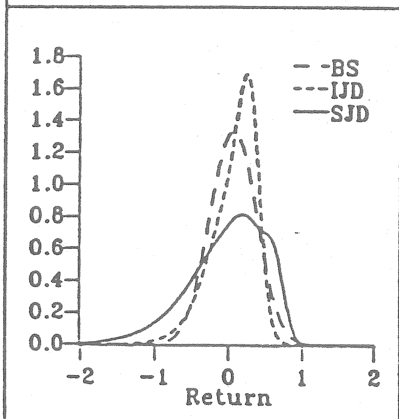
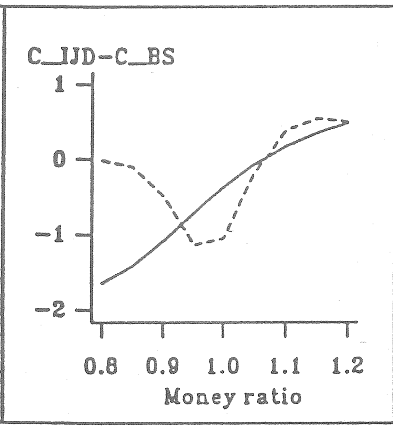
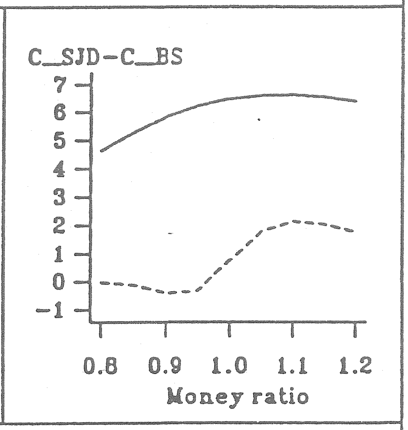
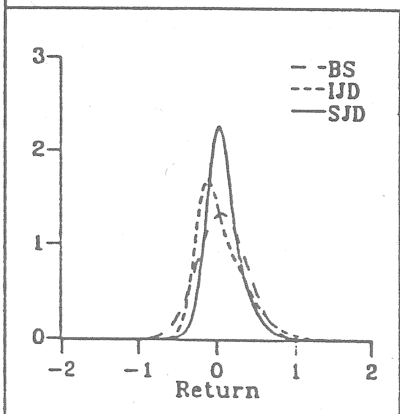
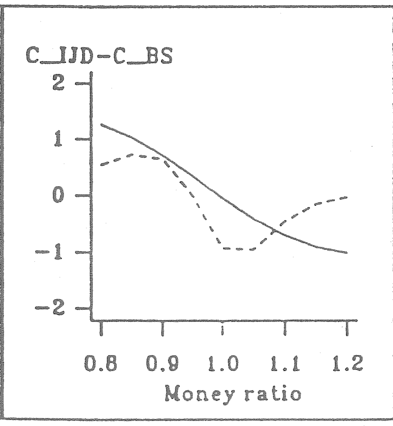
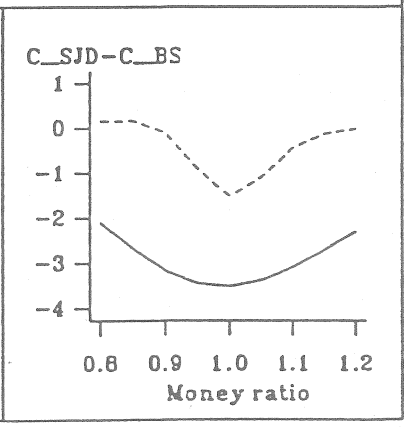
[d] The 'risk-neutral' mean of ln(L+1) is: $\mu_J^* = \mu_J - R\ \sigma_J^2$.

[e] The 'risk-neutral' mean jump size per year.

[f] The 'risk-neutral' volatility is given by: $\text{VOLA}^* = [(\sigma^2 + \lambda^*((\mu_J^*)^2 + \sigma_J^2))]^{1/2}$.

[g] Diff $\equiv C^{SJD}(\mu_J \neq 0) - C^{SJD}(\mu_J = 0)$.

Table 2: Calls written on the market index: Deviations from Black/Scholes-values (Fixed parameters[a]: K=100, r=10%, VOLA=30%, λ=1, γ=0.8)

Shapes of the risk-neutralized return distribution (T=1)	Deviations from Black/Scholes-values if jump risk is idiosyncratic[b] (or R=0)	systematic[b] and R=3
Panel A: $\mu_J = 0$ (i. e. the *actual* return distribution is symmetric)		
		
Panel B: $\mu_J = -0.20$ (i. e. the *actual* return distribution is negatively skewed)		
		
Panel C: $\mu_J = 0.20$ (i. e. the *actual* return distribution is positively skewed)		
		

[a] The resulting risk-neutralized parameters of the SJD-model are the same as in table 1.
[b] Deviations are plotted only for options with one month (dashed line) and one year (solid line) time to maturity, respectively.

