1. **The Real Business Cycle Model**

Consider the following model, where the representative household maximizes

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\ln C_{t+s} + \theta \frac{(1-N_{t+s})^{1-\gamma}}{1-\gamma} \right] \tag{1}$$

subject to

$$Y_t = (A_t N_t)^{\alpha} K_t^{1-\alpha} \tag{2}$$

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t. \tag{3}$$

Log technology follows

$$\ln A_t = \phi \ln A_{t-1} + \varepsilon_t, \tag{4}$$

where $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$.

(a) Define R_{t+1} as the gross rate of return on a one-period investment in capital. Derive the conditions characterizing the optimal consumption-savings and consumption leisure decisions.

(b) Derive the steady-state ratios A/K, Y/K, and C/Y in terms of the model‘s parameters.

(c) Log-linearize the optimality conditions, the production function and the resource constraint around the steady state. Then define vectors $\overline{\boldsymbol{y}}_t$ and $\overline{\boldsymbol{x}}_t$ and matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ so that the system of equations can be written as

$$\boldsymbol{A}\mathbb{E}_t \overline{\boldsymbol{y}}_{t+1} = \boldsymbol{B}\overline{\boldsymbol{y}}_t + \boldsymbol{C}\overline{\boldsymbol{x}}_t. \tag{5}$$

(d) Using $\alpha = 0.67$, $r = 0.015$, $g = 0.005$, $\delta = 0.025$, $N = 0.33$, $\phi = 0.979$ and $\gamma = 2$ write a Matlab program that computes the solution of this system in terms of observables, and plot impulse response functions of y, k, c, and l to a technology shock. You may use the `REDS-SOLDS` package provided on the course website.