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Jump-diffusion models of German stock returns*

A statistical investigation

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This paper discusses the statistical properties of jump-diffusion processes and reports on parameter estimates for the DAX stock index and 48 German stocks with traded options. It is found that a Poisson-type jump-diffusion process can explain the high levels of kurtosis and skewness of observed return distributions of German stocks. Furthermore, we demonstrate that the return dynamics of the DAX include a statistically significant jump component except for a few sample subperiods. This finding is seen to be inconsistent with asset pricing models assuming that the jump component of the stock's return is unsystematic and diversifiable in the market portfolio.

1 Introduction

Continuous time stochastic processes with discontinuous sample path are popular processes to model securities prices for option valuation. These processes constitute an important alternative to the standard diffusion model of Black and Scholes (1973) and were first studied by Press (1967) and incorporated by Merton (1976a, 1976b) into the theory of option valuation. More recent applications of such processes in the option valuation context include the papers of Jones (1984), Naik and Lee (1990), Ahn (1991), Amin (1991) and Jarrow and Madan (1991). The popularity of these jump-diffusion type continuous time stochastic processes stems from at least two facts. First, as distinguished from pure diffusion processes, these processes can explain the observed empirical characteristics of stock return distributions, such as high levels of kurtosis and skewness. Second, they are economically appealing because they allow that stock prices change by significant amounts in a very short time ('jumps') – a reasonable assumption for an efficient stock market – while the probability of such jumps is zero in diffusion processes.

Statistical investigations of such mixtures of a diffusion process and a compound jump process for American stock prices may be found in Ball and Torous (1983, 1985), Jarrow and Rosenfeld (1984) and Akgiray and Booth (1986). Akgiray, Booth and Loistl (1989) provide some evidence for the general form of the mixed process by examining the weekly returns of a portfolio consisting of 48 stocks actively traded on the Frankfurt Stock

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Exchange. However, a complete statistical investigation of such processes for German stock prices has not yet appeared.

This paper extends the work of Akgiray, Booth and Loistl (1989) in at least two ways. First, the general jump-diffusion process is tested on 48 individual stocks as well as on the DAX stock index. Second, we examine daily as well as weekly returns over a reasonably long sample period. Furthermore, the paper reports on the ability of a simplified jump-diffusion process to describe empirical return distributions. The paper is organized as follows. The next section contains a rigorous description of the Poisson-type process and the resulting terminal density function to be estimated. Section 3 describes the methodology of parameter estimation and Section 4 presents the empirical findings. Section 5 concludes with some implications for valuing stock options and stock index options.

2 Description of Jump-Diffusion Processes

Jump-diffusion processes are popular processes to model stock prices since they have an intuitive interpretation. The jump component is an attempt to incorporate the arrival of very important (*abnormal*) new information while the diffusion component models the arrival of less important (*normal*) new information. A rather general jump-diffusion process with independent increments is a Brownian motion superimposed by a compound jump process of the Poisson type. The Poisson process is assumed to be homogeneous (with respect to time and state) and independent of the Brownian motion. Letting S_t denote stock price at time t and S_{t-} the stock price an instant before time t, the dynamics of the stock price process $S \equiv \{S_t; t \geq 0\}$ can be represented by the following stochastic differential equation

$$\frac{dS_t}{S_{t-}} = \alpha dt + \sigma dB_t + I_t dN_t \; , \tag{1}$$

where

i) $B \equiv \{B_t; t \geq 0\}$ is a standard Brownian motion, α is the drift parameter, and $\sigma > 0$ is the diffusion parameter,

ii) $N \equiv \{N_t; t \geq 0\}$ is a Poisson counting process with parameter $\lambda \geq 0$, i.e. the jump intensity per unit time,

iii) $I \equiv \{I_t; t \geq 0\}$ is a process with left-continuous sample paths describing the stochastic size of the jump occuring next: $I_t = \sum_{n=1}^{\infty} L_n 1_{(T_{n-1},T_n]}(t)$ where $L = (L_1, L_2, \ldots)$ is an i.i.d. sequence of random variables with $L_n > -1$ representing the percentage change of S due to a jump (jump size) occuring at time T_n: $L_n \equiv (S_{T_n} - S_{T_n -})/S_{T_n -}$, $T_0 \equiv 0$ and $\{T_1, T_2, \ldots\} = \{t \geq 0 | N_t - N_{t-} = 1\}$ is the set of arrival times of the jumps. The expression $I_t dN_t$ symbolizes a compounded Poisson process,

iv) B, N, I are independent ,

v) B, N and I are adapted to the filtration $\{\mathcal{F}_t; t \geq 0\}$, i.e. N_t, B_t and I_t are $\mathcal{F}_t$-measurable random variables. The filtration will be assumed to satisfy the usual conditions[1].