A comparison between IJD-values and the SJD-values (with $R = 3$) for symmetric return distributions (numbers in bold face in Table 1) shows, that for ITM and ATM options the SJD-value is higher than the IJD-value. The same is true for long-term OTM options, while for short-term OTM calls the IJD-value exceeds the SJD-value. According to Amin/Ng (1993), the interaction between the so called '*drift-effect*' and '*discounting-effect*' explains this difference. As long as the correlation between stock price jumps and wealth jumps is positive, the stock price drifts upwards at a faster rate since the stock return premium is higher under systematic jump risk relative to the diversifiable jump risk case. Therefore, the 'drift-effect' causes the options to be worth more under systematic jump risk than under idiosyncratic jump risk. In contrast, the 'discounting effect' causes the value under systematic jump risk to be smaller than the value under diversifiable jump risk. Wealth tends to jump with the stock return when jump risk is systematic, while jumps in the stock price that increase the value of the call tend to decrease the interest rate.[8] The direction of these effects depends on the assumption about the sign of the correlation between wealth jumps and asset jumps. Therefore, the above effects reverse themselves when a negative correlation is assumed.

Skewness of the actual return distribution changes the sign and magnitude of the differences between the BS-value and the IJD- or SJD-value.[9] According to Tables 1 and 2, the v-shaped relationship between the BS-value and IJD-value still exists for short-term options, but depending upon the sign of the skewness the v-shape is tilt to the right or to the left. For long-term calls the difference in value increases (decreases) with the money ratio if the skewness is negative (positive). In contrast to the IJD-value, the SJD-value increases slightly if the return distribution becomes negatively skewed, except for short-term OTM calls. For positively skewed return distributions the SJD-value is dramatically lower compared to the symmetric case. This is mainly due to the volatility effect; the risk-neutralized volatility is significantly smaller compared to that of the actual return distribution. The shape of the risk-neutral return distribution, as depicted in Table 2 column 1, serves as an explanation.

While the foregoing analysis is restricted to European calls, the same differences in model values also hold for otherwise identical European puts because of the put-call parity.

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[8] According to Cox/Ingersoll/Ross (1985), the endogenous riskless interest rate is equal the negative of the expected rate of change in the marginal utility of wealth.

[9] This is explained by the shape of skewed return distributions. Return distributions with a negative (positive) skewness exhibit a larger probability for returns far below (above) the mean than for returns far above (below) the mean. This implies that the mean return lies above (below) the median if the return distribution has a negative (positive) skewness.

3. Historical jump risk impact on option values

3.1 Data and estimation

A substantial jump risk impact on option values can only be expected if the estimated model parameters reflect the statistical and economic significance of the jump risk. We therefore pay particular attention to the values of calls whose underlying parameter estimates are based on the extremely volatile stock returns around the crash periods in October 1987 and October 1989.[10] The option price data consist of options written on five actively traded stocks: *Daimler Benz, Deutsche Bank, Siemens, Thyssen and VW.* Henceforth this sample is called BIG5. More precisely, we examine only the subsample BIG5/NODIV since price observations are eliminated if dividends were paid, or stock splits took place, during the lifetime of the option. All option prices, stock prices, dividend and split data, as well as daily stock returns, were taken, or generated, from the *Karlsruher Kapitalmarktdatenbank.* The riskless interest rate appropriate to each option was estimated from the interest rate on three-month inter-bank time deposits.[11]

The time-consuming parameter estimation for the Poisson jump-diffusion process, which was performed only once a month during the sample period, is based on 250 daily returns preceding the estimation date.[12] Figure 1 shows the monthly reestimated parameter values of the skewness and kurtosis according to

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[10] The total sample covers the period from April 1983 to December 1991. During the post-crash periods, to which particular attention was paid, trading took place on the Frankfurt Options Market (FOM). But, since the opening of the Deutsche Terminbörse (DTB) in January 1990, trading in the most liquid 'blue chips' is no longer possible on the older Frankfurt Options Market (FOM).

[11] Recall that in the SJD-model the riskless interest rate is endogenously determined. In order to be able to compare the SJD-model with models whose r is exogenously given, we endogenize instead the drift rate of wealth: the choice of αW is such that the 'endogenous' r is equal to the observable r.

[12] The parameters of the pure diffusion process were estimated daily, using the 250 preceding daily stock returns and DAX-returns, respectively. Although the SJD-model requires a simultaneous estimation of the jump diffusion parameters of all stocks' returns and the returns on the market proxy, we estimate them independently. The required correlation structure of jumps in the stock price, and in the wealth, is estimated with the following two-step procedure: (1) We identify the return of the DAX or an individual stock as a 'jump return' if at least one of the two returns either exceeds 3% or is lower than –3%, and (2) the correlation between these selected returns serves as proxy for the true correlation between jumps in stock return and aggregate wealth. Typically there is a strong positive correlation between individual stock returns and DAX-returns.

the jump diffusion process for SIEMENS returns. The historical return distributions are negatively skewed during the crash month October 1989 and during the year 1992, thus reflecting the stock price decline in both periods. Beinert/Trautmann (1991) show that the estimated jump component is statistically significant for returns of single stocks as well as for DAX-returns.

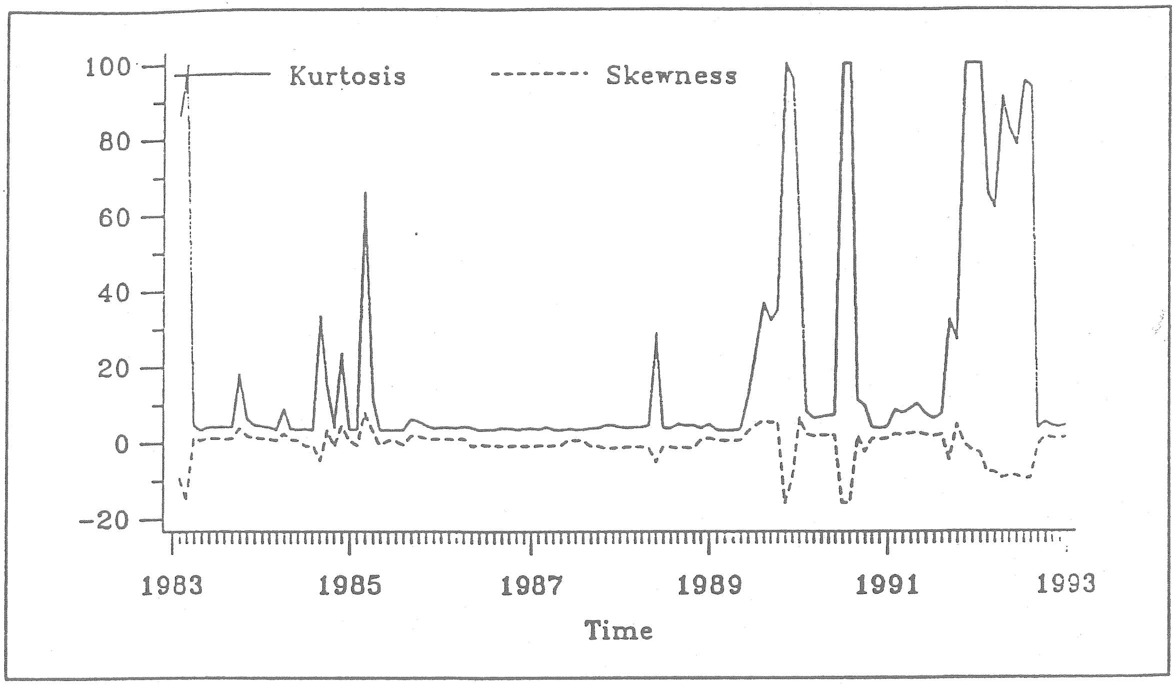

Figure 1: Skewness and kurtosis resulting from the parameter estimates of the Poisson-jump diffusion process of Siemens returns

3.2 Results

Figure 2 depicts the mean differences (in DM) between *BS-values and IJD-values* with respect to different money ratios (left hand scale) for the post-crash period from November 1987 to January 1988 and from November 1989 to January 1990.[13] In these post-crash periods, the pattern resembles that of the *positively* skewed returns which are depicted in Table 2. This *unexpected* result is due to the large positive returns after the October 1989 market crash which obviously cause a positive skewness of the estimated return distributions for some stocks. Figure 1 confirms the positive skewness of the Siemens return distribution after the October 1989 crash. The expected v-shaped relationship obtains for short-term and medium-term options, but the differences are

[13] The right hand scale corresponds to the plotted frequency distribution of observed money ratios.

significant only for OTM options. On an average basis, the differences between BS-values and IJD-values for OTM calls with a short, medium, and long maturity are 0.30 DM (9.5%), 0.61 DM (3.9%), and 0.76 DM (3.3%), respectively. For ATM calls the BS-values are 0.20 DM (0.5%) higher than the IJD-values. For ITM calls with a long maturity the mean IJD-value exceeds the mean BS-value, but the percentage difference is negligible. This deviation pattern contradicts those reported in Tables 1 and 2 when the return distribution is positively skewed. The answer to this puzzle reads as follows: the call values in the period from November 1987 to January 1988 and November 1989 to January 1990 are based on 30 (5x6) different sets of parameter estimates. Unfortunately, some sets of parameter estimates imply a positively skewed return distribution while other sets result in a negatively skewed return distribution.

Figure 3 shows the mean differences (in DM) between *IJD-values and SJD-values* (for R=3) for different money ratios in the post-crash period. The SJD-values substantially exceed the corresponding IJD-values.[14] According to the findings in the foregoing section, the 'drift effect' leads to extremely high SJD-values when the underlying return distributions are symmetric, or negatively skewed, while the opposite is true for return distributions with positive skewness. The observed pattern is therefore consistent with symmetric or negatively skewed return distributions. In contrast to Figure 2, the estimated positive skewness of some of the five underlyings does not influence the expected result. The differences between BS-values and SJD-values are therefore larger than the differences between BS-values and IJD-values, with one exception. Thus, in the case of OTM calls with a short maturity, the IJD-value exceeds the SJD-value and therefore reduces the deviation from the BS-value. This effect is due to the 'discounting effect' as explained in the previous section. On average, the SJD-value is 0.66 DM (5.08%) higher than the BS-value. The percentage difference is 11.39% for OTM calls while for ATM and ITM options the differences are only 1.70% and 0.60%, respectively.

Recall that the put-call parity guarantees that the differences between the BS-value and alternative values for a European call are the same as for an otherwise identical European put. But even if the American puts examined were to be of a European type, we could not expect the same deviation pattern with respect to S/K unless for each call there is an otherwise identical put (and conversely) in the sample. Nonetheless, the deviations are very similar and are therefore not presented here.[15]

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[14] The differences are much smaller for R=1, that is, for a representative investor with logarithmic utility function (figures are not presented here).

[15] More detailed empirical results for calls, as well as for puts, can be found in Trautmann/Beinert (1995).