[1] See, e.g., Karatzas and Shreve (1988, p.10)

An equivalent representation of relationship (1) reads as follows:

$$dS_t = \alpha S_{t-}dt + \sigma S_{t-}dB_t + S_{t-}I_t dN_t\,. \tag{2}$$

Accordingly, the stock price change $dS_t = S_{t+dt} - S_{t-}$ is the sum of three components. The component $\alpha S_{t-}dt$ represents the instantaneous expected stock price change conditional on no arrivals of abnormal information. The $\sigma S_{t-}dB_t$ part describes the unanticipated part of the instantaneous stock price change due to the arrival of normal information, and the $S_{t-}I_t dN_t$ part describes the total instantaneous stock price change due to the arrival of *abnormal* information. Application of a fairly general version of Itô's lemma (see, e.g., Rogers and Williams (1987, p.394)) to $\ln(S_t)$ delivers

$$\ln(S_t) = \ln(S_0) + (\alpha - \sigma^2/2)t + \sigma B_t + \sum_{n=1}^{N_t} \ln(1 + L_n)\,. \tag{3}$$

Defining $X_t \equiv \ln(S_t/S_0)$ to be the rate of return over the interval $[0,t]$, $\mu \equiv \alpha - \sigma^2/2$, and $J_n \equiv \ln(1 + L_n)$, we obtain

$$X_t = \mu t + \sigma B_t + \sum_{i=1}^{N_t} J_i \qquad (t \geq 0)\,. \tag{4}$$

In the special case when the $\{J_i\}$ are normally distributed with parameters μ_J and σ_J^2, the rate of return over the unit interval $[0,1]$, X_1, is then distributed as

$$\begin{aligned} F(x) &= P(X_1 \leq x) = \mathrm{E}P[X_1 \leq x | N_1] \\ &= \mathrm{E}P(\mu + \sigma B_1 + J_1 + \ldots J_n \leq x)|_{n=N_1} = \mathrm{E}\Phi(x|\mu + N_1\mu_J, \sigma^2 + N_1\sigma_J^2) \\ &= \sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!}\, \Phi(x|\mu + n\mu_J, \sigma^2 + n\sigma_J^2)\,. \end{aligned} \tag{5}$$

where $\Phi(\cdot)$ denotes the cumulated normal density function. The corresponding density function is easily obtained as

$$f(x) = \sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!}\, \varphi(x|\mu + n\mu_J, \sigma^2 + n\sigma_J^2) \tag{6}$$

where $\varphi(\cdot)$ denotes the normal density function. In a similar manner we can get the unconditional expected rate of return per unit time and the unconditional variance of the rate of return per unit time, respectively:

$$\begin{aligned} \mathrm{E}(X_1) &= \mathrm{E}\mathrm{E}[X_1|N_1] \\ &= \mathrm{E}\mathrm{E}(\mu + \sigma B_1 + J_1 + \ldots + J_n)|_{n=N_1} \\ &= \mathrm{E}(\mu + N_1\mu_J) \\ &= \mu + \lambda\mu_J\,, \end{aligned} \tag{7}$$

$$\begin{aligned} \mathrm{Var}(X_1) &= \mathrm{E}\mathrm{Var}[X_1|N_1] + \mathrm{Var}\mathrm{E}[X_1|N_1] \\ &= \mathrm{E}(\sigma^2 + \sigma_J^2 N_1) + \mathrm{Var}(\mu + \mu_J N_1) \\ &= \sigma^2 + \sigma_J^2\lambda + \mu_J^2\lambda \\ &= \sigma^2 + \lambda(\sigma_J^2 + \mu_J^2)\,. \end{aligned} \tag{8}$$

Ball and Torous (1983) suggest a Bernoulli-type jump-diffusion process as an appropriate model for information arrivals and as such, stock price jumps. The distinguishing feature of this simpler process is that over a fixed period of time (i.e. unit time) either no important new information arrives, or at most one abnormal information arrival occurs. The corresponding stock return density reads

$$b(x) = (1-\lambda)\varphi(x|\mu,\sigma^2) + \lambda\varphi(x|\mu+\mu_J,\sigma^2+\sigma_J^2) \tag{9}$$

where $\varphi(\cdot)$ denotes the normal density function. For small values of λ, the jump intensity per unit time, however, $b(x)$ approximates the more general density $f(x)$ very well ($|f(x)-b(x)| = o(\lambda)$). Therefore, we may view $b(x)$, up to order λ, as a truncation, after two terms, of $f(x)$. Compared to the normal density function, the shape of $f(x)$ and $b(x)$ is always more peaked in the center and has thicker tails as long as $\lambda > 0$. Both density functions are symmetric around μ if $\mu_J = 0$ and skewed otherwise[2]. Therefore a jump-diffusion process with $\lambda > 0$ might explain the observed leptokurtosis and skewness of stock return distributions.