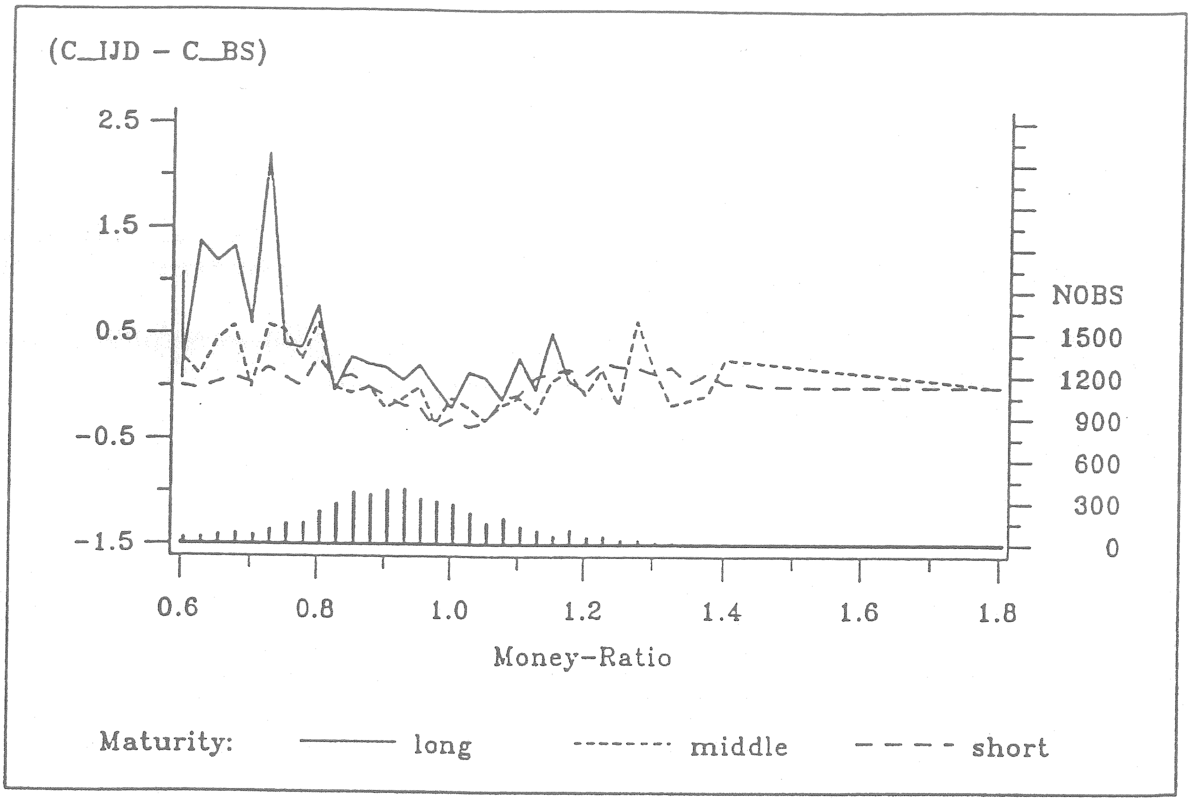

Figure 2: Mean differences (in DM) between B/S-value and IJD-value (BIG5/NODIV Calls, in the post-crash periods 87/11- 88/1 and 89/11 - 90/1)

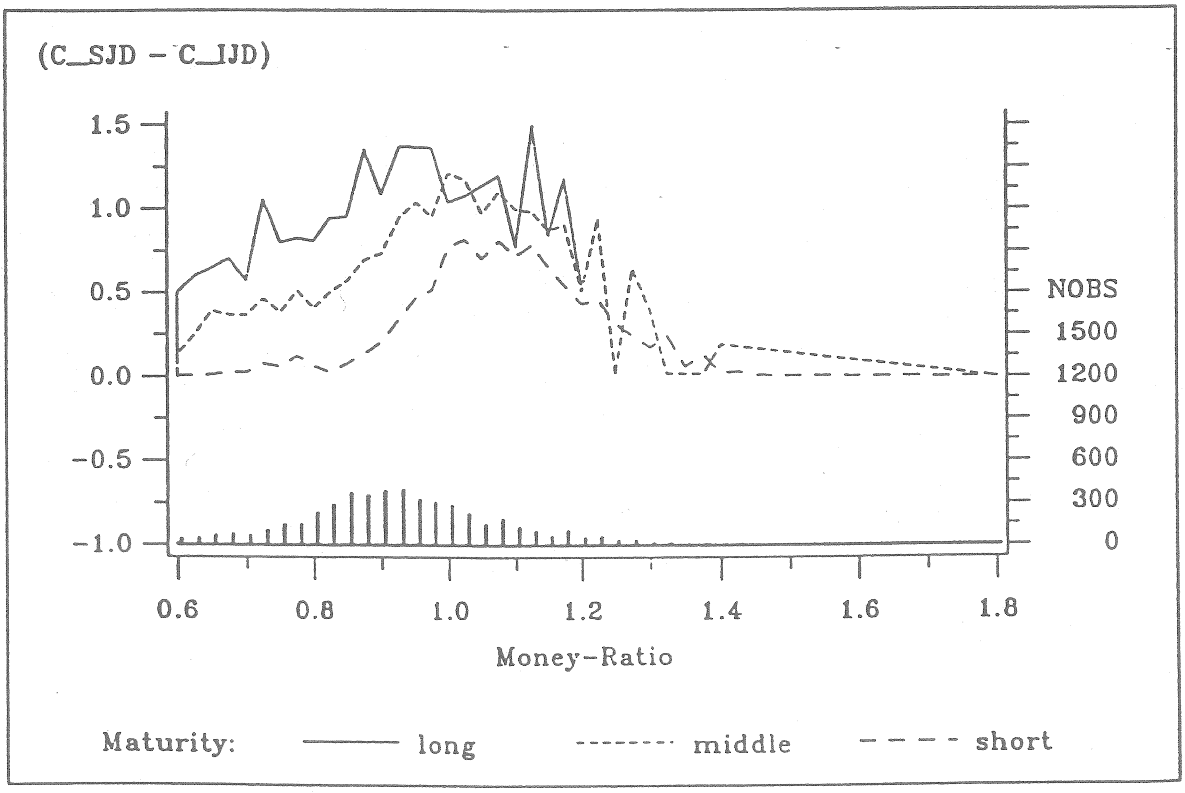

Figure 3: Mean differences (in DM) between the IJD-value and the SJD-value (R=3, BIG5/NODIV Calls, in the post-crash periods 87/11- 88/1 and 89/11 - 90/1)

4. Jump risk implicit in option prices

We now estimate the implicit process parameters by setting observed option prices equal to SJD-values to examine whether stock price jumps were expected by market participants. The subperiods of special interest are therefore the periods before, and after, the market crashes in October 1987 and in October 1989, respectively. Recall that according to the SJD-model a perceived risk of a market crash implies a negatively skewed return distribution which will lead to higher prices for OTM puts and lower prices for OTM calls compared to a symmetric model like the BS-model. Therefore, the parameter estimates, that are implicit in calls and puts according to the SJD-model, give direct effect to market participants' crash fears or hopes of a trend reversion. This is the main advantage of the SJD-model compared to alternative models based on symmetric return distributions, as in the case of the BS-model. Jump intensity, jump size, and risk aversion *implicit* in option prices, determined in accordance with the SJD-model, are reported below.

4.1 Data and estimation

The calculation of SJD-model values reported in the preceding section required either, the seven parameters σ, λ, α_J, σ_J, $\alpha_{J,W}$, $\sigma_{J,W}$, and $\sigma_{J,WS}$ of the *true* stock price distribution and *true* wealth distribution, respectively, as well as the risk aversion parameter R; *or,* the four parameters σ, σ_J, $k^* \equiv \exp\{\alpha_J - R\,\sigma_{J,WS}\}$ and $\lambda^* \equiv \lambda \exp\{-R\,\alpha_{J,W} + (1/2)\,R\,(1+R)\,\sigma^2_{J,W}\}$ characterizing the distribution of the *risk-neutral* terminal stock price. To get parameter estimates of the true distribution a two step procedure is used. First, the parameters of the risk-neutral return distribution are inferred from observed option prices. Second, these estimates of the risk-neutral distribution serve as starting points in inferring the parameters of the true distribution.

The parameters corresponding to the true, or risk-neutral, return distribution are estimated via nonlinear least squares[16]. We minimize

$$\sum_{j=1}^{n} \left[C_j - C^{\text{SJD}}\left(S_j, K_j, T_j, R, \sigma, \lambda, \sigma_J, \mu_J, \sigma_{J,W}, \mu_{J,W}\right)\right]^2, \qquad (7)$$

where C_j denotes the market price of call j=1,..., n. This minimization is done for every trading day during the observation period.[17] This procedure needs at least four price observations. Since this condition is not always fulfilled for the

[16] We used the FORTRAN routine BCLSF available in the IMSL program library.

[17] Since nonlinear least squares do not converge in all cases, we used four different parameter vectors as starting values in order to arrive at the global minimum.

FOM-sample, the corresponding price observations of up to four trading days preceding the trading day under consideration are, if necessary, also used.

4.2 Results

The following figures present the implicit jump frequency per year (IJF), λ, and the implicit jump size per year (IJSY), λk. As long as IJF is small enough, positive and negative values for IJSY imply positive and negative skewness of the implicit return distribution indicating strong crash fears and hopes of a trend reversion, respectively,. Figures 4 and 5 illustrate the parameter estimates implied by the prices of *Deutsche Bank* calls and puts, respectively, for all trading days in the period from July 1987 to December 1987, that is, before and after the market crash on October 19, 1987.

Figure 4 highlights the crash fears in July 1987, confirming the findings of Bates (1991) when examining the implicit distribution of the S&P 500 futures price. The IJSY is quite large while the IJF is relatively small. Therefore the market participants expected a jump with a large negative amplitude (i. e., a market crash). In contrast to the findings of Bates for the US market, German call market participants also expected a crash at the beginning of October 1987. But immediately before the crash on October 1987, crash fears were not strong, since the implicit jump frequency was too high. While the crash fears reflected by the S&P 500 futures option prices returned after the stock market crashed, German option market participants did not expect a further stock price drop. Surprisingly, the IJSY and IJF implicit in call prices indicated an upward stock price correction. The graphs evince this dramatic change in implicit stock price distributions after the crash. The different time series of skewness in return distributions implicit in US and German option prices can be explained by the corresponding historical stock price and index price movement. While the US market peaked in August 1987 after a dramatic upward movement during the preceding twelve months, the German stock market peaked already in December 1985 and declined during the years 1986 and 1987. The October 1987 crash left the US stock market at the end of the year at essentially the same level as in January 1987, while the German market fell back to its January 1985 level. Therefore, while US market participants feared a further drop, the German price level was so low that a further price drop was not expected by the options market participants.