3 Statistical Methodology

In accordance with most of the preceeding studies, we calculate the maximum likelihood estimates (MLEs) of the process parameters. Given a sample of (daily or weekly) stock returns $\mathbf{x} \equiv (x_1, x_2, \ldots, x_m)$, the logarithm of the corresponding likelihood function is defined as

$$\ln L(\mathbf{x}|\theta) = \sum_{i=1}^{m} \ln f(x_i|\theta) \tag{10}$$

where $\theta \equiv (\mu, \sigma^2, \lambda, \mu_J, \sigma_J^2)$, and $f(\cdot|\theta)$ is the density function given in (6) resulting from a Poisson-type jump-diffusion process. Relying on the experimental evidence reported in Ball and Torous (1985, p. 160), we truncate the infinite sum in $f(x_i|\theta)$ at $N = 10$ and maximize the truncated log-likelihood function

$$\ln L_N(\mathbf{x}|\theta) = \sum_{i=1}^{m} \ln \Big(\sum_{n=0}^{N} \frac{e^{-\lambda}\lambda^n}{n!} \, \varphi(x_i|\mu+n\mu_J, \sigma^2+n\sigma_J^2) \Big) \tag{11}$$

with $N = 10$, instead of (10). Necessary conditions for a maximum likelihood estimator θ^* become

$$\frac{\partial \ln L_N(\mathbf{x}|\theta^*)}{\partial \theta_i} = 0\,, \qquad i = 1, \ldots, 5\,, \tag{12}$$

sufficient conditions require the positive definiteness of $-H(\mathbf{x}|\theta^*)$, the 5×5 Hessian matrix $H(\mathbf{x}|\theta)$ being defined by

$$H(\mathbf{x}|\theta)_{ij} = \frac{\partial^2 \ln L_N(\mathbf{x}|\theta)}{\partial\theta_i\,\partial\theta_j}\,, \quad i,j = 1, \ldots, 5\,. \tag{13}$$

[2]Akgiray and Booth (1986, p. 169) show graphs of various density functions $f(\cdot)$ and a standard normal density function $\varphi(\cdot)$ for comparison.

The MLEs of the process parameters are calculated by solving the nonlinear equation system (12) numerically. The employed quasi-Newton procedure[3] is known to converge quickly, provided the initial values of the algorithm are close to the final solution. Confirming Ball and Torous (1985), we found that the Bernoulli jump-diffusion MLEs provide excellent starting values for the quasi-Newton algorithm. Therefore we computed first of all the MLEs for the simpler process by constraining the mean logarithmic jump size equal to zero, $\mu_J = 0$, and by taking arbitrary starting values for the other parameters to be estimated.

Since a diffusion-only model is nested within a combined diffusion and jump model, a likelihood ratio test can be used to test the null-hypothesis H_0: stock and stock index returns are normally distributed. The likelihood ratio statistic

$$\Delta = -2(\ln L(\mathbf{x}|\theta^*) - \ln L(\mathbf{x}|\theta^0)) \tag{14}$$

where θ^* is the MLE under a jump-diffusion specification, and θ^0 is the MLE corresponding to the situation when no jump structure is present (i.e., $\lambda = 0$). Δ is asymptotically χ^2-distributed with $d-2$ degrees of freedom, where d denotes the number of parameters to be estimated[4]. Estimates of the standard errors of θ^* are obtained from the main diagonal of the inverse of the Hessian evaluated at θ^*.

4 Empirical Results

The empirical tests of both types of jump-diffusion processes were performed on the DAX stock index[5] and 48 common and preferred stocks with traded options. The raw data consist of daily share prices (Kassakurse) quoted at the Frankfurt Stock Exchange spanning the 30-year period from January 1, 1961 through December 31, 1990. The data source is a DFDB[6] daily stock price file, wherein cash dividends, issue rights, stock dividends and splits are accounted for by adjusting previous prices downward. The stock's rate of return of trading day t is then defined as $X_t = \ln(S_t/S_{t-1})$ where S_{t-1} is the adjusted share price of the preceding trading day. Accordingly, weekend and holiday returns are treated as overnight returns. A weekly rate of return is defined as the difference between the logarithm of two successive Wednesday prices.

Table 1 summarizes the Poisson jump-diffusion parameter estimates for the DAX stock index across different subperiods. In addition to the 5 parameters to be estimated (instantaneous mean μ and variance σ^2 of the diffusion component, the mean number of abnormal information arrivals (jumps) per unit time λ, the mean μ_J and variance σ_J^2 of the (logarithmic) jump size) the table reports on the annualized total standard deviation

[3]We used a FORTRAN routine (E04JAF) available in the NAG program library. All calculations were done on the IBM 3090 mainframe of the Rechenzentrum der Universität Karlsruhe.

[4]The null hypothesis can be rejected if $\Delta > \chi^2_{(d-2;\alpha)}$ for some significance level α. For $d = 5$ the critical values of Δ are 6.25, 7.81 and 11.35, for α =0.10, 0.05 and 0.01, respectively.

[5]The DAX is a capital weighted index of 30 stocks actively traded on the Frankfurt Stock Exchange. Since 30.12.1987 the DAX is quoted continuously during the trading hours and is supposed to be the most important German stock index.

[6]DFDB (Deutsche Finanzdatenbank) is a German capital market data base maintained with the support from Deutsche Forschungsgemeinschaft (DFG).