The findings for the implicit parameters of the calls are not identical to those of the puts. Figure 5 shows this difference. First of all, the crash fears in July 1987 are insignificant. Furthermore, since the IJF is low immediately after the crash in October 1987 the parameters implicit in puts mainly reflect rebound hopes. Figure 6 depicts the estimated risk aversion parameter implicit in

Deutsche Bank calls. The average value of the implicit risk aversion parameter R is 9.6, indicating that the risk aversion of the representative investor was relatively high. Evidently, the October 1987 crash did not have a substantial influence on the implicit risk aversion. The same is true for Deutsche Bank puts. With an average value of 10.9, the risk aversion is even higher.

Figure 7 shows the differences in root mean squared errors (RMSE) between the BS-model and SJD-model (upper scale) for the pooled sample of Deutsche Bank calls and puts. The lower scale of Figure 7 shows the difference in RMSE when the parameter estimation is based on the *pooled* sample and when the parameters are *separately* estimated for calls and puts, respectively. The sample period starts in April 1987 and ends in December 1987. The observed root mean squared errors are quite large. This is obviously due to the fact that (1) all transaction prices of one day are used to estimate the implicit parameters, and because (2) price data for this period are not time-stamped. Unfortunately, most of the time the SJD-model does not yield a substantially better fit of the market prices compared to the BS-model. Nonetheless, the SJD-model fits the data much better than does the BS-model *after* the October 1987 crash. The difference between the parameter estimates implicit in Deutsche Bank calls and the parameter estimates implicit in Deutsche Bank puts (compare Figures 4 and 5) is reflected in the increase of the RMSE between late August 1987 and October 1987.

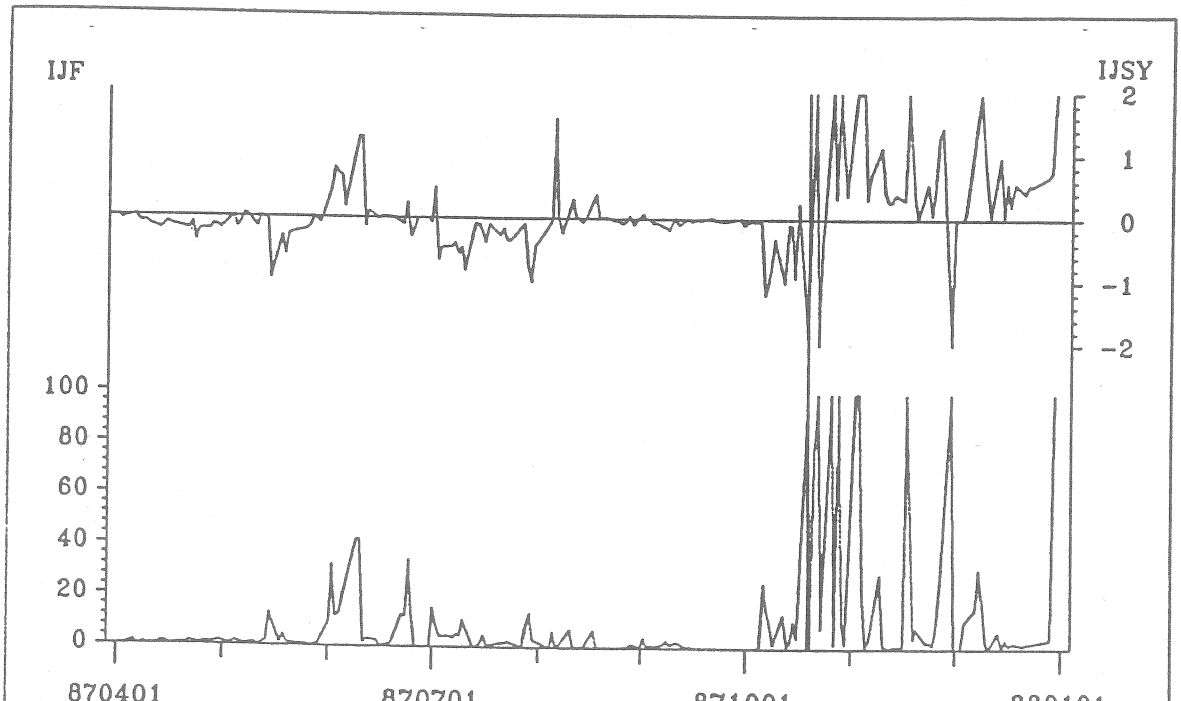

Figure 4: Jump frequency and jump size per year implicit in Deutsche Bank calls observed between April and December in the market crash year 1987

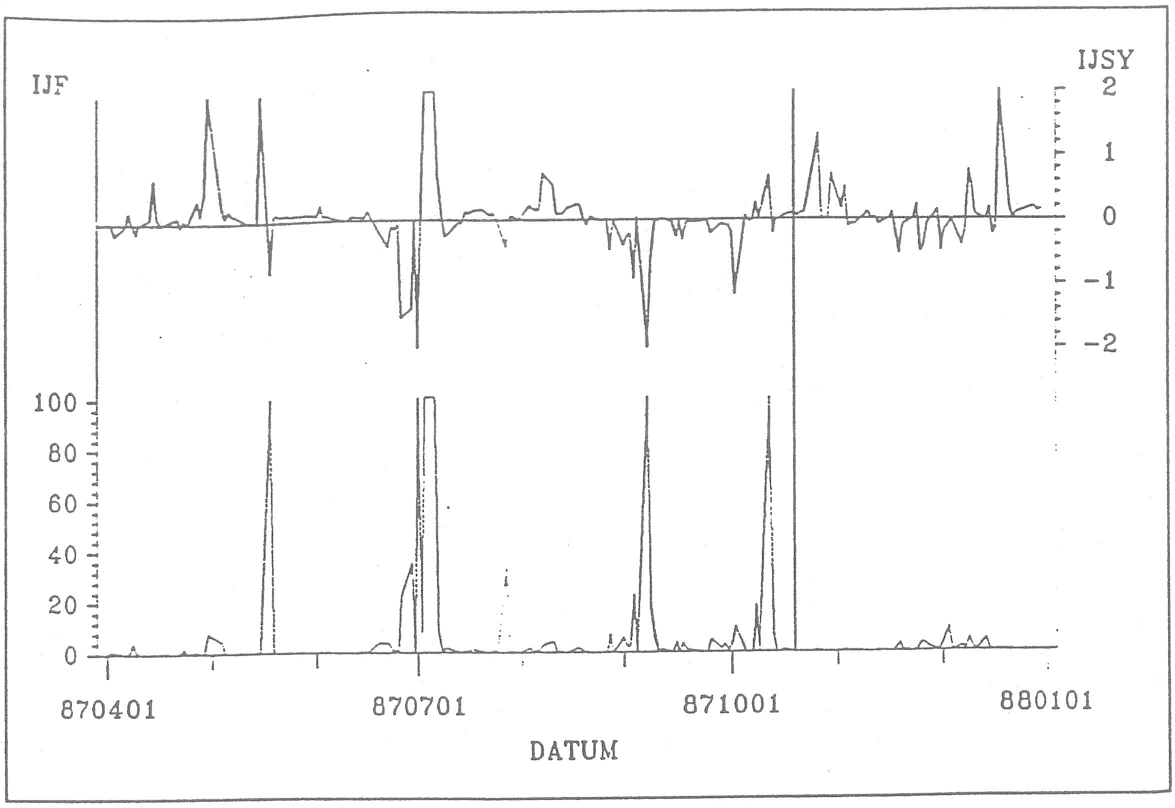

Figure 5: Jump frequency and jump size per year implicit in Deutsche Bank puts observed between April and December in the market crash year 1987

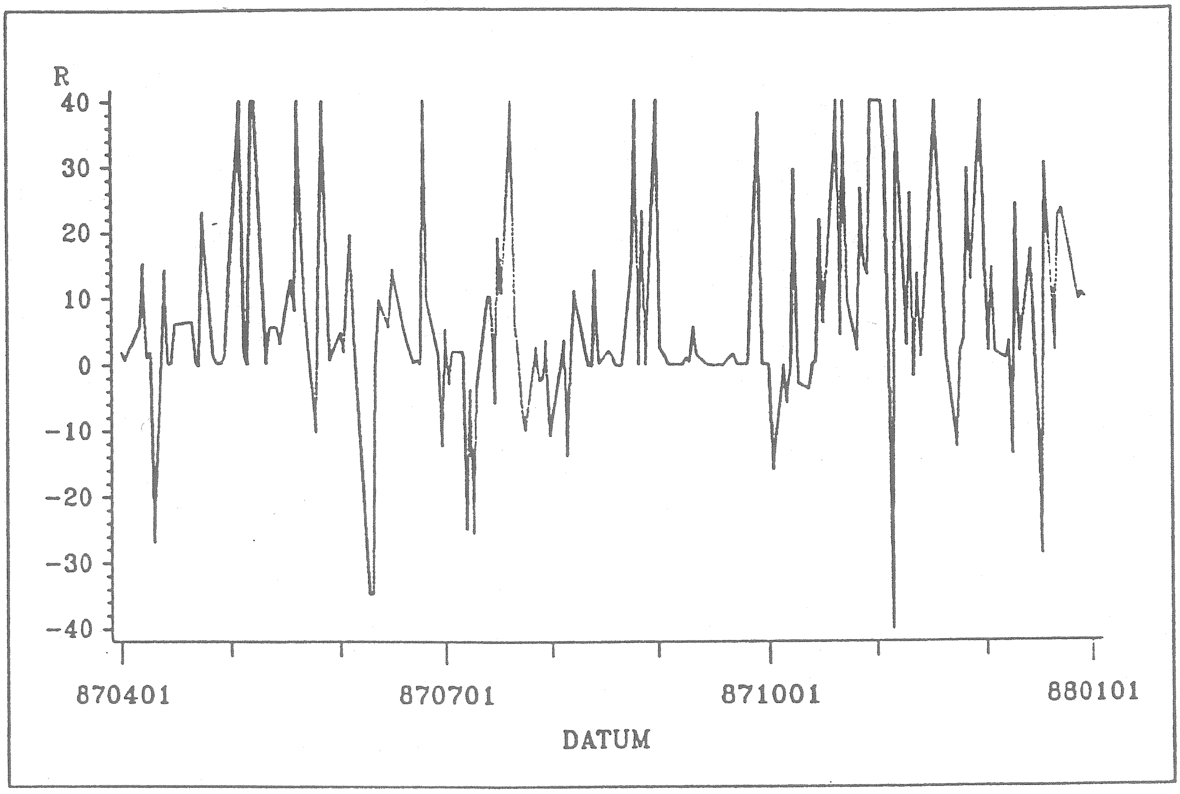

Figure 6: Risk aversion implicit in Deutsche Bank calls observed between April and December in the market crash year 1987