(volatility) of the jump-diffusion process (VOLA)[7], the log-likelihood value and the likelihood ratio test statistic (Δ). Standard errors are not provided, since these are small enough to guarantee the statistical significance of all estimated parameters. Based on the likelihood ratio test, in the majority of cases we have evidence implying the existence of a jump structure in DAX returns. Except for the subperiods in between 1971 and 1980 the null hypothesis of a pure diffusion process is rejected at the 1% significance level[8]. A comparison of the results based on daily returns (panel A) and weekly returns (panel C), shows that the jump component is statistically more significant for daily data.[9] However, when eliminating Monday and Friday returns in daily return series (panel B), the statistical significance of the jump structure is even lower than with weekly data for some subperiods (1966-1970, 1971-1975 and 1971-1980).

Table 2 and 3 report the MLEs of the five parameters for 48 common and preferred stocks based on daily and weekly return data from 1961 to 1990, respectively. The standard errors of the estimates not reported here indicate that most of these estimates are statistically significant. The null hypothesis was rejected in all cases for daily returns and for 44 out of 48 stocks for weekly returns. We found that in the subperiods the likelihood ratio test statistic is always significant for daily returns. For weekly returns, however, the null hypothesis is rejected for 61.1%, 56.8% ,77.8% and 86% of the common and preferred stocks considered in the subperiods 1971-1975, 1976-1980, 1981-1985 and 1971-1980, respectively. In the subperiods 1966-1970 and 1986-1990 this percentage is close to 94%. In all other subperiods the null hypothesis can be rejected for all stocks considered in the sample[10].

Furthermore, we examined the explanatory power of the Bernoulli-type jump-diffusion model under the restrictive assumption $\mu_J = 0$ for the sample period from January 1, 1980 to December 31, 1988. For daily returns this Bernoulli jump-diffusion model has much lower descriptive power than the Poisson jump-diffusion model restricted in the same way.[11]. For weekly returns the Schwarz criterion shows that sometimes the Bernoulli jump-diffusion model is even 'better' than the Poisson jump-diffusion model. The Bernoulli jump-diffusion model is 'better' than the normal distribution as indicated by the likelihood ratio test. The mixture of a pure diffusion process with a jump process either in form of a Bernoulli-type or Poisson-type jump increases the descriptive power of the model.

Figure 1 visualizes the peakedness and the thicker tails of the empirical density function of daily SIEMENS stock returns observed between January 1, 1980 and December 31, 1988.

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[7]VOLA$\equiv \left[(\hat{\sigma}^2 + \hat{\lambda}(\hat{\sigma}_J^2 + \hat{\mu}_J^2)) \cdot n\right]^{1/2}$ where $n = 52$ (weeks a year) and $n = 250$ (trading days per year) when using weekly and daily estimates, respectively.

[8]Akgiray, Booth and Loistl (1989) report a result which seems to be slightly inconsistent with one of ours. Based on weekly returns of a self-constructed index of 48 actively traded stocks they report, unlike our paper, a statistically significant jump structure for the period 1976-1980.

[9]This observation confirms corresponding results for a value-weighted index including all stocks on the New York Stock Exchange and the American Stock Exchange as documented in Jarrow and Rosenfeld (1984).

[10]To compare our results with the results of earlier papers (e.g. Ball and Torous (1985)) the parameters were also estimated under the assumption $\mu_J = 0$. In comparison with the unconstrained model little of the explanatory power of the model was lost, but the values of all other parameters were influenced. Therefore the results obtained for the unconstrained model are more valuable.

[11]To compare the Bernoulli- with the Poisson jump-diffusion model we used the Schwarz criterion (SC) defined as $SC \equiv \ln L(\mathbf{x}|\theta) - d \ln \sqrt{m}$ where d is the number of independent parameters and m is the sample size.

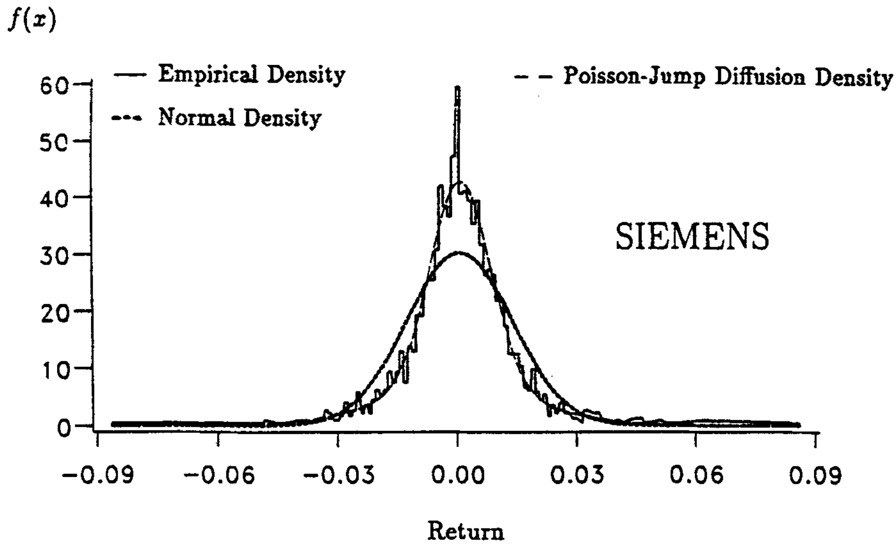

Figure 1: Density functions resulting from different processes based on daily returns