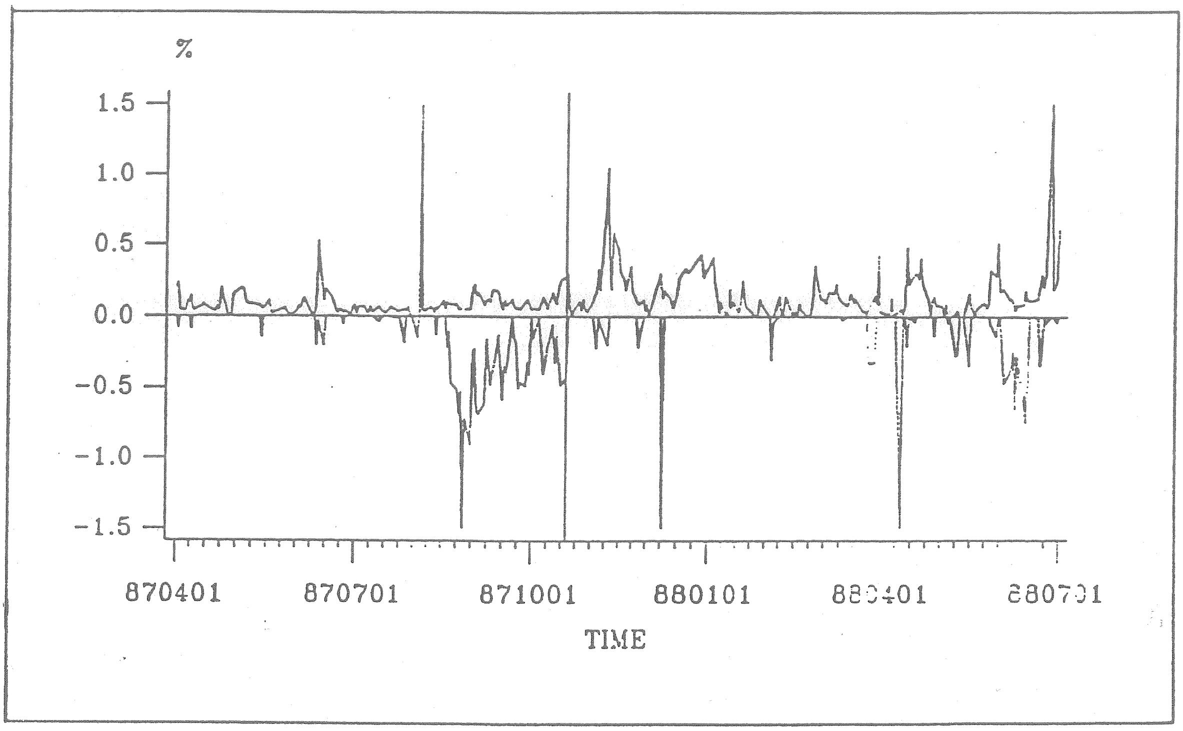

Figure 7: Differences in RMSE as % of the stock price for Deutsche Bank
Upper scale: RMSE (BS-model) minus RMSE (SJD-model) for calls and puts pooled
Lower scale: RMSE (SJD-model calls and puts pooled) minus RMSE(SJD-model unpooled)

5. Conclusions

In conclusion, we list several contributions of this study. In the first place, we have presented a *detailed analysis of the impact of stock price jumps on option values for representative model parameters*. The shapes of the risk-neutral return distributions plotted in Table 2 help to explain the deviation of the jump diffusion values from the BS-values. While the economic rationale behind the difference between SJD-value and IJD-value relies on the interaction between the drift and discounting effect, the interaction between the volatility and skewness effect explains the difference between BS-values and SJD-values.

In the second place, we have examined the *historical impact of jumps on option values*. In the case of options written on 30 NYSE listed common stocks, Ball/Torous (1985) find no operationally significant differences between BS-value and IJD-value, although statistically significant jumps were present in the underlying stock returns. In their sample, the mean percentage deviation for OTM calls is only 2.98%. In contrast to their result, we find a more substantial historical stock price impact. When using the IJD-model in the post-crash period from November 1987 to January 1988, and from November 1989 to January

1990, we find a mean percentage difference between the BS-model and the IJD-model of 6.3% for OTM calls (and –0.81% and 0.19% for ATM calls and ITM calls, respectively). Moreover, the mean percentage difference between the SJD-value (for R=3) and the Black/Scholes value is 7.62% for long-term calls in general and even 11.39% for OTM calls, respectively.

In the third place, we have inferred the *jump intensity, jump size, and risk aversion implicit in option prices* that were quoted before and after market crashes. The time pattern of the implied jump size per year suggests that the market participants feared a stock market crash in July 1987, thus confirming the findings of Bates (1991) for the US market. However, in contrast to Bates, the implicit parameters also suggest crash fears at the beginning of October 1987 and hopes of a stock market rebound after the market crash. Finally, consistent with the findings of Bates (1991), the fit of the SJD-values to the option prices before the October 1987 crash is not significantly better than that of the BS-values (as depicted by the differences in RMSE), while after the October 1987 crash the RMSE for the SJD-model is significantly smaller.

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