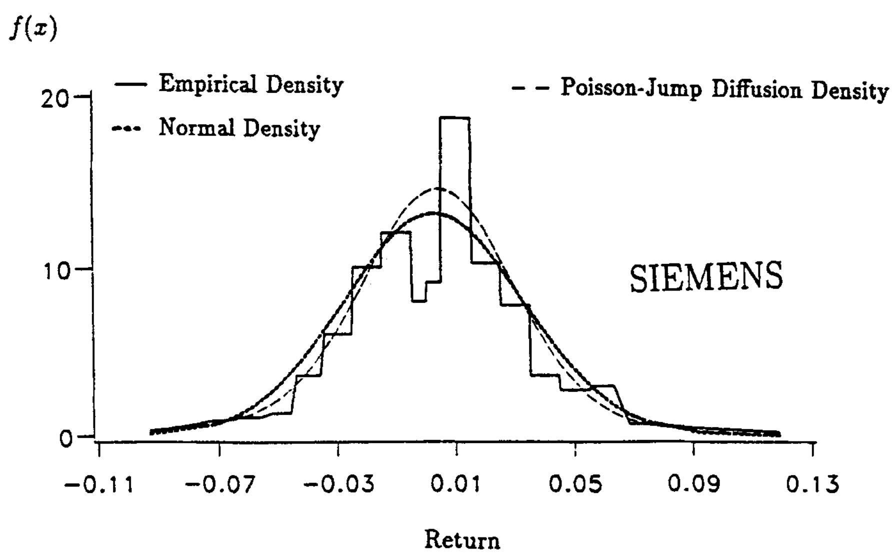

Figure 2: Density functions resulting from different processes based on weekly returns

The two remaining density functions result from the parameter estimates of the normal distribution and of the distribution of the constrained (i.e., with $\mu_J = 0.$) Poisson jump-diffusion process. The density function of the Poisson jump-diffusion process approximates the peakedness of the empirical density function of the returns much better than the normal distribution. The values of kurtosis and skewness are 5.41 and -0.27 respectively,

Panel A: Daily Returns

Period	61-65	66-70	71-75	76-80	81-85	86-90	61-70	71-80	81-90	61-90
m	1253	1251	1249	1252	1249	1247	2505	2501	2496	7501
λ	0.095	0.097	0.097	0.170	0.658	0.060	0.076	0.226	0.071	0.062
$\sigma^2 \times 10^4$	0.567	0.620	0.906	0.366	0.371	1.302	0.609	0.543	0.867	0.693
$\sigma_J^2 \times 10^3$	0.547	0.217	0.075	0.040	0.052	1.497	0.466	0.073	0.780	0.558
$\mu \times 10^3$	-0.414	-0.311	0.427	-0.133	0.825	0.309	-0.320	0.068	0.759	0.168
$\mu_J \times 10^3$	2.350	3.624	-2.292	0.043	0.0004	-4.933	3.218	-0.129	-4.750	-0.662
VOLA	16.52	14.51	15.69	10.40	13.34	23.52	15.58	13.30	18.97	16.11
$\ln L$	4085	4127	3991	4516	4207	3615	8209	8414	7731	24277
Δ	346.0*	99.86*	1.60	5.24	30.98*	279.1*	457.3*	27.96*	514.7*	1177*

Panel B: Daily Returns without Monday and Friday returns

Period	61-65	66-70	71-75	76-80	81-85	86-90	61-70	71-80	81-90	61-90
m	753	752	750	757	753	751	1504	1507	1505	4517
λ	0.072	0.417	0.003	0.074	0.677	0.054	0.062	0.151	0.067	0.056
$\sigma^2 \times 10^4$	0.625	0.519	0.973	0.407	0.395	1.347	0.637	0.606	0.925	0.727
$\sigma_J^2 \times 10^3$	0.730	0.519	0.000	0.060	0.053	1.144	0.491	0.075	0.639	0.501
$\mu \times 10^3$	-0.154	0.232	0.479	-0.047	0.714	0.276	0.108	0.230	0.781	0.383
$\mu_J \times 10^3$	4.631	0.337	-20.06	1.713	0.395	-0.683	3.423	0.040	-2.573	0.117
VOLA	17.05	13.78	15.70	10.60	13.71	22.16	15.40	13.41	18.40	15.87
$\ln L$	2437	2501	2391	2709	2512	2185	4932	5051	4653	14599
Δ	228.2*	7.88	0.20	5.20	16.02*	94.88*	259.2*	8.80	178.5*	508.0*

Panel C: Weekly Returns

Period	61-65	66-70	71-75	76-80	81-85	86-90	61-70	71-80	81-90	61-90
m	251	250	250	253	251	250	501	503	501	1505
λ	0.096	0.201	0.509	1.313	2.407	0.210	0.123	0.201	0.167	0.122
$\sigma^2 \times 10^3$	0.274	0.344	0.439	0.187	0.055	0.520	0.321	0.283	0.356	0.333
$\sigma_J^2 \times 10^2$	0.295	0.106	0.029	0.001	0.011	0.167	0.199	0.057	0.172	0.179
$\mu \times 10^2$	-0.275	-0.058	-0.048	0.301	0.623	0.480	-0.180	-0.048	0.447	0.065
$\mu_J \times 10^2$	1.922	0.458	0.267	-0.275	-0.095	-2.310	1.230	0.303	-1.373	0.024
VOLA	17.55	17.06	17.49	10.44	12.98	22.60	17.31	14.41	18.74	16.93
$\ln L$	614	589	575	712	652	526	1200	1261	1154	3597
Δ	74.8*	15.2*	1.00	0.06	31.7*	38.3*	82.6*	12.6*	86.5*	185.5*

*Indicates significance at 1% level.

Table 1: Poisson Jump-Diffusion Parameter Estimates for the DAX Across Different Subperiods

Stock	m	λ	$\sigma \times 10^4$	$\sigma_J \times 10^3$	$\mu \times 10^3$	$\mu_J \times 10^3$	VOLA	$\ln L$	Δ
AEG	7492	0.1154	1.3470	2.8500	-0.6274	3.6770	34.10	20288.49	40576.98
AGIV	4035	0.7206	0.3176	0.2227	-0.3956	1.3230	21.99	11903.07	1226.84
BABST	7230	0.4252	0.6223	0.4617	-0.3249	0.7251	25.43	20479.57	2513.18
BABVZ	4275	0.4251	0.7407	0.5274	-0.1894	0.5920	27.31	11781.92	1333.20
BASF	7493	0.3159	0.6103	0.2404	-0.0901	0.7817	18.52	23115.66	1016.56
BAYER	7473	0.2720	0.6919	0.2884	-0.3028	1.7147	19.26	22785.88	1164.96
BAYVER	7473	0.4887	0.4284	0.2359	-0.3969	1.3068	19.93	22736.04	1669.62
BBC	7458	0.5896	0.4046	0.2957	-0.1986	0.7855	23.19	21715.00	2093.10
BEKULA	6087	0.2081	0.5226	0.6620	-0.0246	1.2720	21.82	18702.60	26431.98
BHF	7493	0.6998	0.2410	0.1367	-0.1995	0.6120	17.32	23784.16	1458.90
BMW	7473	0.2885	0.9052	0.5823	-0.2119	2.3340	25.50	21041.62	2183.74
COBANK	7474	0.6329	0.3583	0.1962	-0.3891	0.9194	20.03	22646.12	1473.76
COGUMM	7492	0.7135	0.5969	0.3011	-1.2210	1.7790	26.31	20588.63	1336.22
DBENZ	7478	0.2223	0.7894	0.6101	-0.1801	2.2270	23.22	21903.89	2212.00
DEGUSS	7493	0.2404	0.6346	0.4877	-0.2160	1.2531	21.28	22608.24	2340.64
DREBA	7493	0.6261	0.4541	0.1929	-0.5411	1.2572	20.44	22416.04	1185.34
DTBANK	7493	0.3830	0.4734	0.2257	-0.2510	1.4584	18.34	23334.22	1460.40
HARPEN	7444	0.5643	0.5493	0.2832	-0.9915	2.5301	23.36	21420.00	1638.26
HOECHS	7493	0.3501	0.5475	0.2179	-0.1384	0.8795	18.12	23296.37	1076.52
HOESCH	7473	0.4985	0.8366	0.4006	-1.3461	2.9346	26.82	20443.65	1378.16
HYPOBK	7473	0.6078	0.2658	0.1859	-0.3827	0.9882	18.72	23323.99	2100.36
K+S	7488	0.4169	0.7372	0.4088	-0.5610	1.6052	24.76	21178.85	1621.54
KARSTA	7441	0.4903	0.4246	0.2576	-0.3009	0.9934	20.57	22472.70	1906.52
KAUFHF	7465	0.5039	0.5723	0.2665	-0.4971	1.3190	21.93	21882.56	1399.66
KHD	7464	0.4745	0.7365	0.4467	-1.0389	2.2682	26.83	20572.82	2035.18
KLOECK	7472	0.3943	0.7671	0.3726	-1.1165	2.5274	23.78	15704.31	1079.88
LINDE	7479	0.4226	0.5459	0.3065	-0.5279	1.8611	21.54	22216.60	1678.52
LUFTST	4336	0.2508	1.5628	1.0548	-0.2939	1.7760	32.47	11176.52	1190.04
LUFTVZ	4291	0.1850	1.8117	1.3181	-0.1106	2.3348	32.64	11043.15	1317.92
MANNES	7474	0.2475	0.8370	0.4201	-0.3393	2.5027	21.75	22005.03	1411.72
MANST	7446	0.3207	0.8356	0.5783	-0.4535	1.9501	25.99	20892.46	2943.52
MANVZ	3938	0.5317	0.8241	0.3556	-0.6697	1.9476	26.15	10825.79	745.66
MERZHO	3738	0.4966	0.4218	0.3410	0.0417	0.5701	23.00	10983.46	1295.00
METALL	6706	0.6343	0.3737	0.3272	-0.6522	1.1653	24.79	19168.94	4357.78
NXDORF	1632	0.2161	1.0683	1.1073	-0.1781	0.1991	29.42	4469.22	685.96
PREUSS	7492	0.2950	0.8242	0.5524	-0.8037	3.7325	24.97	21309.71	5181.36
RUETGE	7356	1.2588	0.1592	0.1573	-0.2850	0.4837	23.14	21140.97	1511.62
RWEST	7474	0.3910	0.4231	0.2496	-0.3699	1.5905	18.77	23265.70	2063.54
RWEVZ	4399	0.1460	0.6952	1.1714	0.2333	1.5266	24.54	13202.78	2698.78
SCHERI	7493	0.4037	0.4908	0.2404	-0.2535	1.5575	19.18	23020.66	1528.12
SIEMNS	7493	0.3939	0.4656	0.2216	0.1086	0.4925	18.30	23340.34	1505.72
THYSSN	7474	0.6810	0.4817	0.1825	-1.4109	2.1680	20.96	16270.12	704.34
VARTA	5114	0.2418	0.8624	0.8469	-0.3269	2.9240	27.07	14406.64	2399.46
VEBA	4993	0.2352	0.7932	0.3963	0.1049	1.0823	20.78	14912.76	890.80
VEW	4333	0.3295	0.3905	0.4145	-0.1540	2.0026	21.03	13360.86	1993.20
VIAG	1137	0.2966	0.8845	0.6548	-0.2786	3.7088	26.77	3170.47	390.46
VW	5107	0.1671	1.5603	1.1919	0.1061	0.6975	29.80	13613.61	1392.62
WELLA	1800	3.2302	0.1592	0.0550	-7.5850	2.4608	23.08	5036.87	197.54
Index									
DAX	7502	0.0618	0.6932	0.5580	0.1684	-0.6624	16.11	24277.05	1177.38

Table 2: Parameter Estimates Based on Daily Stock Returns (January 1, 1961 - December 31, 1990)

Stock	m	λ	$\sigma \times 10^3$	$\sigma_J \times 10^2$	$\mu \times 10^2$	$\mu_J \times 10^2$	VOLA	$\ln L$	Δ
AEG	1505	0.0922	0.8428	2.3969	-0.1830	0.9348	39.90	2743.89	1657.44
AGIV	754	0.8946	0.1355	0.0886	-0.1862	0.5045	22.24	1614.60	139.42
BABST	1458	0.5653	0.3773	0.1686	-0.1961	0.3400	26.37	2853.05	235.54
BABVZ	754	1.2413	0.1462	0.0815	-0.3478	0.2720	24.63	1512.76	100.50
BASF	1505	0.3428	0.3373	0.0915	-0.0541	0.4024	18.48	3431.65	104.36
BAYER	1503	0.0989	0.4896	0.2340	0.0237	0.6163	19.41	3366.28	132.64
BAYVER	1502	0.7035	0.2253	0.0936	-0.0859	0.2987	21.51	3232.29	195.72
BBC	1499	0.3548	0.3620	0.2002	-0.0365	0.4784	23.70	3129.99	338.40
BEKULA	1220	0.1349	0.3110	0.5030	0.0597	0.4458	22.71	2778.58	2285.48
BHF	1505	1.5761	0.0424	0.0372	-0.1185	0.1457	18.13	3475.41	197.22
BMW	1502	0.7017	0.3184	0.1436	-0.2773	0.7341	26.63	2925.97	275.98
COBANK	1503	1.1132	0.1482	0.0638	-0.2809	0.3418	21.29	3232.05	159.88
COGUM	1505	1.3003	0.1562	0.0907	-0.5022	0.4089	26.57	2907.60	178.46
DBENZ	1502	0.4742	0.3075	0.1715	-0.0697	0.4947	24.27	3099.45	297.40
DEGUSS	1505	0.4553	0.2843	0.1400	-0.2042	0.5489	22.05	3233.42	291.64
DREBA	1505	0.2929	0.4653	0.1631	-0.0708	0.6795	22.30	3170.49	164.84
DTBANK	1505	0.1879	0.4609	0.1774	0.1430	0.0723	20.32	3302.80	134.86
HARPEN	1491	0.4582	0.3336	0.1641	-0.3473	1.2331	24.51	3063.08	316.22
HOECHS	1505	0.1857	0.3880	0.1561	0.0033	0.4636	18.83	3423.64	141.26
HOESCH	1502	0.9940	0.2859	0.0985	-0.7595	0.8275	26.33	2913.71	163.58
HYPOBK	1502	0.0124	0.7307	0.0000	0.2252	-8.9531	20.78	3217.29	58.56
K+S	1505	0.7558	0.3113	0.1377	-0.3175	0.4949	26.70	2917.76	235.90
KARSTA	1496	0.5588	0.2708	0.1161	-0.1462	0.4356	21.99	3197.78	235.32
KAUFHF	1499	0.0131	0.9168	0.0000	0.2205	-9.9108	23.32	3036.92	54.38
KHD	1500	0.6517	0.3991	0.1504	-0.2108	0.3634	26.86	2886.10	210.96
KLOECK	1503	0.3117	0.5844	0.4110	-0.3749	0.9673	31.39	2763.06	466.36
LINDE	1501	0.4703	0.3251	0.1299	0.1017	0.0647	22.06	3193.17	217.44
LUFTST	754	0.2783	0.7306	0.2970	0.0512	0.2723	28.47	1410.40	-241.46
LUFTVZ	754	1.0884	0.3227	0.1081	-0.0004	0.1203	27.94	1407.75	-86.76
MANNES	1503	0.3488	0.4416	0.1680	-0.1192	0.7561	23.34	3114.03	197.10
MANST	1495	1.1531	0.2372	0.0832	-0.3650	0.3953	25.13	2954.06	132.00
MANVZ	754	0.2973	0.5777	0.1846	0.0757	0.2596	24.22	1519.83	78.98
MERZHO	752	0.5148	0.2666	0.1621	0.0413	0.2490	23.96	1564.38	170.38
METALL	754	0.5567	0.3933	0.1839	-0.1724	0.5585	27.31	1453.17	143.06
NXDORF	326	0.2228	0.7234	0.3081	0.4357	-2.2415	28.13	620.02	361.84
PREUSS	1505	0.2506	0.5545	0.3314	-0.0891	0.9584	27.06	2952.38	483.72
RUETGE	1475	0.7888	0.2366	0.1045	0.0280	0.1449	23.51	3042.43	190.82
RWEST	1503	0.6421	0.1980	0.0870	-0.0977	0.3558	19.94	3357.44	223.98
RWEVZ	754	0.6198	0.1735	0.0671	0.0040	0.3592	17.62	1773.03	-288.98
SCHERI	1505	0.3486	0.3389	0.1342	-0.0324	0.6339	20.69	3302.76	238.12
SIEMNS	1505	0.3227	0.3355	0.1292	0.0812	0.2251	19.80	3358.61	204.32
THYSSN	1503	2.1017	0.0888	0.0467	-0.4435	0.2629	23.75	3033.99	78.78
VARTA	1004	0.6487	0.3065	0.1036	0.0609	0.0929	22.56	2099.75	112.22
VEBA	1004	1.5268	0.1258	0.0453	0.0907	0.0603	20.62	2171.90	61.12
VEW	753	0.5734	0.1169	0.0732	-0.0080	0.3058	16.79	1836.97	-189.68
VIAG	228	0.1770	0.6568	0.4316	0.4559	-0.2513	27.19	445.57	42.42
VW	1004	0.5392	0.6630	0.1517	-0.1245	0.5059	27.88	1870.40	70.64
WELLA	362	0.8780	0.2081	0.1044	0.1951	0.0571	24.19	737.16	55.04
Index									
DAX	1505	0.1222	0.3325	0.1793	0.0646	0.0241	16.94	3597.42	185.50

Table 3: Parameter Estimates Based on Weekly Stock Returns (January 1, 1961 – December 31, 1990)

both significant at the 1% significance level[12]. Figure 2 shows the density functions for weekly returns for the same sample period. The density function of the constrained Poisson jump-diffusion process now approximates the skewed empirical density function in a less satisfactory way. This is mainly due to the fact that the constraint $\mu_J = 0$ prevents the estimated density function from being skewed. The values for kurtosis and skewness of the empirical density function are 8.28 and -1.0, respectively, both significant at the 1% significance level. Nonetheless is the density function resulting from the constrained Poisson-type jump-diffusion process compared to the normal density function a slightly better approximation of the empirical density function.

5 Summary and Conclusions

Seminal models of finance assume that stock price movements can be modelled by a pure diffusion process (for example, the Black- Scholes option pricing model and the continuous-time asset pricing models) or assume that jump risk is diversifiable (for example, the jump-diffusion option pricing model of Merton (1976a)). Based on daily and weekly return data, we conclude that all stocks examined contain a statistically significant jump component. The same is true for the DAX although the magnitude of the jump component is smaller. Since the DAX is supposed to be a good proxy for the market portfolio the economic implication is that jump risk is not diversifiable. It seems therefore that Merton's (1976b) jump-diffusion model is not able to remove the wellknown pricing biases of the Black-Scholes model, such as those documented in Trautmann (1986, 1991) for the Frankfurt Options Market. In this case a market equilibrium based option pricing model, such as Naik and Lee (1990) or a complete markets based option pricing model such as Jarrow and Madan (1991), could prove to be more appropriate to predict stock option prices. Research along these lines is currently underway.

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[12]Parts of the sample kurtosis and sample skewness may be due to measurement errors. Geske and Torous (1991) show that robust volatility estimates eliminate the option mispricing with respect to sample skewness and sample kurtosis, and significantly improve the Black-Scholes model's pricing performance with respect to estimated volatility.

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